Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/7.1.4-f-x-^m-d+e-x^2-^p-a+b-arcsinh-c-x-^n

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [663]. This is test number [187].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (663)	% 0. (0)
Mathematica	% 100. (663)	% 0. (0)
Maple	% 76.32 (506)	% 23.68 (157)
Maxima	% 33.94 (225)	% 66.06 (438)
Fricas	% 36.35 (241)	% 63.65 (422)
Sympy	% 24.13 (160)	% 75.87 (503)
Giac	% 26.4 (175)	% 73.6 (488)

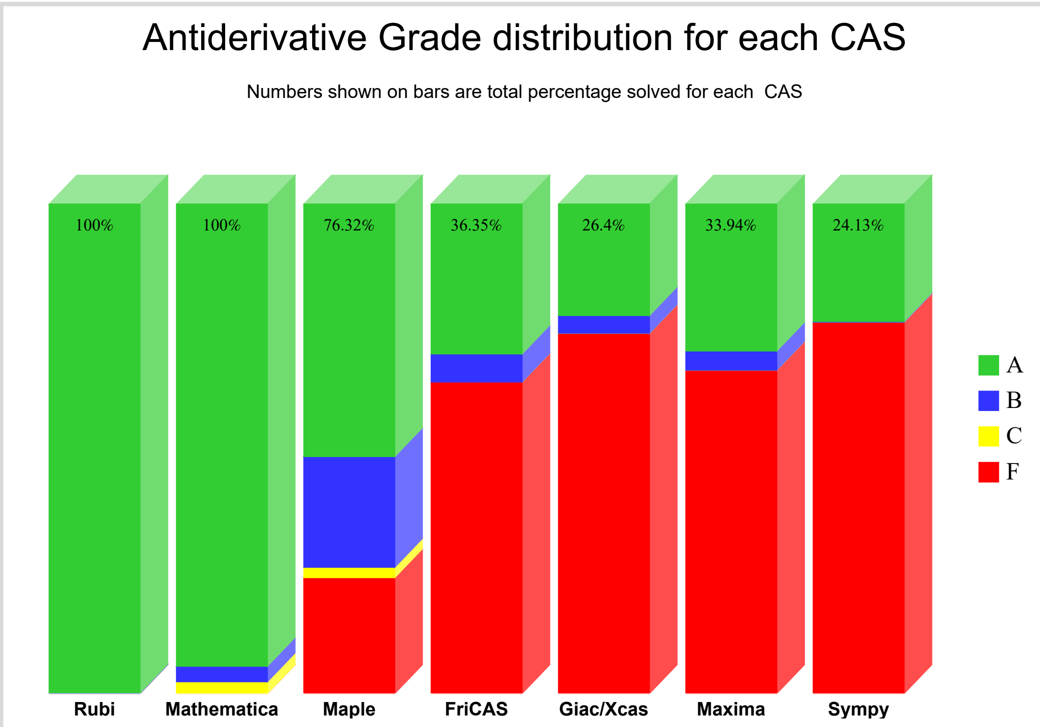
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

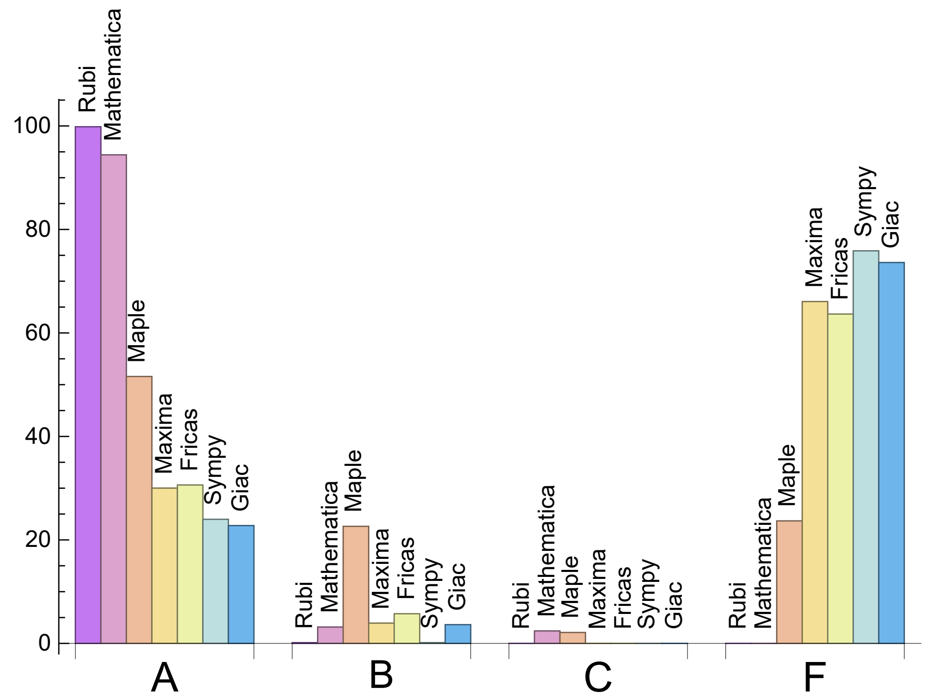
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.85	0.15	0.	0.
Mathematica	94.42	3.17	2.41	0.
Maple	51.58	22.62	2.11	23.68
Maxima	30.02	3.92	0.	66.06
Fricas	30.62	5.73	0.	63.65
Sympy	23.98	0.15	0.	75.87
Giac	22.78	3.62	0.	73.6

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	201.41	0.85	174.	1.
Mathematica	1.74	225.26	0.9	143.	0.91
Maple	0.18	348.92	1.67	181.5	1.4
Maxima	0.59	145.4	0.95	0.	0.
Fricas	1.58	271.74	2.04	134.	2.06
Sympy	7.78	109.33	0.69	0.	0.
Giac	0.54	96.15	0.71	0.	0.

1.4 list of integrals that has no closed form antiderivative

{188, 189, 190, 321, 322, 323, 324, 325, 326, 327, 341, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {6, 8, 15, 17, 24, 26, 203, 205, 212, 214, 221, 223, 263, 265, 271, 273, 279, 281, 297, 299}

Mathematica {6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 59, 61, 67, 69, 75, 77, 86, 88, 96, 98, 107, 109, 117, 119, 124, 132, 134, 140, 151, 153, 161, 163, 172, 174, 182, 184, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 256, 257, 262, 263, 264, 265, 270, 271, 272, 273, 278, 279, 280, 281, 287, 288, 289, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 338, 339, 340, 347, 349, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 477, 481, 485, 486, 490, 494, 495, 496, 497, 501, 502, 507, 508, 545, 551, 564, 565, 574, 575, 580, 581, 586, 587, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

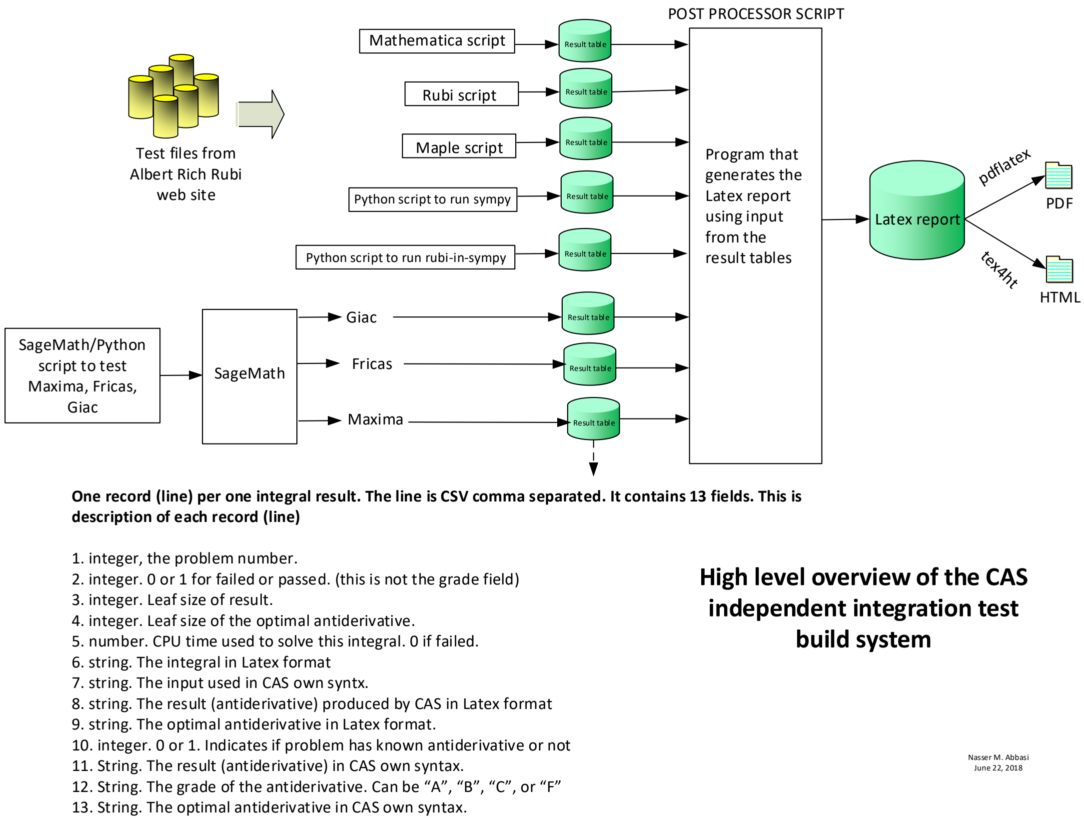
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 73 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 39, 40, 41, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 231, 234, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 588, 589, 590, 592, 593, 596, 597, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 31, 33, 35, 42, 44, 228, 230, 233, 241, 331, 551, 564, 581, 586, 587, 591, 594, 595, 598, 600, 601 }

C grade: { 38, 43, 45, 52, 54, 226, 232, 235, 239, 248, 250, 348, 612, 649, 650, 651 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 59, 61, 63, 64, 66, 67, 69, 71, 72, 74, 75, 77, 79, 80, 81, 82, 83, 84, 86, 88, 96, 98, 107, 109, 112, 113, 114, 115, 116, 117, 119, 124, 126, 129, 132, 134, 137, 139, 140, 142, 144, 150, 151, 153, 158, 161, 162, 163, 172, 174, 177, 178, 179, 180, 181, 182, 184, 188, 189, 190, 198, 199, 200, 201, 202, 204, 206, 207, 208, 209, 210, 211, 213, 215, 216, 217, 218, 219, 220, 222, 224, 226, 228, 252, 282, 283, 284, 285, 286, 287, 288, 289, 300, 302, 305, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 466, 471, 474, 475, 476, 479, 480, 483, 484, 487, 488, 489, 492, 493, 498, 499, 500, 504, 505, 506, 509, 510, 511, 515, 516, 520, 521, 525, 526, 527, 531, 532, 533, 606, 607, 608, 609, 610, 615, 616, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 42, 51, 53, 57, 58, 60, 62, 65, 68, 70, 73, 76, 78, 85, 87, 89, 91, 93, 95, 97, 99, 100, 102, 104, 106, 108, 110, 111, 118, 120, 121, 122, 123, 125, 127, 128, 130, 131, 133, 135, 136, 138, 141, 143, 145, 146, 147, 148, 149, 152, 154, 156, 160, 164, 165, 167, 169, 171, 173, 175, 176, 183, 203, 205, 212, 214, 221, 223, 230, 232, 235, 237, 239, 241, 244, 246, 248, 250, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 303, 304, 307, 309, 310, 311, 312, 313, 314, 315, 317, 319, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 613, 614, 624 }

C grade: { 90, 92, 94, 101, 103, 105, 155, 157, 159, 166, 168, 170, 611, 612 }

F grade: { 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 225, 227, 229, 231, 233, 234, 236, 238, 240, 242, 243, 245, 247, 249, 251, 306, 308, 316, 318, 331, 332, 333, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 477, 478, 481, 482, 485, 486, 490, 491, 494, 495, 496, 497, 501, 502, 503, 507, 508, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 617, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.1.4 Maxima

A grade: { 1, 2, 3, 5, 7, 9, 12, 14, 16, 18, 23, 25, 27, 55, 57, 63, 65, 71, 73, 79, 80, 82, 84, 89, 92, 95, 97, 103, 106, 110, 111, 112, 113, 114, 115, 116, 118, 122, 130, 138, 144, 149, 160, 168, 169, 171, 176, 177, 178, 179, 180, 181, 183, 188, 189, 190, 198, 200, 260, 268, 276, 283, 285, 286, 294, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 341, 343, 345, 346, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 385, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 532, 533, 562, 569, 606, 607, 608, 609, 610, 613, 614, 615, 616, 622, 623, 627, 628, 633, 637, 641, 642, 645, 646, 655, 656, 657, 658, 659, 660, 661, 662, 663

B grade: { 4, 10, 11, 13, 19, 20, 21, 22, 62, 85, 87, 104, 199, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 255 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 86, 88, 90, 91, 93, 94, 96, 98, 99, 100, 101, 102, 105, 107, 108, 109, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 170, 172, 173, 174, 175, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 284, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 632, 634, 635, 636, 638, 639, 640, 643, 644, 647, 648, 649, 650, 651, 652, 653, 654 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 55, 63, 79, 80, 82, 90, 101, 112, 113, 114, 115, 118, 120, 122, 128, 130, 136, 138, 144, 145, 147, 149, 154, 155, 157, 166, 168, 170, 177, 178, 179, 180, 183, 188, 189, 190, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 244, 258, 260, 266, 268, 274, 276, 282, 283, 284, 285, 290, 292, 294, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 341, 342, 343, 344, 345, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 487, 492, 504, 509, 515, 516, 520, 521, 525, 526, 527, 532, 533, 606, 607, 608, 609, 610, 613, 614, 615, 622, 623, 627, 628, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 7, 9, 16, 18, 57, 62, 65, 71, 73, 84, 87, 89, 92, 94, 103, 105, 116, 127, 152, 159, 181, 237, 246, 286, 346, 385, 461, 531, 539, 556, 557, 561, 566, 567, 616, 649, 650, 651 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 56, 58, 59, 60, 61, 64, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 81, 83, 85, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 102, 104, 106, 107, 108, 109, 110, 111, 117, 119, 121, 123, 124, 125, 126, 129, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 146, 148, 150, 151, 153, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 287, 288, 289, 291, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 510, 511, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 558, 559, 560, 562, 563, 564, 565, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 55, 57, 65, 66, 80, 81, 82, 83, 84, 85, 112, 113, 114, 115, 116, 177, 178, 179, 180, 181, 188, 189, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 216, 217, 218, 219, 220, 253, 255, 282, 283, 284, 285, 286, 324, 325, 327, 328, 329, 330, 341, 342, 343, 344, 345, 346, 353, 354, 360, 361, 362, 363, 368, 369, 370, 385, 386, 387, 394, 395, 396, 397, 398, 399, 400, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 457, 458, 459, 461, 466, 471, 475, 480, 488, 493, 499, 515, 516, 527, 531, 532, 533, 606, 607, 608, 609, 610, 613, 614, 615, 616, 622, 632, 633, 636, 641, 645, 647, 648, 652, 653, 654, 656, 657, 658, 659, 660, 661, 662 }

B grade: { 60 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 58, 59, 61, 62, 63, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 326, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 393, 401, 402, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 454, 455, 456, 460, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 634, 635, 637, 638, 639, 640, 642, 643, 644, 646, 649, 650, 651, 655, 663 }

2.1.7 Giac

A grade: { 1, 2, 3, 5, 10, 12, 14, 111, 113, 115, 176, 178, 180, 188, 189, 190, 198, 283, 285, 324, 325, 326, 327, 328, 329, 330, 341, 343, 345, 353, 354, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 409, 410, 415, 416, 417, 418, 423, 424, 425, 426, 431, 432, 433, 434, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 466, 471, 475, 476, 480, 484, 488, 489, 493, 499, 500, 505, 506, 510, 511, 515, 516, 520, 521, 525, 526, 527, 531, 532, 533, 606, 607, 608, 609, 610, 613, 614, 622, 623, 627, 628, 632, 633, 636, 637, 641, 642, 645, 646, 647, 648, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663 }

B grade: { 4, 11, 13, 19, 20, 21, 22, 23, 116, 118, 181, 183, 200, 202, 207, 209, 211, 220, 286, 346, 385, 461, 615, 616 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 114, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 179, 182, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 199, 201, 203, 204, 205, 206, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 347, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 364, 365, 366, 367, 372, 373, 374, 375, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 406, 407, 408, 411, 412, 413, 414, 419, 420, 421, 422, 427, 428, 429, 430, 435, 436, 437, 438, 439, 462, 463, 464, 465, 467, 468, 469, 470, 472, 473, 474, 477, 478, 479, 481, 482, 483, 485, 486, 487, 490, 491, 492, 494, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 517, 518, 519, 522, 523, 524, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 611, 612, 617, 618, 619, 620, 621, 624, 625, 626, 629, 630, 631, 634, 635, 638, 639, 640, 643, 644, 649, 650, 651 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	87	124	248	265	151	238
normalized size	1	1.	0.7	1.	2.	2.14	1.22	1.92
time (sec)	N/A	0.119	0.094	0.009	1.204	2.255	7.977	1.498

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	88	113	257	242	138	273
normalized size	1	1.	0.73	0.94	2.14	2.02	1.15	2.28
time (sec)	N/A	0.098	0.055	0.009	1.155	2.502	5.181	1.581

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	78	105	196	234	126	200
normalized size	1	1.	0.76	1.03	1.92	2.29	1.24	1.96
time (sec)	N/A	0.102	0.083	0.004	1.156	2.709	2.756	1.443

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	94	204	220	117	242
normalized size	1	1.	0.89	1.08	2.34	2.53	1.34	2.78
time (sec)	N/A	0.041	0.052	0.006	1.157	2.666	1.458	1.699

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	86	76	131	184	90	151
normalized size	1	1.	1.15	1.01	1.75	2.45	1.2	2.01
time (sec)	N/A	0.06	0.042	0.006	1.114	2.609	0.729	1.369

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	113	162	0	0	0	0
normalized size	1	1.	1.02	1.46	0.	0.	0.	0.
time (sec)	N/A	0.123	0.061	0.086	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	74	69	89	346	0	0
normalized size	1	1.	1.12	1.05	1.35	5.24	0.	0.
time (sec)	N/A	0.081	0.027	0.009	1.097	2.617	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	111	175	0	0	0	0
normalized size	1	1.	0.87	1.37	0.	0.	0.	0.
time (sec)	N/A	0.126	0.06	0.167	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	93	87	128	396	0	0
normalized size	1	1.	1.16	1.09	1.6	4.95	0.	0.
time (sec)	N/A	0.084	0.034	0.012	1.086	2.675	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	119	167	431	387	230	402
normalized size	1	1.	0.66	0.92	2.38	2.14	1.27	2.22
time (sec)	N/A	0.208	0.097	0.01	1.185	2.346	24.185	1.606

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	115	156	443	373	218	451
normalized size	1	1.	0.64	0.87	2.46	2.07	1.21	2.51
time (sec)	N/A	0.175	0.082	0.009	1.213	2.159	16.442	1.799

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	111	148	352	351	202	347
normalized size	1	1.	0.71	0.94	2.24	2.24	1.29	2.21
time (sec)	N/A	0.169	0.085	0.003	1.189	2.437	9.197	1.714

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	137	365	328	190	405
normalized size	1	1.	0.87	1.14	3.04	2.73	1.58	3.38
time (sec)	N/A	0.065	0.133	0.005	1.053	2.327	5.811	1.776

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	119	262	296	165	281
normalized size	1	1.	0.74	0.93	2.05	2.31	1.29	2.2
time (sec)	N/A	0.102	0.12	0.005	1.194	2.437	2.868	1.569

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	173	231	0	0	0	0
normalized size	1	1.	1.01	1.34	0.	0.	0.	0.
time (sec)	N/A	0.203	0.2	0.116	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	124	114	196	505	0	0
normalized size	1	1.	1.03	0.95	1.63	4.21	0.	0.
time (sec)	N/A	0.157	0.124	0.009	1.171	2.639	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	143	262	0	0	0	0
normalized size	1	1.	0.76	1.4	0.	0.	0.	0.
time (sec)	N/A	0.211	0.343	0.244	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	133	114	190	525	0	0
normalized size	1	1.	1.06	0.9	1.51	4.17	0.	0.
time (sec)	N/A	0.159	0.126	0.012	1.124	3.06	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	143	206	628	509	289	574
normalized size	1	1.	0.63	0.91	2.78	2.25	1.28	2.54
time (sec)	N/A	0.281	0.125	0.017	1.086	2.701	62.296	1.824

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	139	195	644	474	280	633
normalized size	1	1.	0.7	0.98	3.24	2.38	1.41	3.18
time (sec)	N/A	0.169	0.117	0.013	1.183	2.515	42.718	1.896

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	135	187	524	448	265	501
normalized size	1	1.	0.67	0.93	2.59	2.22	1.31	2.48
time (sec)	N/A	0.249	0.103	0.008	1.178	2.533	23.759	1.835

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	128	176	540	424	253	572
normalized size	1	1.	0.88	1.21	3.72	2.92	1.74	3.94
time (sec)	N/A	0.07	0.174	0.004	1.127	2.399	16.028	2.01

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	119	156	406	387	221	416
normalized size	1	1.	0.7	0.92	2.39	2.28	1.3	2.45
time (sec)	N/A	0.161	0.165	0.006	1.143	2.323	8.666	1.676

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	189	284	0	0	0	0
normalized size	1	1.	0.86	1.29	0.	0.	0.	0.
time (sec)	N/A	0.284	0.141	0.135	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	163	151	315	616	0	0
normalized size	1	1.	1.02	0.94	1.97	3.85	0.	0.
time (sec)	N/A	0.221	0.166	0.008	1.245	2.779	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	184	313	0	0	0	0
normalized size	1	1.	0.74	1.26	0.	0.	0.	0.
time (sec)	N/A	0.303	0.436	0.285	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	171	155	286	624	0	0
normalized size	1	1.	0.98	0.89	1.64	3.59	0.	0.
time (sec)	N/A	0.256	0.163	0.01	1.138	3.072	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	170	266	0	0	0	0
normalized size	1	1.	1.09	1.71	0.	0.	0.	0.
time (sec)	N/A	0.241	0.232	0.158	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	181	161	0	0	0	0
normalized size	1	1.	1.34	1.19	0.	0.	0.	0.
time (sec)	N/A	0.195	0.202	0.081	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	121	215	0	0	0	0
normalized size	1	1.	1.12	1.99	0.	0.	0.	0.
time (sec)	N/A	0.14	0.155	0.009	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	167	98	0	0	0	0
normalized size	1	1.	2.29	1.34	0.	0.	0.	0.
time (sec)	N/A	0.117	0.069	0.033	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	135	171	0	0	0	0
normalized size	1	1.	1.93	2.44	0.	0.	0.	0.
time (sec)	N/A	0.064	0.103	0.036	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	207	74	0	0	0	0
normalized size	1	1.	3.39	1.21	0.	0.	0.	0.
time (sec)	N/A	0.117	0.09	0.042	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	182	202	0	0	0	0
normalized size	1	1.	1.8	2.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.163	0.013	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	240	266	0	0	0	0
normalized size	1	1.	2.12	2.35	0.	0.	0.	0.
time (sec)	N/A	0.198	0.272	0.097	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	247	261	0	0	0	0
normalized size	1	1.	1.58	1.67	0.	0.	0.	0.
time (sec)	N/A	0.247	0.213	0.016	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	268	285	0	0	0	0
normalized size	1	1.	1.57	1.67	0.	0.	0.	0.
time (sec)	N/A	0.241	0.339	0.016	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	241	206	0	0	0	0
normalized size	1	1.	1.66	1.42	0.	0.	0.	0.
time (sec)	N/A	0.191	0.219	0.141	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	221	240	0	0	0	0
normalized size	1	1.	1.74	1.89	0.	0.	0.	0.
time (sec)	N/A	0.134	0.264	0.012	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	74	61	0	135	0	0
normalized size	1	1.	1.35	1.11	0.	2.45	0.	0.
time (sec)	N/A	0.05	0.067	0.006	0.	2.393	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	216	234	0	0	0	0
normalized size	1	1.	1.74	1.89	0.	0.	0.	0.
time (sec)	N/A	0.098	0.105	0.01	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	234	283	0	0	0	0
normalized size	1	1.	2.13	2.57	0.	0.	0.	0.
time (sec)	N/A	0.177	0.445	0.085	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	253	267	0	0	0	0
normalized size	1	1.	1.51	1.59	0.	0.	0.	0.
time (sec)	N/A	0.187	0.595	0.018	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	326	311	0	0	0	0
normalized size	1	1.	2.23	2.13	0.	0.	0.	0.
time (sec)	N/A	0.26	0.481	0.112	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	264	311	332	0	0	0	0
normalized size	1	1.1	1.3	1.39	0.	0.	0.	0.
time (sec)	N/A	0.309	0.635	0.021	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	341	313	0	0	0	0
normalized size	1	1.	1.83	1.68	0.	0.	0.	0.
time (sec)	N/A	0.233	0.642	0.016	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	79	108	0	209	0	0
normalized size	1	1.	0.81	1.11	0.	2.15	0.	0.
time (sec)	N/A	0.086	0.145	0.016	0.	2.337	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	340	307	0	0	0	0
normalized size	1	1.	1.85	1.67	0.	0.	0.	0.
time (sec)	N/A	0.176	0.248	0.013	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	56	76	0	207	0	0
normalized size	1	1.	0.7	0.95	0.	2.59	0.	0.
time (sec)	N/A	0.054	0.07	0.006	0.	2.349	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	341	295	0	0	0	0
normalized size	1	1.	1.92	1.66	0.	0.	0.	0.
time (sec)	N/A	0.141	0.152	0.01	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	289	451	0	0	0	0
normalized size	1	1.	1.82	2.84	0.	0.	0.	0.
time (sec)	N/A	0.252	0.633	0.156	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	298	357	0	0	0	0
normalized size	1	1.	1.34	1.61	0.	0.	0.	0.
time (sec)	N/A	0.238	1.219	0.016	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	353	575	0	0	0	0
normalized size	1	1.	1.52	2.48	0.	0.	0.	0.
time (sec)	N/A	0.344	0.846	0.194	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	345	380	425	0	0	0	0
normalized size	1	1.17	1.29	1.44	0.	0.	0.	0.
time (sec)	N/A	0.367	1.046	0.02	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	111	106	164	181	351	221	0
normalized size	1	1.02	0.97	1.5	1.66	3.22	2.03	0.
time (sec)	N/A	0.121	0.195	0.077	1.201	2.408	21.351	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	181	79	170	0	0	0	0
normalized size	1	1.52	0.66	1.43	0.	0.	0.	0.
time (sec)	N/A	0.198	0.168	0.055	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	105	63	108	99	278	141	0
normalized size	1	1.72	1.03	1.77	1.62	4.56	2.31	0.
time (sec)	N/A	0.068	0.117	0.047	1.222	2.542	2.657	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	111	69	112	0	0	0	0
normalized size	1	1.66	1.03	1.67	0.	0.	0.	0.
time (sec)	N/A	0.059	0.137	0.041	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	177	131	171	0	0	0	0
normalized size	1	1.99	1.47	1.92	0.	0.	0.	0.
time (sec)	N/A	0.192	0.188	0.148	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	105	75	155	0	0	110	0
normalized size	1	1.72	1.23	2.54	0.	0.	1.8	0.
time (sec)	N/A	0.108	0.163	0.114	0.	0.	3.199	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	201	185	243	0	0	0	0
normalized size	1	1.78	1.64	2.15	0.	0.	0.	0.
time (sec)	N/A	0.195	3.257	0.195	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	106	78	501	193	486	0	0
normalized size	1	1.71	1.26	8.08	3.11	7.84	0.	0.
time (sec)	N/A	0.09	0.13	0.2	1.199	2.842	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	127	100	195	196	485	0	0
normalized size	1	1.02	0.8	1.56	1.57	3.88	0.	0.
time (sec)	N/A	0.143	0.172	0.092	1.2	2.395	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	254	154	240	0	0	0	0
normalized size	1	1.54	0.93	1.45	0.	0.	0.	0.
time (sec)	N/A	0.321	0.347	0.058	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	146	72	139	115	393	221	0
normalized size	1	1.9	0.94	1.81	1.49	5.1	2.87	0.
time (sec)	N/A	0.087	0.125	0.057	1.166	2.374	126.584	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	180	111	170	0	0	185	0
normalized size	1	1.62	1.	1.53	0.	0.	1.67	0.
time (sec)	N/A	0.112	0.225	0.048	0.	0.	52.345	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	249	180	227	0	0	0	0
normalized size	1	1.86	1.34	1.69	0.	0.	0.	0.
time (sec)	N/A	0.304	0.353	0.172	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	177	122	222	0	0	0	0
normalized size	1	1.64	1.13	2.06	0.	0.	0.	0.
time (sec)	N/A	0.168	0.288	0.148	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	270	292	295	0	0	0	0
normalized size	1	1.74	1.88	1.9	0.	0.	0.	0.
time (sec)	N/A	0.302	1.712	0.371	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	184	125	622	0	0	0	0
normalized size	1	1.6	1.09	5.41	0.	0.	0.	0.
time (sec)	N/A	0.219	0.227	0.221	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	143	108	226	211	613	0	0
normalized size	1	1.01	0.77	1.6	1.5	4.35	0.	0.
time (sec)	N/A	0.153	0.198	0.1	1.209	2.48	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	337	196	301	0	0	0	0
normalized size	1	1.58	0.92	1.41	0.	0.	0.	0.
time (sec)	N/A	0.458	0.566	0.093	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	193	80	170	130	508	0	0
normalized size	1	2.08	0.86	1.83	1.4	5.46	0.	0.
time (sec)	N/A	0.087	0.145	0.063	1.201	2.604	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	254	153	228	0	0	0	0
normalized size	1	1.54	0.93	1.38	0.	0.	0.	0.
time (sec)	N/A	0.165	0.379	0.052	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	329	257	284	0	0	0	0
normalized size	1	1.84	1.44	1.59	0.	0.	0.	0.
time (sec)	N/A	0.43	0.35	0.235	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	257	168	283	0	0	0	0
normalized size	1	1.64	1.07	1.8	0.	0.	0.	0.
time (sec)	N/A	0.236	0.395	0.191	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	355	349	356	0	0	0	0
normalized size	1	1.73	1.7	1.74	0.	0.	0.	0.
time (sec)	N/A	0.431	1.894	0.421	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	266	179	692	0	0	0	0
normalized size	1	1.6	1.08	4.17	0.	0.	0.	0.
time (sec)	N/A	0.294	0.399	0.24	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	38	115	0	0
normalized size	1	1.	0.88	0.81	1.19	3.59	0.	0.
time (sec)	N/A	0.03	0.013	0.024	1.658	2.35	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	215	108	193	235	363	182	0
normalized size	1	1.44	0.72	1.3	1.58	2.44	1.22	0.
time (sec)	N/A	0.257	0.185	0.092	1.172	2.606	26.581	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	170	111	188	0	0	185	0
normalized size	1	1.35	0.88	1.49	0.	0.	1.47	0.
time (sec)	N/A	0.226	0.237	0.085	0.	0.	14.462	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	142	82	133	158	286	122	0
normalized size	1	1.45	0.84	1.36	1.61	2.92	1.24	0.
time (sec)	N/A	0.157	0.135	0.083	1.142	2.593	5.43	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	97	69	125	0	0	92	0
normalized size	1	1.29	0.92	1.67	0.	0.	1.23	0.
time (sec)	N/A	0.121	0.175	0.055	0.	0.	4.64	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	64	49	72	74	213	60	0
normalized size	1	1.52	1.17	1.71	1.76	5.07	1.43	0.
time (sec)	N/A	0.065	0.079	0.042	1.25	2.39	2.256	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	53	103	0	85	0
normalized size	1	1.	1.	2.12	4.12	0.	3.4	0.
time (sec)	N/A	0.03	0.017	0.037	1.15	0.	2.326	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	96	72	0	0	0	0
normalized size	1	1.	1.71	1.29	0.	0.	0.	0.
time (sec)	N/A	0.119	0.155	0.047	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	63	42	84	150	319	0	0
normalized size	1	1.54	1.02	2.05	3.66	7.78	0.	0.
time (sec)	N/A	0.088	0.109	0.066	1.174	2.841	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	137	185	225	0	0	0	0
normalized size	1	1.19	1.61	1.96	0.	0.	0.	0.
time (sec)	N/A	0.213	2.717	0.079	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	141	99	372	163	495	0	0
normalized size	1	1.45	1.02	3.84	1.68	5.1	0.	0.
time (sec)	N/A	0.182	0.158	0.09	1.192	3.033	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	140	131	224	0	454	0	0
normalized size	1	1.02	0.96	1.64	0.	3.31	0.	0.
time (sec)	N/A	0.172	0.2	0.237	0.	2.968	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	181	147	269	0	0	0	0
normalized size	1	1.38	1.12	2.05	0.	0.	0.	0.
time (sec)	N/A	0.257	0.348	0.178	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	88	87	158	161	382	0	0
normalized size	1	1.02	1.01	1.84	1.87	4.44	0.	0.
time (sec)	N/A	0.142	0.16	0.181	1.757	2.981	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	105	78	196	0	0	0	0
normalized size	1	1.31	0.98	2.45	0.	0.	0.	0.
time (sec)	N/A	0.14	0.279	0.105	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	70	52	103	0	305	0	0
normalized size	1	1.56	1.16	2.29	0.	6.78	0.	0.
time (sec)	N/A	0.072	0.107	0.086	0.	2.882	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	76	66	132	88	0	0	0
normalized size	1	1.49	1.29	2.59	1.73	0.	0.	0.
time (sec)	N/A	0.039	0.093	0.066	1.122	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	119	143	156	0	0	0	0
normalized size	1	1.27	1.52	1.66	0.	0.	0.	0.
time (sec)	N/A	0.222	0.318	0.153	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	95	69	180	161	0	0	0
normalized size	1	1.02	0.74	1.94	1.73	0.	0.	0.
time (sec)	N/A	0.138	0.16	0.097	1.21	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	212	269	234	0	0	0	0
normalized size	1	1.31	1.66	1.44	0.	0.	0.	0.
time (sec)	N/A	0.35	3.947	0.177	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	156	127	601	0	0	0	0
normalized size	1	1.02	0.83	3.93	0.	0.	0.	0.
time (sec)	N/A	0.175	0.217	0.19	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	256	202	970	0	0	0	0
normalized size	1	1.33	1.05	5.05	0.	0.	0.	0.
time (sec)	N/A	0.425	0.453	0.301	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	149	132	231	0	502	0	0
normalized size	1	1.02	0.9	1.58	0.	3.44	0.	0.
time (sec)	N/A	0.181	0.214	0.425	0.	3.242	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	178	166	897	0	0	0	0
normalized size	1	1.28	1.19	6.45	0.	0.	0.	0.
time (sec)	N/A	0.283	0.35	0.203	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	107	93	175	186	435	0	0
normalized size	1	1.02	0.89	1.67	1.77	4.14	0.	0.
time (sec)	N/A	0.147	0.184	0.221	1.85	3.112	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	119	88	707	185	0	0	0
normalized size	1	1.49	1.1	8.84	2.31	0.	0.	0.
time (sec)	N/A	0.129	0.172	0.158	1.27	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	114	72	124	0	389	0	0
normalized size	1	1.52	0.96	1.65	0.	5.19	0.	0.
time (sec)	N/A	0.08	0.135	0.088	0.	3.114	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	147	100	618	170	0	0	0
normalized size	1	1.36	0.93	5.72	1.57	0.	0.	0.
time (sec)	N/A	0.087	0.147	0.09	1.121	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	187	209	220	0	0	0	0
normalized size	1	1.26	1.41	1.49	0.	0.	0.	0.
time (sec)	N/A	0.339	0.822	0.171	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	153	123	778	0	0	0	0
normalized size	1	1.02	0.82	5.19	0.	0.	0.	0.
time (sec)	N/A	0.176	0.208	0.179	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	325	331	314	0	0	0	0
normalized size	1	1.32	1.34	1.27	0.	0.	0.	0.
time (sec)	N/A	0.475	6.156	0.239	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	212	142	1153	319	0	0	0
normalized size	1	1.02	0.68	5.54	1.53	0.	0.	0.
time (sec)	N/A	0.24	0.237	0.158	1.219	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	201	0	0	167
normalized size	1	1.	0.6	1.82	1.	0.	0.	0.84
time (sec)	N/A	0.123	0.172	0.161	1.143	0.	0.	1.376

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	74	138	188	82	0
normalized size	1	1.	0.73	0.86	1.6	2.19	0.95	0.
time (sec)	N/A	0.155	0.046	0.038	1.121	2.581	2.971	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	48	82	80	126	65	85
normalized size	1	1.	0.69	1.17	1.14	1.8	0.93	1.21
time (sec)	N/A	0.111	0.04	0.031	1.145	2.533	1.527	1.34

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	100	146	42	0
normalized size	1	1.	0.86	0.82	2.04	2.98	0.86	0.
time (sec)	N/A	0.103	0.038	0.03	1.16	2.542	0.934	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	35	82	24	51
normalized size	1	1.	1.	1.68	1.25	2.93	0.86	1.82
time (sec)	N/A	0.046	0.027	0.026	1.247	2.505	0.561	1.343

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	51	10	31
normalized size	1	1.	1.	0.92	1.15	3.92	0.77	2.38
time (sec)	N/A	0.022	0.006	0.004	1.088	2.473	0.409	1.398

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	34	34	57	42	0	0	0	0
normalized size	1	1.	1.68	1.24	0.	0.	0.	0.
time (sec)	N/A	0.092	0.096	0.03	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	34	88	0	113
normalized size	1	1.	1.07	2.07	1.26	3.26	0.	4.19
time (sec)	N/A	0.064	0.036	0.056	1.165	2.652	0.	1.389

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0
normalized size	1	1.	1.58	1.88	0.	0.	0.	0.
time (sec)	N/A	0.148	0.671	0.052	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	120	578	0	346	0	0
normalized size	1	1.	0.69	3.3	0.	1.98	0.	0.
time (sec)	N/A	0.161	0.118	0.219	0.	2.653	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	129	320	0	0	0	0
normalized size	1	1.	0.71	1.77	0.	0.	0.	0.
time (sec)	N/A	0.194	0.789	0.148	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	92	321	99	273	0	0
normalized size	1	1.	0.88	3.06	0.94	2.6	0.	0.
time (sec)	N/A	0.067	0.092	0.122	1.136	2.589	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	120	222	0	0	0	0
normalized size	1	1.	1.08	2.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.44	0.091	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	168	331	0	0	0	0
normalized size	1	1.	0.95	1.87	0.	0.	0.	0.
time (sec)	N/A	0.195	0.452	0.146	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	129	263	0	0	0	0
normalized size	1	1.	1.23	2.5	0.	0.	0.	0.
time (sec)	N/A	0.115	0.321	0.128	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	223	377	0	0	0	0
normalized size	1	1.	1.11	1.88	0.	0.	0.	0.
time (sec)	N/A	0.195	2.904	0.178	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	131	946	0	470	0	0
normalized size	1	1.	1.24	8.92	0.	4.43	0.	0.
time (sec)	N/A	0.095	0.186	0.171	0.	3.045	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	130	872	0	460	0	0
normalized size	1	1.	0.6	4.02	0.	2.12	0.	0.
time (sec)	N/A	0.177	0.164	0.211	0.	2.515	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	251	421	0	0	0	0
normalized size	1	1.	0.99	1.66	0.	0.	0.	0.
time (sec)	N/A	0.31	0.713	0.214	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	102	559	115	373	0	0
normalized size	1	1.	0.7	3.83	0.79	2.55	0.	0.
time (sec)	N/A	0.074	0.129	0.149	1.205	2.44	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	200	318	0	0	0	0
normalized size	1	1.	1.11	1.77	0.	0.	0.	0.
time (sec)	N/A	0.109	0.793	0.13	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	248	428	0	0	0	0
normalized size	1	1.	1.	1.72	0.	0.	0.	0.
time (sec)	N/A	0.303	0.74	0.158	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	200	392	0	0	0	0
normalized size	1	1.	1.13	2.21	0.	0.	0.	0.
time (sec)	N/A	0.17	0.778	0.146	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	352	472	0	0	0	0
normalized size	1	1.	1.3	1.75	0.	0.	0.	0.
time (sec)	N/A	0.31	4.079	0.184	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	214	1107	0	0	0	0
normalized size	1	1.	1.16	6.02	0.	0.	0.	0.
time (sec)	N/A	0.231	0.721	0.176	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	140	996	0	585	0	0
normalized size	1	1.	0.53	3.74	0.	2.2	0.	0.
time (sec)	N/A	0.192	0.197	0.216	0.	2.436	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	388	537	0	0	0	0
normalized size	1	1.	1.15	1.59	0.	0.	0.	0.
time (sec)	N/A	0.466	0.94	0.259	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	112	863	130	483	0	0
normalized size	1	1.	0.58	4.47	0.67	2.5	0.	0.
time (sec)	N/A	0.088	0.148	0.18	1.147	2.611	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	317	427	0	0	0	0
normalized size	1	1.	1.25	1.68	0.	0.	0.	0.
time (sec)	N/A	0.16	0.691	0.161	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	361	540	0	0	0	0
normalized size	1	1.	1.1	1.64	0.	0.	0.	0.
time (sec)	N/A	0.439	1.235	0.202	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	270	506	0	0	0	0
normalized size	1	1.	1.05	1.97	0.	0.	0.	0.
time (sec)	N/A	0.238	1.343	0.185	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	467	588	0	0	0	0
normalized size	1	1.	1.32	1.66	0.	0.	0.	0.
time (sec)	N/A	0.447	6.73	0.217	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	287	1316	0	0	0	0
normalized size	1	1.	1.08	4.95	0.	0.	0.	0.
time (sec)	N/A	0.3	0.961	0.208	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	26	38	115	0	0
normalized size	1	1.	0.88	0.81	1.19	3.59	0.	0.
time (sec)	N/A	0.029	0.013	0.	1.657	2.405	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	119	625	0	355	0	0
normalized size	1	1.	0.55	2.91	0.	1.65	0.	0.
time (sec)	N/A	0.264	0.171	0.214	0.	2.544	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	151	347	0	0	0	0
normalized size	1	1.	0.79	1.81	0.	0.	0.	0.
time (sec)	N/A	0.252	0.603	0.253	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	93	358	0	278	0	0
normalized size	1	1.	0.65	2.52	0.	1.96	0.	0.
time (sec)	N/A	0.158	0.133	0.158	0.	2.514	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	121	247	0	0	0	0
normalized size	1	1.	1.02	2.08	0.	0.	0.	0.
time (sec)	N/A	0.146	0.723	0.171	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	74	148	74	205	0	0
normalized size	1	1.	1.16	2.31	1.16	3.2	0.	0.
time (sec)	N/A	0.061	0.102	0.086	1.148	2.394	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	48	77	0	0	0	0
normalized size	1	1.	1.02	1.64	0.	0.	0.	0.
time (sec)	N/A	0.057	0.049	0.036	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	129	234	0	0	0	0
normalized size	1	1.	1.06	1.92	0.	0.	0.	0.
time (sec)	N/A	0.188	0.362	0.124	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	67	183	0	301	0	0
normalized size	1	1.	1.06	2.9	0.	4.78	0.	0.
time (sec)	N/A	0.089	0.159	0.118	0.	3.127	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	203	203	229	380	0	0	0	0
normalized size	1	1.	1.13	1.87	0.	0.	0.	0.
time (sec)	N/A	0.292	3.025	0.187	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	135	791	0	477	0	0
normalized size	1	1.	0.96	5.61	0.	3.38	0.	0.
time (sec)	N/A	0.184	0.209	0.17	0.	3.185	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	220	148	362	0	436	0	0
normalized size	1	1.04	0.7	1.71	0.	2.06	0.	0.
time (sec)	N/A	0.294	0.265	0.206	0.	3.618	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	161	366	0	0	0	0
normalized size	1	1.	0.78	1.78	0.	0.	0.	0.
time (sec)	N/A	0.282	0.472	0.246	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	141	143	260	0	365	0	0
normalized size	1	1.04	1.05	1.91	0.	2.68	0.	0.
time (sec)	N/A	0.182	0.243	0.159	0.	3.315	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	146	232	0	0	0	0
normalized size	1	1.	1.12	1.78	0.	0.	0.	0.
time (sec)	N/A	0.168	0.195	0.151	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	82	164	0	288	0	0
normalized size	1	1.	1.17	2.34	0.	4.11	0.	0.
time (sec)	N/A	0.07	0.162	0.083	0.	3.18	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	100	143	88	0	0	0
normalized size	1	1.	1.32	1.88	1.16	0.	0.	0.
time (sec)	N/A	0.038	0.159	0.085	1.245	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	231	274	0	0	0	0
normalized size	1	1.	1.19	1.41	0.	0.	0.	0.
time (sec)	N/A	0.303	0.883	0.138	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	163	205	0	0	0	0
normalized size	1	1.	1.14	1.43	0.	0.	0.	0.
time (sec)	N/A	0.154	0.279	0.118	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	381	389	0	0	0	0
normalized size	1	1.	1.33	1.36	0.	0.	0.	0.
time (sec)	N/A	0.431	6.368	0.172	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	216	965	0	0	0	0
normalized size	1	1.	0.95	4.23	0.	0.	0.	0.
time (sec)	N/A	0.289	0.299	0.171	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	222	1607	0	0	0	0
normalized size	1	1.	0.79	5.72	0.	0.	0.	0.
time (sec)	N/A	0.435	1.003	0.341	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	225	154	394	0	483	0	0
normalized size	1	1.07	0.73	1.88	0.	2.3	0.	0.
time (sec)	N/A	0.309	0.272	0.194	0.	3.315	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	191	1430	0	0	0	0
normalized size	1	1.	0.94	7.04	0.	0.	0.	0.
time (sec)	N/A	0.296	0.532	0.238	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	149	151	262	186	416	0	0
normalized size	1	1.03	1.05	1.82	1.29	2.89	0.	0.
time (sec)	N/A	0.182	0.24	0.151	1.782	3.234	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	1153	185	0	0	0
normalized size	1	1.	0.99	9.69	1.55	0.	0.	0.
time (sec)	N/A	0.125	0.196	0.161	1.25	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	130	198	0	370	0	0
normalized size	1	1.	1.14	1.74	0.	3.25	0.	0.
time (sec)	N/A	0.078	0.183	0.114	0.	3.128	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	143	1005	170	0	0	0
normalized size	1	1.	0.97	6.84	1.16	0.	0.	0.
time (sec)	N/A	0.079	0.193	0.109	1.283	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	247	364	0	0	0	0
normalized size	1	1.	0.94	1.39	0.	0.	0.	0.
time (sec)	N/A	0.415	1.234	0.146	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	227	1257	0	0	0	0
normalized size	1	1.	1.06	5.87	0.	0.	0.	0.
time (sec)	N/A	0.213	0.311	0.151	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	400	400	437	546	0	0	0	0
normalized size	1	1.	1.09	1.36	0.	0.	0.	0.
time (sec)	N/A	0.56	6.57	0.204	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	267	1790	0	0	0	0
normalized size	1	1.	0.9	6.03	0.	0.	0.	0.
time (sec)	N/A	0.372	0.357	0.175	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	121	363	201	0	0	167
normalized size	1	1.	0.6	1.82	1.	0.	0.	0.84
time (sec)	N/A	0.112	0.141	0.	1.269	0.	0.	1.484

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	63	74	138	188	82	0
normalized size	1	1.	0.73	0.86	1.6	2.19	0.95	0.
time (sec)	N/A	0.146	0.045	0.	1.154	2.716	3.317	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	48	82	80	126	65	85
normalized size	1	1.	0.69	1.17	1.14	1.8	0.93	1.21
time (sec)	N/A	0.103	0.041	0.	1.244	2.573	1.664	1.359

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	40	100	146	42	0
normalized size	1	1.	0.86	0.82	2.04	2.98	0.86	0.
time (sec)	N/A	0.081	0.039	0.	1.298	2.558	0.957	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	47	35	82	24	51
normalized size	1	1.	1.	1.68	1.25	2.93	0.86	1.82
time (sec)	N/A	0.042	0.026	0.	1.242	3.104	0.539	1.418

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	51	10	31
normalized size	1	1.	1.	0.92	1.15	3.92	0.77	2.38
time (sec)	N/A	0.02	0.006	0.	1.154	3.004	0.414	1.379

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	34	34	57	42	0	0	0	0
normalized size	1	1.	1.68	1.24	0.	0.	0.	0.
time (sec)	N/A	0.085	0.041	0.	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	56	34	88	0	113
normalized size	1	1.	1.07	2.07	1.26	3.26	0.	4.19
time (sec)	N/A	0.059	0.037	0.	1.155	2.642	0.	1.452

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	126	150	0	0	0	0
normalized size	1	1.	1.58	1.88	0.	0.	0.	0.
time (sec)	N/A	0.144	0.026	0.	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	257	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	2.172	0.517	2.484	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	188	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.297	0.079	2.119	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	118	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.069	1.754	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	3.67	0.38	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	115	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	5.408	0.435	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	5.663	0.47	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	618	332	0	0	0	0	0
normalized size	1	1.	0.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.556	1.319	1.312	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	233	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.33	0.528	1.136	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	179	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.071	1.042	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	129	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.056	0.323	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	206	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.331	0.224	0.424	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	286	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.471	0.421	0.433	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	97	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.031	0.227	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	201	342	595	613	388	599
normalized size	1	1.	0.71	1.21	2.1	2.17	1.37	2.12
time (sec)	N/A	0.477	0.271	0.043	1.186	2.796	15.72	2.439

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	186	298	680	525	332	0
normalized size	1	1.	0.94	1.51	3.43	2.65	1.68	0.
time (sec)	N/A	0.568	0.259	0.039	1.253	2.699	10.761	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	177	260	467	509	313	501
normalized size	1	1.	0.86	1.26	2.27	2.47	1.52	2.43
time (sec)	N/A	0.342	0.235	0.037	1.188	2.709	5.562	2.2

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	155	216	552	447	269	0
normalized size	1	1.	1.15	1.6	4.09	3.31	1.99	0.
time (sec)	N/A	0.134	0.258	0.036	1.227	2.691	3.394	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	135	172	311	387	224	383
normalized size	1	1.	1.08	1.38	2.49	3.1	1.79	3.06
time (sec)	N/A	0.144	0.201	0.031	1.104	2.714	1.561	2.063

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	166	165	209	425	0	0	0	0
normalized size	1	0.99	1.26	2.56	0.	0.	0.	0.
time (sec)	N/A	0.246	0.374	0.125	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	192	252	0	0	0	0
normalized size	1	1.	1.47	1.92	0.	0.	0.	0.
time (sec)	N/A	0.318	0.427	0.105	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	180	179	212	515	0	0	0	0
normalized size	1	0.99	1.18	2.86	0.	0.	0.	0.
time (sec)	N/A	0.31	0.366	0.246	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	245	278	0	0	0	0
normalized size	1	1.	1.55	1.76	0.	0.	0.	0.
time (sec)	N/A	0.393	0.794	0.238	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	251	446	1026	871	563	998
normalized size	1	1.	0.65	1.16	2.66	2.26	1.46	2.59
time (sec)	N/A	0.739	0.388	0.046	1.353	3.2	44.95	2.731

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	237	406	1154	784	515	0
normalized size	1	1.	0.8	1.37	3.9	2.65	1.74	0.
time (sec)	N/A	1.037	0.356	0.045	1.338	3.003	31.401	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	227	364	836	755	483	853
normalized size	1	1.	0.75	1.2	2.76	2.49	1.59	2.82
time (sec)	N/A	0.594	0.396	0.046	1.225	2.708	17.336	2.765

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	208	324	964	663	430	0
normalized size	1	1.	1.02	1.59	4.73	3.25	2.11	0.
time (sec)	N/A	0.206	0.487	0.038	1.238	2.74	11.853	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	191	276	617	616	389	690
normalized size	1	1.	0.89	1.29	2.88	2.88	1.82	3.22
time (sec)	N/A	0.256	0.404	0.036	1.156	2.716	5.988	2.442

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	257	256	323	586	0	0	0	0
normalized size	1	1.	1.26	2.28	0.	0.	0.	0.
time (sec)	N/A	0.441	0.38	0.162	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	306	400	0	0	0	0
normalized size	1	1.	1.34	1.75	0.	0.	0.	0.
time (sec)	N/A	0.523	1.151	0.134	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	305	719	0	0	0	0
normalized size	1	1.	1.12	2.64	0.	0.	0.	0.
time (sec)	N/A	0.501	0.879	0.342	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	357	408	0	0	0	0
normalized size	1	1.	1.44	1.65	0.	0.	0.	0.
time (sec)	N/A	0.707	0.904	0.281	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	299	544	1497	1135	702	0
normalized size	1	1.	0.64	1.17	3.22	2.44	1.51	0.
time (sec)	N/A	1.058	0.461	0.092	1.241	2.875	115.345	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	285	508	1669	996	654	0
normalized size	1	1.	0.76	1.35	4.44	2.65	1.74	0.
time (sec)	N/A	1.657	0.46	0.086	1.441	2.783	82.191	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	275	462	1245	963	626	0
normalized size	1	1.	0.72	1.21	3.26	2.52	1.64	0.
time (sec)	N/A	0.861	0.484	0.05	1.334	2.751	49.563	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	256	426	1416	855	573	0
normalized size	1	1.	0.98	1.63	5.43	3.28	2.2	0.
time (sec)	N/A	0.257	0.659	0.043	1.474	2.821	35.235	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	239	372	961	818	524	1025
normalized size	1	1.	0.82	1.28	3.3	2.81	1.8	3.52
time (sec)	N/A	0.403	0.536	0.041	1.297	2.787	18.653	2.986

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	337	336	416	706	0	0	0	0
normalized size	1	1.	1.23	2.09	0.	0.	0.	0.
time (sec)	N/A	0.684	0.658	0.199	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	466	516	0	0	0	0
normalized size	1	1.	1.52	1.68	0.	0.	0.	0.
time (sec)	N/A	0.739	1.414	0.163	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	354	355	459	838	0	0	0	0
normalized size	1	1.	1.3	2.37	0.	0.	0.	0.
time (sec)	N/A	0.75	1.175	0.411	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	461	528	0	0	0	0
normalized size	1	1.	1.41	1.62	0.	0.	0.	0.
time (sec)	N/A	1.021	1.111	0.331	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	365	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.553	1.29	0.25	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	199	199	279	380	0	0	0	0
normalized size	1	1.	1.4	1.91	0.	0.	0.	0.
time (sec)	N/A	0.408	0.458	0.114	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	293	0	0	0	0	0
normalized size	1	1.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	0.571	0.097	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	281	223	0	0	0	0
normalized size	1	1.	2.68	2.12	0.	0.	0.	0.
time (sec)	N/A	0.184	0.238	0.049	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	274	0	0	0	0	0
normalized size	1	1.	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.24	0.128	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	400	354	0	0	0	0
normalized size	1	1.	3.45	3.05	0.	0.	0.	0.
time (sec)	N/A	0.208	0.366	0.074	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	363	0	0	0	0	0
normalized size	1	1.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	1.061	0.201	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	419	719	0	0	0	0
normalized size	1	1.	2.16	3.71	0.	0.	0.	0.
time (sec)	N/A	0.388	1.063	0.148	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	602	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.639	8.019	0.228	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	482	0	0	0	0	0
normalized size	1	1.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.531	2.191	0.26	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	320	499	0	0	0	0
normalized size	1	1.	1.5	2.34	0.	0.	0.	0.
time (sec)	N/A	0.405	1.063	0.197	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	385	0	0	0	0	0
normalized size	1	1.	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.298	1.802	0.174	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	145	222	0	397	0	0
normalized size	1	1.	1.71	2.61	0.	4.67	0.	0.
time (sec)	N/A	0.107	0.219	0.063	0.	2.684	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	403	0	0	0	0	0
normalized size	1	1.	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	1.358	0.095	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	428	724	0	0	0	0
normalized size	1	1.	2.22	3.75	0.	0.	0.	0.
time (sec)	N/A	0.34	1.806	0.128	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	549	0	0	0	0	0
normalized size	1	1.	1.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.544	7.195	0.255	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	253	253	594	799	0	0	0	0
normalized size	1	1.	2.35	3.16	0.	0.	0.	0.
time (sec)	N/A	0.581	0.925	0.157	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	401	401	764	0	0	0	0	0
normalized size	1	1.	1.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.963	8.731	0.338	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	552	0	0	0	0	0
normalized size	1	1.	1.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.545	3.193	0.421	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	186	523	0	605	0	0
normalized size	1	1.	1.11	3.13	0.	3.62	0.	0.
time (sec)	N/A	0.337	0.395	0.248	0.	2.849	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	318	318	550	0	0	0	0	0
normalized size	1	1.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.417	2.396	0.326	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	152	432	0	590	0	0
normalized size	1	1.	1.05	2.98	0.	4.07	0.	0.
time (sec)	N/A	0.146	0.241	0.095	0.	2.831	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	546	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.364	2.233	0.168	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	560	1129	0	0	0	0
normalized size	1	1.	2.04	4.11	0.	0.	0.	0.
time (sec)	N/A	0.507	3.676	0.237	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	389	389	716	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.767	7.619	0.36	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	381	381	759	1436	0	0	0	0
normalized size	1	1.	1.99	3.77	0.	0.	0.	0.
time (sec)	N/A	0.803	9.572	0.289	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	529	529	937	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	1.307	10.159	0.392	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	420	284	486	0	0	0	0
normalized size	1	1.4	0.95	1.62	0.	0.	0.	0.
time (sec)	N/A	0.382	0.952	0.103	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	294	202	350	0	0	405	0
normalized size	1	1.4	0.96	1.67	0.	0.	1.93	0.
time (sec)	N/A	0.227	0.621	0.081	0.	0.	151.738	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	184	124	213	0	0	0	0
normalized size	1	1.51	1.02	1.75	0.	0.	0.	0.
time (sec)	N/A	0.113	0.345	0.071	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	72	230	0	88	0
normalized size	1	1.	1.	2.88	9.2	0.	3.52	0.
time (sec)	N/A	0.05	0.024	0.063	1.363	0.	3.129	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	179	153	306	0	0	0	0
normalized size	1	1.72	1.47	2.94	0.	0.	0.	0.
time (sec)	N/A	0.177	0.388	0.129	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	292	293	1730	0	0	0	0
normalized size	1	1.43	1.44	8.48	0.	0.	0.	0.
time (sec)	N/A	0.282	0.624	0.224	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	222	1162	0	694	0	0
normalized size	1	1.	0.62	3.25	0.	1.94	0.	0.
time (sec)	N/A	0.479	0.268	0.349	0.	2.909	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	207	701	0	0	0	0
normalized size	1	1.	0.71	2.41	0.	0.	0.	0.
time (sec)	N/A	0.372	1.312	0.241	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	166	657	247	531	0	0
normalized size	1	1.	0.92	3.65	1.37	2.95	0.	0.
time (sec)	N/A	0.152	0.271	0.177	1.263	2.712	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	200	482	0	0	0	0
normalized size	1	1.	1.09	2.62	0.	0.	0.	0.
time (sec)	N/A	0.12	1.087	0.146	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	352	823	0	0	0	0
normalized size	1	1.	1.04	2.43	0.	0.	0.	0.
time (sec)	N/A	0.343	1.162	0.277	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	232	625	0	0	0	0
normalized size	1	1.	1.11	2.99	0.	0.	0.	0.
time (sec)	N/A	0.255	1.152	0.241	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	446	870	0	0	0	0
normalized size	1	1.	1.25	2.43	0.	0.	0.	0.
time (sec)	N/A	0.383	5.069	0.318	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	240	2557	0	0	0	0
normalized size	1	1.	0.82	8.7	0.	0.	0.	0.
time (sec)	N/A	0.285	0.779	0.298	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	251	1766	0	942	0	0
normalized size	1	1.	0.52	3.66	0.	1.95	0.	0.
time (sec)	N/A	0.811	0.384	0.38	0.	2.981	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	508	934	0	0	0	0
normalized size	1	1.	1.25	2.31	0.	0.	0.	0.
time (sec)	N/A	0.664	1.197	0.358	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	198	1149	311	743	0	0
normalized size	1	1.	0.74	4.3	1.16	2.78	0.	0.
time (sec)	N/A	0.219	0.361	0.266	1.289	3.055	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	329	709	0	0	0	0
normalized size	1	1.	1.12	2.41	0.	0.	0.	0.
time (sec)	N/A	0.246	1.938	0.22	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	498	498	520	1053	0	0	0	0
normalized size	1	1.	1.04	2.11	0.	0.	0.	0.
time (sec)	N/A	0.595	2.438	0.291	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	398	398	369	954	0	0	0	0
normalized size	1	1.	0.93	2.4	0.	0.	0.	0.
time (sec)	N/A	0.422	3.031	0.259	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	541	541	771	1131	0	0	0	0
normalized size	1	1.	1.43	2.09	0.	0.	0.	0.
time (sec)	N/A	0.654	7.872	0.349	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	458	2796	0	0	0	0
normalized size	1	1.	1.21	7.4	0.	0.	0.	0.
time (sec)	N/A	0.584	1.374	0.319	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	277	2014	0	1181	0	0
normalized size	1	1.	0.44	3.22	0.	1.89	0.	0.
time (sec)	N/A	1.238	0.446	0.388	0.	3.208	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	619	1204	0	0	0	0
normalized size	1	1.	1.15	2.25	0.	0.	0.	0.
time (sec)	N/A	1.047	2.047	0.426	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	224	1773	370	980	0	0
normalized size	1	1.	0.61	4.84	1.01	2.68	0.	0.
time (sec)	N/A	0.294	0.41	0.34	1.339	3.063	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	499	966	0	0	0	0
normalized size	1	1.	1.19	2.3	0.	0.	0.	0.
time (sec)	N/A	0.404	1.533	0.286	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	635	635	710	1321	0	0	0	0
normalized size	1	1.	1.12	2.08	0.	0.	0.	0.
time (sec)	N/A	0.908	4.386	0.361	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	530	530	550	1223	0	0	0	0
normalized size	1	1.	1.04	2.31	0.	0.	0.	0.
time (sec)	N/A	0.613	2.257	0.338	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	687	687	990	1404	0	0	0	0
normalized size	1	1.	1.44	2.04	0.	0.	0.	0.
time (sec)	N/A	0.988	8.002	0.415	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	561	561	616	3311	0	0	0	0
normalized size	1	1.	1.1	5.9	0.	0.	0.	0.
time (sec)	N/A	0.851	2.38	0.375	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	125	0	297	146	0
normalized size	1	1.	0.64	0.82	0.	1.94	0.95	0.
time (sec)	N/A	0.29	0.069	0.059	0.	3.122	5.759	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	79	113	136	223	121	154
normalized size	1	1.	0.65	0.93	1.11	1.83	0.99	1.26
time (sec)	N/A	0.215	0.061	0.058	1.221	3.172	3.073	1.399

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	72	69	0	239	78	0
normalized size	1	1.	0.83	0.79	0.	2.75	0.9	0.
time (sec)	N/A	0.154	0.046	0.046	0.	2.826	1.713	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	65	157	49	100
normalized size	1	1.	0.92	1.23	1.25	3.02	0.94	1.92
time (sec)	N/A	0.078	0.033	0.043	1.212	3.112	0.987	1.384

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	51	10	31
normalized size	1	1.	1.	0.92	1.15	3.92	0.77	2.38
time (sec)	N/A	0.034	0.007	0.004	1.135	2.93	0.57	1.439

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	100	144	0	0	0	0
normalized size	1	1.	1.47	2.12	0.	0.	0.	0.
time (sec)	N/A	0.149	0.111	0.052	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	65	132	0	0	0	0
normalized size	1	1.	0.98	2.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.321	0.078	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	188	233	0	0	0	0
normalized size	1	1.	1.39	1.73	0.	0.	0.	0.
time (sec)	N/A	0.262	1.2	0.116	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	230	1227	0	711	0	0
normalized size	1	1.	0.6	3.2	0.	1.86	0.	0.
time (sec)	N/A	0.554	0.366	0.355	0.	3.316	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	268	760	0	0	0	0
normalized size	1	1.	0.83	2.35	0.	0.	0.	0.
time (sec)	N/A	0.479	0.835	0.38	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	176	706	0	536	0	0
normalized size	1	1.	0.66	2.66	0.	2.02	0.	0.
time (sec)	N/A	0.33	0.28	0.267	0.	3.122	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	198	530	0	0	0	0
normalized size	1	1.	0.97	2.6	0.	0.	0.	0.
time (sec)	N/A	0.275	0.881	0.262	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	127	296	169	385	0	0
normalized size	1	1.	0.92	2.14	1.22	2.79	0.	0.
time (sec)	N/A	0.124	0.284	0.155	1.274	3.201	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	62	120	0	0	0	0
normalized size	1	1.	1.32	2.55	0.	0.	0.	0.
time (sec)	N/A	0.095	0.13	0.052	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	266	564	0	0	0	0
normalized size	1	1.	1.19	2.53	0.	0.	0.	0.
time (sec)	N/A	0.342	0.88	0.234	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	168	526	0	0	0	0
normalized size	1	1.	1.01	3.15	0.	0.	0.	0.
time (sec)	N/A	0.227	0.449	0.219	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	455	901	0	0	0	0
normalized size	1	1.	1.26	2.5	0.	0.	0.	0.
time (sec)	N/A	0.57	5.442	0.342	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	278	2147	0	0	0	0
normalized size	1	1.	0.93	7.18	0.	0.	0.	0.
time (sec)	N/A	0.427	0.629	0.309	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	427	933	0	0	0	0
normalized size	1	1.	0.83	1.81	0.	0.	0.	0.
time (sec)	N/A	0.793	0.606	0.369	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	400	400	288	816	0	0	0	0
normalized size	1	1.	0.72	2.04	0.	0.	0.	0.
time (sec)	N/A	0.665	1.93	0.386	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	383	383	318	703	0	0	0	0
normalized size	1	1.	0.83	1.84	0.	0.	0.	0.
time (sec)	N/A	0.455	0.46	0.319	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	215	478	0	0	0	0
normalized size	1	1.	0.92	2.05	0.	0.	0.	0.
time (sec)	N/A	0.383	1.032	0.257	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	217	446	0	0	0	0
normalized size	1	1.	1.15	2.37	0.	0.	0.	0.
time (sec)	N/A	0.187	0.455	0.151	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	152	343	0	0	0	0
normalized size	1	1.	0.85	1.92	0.	0.	0.	0.
time (sec)	N/A	0.184	0.455	0.17	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	412	412	568	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.595	1.565	0.255	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	296	660	0	0	0	0
normalized size	1	1.	0.97	2.16	0.	0.	0.	0.
time (sec)	N/A	0.464	0.898	0.227	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	573	573	884	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.926	7.495	0.338	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	452	452	438	2609	0	0	0	0
normalized size	1	1.	0.97	5.77	0.	0.	0.	0.
time (sec)	N/A	0.841	0.849	0.314	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	333	1040	0	0	0	0
normalized size	1	1.	0.65	2.03	0.	0.	0.	0.
time (sec)	N/A	0.881	1.739	0.342	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	398	398	359	3705	0	0	0	0
normalized size	1	1.	0.9	9.31	0.	0.	0.	0.
time (sec)	N/A	0.761	1.243	0.378	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	301	705	0	0	0	0
normalized size	1	1.	0.98	2.3	0.	0.	0.	0.
time (sec)	N/A	0.505	1.094	0.286	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	280	3112	0	0	0	0
normalized size	1	1.	0.9	9.97	0.	0.	0.	0.
time (sec)	N/A	0.364	0.962	0.281	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	254	591	0	0	0	0
normalized size	1	1.	0.94	2.19	0.	0.	0.	0.
time (sec)	N/A	0.217	1.026	0.184	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	236	2729	0	0	0	0
normalized size	1	1.	0.81	9.35	0.	0.	0.	0.
time (sec)	N/A	0.294	1.307	0.194	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	547	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.863	4.124	0.257	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	408	3517	0	0	0	0
normalized size	1	1.	0.97	8.35	0.	0.	0.	0.
time (sec)	N/A	0.655	1.857	0.276	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	687	687	983	0	0	0	0	0
normalized size	1	1.	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	1.259	7.979	0.388	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	506	506	417	4955	0	0	0	0
normalized size	1	1.	0.82	9.79	0.	0.	0.	0.
time (sec)	N/A	1.094	3.024	0.33	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	366	366	178	570	0	0	0	0
normalized size	1	1.	0.49	1.56	0.	0.	0.	0.
time (sec)	N/A	0.353	0.762	0.178	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	935	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	2.724	1.248	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	487	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.42	0.997	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.295	0.852	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	2.79	0.285	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	4.227	0.444	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	4.315	0.433	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.485	0.249	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	169	270	373	603	355	333
normalized size	1	1.	0.47	0.75	1.04	1.68	0.99	0.93
time (sec)	N/A	0.728	0.258	0.055	1.228	2.248	25.476	1.674

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	137	200	284	467	262	273
normalized size	1	1.	0.52	0.75	1.07	1.76	0.99	1.03
time (sec)	N/A	0.421	0.144	0.046	1.212	2.172	9.066	1.722

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	99	128	167	315	150	196
normalized size	1	1.	0.65	0.84	1.09	2.06	0.98	1.28
time (sec)	N/A	0.217	0.082	0.036	1.21	2.1	2.467	1.678

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	454	0	0	0	0	0
normalized size	1	1.	2.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.219	0.086	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	568	0	0	0	0	0
normalized size	1	1.	1.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	2.537	0.126	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	409	409	654	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.517	5.364	0.227	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	177	802	0	0	0	0
normalized size	1	1.	0.35	1.58	0.	0.	0.	0.
time (sec)	N/A	0.595	0.685	0.176	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	136	484	0	0	0	0
normalized size	1	1.	0.39	1.39	0.	0.	0.	0.
time (sec)	N/A	0.348	0.281	0.135	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	86	231	0	0	0	0
normalized size	1	1.	0.42	1.13	0.	0.	0.	0.
time (sec)	N/A	0.176	0.141	0.118	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	0	0	0
normalized size	1	1.	1.	0.98	0.	0.	0.	0.
time (sec)	N/A	0.078	0.037	0.032	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	133	262	0	0	0	0
normalized size	1	1.	0.61	1.2	0.	0.	0.	0.
time (sec)	N/A	0.188	0.26	0.121	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	195	550	0	0	0	0
normalized size	1	1.	0.54	1.52	0.	0.	0.	0.
time (sec)	N/A	0.329	0.535	0.184	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	297	888	0	0	0	0
normalized size	1	1.	0.58	1.72	0.	0.	0.	0.
time (sec)	N/A	0.526	0.66	0.226	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.475	0.22	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	121	156	0	383	185	0
normalized size	1	1.	0.65	0.83	0.	2.05	0.99	0.
time (sec)	N/A	0.498	0.077	0.061	0.	2.167	9.194	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	164	171	296	148	193
normalized size	1	1.	0.64	1.07	1.12	1.93	0.97	1.26
time (sec)	N/A	0.338	0.068	0.054	1.16	2.143	4.802	1.43

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	83	84	0	293	100	0
normalized size	1	1.	0.79	0.8	0.	2.79	0.95	0.
time (sec)	N/A	0.225	0.067	0.052	0.	2.118	3.077	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	58	90	82	209	61	136
normalized size	1	1.	0.91	1.41	1.28	3.27	0.95	2.12
time (sec)	N/A	0.11	0.033	0.045	1.16	2.109	1.476	1.48

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	51	10	31
normalized size	1	1.	1.	0.92	1.15	3.92	0.77	2.38
time (sec)	N/A	0.031	0.006	0.004	1.086	2.081	0.843	1.452

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	146	197	0	0	0	0
normalized size	1	1.	1.43	1.93	0.	0.	0.	0.
time (sec)	N/A	0.162	0.129	0.056	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	97	187	0	0	0	0
normalized size	1	1.	1.1	2.12	0.	0.	0.	0.
time (sec)	N/A	0.189	0.199	0.083	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	304	377	0	0	0	0
normalized size	1	1.	1.45	1.8	0.	0.	0.	0.
time (sec)	N/A	0.363	4.463	0.133	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	42	0	0	0	0
normalized size	1	1.	0.64	0.63	0.	0.	0.	0.
time (sec)	N/A	0.115	0.113	0.04	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	33	0	0	0	0
normalized size	1	1.	0.68	0.66	0.	0.	0.	0.
time (sec)	N/A	0.1	0.076	0.033	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	22	0	0	0	0
normalized size	1	1.	0.79	0.76	0.	0.	0.	0.
time (sec)	N/A	0.071	0.013	0.028	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.347	0.079	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	1.545	0.121	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	152	199	0	0	0	0
normalized size	1	1.	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.508	0.352	0.262	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	179	135	178	0	0	0	0
normalized size	1	0.98	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.532	0.286	0.168	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	79	0	0	0	0
normalized size	1	1.	0.79	0.96	0.	0.	0.	0.
time (sec)	N/A	0.273	0.175	0.106	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	117	91	118	0	0	0	0
normalized size	1	0.97	0.75	0.98	0.	0.	0.	0.
time (sec)	N/A	0.313	0.193	0.092	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	79	0	0	0	0
normalized size	1	1.	0.77	0.96	0.	0.	0.	0.
time (sec)	N/A	0.183	0.127	0.08	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.426	1.886	0.155	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	44	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	1.102	0.162	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	4.624	0.333	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.844	0.399	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	241	179	238	0	0	0	0
normalized size	1	0.98	0.73	0.97	0.	0.	0.	0.
time (sec)	N/A	0.616	0.654	0.269	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	152	199	0	0	0	0
normalized size	1	1.	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.495	0.481	0.208	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	179	136	178	0	0	0	0
normalized size	1	0.98	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.417	0.485	0.175	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	109	139	0	0	0	0
normalized size	1	1.	0.76	0.97	0.	0.	0.	0.
time (sec)	N/A	0.287	0.253	0.144	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	138	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.853	1.955	0.152	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	105	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.604	1.395	0.143	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	4.596	0.44	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.816	0.369	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	241	180	238	0	0	0	0
normalized size	1	0.98	0.73	0.97	0.	0.	0.	0.
time (sec)	N/A	0.577	0.969	0.35	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	197	259	0	0	0	0
normalized size	1	1.	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.619	0.866	0.316	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	241	180	238	0	0	0	0
normalized size	1	0.98	0.73	0.97	0.	0.	0.	0.
time (sec)	N/A	0.491	0.809	0.262	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	153	199	0	0	0	0
normalized size	1	1.	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.359	0.542	0.225	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.28	1.943	0.22	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.967	1.304	0.171	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	4.569	0.542	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.899	0.429	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	36	0	0	0	0
normalized size	1	1.	0.76	0.88	0.	0.	0.	0.
time (sec)	N/A	0.161	0.071	0.052	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	23	0	0	0	0
normalized size	1	1.	0.81	0.85	0.	0.	0.	0.
time (sec)	N/A	0.155	0.077	0.043	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	0	0	0	0
normalized size	1	1.	0.81	0.89	0.	0.	0.	0.
time (sec)	N/A	0.145	0.068	0.041	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	0	0	0	0
normalized size	1	1.	0.81	0.89	0.	0.	0.	0.
time (sec)	N/A	0.144	0.023	0.	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0
normalized size	1	1.	1.	1.11	0.	0.	0.	0.
time (sec)	N/A	0.08	0.026	0.034	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	50	7	30
normalized size	1	1.	1.	1.11	1.33	5.56	0.78	3.33
time (sec)	N/A	0.037	0.015	0.004	1.119	2.034	0.532	1.325

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	1.053	0.078	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.131	0.092	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	179	136	178	0	0	0	0
normalized size	1	0.98	0.74	0.97	0.	0.	0.	0.
time (sec)	N/A	0.459	0.342	0.125	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	109	139	0	0	0	0
normalized size	1	1.	0.76	0.97	0.	0.	0.	0.
time (sec)	N/A	0.403	0.218	0.151	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	117	92	118	0	0	0	0
normalized size	1	0.97	0.76	0.98	0.	0.	0.	0.
time (sec)	N/A	0.386	0.216	0.095	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	79	0	0	0	0
normalized size	1	1.	0.79	0.96	0.	0.	0.	0.
time (sec)	N/A	0.292	0.18	0.083	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	50	46	58	0	0	0	0
normalized size	1	0.93	0.85	1.07	0.	0.	0.	0.
time (sec)	N/A	0.185	0.111	0.034	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	63	0	39
normalized size	1	1.	1.	1.06	1.38	3.94	0.	2.44
time (sec)	N/A	0.05	0.025	0.006	1.087	2.135	0.	1.412

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	1.694	0.105	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	1.166	0.085	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	2.055	0.161	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	1.066	0.121	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.063	0.098	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	1.453	0.249	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	1.083	0.158	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.882	0.766	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.49	0.683	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.115	0.605	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.333	0.157	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.587	0.326	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	106	0	0	0	0
normalized size	1	1.	0.87	1.13	0.	0.	0.	0.
time (sec)	N/A	0.184	0.49	0.052	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	69	84	0	0	0	0
normalized size	1	1.	0.9	1.09	0.	0.	0.	0.
time (sec)	N/A	0.17	0.383	0.033	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	60	0	0	0	0
normalized size	1	1.	1.	1.11	0.	0.	0.	0.
time (sec)	N/A	0.135	0.212	0.036	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	1.475	0.08	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	3.636	0.132	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	209	175	633	0	0	0	0
normalized size	1	0.98	0.82	2.97	0.	0.	0.	0.
time (sec)	N/A	0.711	0.668	0.25	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	248	0	0	0	0
normalized size	1	1.	0.88	2.67	0.	0.	0.	0.
time (sec)	N/A	0.562	0.31	0.145	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	198	126	364	0	0	0	0
normalized size	1	1.33	0.85	2.44	0.	0.	0.	0.
time (sec)	N/A	0.412	0.362	0.135	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	192	0	0	0	0
normalized size	1	1.	0.86	2.26	0.	0.	0.	0.
time (sec)	N/A	0.181	0.167	0.126	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	10.363	0.263	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	55	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	2.67	0.177	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	18.291	0.385	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	3.517	0.472	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	273	399	958	0	0	0	0
normalized size	1	0.99	1.44	3.46	0.	0.	0.	0.
time (sec)	N/A	1.006	0.8	0.369	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	306	704	0	0	0	0
normalized size	1	1.	1.4	3.21	0.	0.	0.	0.
time (sec)	N/A	0.716	0.731	0.298	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	209	295	633	0	0	0	0
normalized size	1	0.98	1.38	2.97	0.	0.	0.	0.
time (sec)	N/A	0.732	0.607	0.233	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	122	420	0	0	0	0
normalized size	1	1.	0.82	2.82	0.	0.	0.	0.
time (sec)	N/A	0.318	0.479	0.191	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	174	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	7.291	0.191	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	4.114	0.345	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	14.038	0.813	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	2.687	0.45	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	273	408	1070	0	0	0	0
normalized size	1	0.99	1.47	3.86	0.	0.	0.	0.
time (sec)	N/A	1.215	1.338	0.452	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	413	1044	0	0	0	0
normalized size	1	1.	1.47	3.72	0.	0.	0.	0.
time (sec)	N/A	1.072	1.201	0.416	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	271	404	958	0	0	0	0
normalized size	1	0.99	1.47	3.48	0.	0.	0.	0.
time (sec)	N/A	0.97	1.008	0.346	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	311	704	0	0	0	0
normalized size	1	1.	1.44	3.26	0.	0.	0.	0.
time (sec)	N/A	0.446	0.719	0.301	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	232	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.568	9.042	0.267	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.318	3.263	0.208	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	14.017	0.807	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	2.934	0.72	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	200	158	633	0	0	0	0
normalized size	1	0.98	0.77	3.1	0.	0.	0.	0.
time (sec)	N/A	0.48	0.33	0.179	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	117	420	0	0	0	0
normalized size	1	1.	0.83	2.98	0.	0.	0.	0.
time (sec)	N/A	0.395	0.28	0.195	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	138	113	364	0	0	0	0
normalized size	1	0.97	0.8	2.56	0.	0.	0.	0.
time (sec)	N/A	0.367	0.261	0.128	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	70	192	0	0	0	0
normalized size	1	1.	0.89	2.43	0.	0.	0.	0.
time (sec)	N/A	0.258	0.36	0.107	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	151	0	0	0	0
normalized size	1	1.	0.82	2.07	0.	0.	0.	0.
time (sec)	N/A	0.161	0.123	0.066	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	66	0	41
normalized size	1	1.	1.	1.06	1.33	3.67	0.	2.28
time (sec)	N/A	0.045	0.012	0.007	1.164	2.243	0.	1.358

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	5.126	0.097	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	46	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	1.197	0.06	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	7.7	0.347	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	3.38	0.174	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	5.086	0.129	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	2.336	0.102	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	7.473	0.326	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	19.027	0.185	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	13.868	0.547	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	5.631	0.412	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	10.686	0.26	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	2.775	0.182	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	12.43	0.617	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	11.42	0.7	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.963	0.737	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.538	0.673	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.119	0.591	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.342	0.177	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.633	0.332	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.849	0.4	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	55	12	31
normalized size	1	1.	1.	0.92	1.15	4.23	0.92	2.38
time (sec)	N/A	0.035	0.007	0.008	1.321	2.628	1.253	1.265

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	232	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	1.308	0.842	0.172	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	335	335	436	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	1.379	0.696	0.226	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	236	227	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.767	0.445	0.128	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	228	228	295	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.525	0.873	0.139	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	174	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.796	4.072	0.184	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	474	474	462	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	1.527	0.983	0.313	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	457	457	577	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	1.78	1.557	0.366	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	358	358	351	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	1.33	0.663	0.231	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	346	346	440	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.729	2.022	0.204	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	374	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.441	3.129	0.243	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	142	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.408	0.189	0.187	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	104	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.1	0.207	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0
normalized size	1	1.	1.	0.86	0.	0.	0.	0.
time (sec)	N/A	0.077	0.038	0.037	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.587	0.205	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	1.366	0.234	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	186	0	0	0	0	0
normalized size	1	1.	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.565	0.319	0.155	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	126	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.273	0.271	0.218	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0
normalized size	1	1.	1.	0.86	0.	0.	0.	0.
time (sec)	N/A	0.074	0.042	0.039	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.636	0.172	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	514	514	201	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.765	0.345	0.164	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	135	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.301	0.291	0.207	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	0	0	0
normalized size	1	1.	1.	0.86	0.	0.	0.	0.
time (sec)	N/A	0.072	0.036	0.036	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.611	0.177	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	156	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	0.175	0.198	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	110	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.087	0.224	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	54	0	0
normalized size	1	1.	1.	0.87	0.	1.38	0.	0.
time (sec)	N/A	0.062	0.028	0.043	0.	2.546	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.339	0.178	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	171	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.923	0.218	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	433	433	210	0	0	0	0	0
normalized size	1	1.	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.548	0.301	0.157	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	133	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	0.245	0.214	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	0	54	0	0
normalized size	1	1.	1.	0.87	0.	1.38	0.	0.
time (sec)	N/A	0.064	0.03	0.039	0.	2.481	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.371	0.161	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	34	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.068	0.105	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	197	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.308	0.321	0.164	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	141	0	0	0	0	0
normalized size	1	1.	0.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	0.184	0.162	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	101	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	0.097	0.22	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	0	0	0
normalized size	1	1.	1.	0.9	0.	0.	0.	0.
time (sec)	N/A	0.072	0.042	0.036	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.721	0.178	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	1.598	0.24	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	399	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	1.066	0.159	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	225	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	0.367	0.164	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.185	0.229	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	131	0	0
normalized size	1	1.	1.	0.9	0.	3.28	0.	0.
time (sec)	N/A	0.071	0.037	0.039	0.	2.584	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.71	0.178	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	1.467	0.238	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	296	262	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.377	0.397	0.162	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	122	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.145	0.21	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	0	134	0	0
normalized size	1	1.	1.	0.86	0.	3.19	0.	0.
time (sec)	N/A	0.07	0.04	0.042	0.	2.497	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.702	0.174	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	1.479	0.228	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	170	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.454	0.946	0.241	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	229	0	0	0	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.473	0.924	0.19	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	160	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.298	0.581	0.154	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	198	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.208	0.194	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.208	0.185	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	616	616	429	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.833	3.087	0.188	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	390	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.631	1.743	0.158	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	287	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.417	1.444	0.137	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	389	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.244	0.162	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	272	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.618	0.163	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	816	816	667	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.975	6.975	0.205	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	745	745	685	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.785	2.93	0.161	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	632	632	529	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.6	5.306	0.132	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	755	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	0.27	0.152	0.	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	454	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.639	0.162	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.477	0.114	0.	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	100	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	0.19	0.187	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.206	0.174	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.072	0.087	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	212	34	39
normalized size	1	1.	1.	1.06	0.	12.47	2.	2.29
time (sec)	N/A	0.038	0.01	0.005	0.	2.527	1.053	1.398

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	5.666	0.091	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	1.832	0.122	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	361	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.597	1.422	0.379	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.343	1.543	0.252	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	233	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.568	0.252	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.298	0.588	0.296	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	283	0	0	0	0	0
normalized size	1	1.	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.401	1.435	0.308	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	141	0	0	1332	0	0
normalized size	1	1.	0.75	0.	0.	7.12	0.	0.
time (sec)	N/A	0.306	0.458	0.295	0.	3.134	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0
normalized size	1	1.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.441	1.687	0.247	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	352	0	0	0	0	0
normalized size	1	1.	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.879	0.243	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	273	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.341	1.549	0.259	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.465	1.057	0.284	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	514	0	0	0	0	0
normalized size	1	1.	1.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.474	2.269	0.295	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	364	364	706	0	0	0	0	0
normalized size	1	1.	1.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.384	5.687	0.292	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	481	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	1.156	0.246	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	683	0	0	0	0	0
normalized size	1	1.	1.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.426	1.597	0.253	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	565	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.571	1.187	0.267	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.628	1.631	0.29	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	779	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.416	3.843	0.292	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	472	472	1005	0	0	0	0	0
normalized size	1	1.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.444	7.294	0.296	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	465	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.629	1.667	0.293	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	344	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.466	1.028	0.288	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	227	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	0.583	0.298	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	113	0	0	0	0	0
normalized size	1	1.	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	0.439	0.237	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	1011	0	0
normalized size	1	1.	1.02	0.	0.	9.11	0.	0.
time (sec)	N/A	0.241	0.397	0.255	0.	2.312	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	143	0	0	1386	0	0
normalized size	1	1.	0.48	0.	0.	4.7	0.	0.
time (sec)	N/A	0.337	0.496	0.26	0.	2.918	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	781	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	3.887	0.306	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	515	0	0	0	0	0
normalized size	1	1.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.478	2.351	0.3	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	285	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.4	1.209	0.299	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	94	0	0	1008	0	0
normalized size	1	1.	0.84	0.	0.	9.	0.	0.
time (sec)	N/A	0.249	0.403	0.247	0.	2.428	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	118	0	112	0	0	0
normalized size	1	1.	1.15	0.	1.09	0.	0.	0.
time (sec)	N/A	0.205	0.465	0.243	1.222	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	201	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.306	0.607	0.25	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	470	470	1331	0	0	0	0	0
normalized size	1	1.	2.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.439	8.667	0.309	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	706	0	0	0	0	0
normalized size	1	1.	1.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.392	5.632	0.291	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	131	0	0	1330	0	0
normalized size	1	1.	0.71	0.	0.	7.19	0.	0.
time (sec)	N/A	0.306	0.419	0.313	0.	2.732	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	0	1384	0	0
normalized size	1	1.	0.47	0.	0.	4.71	0.	0.
time (sec)	N/A	0.333	0.439	0.249	0.	2.927	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	202	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.299	0.601	0.245	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	193	0	215	0	0	0
normalized size	1	1.	0.95	0.	1.06	0.	0.	0.
time (sec)	N/A	0.245	0.553	0.254	1.248	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	1.108	2.439	0.288	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.634	1.814	0.283	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	352	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	1.108	0.286	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.508	1.144	0.331	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	594	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	1.005	4.154	0.272	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	783	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	1.149	8.062	0.323	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.858	3.261	0.269	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	524	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.482	1.764	0.263	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	705	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.649	1.82	0.289	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	532	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.633	2.183	0.316	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	752	752	1174	0	0	0	0	0
normalized size	1	1.	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	1.128	9.045	0.264	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	580	580	1609	0	0	0	0	0
normalized size	1	1.	2.77	0.	0.	0.	0.	0.
time (sec)	N/A	1.204	10.047	0.261	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	735	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.612	2.247	0.266	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	774	1084	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.833	3.115	0.269	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	890	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	1.063	2.366	0.284	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.781	3.606	0.323	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	972	972	2492	0	0	0	0	0
normalized size	1	1.	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	1.364	11.613	0.274	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	790	790	2622	0	0	0	0	0
normalized size	1	1.	3.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.365	13.471	0.266	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	615	615	723	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.785	3.358	0.323	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	529	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.669	2.148	0.308	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	315	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.511	1.119	0.323	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	168	0	0	0	0	0
normalized size	1	1.	2.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.296	0.815	0.261	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	464	464	508	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.753	2.19	0.286	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	942	942	524	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	1.332	6.875	0.286	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	972	972	2323	0	0	0	0	0
normalized size	1	1.	2.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.37	14.289	0.257	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	752	752	1526	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	1.133	10.815	0.262	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	530	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.988	5.454	0.268	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	464	464	511	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.704	2.218	0.276	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	488	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.423	1.369	0.277	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	743	743	754	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.909	8.257	0.276	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	794	794	2552	0	0	0	0	0
normalized size	1	1.	3.21	0.	0.	0.	0.	0.
time (sec)	N/A	1.396	13.648	0.273	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	584	584	1617	0	0	0	0	0
normalized size	1	1.	2.77	0.	0.	0.	0.	0.
time (sec)	N/A	1.205	10.073	0.266	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	788	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	1.155	8.207	0.324	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	942	942	528	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	1.301	6.602	0.273	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	743	743	757	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.897	8.208	0.269	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	386	386	642	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.531	7.821	0.278	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	260	451	560	803	593	547
normalized size	1	1.	0.83	1.45	1.79	2.57	1.9	1.75
time (sec)	N/A	0.351	0.343	0.014	1.227	2.488	24.291	1.842

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	187	316	387	559	389	396
normalized size	1	1.	0.85	1.43	1.75	2.53	1.76	1.79
time (sec)	N/A	0.261	0.273	0.005	1.241	2.549	8.749	1.844

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	204	243	370	240	262
normalized size	1	1.	0.85	1.39	1.65	2.52	1.63	1.78
time (sec)	N/A	0.143	0.163	0.005	1.176	2.363	2.788	1.625

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	109	123	208	109	146
normalized size	1	1.	0.88	1.35	1.52	2.57	1.35	1.8
time (sec)	N/A	0.07	0.069	0.005	1.122	2.479	0.668	1.555

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	41	95	26	55
normalized size	1	1.	1.	1.03	1.37	3.17	0.87	1.83
time (sec)	N/A	0.014	0.008	0.01	1.23	2.349	0.151	1.46

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	485	485	434	224	0	0	0	0
normalized size	1	1.	0.89	0.46	0.	0.	0.	0.
time (sec)	N/A	0.832	0.426	0.424	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	707	707	622	1745	0	0	0	0
normalized size	1	1.	0.88	2.47	0.	0.	0.	0.
time (sec)	N/A	1.075	1.606	0.668	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	559	559	443	1166	923	1338	989	986
normalized size	1	1.	0.79	2.09	1.65	2.39	1.77	1.76
time (sec)	N/A	0.966	0.596	0.115	1.259	2.586	18.826	3.024

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	289	620	579	845	595	656
normalized size	1	1.	0.88	1.88	1.76	2.57	1.81	1.99
time (sec)	N/A	0.582	0.426	0.071	1.193	2.923	6.181	2.552

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	164	271	294	454	279	373
normalized size	1	1.	1.07	1.77	1.92	2.97	1.82	2.44
time (sec)	N/A	0.277	0.213	0.046	1.175	2.792	1.626	2.073

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	97	212	82	150
normalized size	1	1.	1.61	1.57	2.11	4.61	1.78	3.26
time (sec)	N/A	0.064	0.059	0.027	1.1	2.74	0.32	1.706

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	739	739	985	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	1.315	0.681	0.296	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	670	658	444	654	0	0	0	0
normalized size	1	0.98	0.66	0.98	0.	0.	0.	0.
time (sec)	N/A	1.344	0.974	0.221	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	380	253	380	0	0	0	0
normalized size	1	0.98	0.65	0.98	0.	0.	0.	0.
time (sec)	N/A	0.786	0.541	0.131	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	176	126	178	0	0	0	0
normalized size	1	0.98	0.7	0.99	0.	0.	0.	0.
time (sec)	N/A	0.367	0.24	0.086	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	45	56	0	0	0	0
normalized size	1	1.	0.83	1.04	0.	0.	0.	0.
time (sec)	N/A	0.07	0.019	0.031	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.678	0.237	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	3.29	0.305	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	483	356	1036	0	0	0	0
normalized size	1	0.98	0.72	2.09	0.	0.	0.	0.
time (sec)	N/A	0.875	2.029	0.221	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	239	190	438	0	0	0	0
normalized size	1	0.97	0.77	1.77	0.	0.	0.	0.
time (sec)	N/A	0.476	0.935	0.115	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	81	71	118	0	0	0	0
normalized size	1	0.95	0.84	1.39	0.	0.	0.	0.
time (sec)	N/A	0.188	0.142	0.043	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	11.365	0.216	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	27.445	0.344	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	672	672	535	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.879	6.177	0.35	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	319	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.932	2.78	0.178	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	0.095	0.	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	7.048	0.178	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	15.953	0.365	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	770	0	0	0	0	0
normalized size	1	1.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.264	4.197	0.181	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.161	0.	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	2.74	0.178	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	8.914	0.392	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	1.195	1.171	0.324	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	218	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.572	0.662	0.192	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.104	0.	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.133	0.186	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.249	0.361	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	303	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.691	1.447	0.185	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.115	0.	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.141	0.184	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.272	0.369	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	4.968	0.636	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	3.755	0.447	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	75	0	0	736	0	0
normalized size	1	1.	1.07	0.	0.	10.51	0.	0.
time (sec)	N/A	0.101	0.112	0.295	0.	2.753	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	139	0	0	1503	0	0
normalized size	1	1.	0.95	0.	0.	10.29	0.	0.
time (sec)	N/A	0.169	0.335	0.325	0.	3.297	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	191	0	0	2782	0	0
normalized size	1	1.	0.84	0.	0.	12.26	0.	0.
time (sec)	N/A	0.82	0.427	0.412	0.	4.291	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	13.704	0.242	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	10.862	0.224	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	3.008	0.174	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	5.72	0.182	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	1.172	0.218	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	1.034	0.201	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	1.489	0.151	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	3.603	0.152	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	4.419	0.209	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	6.223	0.197	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	16.394	0.152	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	28.172	0.153	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [250] had the largest ratio of [0.7308]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.	22	0.227
2	A	6	6	1.	22	0.273
3	A	5	5	1.	22	0.227
4	A	4	3	1.	20	0.15
5	A	5	4	1.	19	0.21
6	A	8	8	1.	22	0.364
7	A	6	7	1.	22	0.318
8	A	8	8	1.	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	6	7	1.	22	0.318
10	A	6	6	1.	24	0.25
11	A	7	8	1.	24	0.333
12	A	5	5	1.	24	0.208
13	A	5	3	1.	22	0.136
14	A	5	5	1.	21	0.238
15	A	12	8	1.	24	0.333
16	A	7	7	1.	24	0.292
17	A	12	10	1.	24	0.417
18	A	7	8	1.	24	0.333
19	A	5	5	1.	24	0.208
20	A	8	7	1.	24	0.292
21	A	5	5	1.	24	0.208
22	A	6	3	1.	22	0.136
23	A	5	5	1.	21	0.238
24	A	17	8	1.	24	0.333
25	A	7	7	1.	24	0.292
26	A	17	10	1.	24	0.417
27	A	8	8	1.	24	0.333
28	A	12	8	1.	24	0.333
29	A	8	8	1.	24	0.333
30	A	8	6	1.	24	0.25
31	A	5	5	1.	22	0.227
32	A	6	4	1.	21	0.19
33	A	7	5	1.	24	0.208
34	A	10	8	1.	24	0.333
35	A	9	7	1.	24	0.292
36	A	15	9	1.	24	0.375
37	A	12	9	1.	24	0.375
38	A	8	8	1.	24	0.333
39	A	8	6	1.	24	0.25
40	A	2	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	8	6	1.	21	0.286
42	A	9	7	1.	24	0.292
43	A	13	11	1.	24	0.458
44	A	12	9	1.	24	0.375
45	A	19	11	1.1	24	0.458
46	A	12	8	1.	24	0.333
47	A	4	3	1.	24	0.125
48	A	10	7	1.	24	0.292
49	A	3	3	1.	22	0.136
50	A	10	6	1.	21	0.286
51	A	12	8	1.	24	0.333
52	A	16	11	1.	24	0.458
53	A	16	10	1.	24	0.417
54	A	23	11	1.17	24	0.458
55	A	3	4	1.02	26	0.154
56	A	5	4	1.52	26	0.154
57	A	2	1	1.72	24	0.042
58	A	3	3	1.66	23	0.13
59	A	8	6	1.99	26	0.231
60	A	3	3	1.72	26	0.115
61	A	8	6	1.78	26	0.231
62	A	3	2	1.71	26	0.077
63	A	4	5	1.02	26	0.192
64	A	8	6	1.54	26	0.231
65	A	3	2	1.9	24	0.083
66	A	6	5	1.62	23	0.217
67	A	10	7	1.86	26	0.269
68	A	6	5	1.64	26	0.192
69	A	11	8	1.74	26	0.308
70	A	6	5	1.6	26	0.192
71	A	4	5	1.01	26	0.192
72	A	12	8	1.58	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	B	3	2	2.08	24	0.083
74	A	8	6	1.54	23	0.261
75	A	13	8	1.84	26	0.308
76	A	10	8	1.64	26	0.308
77	A	13	9	1.73	26	0.346
78	A	10	7	1.6	26	0.269
79	A	3	3	1.	12	0.25
80	A	6	4	1.44	26	0.154
81	A	5	3	1.35	26	0.115
82	A	4	4	1.45	26	0.154
83	A	3	3	1.29	26	0.115
84	A	2	2	1.52	24	0.083
85	A	1	1	1.	23	0.043
86	A	6	4	1.	26	0.154
87	A	2	2	1.54	26	0.077
88	A	8	6	1.19	26	0.231
89	A	4	4	1.45	26	0.154
90	A	4	5	1.02	26	0.192
91	A	7	6	1.38	26	0.231
92	A	3	5	1.02	26	0.192
93	A	3	3	1.31	26	0.115
94	A	2	2	1.56	24	0.083
95	A	2	2	1.49	23	0.087
96	A	8	6	1.27	26	0.231
97	A	4	5	1.02	26	0.192
98	A	11	8	1.31	26	0.308
99	A	5	6	1.02	26	0.231
100	A	11	6	1.33	26	0.231
101	A	5	7	1.02	26	0.269
102	A	7	5	1.28	26	0.192
103	A	4	6	1.02	26	0.231
104	A	4	3	1.49	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	3	3	1.52	24	0.125
106	A	4	4	1.36	23	0.174
107	A	11	7	1.26	26	0.269
108	A	5	7	1.02	26	0.269
109	A	15	10	1.32	26	0.385
110	A	5	7	1.02	26	0.269
111	A	6	4	1.	19	0.21
112	A	5	3	1.	21	0.143
113	A	4	4	1.	21	0.19
114	A	3	3	1.	21	0.143
115	A	2	2	1.	19	0.105
116	A	1	1	1.	18	0.056
117	A	6	4	1.	21	0.19
118	A	2	2	1.	21	0.095
119	A	8	6	1.	21	0.286
120	A	6	4	1.	26	0.154
121	A	5	4	1.	26	0.154
122	A	2	1	1.	24	0.042
123	A	3	3	1.	23	0.13
124	A	8	6	1.	26	0.231
125	A	3	3	1.	26	0.115
126	A	8	6	1.	26	0.231
127	A	3	2	1.	26	0.077
128	A	7	5	1.	26	0.192
129	A	8	6	1.	26	0.231
130	A	3	2	1.	24	0.083
131	A	6	5	1.	23	0.217
132	A	10	7	1.	26	0.269
133	A	6	5	1.	26	0.192
134	A	11	8	1.	26	0.308
135	A	6	5	1.	26	0.192
136	A	7	5	1.	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	12	8	1.	26	0.308
138	A	3	2	1.	24	0.083
139	A	8	6	1.	23	0.261
140	A	13	8	1.	26	0.308
141	A	10	8	1.	26	0.308
142	A	13	9	1.	26	0.346
143	A	10	7	1.	26	0.269
144	A	3	3	1.	12	0.25
145	A	6	4	1.	26	0.154
146	A	6	4	1.	26	0.154
147	A	4	4	1.	26	0.154
148	A	4	4	1.	26	0.154
149	A	2	2	1.	24	0.083
150	A	2	2	1.	23	0.087
151	A	7	5	1.	26	0.192
152	A	2	2	1.	26	0.077
153	A	9	7	1.	26	0.269
154	A	4	4	1.	26	0.154
155	A	8	7	1.04	26	0.269
156	A	8	7	1.	26	0.269
157	A	5	5	1.04	26	0.192
158	A	4	4	1.	26	0.154
159	A	2	2	1.	24	0.083
160	A	2	2	1.	23	0.087
161	A	9	7	1.	26	0.269
162	A	7	7	1.	26	0.269
163	A	12	9	1.	26	0.346
164	A	11	8	1.	26	0.308
165	A	12	7	1.	26	0.269
166	A	9	6	1.07	26	0.231
167	A	8	6	1.	26	0.231
168	A	5	4	1.03	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
169	A	4	3	1.	26	0.115
170	A	3	3	1.	24	0.125
171	A	4	4	1.	23	0.174
172	A	12	8	1.	26	0.308
173	A	8	7	1.	26	0.269
174	A	16	11	1.	26	0.423
175	A	12	7	1.	26	0.269
176	A	6	4	1.	19	0.21
177	A	5	3	1.	21	0.143
178	A	4	4	1.	21	0.19
179	A	3	3	1.	21	0.143
180	A	2	2	1.	19	0.105
181	A	1	1	1.	18	0.056
182	A	6	4	1.	21	0.19
183	A	2	2	1.	21	0.095
184	A	8	6	1.	21	0.286
185	A	6	7	1.	24	0.292
186	A	5	6	1.	24	0.25
187	A	4	5	1.	22	0.227
188	A	0	0	0.	0	0.
189	A	0	0	0.	0	0.
190	A	0	0	0.	0	0.
191	A	9	6	1.	26	0.231
192	A	6	5	1.	26	0.192
193	A	3	3	1.	26	0.115
194	A	2	2	1.	26	0.077
195	A	4	4	1.	26	0.154
196	A	6	4	1.	26	0.154
197	A	1	1	1.	21	0.048
198	A	11	10	1.	24	0.417
199	A	14	6	1.	24	0.25
200	A	9	10	1.	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	7	6	1.	22	0.273
202	A	6	4	1.	21	0.19
203	A	10	10	0.99	24	0.417
204	A	12	9	1.	24	0.375
205	A	10	10	0.99	24	0.417
206	A	16	8	1.	24	0.333
207	A	16	11	1.	26	0.423
208	A	25	7	1.	26	0.269
209	A	14	11	1.	26	0.423
210	A	9	7	1.	24	0.292
211	A	10	5	1.	23	0.217
212	A	17	12	1.	26	0.462
213	A	17	11	1.	26	0.423
214	A	17	12	1.	26	0.462
215	A	24	10	1.	26	0.385
216	A	21	11	1.	26	0.423
217	A	40	9	1.	26	0.346
218	A	19	11	1.	26	0.423
219	A	11	7	1.	24	0.292
220	A	14	5	1.	23	0.217
221	A	26	13	1.	26	0.5
222	A	24	12	1.	26	0.462
223	A	28	15	1.	26	0.577
224	A	31	12	1.	26	0.462
225	A	16	10	1.	26	0.385
226	A	10	10	1.	26	0.385
227	A	11	8	1.	26	0.308
228	A	6	6	1.	24	0.25
229	A	8	5	1.	23	0.217
230	A	9	6	1.	26	0.231
231	A	15	10	1.	26	0.385
232	A	12	9	1.	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	24	11	1.	26	0.423
234	A	15	14	1.	26	0.538
235	A	10	9	1.	26	0.346
236	A	11	8	1.	26	0.308
237	A	3	3	1.	24	0.125
238	A	11	8	1.	23	0.348
239	A	12	9	1.	26	0.346
240	A	20	14	1.	26	0.538
241	A	17	15	1.	26	0.577
242	A	32	15	1.	26	0.577
243	A	16	13	1.	26	0.5
244	A	8	6	1.	26	0.231
245	A	15	10	1.	26	0.385
246	A	5	5	1.	24	0.208
247	A	15	9	1.	23	0.391
248	A	17	11	1.	26	0.423
249	A	27	15	1.	26	0.577
250	A	23	19	1.	26	0.731
251	A	43	17	1.	26	0.654
252	A	16	8	1.4	25	0.32
253	A	10	8	1.4	25	0.32
254	A	5	5	1.51	25	0.2
255	A	1	1	1.	25	0.04
256	A	6	6	1.72	25	0.24
257	A	9	9	1.43	25	0.36
258	A	14	8	1.	28	0.286
259	A	10	6	1.	28	0.214
260	A	5	4	1.	26	0.154
261	A	5	5	1.	25	0.2
262	A	12	8	1.	28	0.286
263	A	7	7	1.	28	0.25
264	A	13	10	1.	28	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	9	9	1.	28	0.321
266	A	20	14	1.	28	0.5
267	A	17	11	1.	28	0.393
268	A	6	6	1.	26	0.231
269	A	10	8	1.	25	0.32
270	A	17	12	1.	28	0.429
271	A	14	13	1.	28	0.464
272	A	18	15	1.	28	0.536
273	A	16	11	1.	28	0.393
274	A	27	18	1.	28	0.643
275	A	25	14	1.	28	0.5
276	A	6	6	1.	26	0.231
277	A	16	8	1.	25	0.32
278	A	23	16	1.	28	0.571
279	A	23	15	1.	28	0.536
280	A	25	20	1.	28	0.714
281	A	27	15	1.	28	0.536
282	A	10	5	1.	23	0.217
283	A	8	7	1.	23	0.304
284	A	5	5	1.	23	0.217
285	A	3	3	1.	21	0.143
286	A	1	1	1.	20	0.05
287	A	8	5	1.	23	0.217
288	A	6	6	1.	23	0.261
289	A	13	10	1.	23	0.435
290	A	14	7	1.	28	0.25
291	A	11	6	1.	28	0.214
292	A	9	7	1.	28	0.25
293	A	6	6	1.	28	0.214
294	A	4	3	1.	26	0.115
295	A	2	2	1.	25	0.08
296	A	9	6	1.	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	6	6	1.	28	0.214
298	A	14	11	1.	28	0.393
299	A	9	9	1.	28	0.321
300	A	22	13	1.	28	0.464
301	A	15	13	1.	28	0.464
302	A	13	9	1.	28	0.321
303	A	8	8	1.	28	0.286
304	A	7	5	1.	26	0.192
305	A	6	6	1.	25	0.24
306	A	16	11	1.	28	0.393
307	A	14	10	1.	28	0.357
308	A	27	15	1.	28	0.536
309	A	24	11	1.	28	0.393
310	A	26	11	1.	28	0.393
311	A	17	10	1.	28	0.357
312	A	16	7	1.	28	0.25
313	A	9	9	1.	28	0.321
314	A	9	7	1.	26	0.269
315	A	9	9	1.	25	0.36
316	A	25	13	1.	28	0.464
317	A	19	14	1.	28	0.5
318	A	39	18	1.	28	0.643
319	A	32	15	1.	28	0.536
320	A	13	10	1.	21	0.476
321	A	0	0	0.	0	0.
322	A	0	0	0.	0	0.
323	A	0	0	0.	0	0.
324	A	0	0	0.	0	0.
325	A	0	0	0.	0	0.
326	A	0	0	0.	0	0.
327	A	0	0	0.	0	0.
328	A	24	13	1.	19	0.684

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	17	11	1.	19	0.579
330	A	10	7	1.	17	0.412
331	A	10	6	1.	19	0.316
332	A	18	10	1.	19	0.526
333	A	28	11	1.	19	0.579
334	A	24	9	1.	21	0.429
335	A	14	8	1.	21	0.381
336	A	6	5	1.	21	0.238
337	A	2	2	1.	21	0.095
338	A	7	7	1.	21	0.333
339	A	11	10	1.	21	0.476
340	A	17	11	1.	21	0.524
341	A	0	0	0.	0	0.
342	A	13	4	1.	23	0.174
343	A	10	6	1.	23	0.261
344	A	6	4	1.	23	0.174
345	A	4	3	1.	21	0.143
346	A	1	1	1.	20	0.05
347	A	10	6	1.	23	0.261
348	A	7	7	1.	23	0.304
349	A	18	10	1.	23	0.435
350	A	7	3	1.	19	0.158
351	A	6	3	1.	19	0.158
352	A	5	3	1.	17	0.176
353	A	0	0	0.	0	0.
354	A	0	0	0.	0	0.
355	A	12	5	1.	27	0.185
356	A	12	5	0.98	27	0.185
357	A	6	5	1.	27	0.185
358	A	9	5	0.97	25	0.2
359	A	6	5	1.	24	0.208
360	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
361	A	0	0	0.	0	0.
362	A	0	0	0.	0	0.
363	A	0	0	0.	0	0.
364	A	15	5	0.98	27	0.185
365	A	12	5	1.	27	0.185
366	A	12	5	0.98	25	0.2
367	A	9	5	1.	24	0.208
368	A	0	0	0.	0	0.
369	A	0	0	0.	0	0.
370	A	0	0	0.	0	0.
371	A	0	0	0.	0	0.
372	A	15	5	0.98	27	0.185
373	A	15	5	1.	27	0.185
374	A	15	5	0.98	25	0.2
375	A	12	5	1.	24	0.208
376	A	0	0	0.	0	0.
377	A	0	0	0.	0	0.
378	A	0	0	0.	0	0.
379	A	0	0	0.	0	0.
380	A	5	3	1.	23	0.13
381	A	5	3	1.	23	0.13
382	A	4	3	1.	23	0.13
383	A	4	3	1.	23	0.13
384	A	2	2	1.	21	0.095
385	A	1	1	1.	20	0.05
386	A	0	0	0.	0	0.
387	A	0	0	0.	0	0.
388	A	12	5	0.98	27	0.185
389	A	9	5	1.	27	0.185
390	A	9	5	0.97	27	0.185
391	A	6	5	1.	27	0.185
392	A	4	4	0.93	25	0.16

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
393	A	1	1	1.	24	0.042
394	A	0	0	0.	0	0.
395	A	0	0	0.	0	0.
396	A	0	0	0.	0	0.
397	A	0	0	0.	0	0.
398	A	0	0	0.	0	0.
399	A	0	0	0.	0	0.
400	A	0	0	0.	0	0.
401	A	0	0	0.	0	0.
402	A	0	0	0.	0	0.
403	A	0	0	0.	0	0.
404	A	0	0	0.	0	0.
405	A	0	0	0.	0	0.
406	A	8	4	1.	19	0.21
407	A	7	4	1.	19	0.21
408	A	6	4	1.	17	0.235
409	A	0	0	0.	0	0.
410	A	0	0	0.	0	0.
411	A	22	6	0.98	27	0.222
412	A	16	7	1.	27	0.259
413	A	14	7	1.33	25	0.28
414	A	7	7	1.	24	0.292
415	A	0	0	0.	0	0.
416	A	0	0	0.	0	0.
417	A	0	0	0.	0	0.
418	A	0	0	0.	0	0.
419	A	28	6	0.99	27	0.222
420	A	19	6	1.	27	0.222
421	A	22	8	0.98	25	0.32
422	A	10	6	1.	24	0.25
423	A	0	0	0.	0	0.
424	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
425	A	0	0	0.	0	0.
426	A	0	0	0.	0	0.
427	A	34	6	0.99	27	0.222
428	A	28	6	1.	27	0.222
429	A	28	8	0.99	25	0.32
430	A	13	6	1.	24	0.25
431	A	0	0	0.	0	0.
432	A	0	0	0.	0	0.
433	A	0	0	0.	0	0.
434	A	0	0	0.	0	0.
435	A	13	6	0.98	27	0.222
436	A	10	6	1.	27	0.222
437	A	10	6	0.97	27	0.222
438	A	7	7	1.	27	0.259
439	A	5	5	1.	25	0.2
440	A	1	1	1.	24	0.042
441	A	0	0	0.	0	0.
442	A	0	0	0.	0	0.
443	A	0	0	0.	0	0.
444	A	0	0	0.	0	0.
445	A	0	0	0.	0	0.
446	A	0	0	0.	0	0.
447	A	0	0	0.	0	0.
448	A	0	0	0.	0	0.
449	A	0	0	0.	0	0.
450	A	0	0	0.	0	0.
451	A	0	0	0.	0	0.
452	A	0	0	0.	0	0.
453	A	0	0	0.	0	0.
454	A	0	0	0.	0	0.
455	A	0	0	0.	0	0.
456	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
457	A	0	0	0.	0	0.
458	A	0	0	0.	0	0.
459	A	0	0	0.	0	0.
460	A	0	0	0.	0	0.
461	A	1	1	1.	20	0.05
462	A	27	7	1.	26	0.269
463	A	32	7	1.	26	0.269
464	A	17	9	1.	24	0.375
465	A	14	7	1.	23	0.304
466	A	0	0	0.	0	0.
467	A	32	7	1.	28	0.25
468	A	42	7	1.	28	0.25
469	A	32	9	1.	26	0.346
470	A	19	7	1.	25	0.28
471	A	0	0	0.	0	0.
472	A	24	11	1.	23	0.478
473	A	10	9	1.	23	0.391
474	A	2	2	1.	23	0.087
475	A	0	0	0.	0	0.
476	A	0	0	0.	0	0.
477	A	26	12	1.	23	0.522
478	A	11	9	1.	23	0.391
479	A	2	2	1.	23	0.087
480	A	0	0	0.	0	0.
481	A	39	14	1.	23	0.609
482	A	13	11	1.	23	0.478
483	A	2	2	1.	23	0.087
484	A	0	0	0.	0	0.
485	A	24	11	1.	22	0.5
486	A	10	9	1.	22	0.409
487	A	2	2	1.	22	0.091
488	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
489	A	0	0	0.	0	0.
490	A	26	12	1.	22	0.546
491	A	11	9	1.	22	0.409
492	A	2	2	1.	22	0.091
493	A	0	0	0.	0	0.
494	A	6	5	1.	17	0.294
495	A	19	7	1.	23	0.304
496	A	14	7	1.	23	0.304
497	A	9	7	1.	23	0.304
498	A	2	2	1.	23	0.087
499	A	0	0	0.	0	0.
500	A	0	0	0.	0	0.
501	A	19	7	1.	23	0.304
502	A	14	7	1.	23	0.304
503	A	9	8	1.	23	0.348
504	A	2	2	1.	23	0.087
505	A	0	0	0.	0	0.
506	A	0	0	0.	0	0.
507	A	18	10	1.	23	0.435
508	A	7	6	1.	23	0.261
509	A	2	2	1.	23	0.087
510	A	0	0	0.	0	0.
511	A	0	0	0.	0	0.
512	A	7	5	1.	28	0.179
513	A	10	5	1.	26	0.192
514	A	7	5	1.	25	0.2
515	A	0	0	0.	0	0.
516	A	0	0	0.	0	0.
517	A	13	5	1.	28	0.179
518	A	13	5	1.	26	0.192
519	A	10	5	1.	25	0.2
520	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
521	A	0	0	0.	0	0.
522	A	16	5	1.	28	0.179
523	A	16	5	1.	26	0.192
524	A	13	5	1.	25	0.2
525	A	0	0	0.	0	0.
526	A	0	0	0.	0	0.
527	A	0	0	0.	0	0.
528	A	9	4	1.	23	0.174
529	A	6	4	1.	23	0.174
530	A	4	3	1.	21	0.143
531	A	1	1	1.	20	0.05
532	A	0	0	0.	0	0.
533	A	0	0	0.	0	0.
534	A	13	8	1.	35	0.229
535	A	8	6	1.	35	0.171
536	A	4	4	1.	35	0.114
537	A	6	5	1.	35	0.143
538	A	8	8	1.	35	0.229
539	A	6	6	1.	35	0.171
540	A	12	9	1.	35	0.257
541	A	7	6	1.	35	0.171
542	A	8	6	1.	35	0.171
543	A	9	7	1.	35	0.2
544	A	10	10	1.	35	0.286
545	A	9	9	1.	35	0.257
546	A	9	7	1.	35	0.2
547	A	12	9	1.	35	0.257
548	A	13	8	1.	35	0.229
549	A	13	7	1.	35	0.2
550	A	7	9	1.	35	0.257
551	A	10	8	1.	35	0.229
552	A	13	7	1.	35	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
553	A	9	7	1.	35	0.2
554	A	6	5	1.	35	0.143
555	A	2	2	1.	35	0.057
556	A	5	6	1.	35	0.171
557	A	8	8	1.	35	0.229
558	A	7	9	1.	35	0.257
559	A	10	10	1.	35	0.286
560	A	8	8	1.	35	0.229
561	A	5	6	1.	35	0.171
562	A	3	3	1.	35	0.086
563	A	8	8	1.	35	0.229
564	A	10	8	1.	35	0.229
565	A	9	9	1.	35	0.257
566	A	6	6	1.	35	0.171
567	A	8	8	1.	35	0.229
568	A	8	8	1.	35	0.229
569	A	5	5	1.	35	0.143
570	A	23	13	1.	37	0.351
571	A	13	11	1.	37	0.297
572	A	6	6	1.	37	0.162
573	A	8	6	1.	37	0.162
574	A	19	13	1.	37	0.351
575	A	20	12	1.	37	0.324
576	A	19	15	1.	37	0.405
577	A	11	9	1.	37	0.243
578	A	13	11	1.	37	0.297
579	A	11	9	1.	37	0.243
580	A	23	15	1.	37	0.405
581	A	21	13	1.	37	0.351
582	A	17	9	1.	37	0.243
583	A	19	15	1.	37	0.405
584	A	23	13	1.	37	0.351

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	17	10	1.	37	0.27
586	A	28	19	1.	37	0.514
587	A	25	16	1.	37	0.432
588	A	17	10	1.	37	0.27
589	A	11	9	1.	37	0.243
590	A	8	6	1.	37	0.162
591	A	2	2	1.	37	0.054
592	A	16	11	1.	37	0.297
593	A	30	18	1.	37	0.486
594	A	28	19	1.	37	0.514
595	A	23	15	1.	37	0.405
596	A	19	13	1.	37	0.351
597	A	16	11	1.	37	0.297
598	A	7	7	1.	37	0.189
599	A	21	14	1.	37	0.378
600	A	25	16	1.	37	0.432
601	A	21	13	1.	37	0.351
602	A	20	12	1.	37	0.324
603	A	30	18	1.	37	0.486
604	A	21	14	1.	37	0.378
605	A	10	10	1.	37	0.27
606	A	5	5	1.	18	0.278
607	A	5	5	1.	18	0.278
608	A	5	5	1.	18	0.278
609	A	4	3	1.	16	0.188
610	A	3	2	1.	8	0.25
611	A	18	6	1.	18	0.333
612	A	26	9	1.	18	0.5
613	A	26	7	1.	20	0.35
614	A	17	7	1.	20	0.35
615	A	10	7	1.	18	0.389
616	A	3	3	1.	10	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
617	A	22	7	1.	20	0.35
618	A	42	7	0.98	20	0.35
619	A	27	7	0.98	20	0.35
620	A	15	7	0.98	18	0.389
621	A	4	4	1.	10	0.4
622	A	0	0	0.	0	0.
623	A	0	0	0.	0	0.
624	A	26	7	0.98	20	0.35
625	A	15	7	0.97	18	0.389
626	A	5	5	0.95	10	0.5
627	A	0	0	0.	0	0.
628	A	0	0	0.	0	0.
629	A	42	9	1.	22	0.409
630	A	23	9	1.	20	0.45
631	A	7	6	1.	12	0.5
632	A	0	0	0.	0	0.
633	A	0	0	0.	0	0.
634	A	32	12	1.	20	0.6
635	A	8	7	1.	12	0.583
636	A	0	0	0.	0	0.
637	A	0	0	0.	0	0.
638	A	39	8	1.	22	0.364
639	A	21	8	1.	20	0.4
640	A	6	5	1.	12	0.417
641	A	0	0	0.	0	0.
642	A	0	0	0.	0	0.
643	A	21	8	1.	20	0.4
644	A	7	6	1.	12	0.5
645	A	0	0	0.	0	0.
646	A	0	0	0.	0	0.
647	A	0	0	0.	0	0.
648	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
649	A	6	7	1.	20	0.35
650	A	7	9	1.	20	0.45
651	A	8	10	1.	20	0.5
652	A	0	0	0.	0	0.
653	A	0	0	0.	0	0.
654	A	0	0	0.	0	0.
655	A	0	0	0.	0	0.
656	A	0	0	0.	0	0.
657	A	0	0	0.	0	0.
658	A	0	0	0.	0	0.
659	A	0	0	0.	0	0.
660	A	0	0	0.	0	0.
661	A	0	0	0.	0	0.
662	A	0	0	0.	0	0.
663	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^4 \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=124

$$\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd\left(c^2x^2+1\right)^{7/2}}{49c^5}+\frac{8bd\left(c^2x^2+1\right)^{5/2}}{175c^5}-\frac{bd\left(c^2x^2+1\right)^{3/2}}{105c^5}-\frac{2bd\sqrt{c}}{35}$$

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(35*c^5) - (b*d*(1 + c^2*x^2)^(3/2))/(105*c^5) +
 (8*b*d*(1 + c^2*x^2)^(5/2))/(175*c^5) - (b*d*(1 + c^2*x^2)^(7/2))/(49*c^5)
 + (d*x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*d*x^7*(a + b*ArcSinh[c*x]))/7
```

Rubi [A] time = 0.118849, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5730, 12, 446, 77}

$$\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd\left(c^2x^2+1\right)^{7/2}}{49c^5}+\frac{8bd\left(c^2x^2+1\right)^{5/2}}{175c^5}-\frac{bd\left(c^2x^2+1\right)^{3/2}}{105c^5}-\frac{2bd\sqrt{c}}{35}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(35*c^5) - (b*d*(1 + c^2*x^2)^(3/2))/(105*c^5) +
 (8*b*d*(1 + c^2*x^2)^(5/2))/(175*c^5) - (b*d*(1 + c^2*x^2)^(7/2))/(49*c^5)
 + (d*x^5*(a + b*ArcSinh[c*x]))/5 + (c^2*d*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx^5\left(7+5c^2x^2\right.}{35\sqrt{1+c^2x^2}}$$

$$=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{35}(bcd)\int\frac{x^5\left(7+5c\right.}{\sqrt{1+c^2}}$$

$$=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{70}(bcd)\,\text{Subst}\left(\int\frac{x^2}{}\right.$$

$$=\frac{1}{5}dx^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^2dx^7\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{70}(bcd)\,\text{Subst}\left(\int\left(\frac{}{c}\right.\right.$$

$$=-\frac{2bd\sqrt{1+c^2x^2}}{35c^5}-\frac{bd\left(1+c^2x^2\right)^{3/2}}{105c^5}+\frac{8bd\left(1+c^2x^2\right)^{5/2}}{175c^5}-\frac{bd\left(1+c^2x^2\right)^{7/2}}{49c^5}+\frac{1}{5}$$

Mathematica [A] time = 0.0937715, size = 87, normalized size = 0.7

$$\frac{d\left(105ax^5\left(5c^2x^2+7\right)-\frac{b\sqrt{c^2x^2+1}\left(75c^6x^6+57c^4x^4-76c^2x^2+152\right)}{c^5}+105bx^5\left(5c^2x^2+7\right)\sinh^{-1}(cx)\right)}{3675}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(105*a*x^5*(7 + 5*c^2*x^2) - (b*Sqrt[1 + c^2*x^2]*(152 - 76*c^2*x^2 + 57
*c^4*x^4 + 75*c^6*x^6))/c^5 + 105*b*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]))/3675
```

Maple [A] time = 0.009, size = 124, normalized size = 1.

$$\frac{1}{c^5}\left(da\left(\frac{c^7x^7}{7}+\frac{c^5x^5}{5}\right)+db\left(\frac{\text{Arcsinh}\,(cx)\,c^7x^7}{7}+\frac{\text{Arcsinh}\,(cx)\,c^5x^5}{5}-\frac{c^6x^6}{49}\sqrt{c^2x^2+1}-\frac{19\,c^4x^4}{1225}\sqrt{c^2x^2+1}+\frac{76\,c^2x^2}{3675}\sqrt{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^5*(d*a*(1/7*c^7*x^7+1/5*c^5*x^5)+d*b*(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcs
inh(c*x)*c^5*x^5-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)
```

```
^(1/2)+76/3675*c^2*x^2*(c^2*x^2+1)^(1/2)-152/3675*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.20355, size = 248, normalized size = 2.

$$\frac{1}{7}\,ac^2dx^7 + \frac{1}{5}\,adx^5 + \frac{1}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx) - \left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2} - \frac{6\,\sqrt{c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{c^2x^2+1}x^2}{c^6} - \frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right)bc^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^
2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6
- 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d
```

Fricas [A] time = 2.255, size = 265, normalized size = 2.14

$$\frac{525\,ac^7dx^7 + 735\,ac^5dx^5 + 105\left(5\,bc^7dx^7 + 7\,bc^5dx^5\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(75\,bc^6dx^6 + 57\,bc^4dx^4 - 76\,bc^2dx^2 + 152\,bc\right.}{3675\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d*x^7 + 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 + 7*b*c^5*d*
x^5)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d*x^6 + 57*b*c^4*d*x^4 - 76*b
*c^2*d*x^2 + 152*b*d)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 7.9766, size = 151, normalized size = 1.22

$$\begin{cases} \frac{ac^2dx^7}{7} + \frac{adx^5}{5} + \frac{bc^2dx^7\,\mathrm{asinh}\,(cx)}{7} - \frac{bcdx^6\sqrt{c^2x^2+1}}{49} + \frac{bdx^5\,\mathrm{asinh}\,(cx)}{5} - \frac{19bdx^4\sqrt{c^2x^2+1}}{1225c} + \frac{76bdx^2\sqrt{c^2x^2+1}}{3675c^3} - \frac{152bd\sqrt{c^2x^2+1}}{3675c^5} & \text{for}\ c \neq 0 \\ \frac{adx^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**7/7 + a*d*x**5/5 + b*c**2*d*x**7*asinh(c*x)/7 - b*c*
d*x**6*sqrt(c**2*x**2 + 1)/49 + b*d*x**5*asinh(c*x)/5 - 19*b*d*x**4*sqrt(c*
*2*x**2 + 1)/(1225*c) + 76*b*d*x**2*sqrt(c**2*x**2 + 1)/(3675*c**3) - 152*b
*d*sqrt(c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

Giac [A] time = 1.49798, size = 238, normalized size = 1.92

$$\frac{1}{7}ac^2dx^7+\frac{1}{5}adx^5+\frac{1}{245}\left(35x^7\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{5\left(c^2x^2+1\right)^{\frac{7}{2}}-21\left(c^2x^2+1\right)^{\frac{5}{2}}+35\left(c^2x^2+1\right)^{\frac{3}{2}}-35\sqrt{c^2x^2+1}}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 + 1/245*(35*x^7*log(c*x + sqrt(c^2*x^2 + 1))
- (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1)^(3/2)
- 35*sqrt(c^2*x^2 + 1))/c^7)*b*c^2*d + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2
+ 1)) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 +
 1))/c^5)*b*d
```

3.2 $\int x^3 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(c x)\right) \, dx$

Optimal. Leaf size=120

$$\frac{1}{6} c^2 d x^6 \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{4} d x^4 \left(a+b \sinh^{-1}(c x)\right)-\frac{1}{36} b c d x^5 \sqrt{c^2 x^2+1}-\frac{b d x^3 \sqrt{c^2 x^2+1}}{36 c}+\frac{b d x \sqrt{c^2 x^2+1}}{24 c^3}-\frac{b d \sinh^{-1}}{24 c^4}$$

```
[Out] (b*d*x*Sqrt[1 + c^2*x^2])/(24*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2])/(36*c) - (
b*c*d*x^5*Sqrt[1 + c^2*x^2])/36 - (b*d*ArcSinh[c*x])/(24*c^4) + (d*x^4*(a +
 b*ArcSinh[c*x]))/4 + (c^2*d*x^6*(a + b*ArcSinh[c*x]))/6
```

Rubi [A] time = 0.0981831, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {14, 5730, 12, 459, 321, 215}

$$\frac{1}{6} c^2 d x^6 \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{4} d x^4 \left(a+b \sinh^{-1}(c x)\right)-\frac{1}{36} b c d x^5 \sqrt{c^2 x^2+1}-\frac{b d x^3 \sqrt{c^2 x^2+1}}{36 c}+\frac{b d x \sqrt{c^2 x^2+1}}{24 c^3}-\frac{b d \sinh^{-1}}{24 c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[1 + c^2*x^2])/(24*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2])/(36*c) - (
b*c*d*x^5*Sqrt[1 + c^2*x^2])/36 - (b*d*ArcSinh[c*x])/(24*c^4) + (d*x^4*(a +
 b*ArcSinh[c*x]))/4 + (c^2*d*x^6*(a + b*ArcSinh[c*x]))/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-(bc)\int\frac{dx^4\left(3+2c^2x^2\right.}{12\sqrt{1+c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x^4\left(3+2c\right.}{\sqrt{1+c^2}}$$

$$=-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a+b\sinh^{-1}(cx)\right)-$$

$$=-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}c^2dx^6\left(a\right.$$

$$=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}+\frac{1}{4}dx^4\left(a+b\sinh^{-1}(c\right.$$

$$=\frac{bdx\sqrt{1+c^2x^2}}{24c^3}-\frac{bdx^3\sqrt{1+c^2x^2}}{36c}-\frac{1}{36}bcdx^5\sqrt{1+c^2x^2}-\frac{bd\sinh^{-1}(cx)}{24c^4}+\frac{1}{4}dx$$

Mathematica [A] time = 0.0547256, size = 88, normalized size = 0.73

$$\frac{d\left(6ac^4x^4\left(2c^2x^2+3\right)+bcx\sqrt{c^2x^2+1}\left(-2c^4x^4-2c^2x^2+3\right)+3b\left(4c^6x^6+6c^4x^4-1\right)\sinh^{-1}(cx)\right)}{72c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(6*a*c^4*x^4*(3 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 - 2*c^2*x^2 -
2*c^4*x^4) + 3*b*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]))/(72*c^4)
```

Maple [A] time = 0.009, size = 113, normalized size = 0.9

$$\frac{1}{c^4}\left(da\left(\frac{c^6x^6}{6}+\frac{c^4x^4}{4}\right)+db\left(\frac{\operatorname{Arcsinh}(cx)\,c^6x^6}{6}+\frac{\operatorname{Arcsinh}(cx)\,c^4x^4}{4}-\frac{c^5x^5}{36}\sqrt{c^2x^2+1}-\frac{c^3x^3}{36}\sqrt{c^2x^2+1}+\frac{cx}{24}\sqrt{c^2x^2+1}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^4*(d*a*(1/6*c^6*x^6+1/4*c^4*x^4)+d*b*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcs
inh(c*x)*c^4*x^4-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1
/2)+1/24*c*x*(c^2*x^2+1)^(1/2)-1/24*arcsinh(c*x)))
```

Maxima [A] time = 1.15503, size = 257, normalized size = 2.14

$$\frac{1}{6}ac^2dx^6+\frac{1}{4}adx^4+\frac{1}{288}\left(48\,x^6\operatorname{arsinh}(cx)-\left(\frac{8\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{c^2x^2+1}x}{c^6}-\frac{15\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^6}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^
2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6
- 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^2*d + 1/32*(8*x^4*arc
```

```
sinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*ar
csinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d
```

Fricas [A] time = 2.50227, size = 242, normalized size = 2.02

$$\frac{12\,ac^6dx^6 + 18\,ac^4dx^4 + 3\left(4\,bc^6dx^6 + 6\,bc^4dx^4 - bd\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(2\,bc^5dx^5 + 2\,bc^3dx^3 - 3\,bcdx\right)\sqrt{c^2x^2+1}}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/72*(12*a*c^6*d*x^6 + 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 + 6*b*c^4*d*x^4 -
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^5*d*x^5 + 2*b*c^3*d*x^3 - 3*b*c*
d*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 5.18128, size = 138, normalized size = 1.15

$$\begin{cases} \frac{ac^2dx^6}{6} + \frac{adx^4}{4} + \frac{bc^2dx^6\operatorname{asinh}(cx)}{6} - \frac{bcdx^5\sqrt{c^2x^2+1}}{36} + \frac{bdx^4\operatorname{asinh}(cx)}{4} - \frac{bdx^3\sqrt{c^2x^2+1}}{36c} + \frac{bdx\sqrt{c^2x^2+1}}{24c^3} - \frac{bd\operatorname{asinh}(cx)}{24c^4} & \text{for}\ c \neq 0 \\ \frac{adx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**6/6 + a*d*x**4/4 + b*c**2*d*x**6*asinh(c*x)/6 - b*c*
d*x**5*sqrt(c**2*x**2 + 1)/36 + b*d*x**4*asinh(c*x)/4 - b*d*x**3*sqrt(c**2*
x**2 + 1)/(36*c) + b*d*x*sqrt(c**2*x**2 + 1)/(24*c**3) - b*d*asinh(c*x)/(24
*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

Giac [A] time = 1.58097, size = 273, normalized size = 2.28

$$\frac{1}{6}\,ac^2dx^6 + \frac{1}{4}\,adx^4 + \frac{1}{288}\left(48\,x^6\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(\sqrt{c^2x^2+1}\left(2\,x^2\left(\frac{4\,x^2}{c^2} - \frac{5}{c^4}\right) + \frac{15}{c^6}\right)x + \frac{15\,\log\left(\left|-x|c| + \sqrt{c^2x^2+\right.\right.}{c^6|c|}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 + 1/288*(48*x^6*log(c*x + sqrt(c^2*x^2 + 1))
- (sqrt(c^2*x^2 + 1)*(2*x^2*(4*x^2/c^2 - 5/c^4) + 15/c^6)*x + 15*log(abs(-x
*abs(c) + sqrt(c^2*x^2 + 1)))/(c^6*abs(c)))*c)*b*c^2*d + 1/32*(8*x^4*log(c*
x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*x*(2*x^2/c^2 - 3/c^4) - 3*log(a
bs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4*abs(c)))*c)*b*d
```

3.3 $\int x^2 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(c x)\right) \, dx$

Optimal. Leaf size=102

$$\frac{1}{5} c^2 d x^5 \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{3} d x^3 \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{5/2}}{25 c^3}+\frac{b d \left(c^2 x^2+1\right)^{3/2}}{45 c^3}+\frac{2 b d \sqrt{c^2 x^2+1}}{15 c^3}$$

```
[Out] (2*b*d*Sqrt[1 + c^2*x^2])/(15*c^3) + (b*d*(1 + c^2*x^2)^(3/2))/(45*c^3) - (
b*d*(1 + c^2*x^2)^(5/2))/(25*c^3) + (d*x^3*(a + b*ArcSinh[c*x]))/3 + (c^2*d
*x^5*(a + b*ArcSinh[c*x]))/5
```

Rubi [A] time = 0.101588, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5730, 12, 446, 77}

$$\frac{1}{5} c^2 d x^5 \left(a+b \sinh^{-1}(c x)\right)+\frac{1}{3} d x^3 \left(a+b \sinh^{-1}(c x)\right)-\frac{b d \left(c^2 x^2+1\right)^{5/2}}{25 c^3}+\frac{b d \left(c^2 x^2+1\right)^{3/2}}{45 c^3}+\frac{2 b d \sqrt{c^2 x^2+1}}{15 c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*d*Sqrt[1 + c^2*x^2])/(15*c^3) + (b*d*(1 + c^2*x^2)^(3/2))/(45*c^3) - (
b*d*(1 + c^2*x^2)^(5/2))/(25*c^3) + (d*x^3*(a + b*ArcSinh[c*x]))/3 + (c^2*d
*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-(bc)\int \frac{dx^3\left(5+3c^2x^2\right)}{15\sqrt{1+c^2x^2}}$$

$$= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{15}(bcd)\int \frac{x^3\left(5+3c^2x\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}(bcd)\,\text{Subst}\left(\int \frac{x(5}{\sqrt{}}\right.$$

$$= \frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^2dx^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}(bcd)\,\text{Subst}\left(\int\left(-\frac{}{c}\right.\right.$$

$$= \frac{2bd\sqrt{1+c^2x^2}}{15c^3}+\frac{bd\left(1+c^2x^2\right)^{3/2}}{45c^3}-\frac{bd\left(1+c^2x^2\right)^{5/2}}{25c^3}+\frac{1}{3}dx^3\left(a+b\sinh^{-1}(cx)\right)$$

Mathematica [A] time = 0.0825324, size = 78, normalized size = 0.76

$$\frac{1}{225}d\left(15ax^3\left(3c^2x^2+5\right)+\frac{b\sqrt{c^2x^2+1}\left(-9c^4x^4-13c^2x^2+26\right)}{c^3}+15bx^3\left(3c^2x^2+5\right)\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(15*a*x^3*(5 + 3*c^2*x^2) + (b*Sqrt[1 + c^2*x^2]*(26 - 13*c^2*x^2 - 9*c^
4*x^4))/c^3 + 15*b*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]))/225
```

Maple [A] time = 0.004, size = 105, normalized size = 1.

$$\frac{1}{c^3}\left(da\left(\frac{c^5x^5}{5}+\frac{c^3x^3}{3}\right)+db\left(\frac{\operatorname{Arcsinh}(cx)\,c^5x^5}{5}+\frac{\operatorname{Arcsinh}(cx)\,c^3x^3}{3}-\frac{c^4x^4}{25}\sqrt{c^2x^2+1}-\frac{13\,c^2x^2}{225}\sqrt{c^2x^2+1}+\frac{26}{225}\sqrt{c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^3*(d*a*(1/5*c^5*x^5+1/3*c^3*x^3)+d*b*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcs
inh(c*x)*c^3*x^3-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^
(1/2)+26/225*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.15596, size = 196, normalized size = 1.92

$$\frac{1}{5}\,ac^2dx^5+\frac{1}{75}\left(15\,x^5\operatorname{arsinh}(cx)-\left(\frac{3\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^2d+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^2*d*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2
- 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^2*d + 1/3*a
*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^
2*x^2 + 1)/c^4))*b*d
```

Fricas [A] time = 2.70901, size = 234, normalized size = 2.29

$$\frac{45\,ac^5dx^5+75\,ac^3dx^3+15\left(3\,bc^5dx^5+5\,bc^3dx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4dx^4+13\,bc^2dx^2-26\,bd\right)\sqrt{c^2x^2+1}}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d*x^5 + 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 + 5*b*c^3*d*x^3)
*log(c*x + sqrt(c^2*x^2 + 1)) - (9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 26*b*d)*s
qrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 2.75605, size = 126, normalized size = 1.24

$$\begin{cases} \frac{ac^2dx^5}{5} + \frac{adx^3}{3} + \frac{bc^2dx^5 \operatorname{asinh}(cx)}{5} - \frac{bcdx^4\sqrt{c^2x^2+1}}{25} + \frac{bdx^3 \operatorname{asinh}(cx)}{3} - \frac{13bdx^2\sqrt{c^2x^2+1}}{225c} + \frac{26bd\sqrt{c^2x^2+1}}{225c^3} & \text{for } c \neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**5/5 + a*d*x**3/3 + b*c**2*d*x**5*asinh(c*x)/5 - b*c*
d*x**4*sqrt(c**2*x**2 + 1)/25 + b*d*x**3*asinh(c*x)/3 - 13*b*d*x**2*sqrt(c*
*2*x**2 + 1)/(225*c) + 26*b*d*sqrt(c**2*x**2 + 1)/(225*c**3), Ne(c, 0)), (a
*d*x**3/3, True))
```

Giac [A] time = 1.44274, size = 200, normalized size = 1.96

$$\frac{1}{5}\,ac^2dx^5 + \frac{1}{75}\left(15\,x^5\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{3\left(c^2x^2+1\right)^{\frac{5}{2}} - 10\left(c^2x^2+1\right)^{\frac{3}{2}} + 15\sqrt{c^2x^2+1}}{c^5}\right)bc^2d + \frac{1}{3}\,adx^3 + \frac{1}{9}\left(3\,x^3\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c^2*d*x^5 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 +
 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*b*c^2*d + 1
/3*a*d*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2)
 - 3*sqrt(c^2*x^2 + 1))/c^3)*b*d
```

3.4 $\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{bdx\left(c^2x^2+1\right)^{3/2}}{16c}-\frac{3bdx\sqrt{c^2x^2+1}}{32c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}$$

```
[Out] (-3*b*d*x*Sqrt[1 + c^2*x^2])/(32*c) - (b*d*x*(1 + c^2*x^2)^(3/2))/(16*c) -
(3*b*d*ArcSinh[c*x])/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4
*c^2)
```

Rubi [A] time = 0.0413919, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5717, 195, 215}

$$\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{bdx\left(c^2x^2+1\right)^{3/2}}{16c}-\frac{3bdx\sqrt{c^2x^2+1}}{32c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[1 + c^2*x^2])/(32*c) - (b*d*x*(1 + c^2*x^2)^(3/2))/(16*c) -
(3*b*d*ArcSinh[c*x])/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(4
*c^2)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx = \frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(bd)\int\left(1+c^2x^2\right)^{3/2}dx}{4c}$$

$$=-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{16c}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(3bd)\int\sqrt{1+c^2x^2}\,dx}{16c}$$

$$=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{16c}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{4c^2}-\frac{(3bd)}{}$$

$$=-\frac{3bdx\sqrt{1+c^2x^2}}{32c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{16c}-\frac{3bd\sinh^{-1}(cx)}{32c^2}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sin\right.}{4c^2}$$

Mathematica [A] time = 0.0520749, size = 77, normalized size = 0.89

$$\frac{d\left(cx\left(8acx\left(c^2x^2+2\right)-b\sqrt{c^2x^2+1}\left(2c^2x^2+5\right)\right)+b\left(8c^4x^4+16c^2x^2+5\right)\sinh^{-1}(cx)\right)}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*(c*x*(8*a*c*x*(2 + c^2*x^2) - b*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)) + b*(
5 + 16*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x]))/(32*c^2)
```

Maple [A] time = 0.006, size = 94, normalized size = 1.1

$$\frac{1}{c^2}\left(da\left(\frac{c^4x^4}{4}+\frac{c^2x^2}{2}\right)+db\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^4x^4}{4}+\frac{\mathrm{Arcsinh}\,(cx)\,c^2x^2}{2}-\frac{c^3x^3}{16}\sqrt{c^2x^2+1}-\frac{5\,cx}{32}\sqrt{c^2x^2+1}+\frac{5\,\mathrm{Arcsinh}\,(cx)}{32}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^2*(d*a*(1/4*c^4*x^4+1/2*c^2*x^2)+d*b*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcs
inh(c*x)*c^2*x^2-1/16*c^3*x^3*(c^2*x^2+1)^(1/2)-5/32*c*x*(c^2*x^2+1)^(1/2)+
5/32*arcsinh(c*x)))
```

Maxima [B] time = 1.15665, size = 204, normalized size = 2.34

$$\frac{1}{4}\,ac^2dx^4+\frac{1}{32}\left(8\,x^4\operatorname{arsinh}(cx)-\left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\,\sqrt{c^2x^2+1}x}{c^4}+\frac{3\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^4}\right)c\right)bc^2d+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\operatorname{arsinh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d*x^4 + 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 -
 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)
*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c
^2 - arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d
```

Fricas [A] time = 2.66611, size = 220, normalized size = 2.53

$$\frac{8\,ac^4dx^4+16\,ac^2dx^2+\left(8\,bc^4dx^4+16\,bc^2dx^2+5\,bd\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(2\,bc^3dx^3+5\,bcdx\right)\sqrt{c^2x^2+1}}{32\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*d*x^4 + 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 + 16*b*c^2*d*x^2 + 5*
b*d)*log(c*x + sqrt(c^2*x^2 + 1)) - (2*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(c^2*x^
2 + 1))/c^2
```

Sympy [A] time = 1.45835, size = 117, normalized size = 1.34

$$\begin{cases} \frac{ac^2dx^4}{4} + \frac{adx^2}{2} + \frac{bc^2dx^4 \operatorname{asinh}(cx)}{4} - \frac{bcdx^3\sqrt{c^2x^2+1}}{16} + \frac{bdx^2 \operatorname{asinh}(cx)}{2} - \frac{5bdx\sqrt{c^2x^2+1}}{32c} + \frac{5bd \operatorname{asinh}(cx)}{32c^2} & \text{for } c \neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**4/4 + a*d*x**2/2 + b*c**2*d*x**4*asinh(c*x)/4 - b*c*
d*x**3*sqrt(c**2*x**2 + 1)/16 + b*d*x**2*asinh(c*x)/2 - 5*b*d*x*sqrt(c**2*x
**2 + 1)/(32*c) + 5*b*d*asinh(c*x)/(32*c**2), Ne(c, 0)), (a*d*x**2/2, True)
)
```

Giac [B] time = 1.69869, size = 242, normalized size = 2.78

$$\frac{1}{4}\,ac^2dx^4 + \frac{1}{32}\left(8\,x^4\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(\sqrt{c^2x^2+1}x\left(\frac{2\,x^2}{c^2} - \frac{3}{c^4}\right) - \frac{3\,\log\left(\left|-x|c| + \sqrt{c^2x^2+1}\right|\right)}{c^4|c|}\right)c\right)bc^2d + \frac{1}{2}\,adx^2 + \frac{1}{4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*c^2*d*x^4 + 1/32*(8*x^4*log(c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2
+ 1)*x*(2*x^2/c^2 - 3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4
*abs(c)))*c)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)
) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c
^2*abs(c))))*b*d
```

3.5 $\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=75

$$\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)+dx\left(a+b\sinh^{-1}(cx)\right)-\frac{bd\left(c^2x^2+1\right)^{3/2}}{9c}-\frac{2bd\sqrt{c^2x^2+1}}{3c}$$

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(3*c) - (b*d*(1 + c^2*x^2)^(3/2))/(9*c) + d*x*(a
 + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

Rubi [A] time = 0.0599111, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5679, 12, 444, 43}

$$\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)+dx\left(a+b\sinh^{-1}(cx)\right)-\frac{bd\left(c^2x^2+1\right)^{3/2}}{9c}-\frac{2bd\sqrt{c^2x^2+1}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*Sqrt[1 + c^2*x^2])/(3*c) - (b*d*(1 + c^2*x^2)^(3/2))/(9*c) + d*x*(a
 + b*ArcSinh[c*x]) + (c^2*d*x^3*(a + b*ArcSinh[c*x]))/3
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx = dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-(bc)\int \frac{dx\left(3+c^2x^2\right)}{3\sqrt{1+c^2x^2}}\,dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{3}(bcd)\int \frac{x\left(3+c^2x^2\right)}{\sqrt{1+c^2x^2}}\,dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{6}(bcd)\,\text{Subst}\left(\int \frac{3+c^2x}{\sqrt{1+c^2x}}\,d\right.$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{6}(bcd)\,\text{Subst}\left(\int \left(\frac{2}{\sqrt{1+c^2x}}\right.\right.$$

$$= -\frac{2bd\sqrt{1+c^2x^2}}{3c}-\frac{bd\left(1+c^2x^2\right)^{3/2}}{9c}+dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2dx^3\left(a+b\sinh^{-1}\right.$$

Mathematica [A] time = 0.0418292, size = 86, normalized size = 1.15

$$\frac{1}{3}ac^2dx^3+adx-\frac{1}{9}bcdx^2\sqrt{c^2x^2+1}-\frac{7bd\sqrt{c^2x^2+1}}{9c}+\frac{1}{3}bc^2dx^3\sinh^{-1}(cx)+bdx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*d*x + (a*c^2*d*x^3)/3 - (7*b*d*Sqrt[1 + c^2*x^2])/(9*c) - (b*c*d*x^2*Sqrt
[1 + c^2*x^2])/9 + b*d*x*ArcSinh[c*x] + (b*c^2*d*x^3*ArcSinh[c*x])/3
```

Maple [A] time = 0.006, size = 76, normalized size = 1.

$$\frac{1}{c}\left(da\left(\frac{c^3x^3}{3}+cx\right)+db\left(\frac{\text{Arcsinh}\,(cx)\,c^3x^3}{3}+\text{Arcsinh}\,(cx)\,cx-\frac{c^2x^2}{9}\sqrt{c^2x^2+1}-\frac{7}{9}\sqrt{c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(d*a*(1/3*c^3*x^3+c*x)+d*b*(1/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1
/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.11389, size = 131, normalized size = 1.75

$$\frac{1}{3}\,ac^2dx^3+\frac{1}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^2d+adx+\frac{\left(cx\,\mathrm{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*b*d/c
```

Fricas [A] time = 2.60895, size = 184, normalized size = 2.45

$$\frac{3\,ac^3dx^3+9\,acdx+3\left(bc^3dx^3+3\,bcdx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^2dx^2+7\,bd\right)\sqrt{c^2x^2+1}}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*d*x^3 + 9*a*c*d*x + 3*(b*c^3*d*x^3 + 3*b*c*d*x)*log(c*x + sqrt
(c^2*x^2 + 1)) - (b*c^2*d*x^2 + 7*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] time = 0.729231, size = 90, normalized size = 1.2

$$\begin{cases}\frac{ac^2dx^3}{3}+adx+\frac{bc^2dx^3\,\mathrm{asinh}\,(cx)}{3}-\frac{bcdx^2\sqrt{c^2x^2+1}}{9}+bdx\,\mathrm{asinh}\,(cx)-\frac{7bd\sqrt{c^2x^2+1}}{9c} & \text{for } c\neq 0\\ adx & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d*x**3/3 + a*d*x + b*c**2*d*x**3*asinh(c*x)/3 - b*c*d*x**
2*sqrt(c**2*x**2 + 1)/9 + b*d*x*asinh(c*x) - 7*b*d*sqrt(c**2*x**2 + 1)/(9*c
), Ne(c, 0)), (a*d*x, True))
```

Giac [A] time = 1.36874, size = 151, normalized size = 2.01

$$\frac{1}{3}\,ac^2dx^3 + \frac{1}{9}\left(3\,x^3\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\left(c^2x^2+1\right)^{\frac{3}{2}} - 3\,\sqrt{c^2x^2+1}}{c^3}\right)bc^2d + \left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)bd + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*c^2*d*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^
(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*c^2*d + (x*log(c*x + sqrt(c^2*x^2 + 1))
 - sqrt(c^2*x^2 + 1)/c)*b*d + a*d*x
```

3.6 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=111

$$-\frac{1}{2}bd\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d\log\left(1-e^{-2\sinh^{-1}(cx)}\right)(a$$

```
[Out] -(b*c*d*x*Sqrt[1 + c^2*x^2])/4 - (b*d*ArcSinh[c*x])/4 + (d*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/2 + (d*(a + b*ArcSinh[c*x])^2)/(2*b) + d*(a + b*ArcSinh
[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*d*PolyLog[2, E^(-2*ArcSinh[c*x])])
/2
```

Rubi [A] time = 0.122861, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5726, 5659, 3716, 2190, 2279, 2391, 195, 215}

$$\frac{1}{2}bd\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+d\log\left(1-e^{2\sinh^{-1}(cx)}\right)(a+b$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -(b*c*d*x*Sqrt[1 + c^2*x^2])/4 - (b*d*ArcSinh[c*x])/4 + (d*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/2 - (d*(a + b*ArcSinh[c*x])^2)/(2*b) + d*(a + b*ArcSinh
[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{a+b\sinh^{-1}(cx)}{x}\,dx-\frac{1}{2}(bcd)\int \sqrt{1+c^2$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+d\,\text{Subst}\left(\int (a+bx)\cot$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+\right.}{}$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+\right.}{}$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+\right.}{}$$

$$= -\frac{1}{4}bcdx\sqrt{1+c^2x^2}-\frac{1}{4}bd\sinh^{-1}(cx)+\frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+\right.}{}$$

Mathematica [A] time = 0.0606308, size = 113, normalized size = 1.02

$$\frac{1}{2}bd\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}ac^2dx^2+ad\log(x)-\frac{1}{4}bcdx\sqrt{c^2x^2+1}+\frac{1}{2}bc^2dx^2\sinh^{-1}(cx)-\frac{1}{2}bd\sinh^{-1}(cx)^2+\frac{1}{4}bd$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*c^2*d*x^2)/2 - (b*c*d*x*Sqrt[1 + c^2*x^2])/4 + (b*d*ArcSinh[c*x])/4 + (b
*c^2*d*x^2*ArcSinh[c*x])/2 - (b*d*ArcSinh[c*x]^2)/2 + b*d*ArcSinh[c*x]*Log[
1 - E^(2*ArcSinh[c*x])] + a*d*Log[x] + (b*d*PolyLog[2, E^(2*ArcSinh[c*x])])
/2
```

Maple [A] time = 0.086, size = 162, normalized size = 1.5

$$\frac{dac^2x^2}{2}+da\ln(cx)-\frac{db\left(\text{Arcsinh}(cx)\right)^2}{2}+\frac{db\text{Arcsinh}(cx)\,c^2x^2}{2}-\frac{dbcx}{4}\sqrt{c^2x^2+1}+\frac{bd\text{Arcsinh}(cx)}{4}+db\text{Arcsinh}(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/2*d*a*c^2*x^2+d*a*ln(c*x)-1/2*d*b*arcsinh(c*x)^2+1/2*d*b*arcsinh(c*x)*c^2
*x^2-1/4*b*c*d*x*(c^2*x^2+1)^(1/2)+1/4*b*d*arcsinh(c*x)+d*b*arcsinh(c*x)*ln
(1+c*x+(c^2*x^2+1)^(1/2))+d*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d*b*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+d*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,ac^2dx^2 + ad\log\left(x\right) + \int bc^2dx\log\left(cx + \sqrt{c^2x^2+1}\right) + \frac{bd\log\left(cx + \sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^2*d*x^2 + a*d*log(x) + integrate(b*c^2*d*x*log(c*x + sqrt(c^2*x^2 +
 1)) + b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right)\operatorname{arsinh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a}{x}\,dx + \int ac^2x\,dx + \int\frac{b\operatorname{asinh}\left(cx\right)}{x}\,dx + \int bc^2x\operatorname{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x,x)
```

```
[Out] d*(Integral(a/x, x) + Integral(a*c**2*x, x) + Integral(b*asinh(c*x)/x, x) +
 Integral(b*c**2*x*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2dx^2 + d)(b\,\mathrm{arsinh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

3.7 $\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=66

$$c^2dx\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}-bcd\sqrt{c^2x^2+1}-bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2]) - (d*(a + b*ArcSinh[c*x]))/x + c^2*d*x*(a + b*Ar
cSinh[c*x]) - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rubi [A] time = 0.0814023, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5730, 12, 446, 80, 63, 208}

$$c^2dx\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x}-bcd\sqrt{c^2x^2+1}-bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2]) - (d*(a + b*ArcSinh[c*x]))/x + c^2*d*x*(a + b*Ar
cSinh[c*x]) - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2dx\left(a+b\sinh^{-1}(cx)\right) - (bc)\int \frac{d\left(-1+c^2x^2\right)}{x\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2dx\left(a+b\sinh^{-1}(cx)\right) - (bcd)\int \frac{-1+c^2x^2}{x\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2dx\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{2}(bcd)\,\mathrm{Subst}\left(\int \frac{-1+c^2x}{x\sqrt{1+c^2x}}\,d\right.$$

$$= -bcd\sqrt{1+c^2x^2} - \frac{d\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2dx\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}(bcd)\,\mathrm{Subst}\left(\right.$$

$$= -bcd\sqrt{1+c^2x^2} - \frac{d\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2dx\left(a+b\sinh^{-1}(cx)\right) + \frac{(bd)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -bcd\sqrt{1+c^2x^2} - \frac{d\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2dx\left(a+b\sinh^{-1}(cx)\right) - bcd\tanh^{-1}\left(\sqrt{\right.$$

Mathematica [A] time = 0.0265772, size = 74, normalized size = 1.12

$$ac^2dx - \frac{ad}{x} - bcd\sqrt{c^2x^2+1} - bcd\tanh^{-1}\left(\sqrt{c^2x^2+1}\right) + bc^2dx\sinh^{-1}(cx) - \frac{bd\sinh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c^2*d*x - b*c*d*Sqrt[1 + c^2*x^2] - (b*d*ArcSinh[c*x])/x + b
*c^2*d*x*ArcSinh[c*x] - b*c*d*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Maple [A] time = 0.009, size = 69, normalized size = 1.1

$$c\left(da\left(cx-\frac{1}{cx}\right)+db\left(\mathrm{Arcsinh}\,(cx)\,cx-\frac{\mathrm{Arcsinh}\,(cx)}{cx}-\sqrt{c^2x^2+1}-\mathrm{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] c*(d*a*(c*x-1/c/x)+d*b*(arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-(c^2*x^2+1)^(1/2)
-arctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.09682, size = 89, normalized size = 1.35

$$ac^2dx + \left(cx \operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bcd - \left(c \operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bd - \frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] a*c^2*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c*d - (c*arcsinh(1/(sq
rt(c^2)*abs(x))) + arcsinh(c*x)/x)*b*d - a*d/x
```

Fricas [B] time = 2.61708, size = 346, normalized size = 5.24

$$\frac{ac^2dx^2 - bcdx\log\left(-cx + \sqrt{c^2x^2+1} + 1\right) + bcdx\log\left(-cx + \sqrt{c^2x^2+1} - 1\right) - \sqrt{c^2x^2+1}bcdx - \left(bc^2 - b\right)dx\log\left(-cx + \sqrt{\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (a*c^2*d*x^2 - b*c*d*x*log(-c*x + sqrt(c^2*x^2 + 1) + 1) + b*c*d*x*log(-c*x
 + sqrt(c^2*x^2 + 1) - 1) - sqrt(c^2*x^2 + 1)*b*c*d*x - (b*c^2 - b)*d*x*log
(-c*x + sqrt(c^2*x^2 + 1)) - a*d + (b*c^2*d*x^2 - (b*c^2 - b)*d*x - b*d)*lo
g(c*x + sqrt(c^2*x^2 + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int ac^2\,dx + \int \frac{a}{x^2}\,dx + \int bc^2 \operatorname{asinh}(cx)\,dx + \int \frac{b \operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(b*c**2*asinh(c*x),
x) + Integral(b*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)(b\operatorname{arsinh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.8 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=128

$$-\frac{1}{2}bc^2d\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+c^2d\log\left(1-e^{-2\sinh^{-1}(cx)}\right.$$

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2])/(2*x) + (b*c^2*d*ArcSinh[c*x])/2 - (d*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x]))/(2*x^2) + (c^2*d*(a + b*ArcSinh[c*x])^2)/(2*b) +
c^2*d*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*c^2*d*PolyLog[
2, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.126375, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5728, 277, 215, 5659, 3716, 2190, 2279, 2391}

$$\frac{1}{2}bc^2d\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+c^2d\log\left(1-e^{2\sinh^{-1}(cx)}\right)(a$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2])/(2*x) + (b*c^2*d*ArcSinh[c*x])/2 - (d*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*d*(a + b*ArcSinh[c*x])^2)/(2*b) +
c^2*d*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (b*c^2*d*PolyLog[2
, E^(2*ArcSinh[c*x])])/2
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}(bcd)\int\frac{\sqrt{1+c^2x^2}}{x^2}\,dx+\left(c^2d\right)\int\frac{a+b\sinh}{x}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\left(c^2d\right)\text{Subst}\left(\int(a+bx)\cot\right.$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+\right.}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+\right.}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+\right.}{}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{2x}+\frac{1}{2}bc^2d\sinh^{-1}(cx)-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+\right.}{}$$

Mathematica [A] time = 0.0599271, size = 111, normalized size = 0.87

$$\frac{1}{2}bc^2d\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+ac^2d\log(x)-\frac{ad}{2x^2}-\frac{bcd\sqrt{c^2x^2+1}}{2x}-\frac{1}{2}bc^2d\sinh^{-1}(cx)^2+bc^2d\sinh^{-1}(cx)\log\left(1-e^{2\sinh}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(a*d)/(2*x^2) - (b*c*d*Sqrt[1 + c^2*x^2])/(2*x) - (b*d*ArcSinh[c*x])/(2*x^
2) - (b*c^2*d*ArcSinh[c*x]^2)/2 + b*c^2*d*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh
[c*x])] + a*c^2*d*Log[x] + (b*c^2*d*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Maple [A] time = 0.167, size = 175, normalized size = 1.4

$$c^2da\ln(cx)-\frac{da}{2x^2}-\frac{c^2db\left(\text{Arcsinh}(cx)\right)^2}{2}-\frac{bcd}{2x}\sqrt{c^2x^2+1}+\frac{c^2db}{2}-\frac{db\text{Arcsinh}(cx)}{2x^2}+c^2db\text{Arcsinh}(cx)\ln\left(1+cx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] c^2*d*a*ln(c*x)-1/2*d*a/x^2-1/2*c^2*d*b*arcsinh(c*x)^2-1/2*b*c*d*(c^2*x^2+1
)^(1/2)/x+1/2*c^2*d*b-1/2*d*b*arcsinh(c*x)/x^2+c^2*d*b*arcsinh(c*x)*ln(1+c*
x+(c^2*x^2+1)^(1/2))+c^2*d*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+c^2*d*b*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+c^2*d*b*polylog(2,c*x+(c^2*x^2+1)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$bc^2d\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx + ac^2d\log\left(x\right) - \frac{1}{2}\,bd\left(\frac{\sqrt{c^2x^2+1}c}{x} + \frac{\operatorname{arsinh}\left(cx\right)}{x^2}\right) - \frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] b*c^2*d*integrate(log(c*x + sqrt(c^2*x^2 + 1))/x, x) + a*c^2*d*log(x) - 1/2
*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right)\operatorname{arsinh}\left(cx\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{a}{x^3}\,dx + \int \frac{ac^2}{x}\,dx + \int \frac{b\operatorname{asinh}\left(cx\right)}{x^3}\,dx + \int \frac{bc^2\operatorname{asinh}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d*(Integral(a/x**3, x) + Integral(a*c**2/x, x) + Integral(b*asinh(c*x)/x**3
, x) + Integral(b*c**2*asinh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2dx^2 + d)(b \operatorname{arsinh}(cx) + a)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.9 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=80

$$-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{c^2x^2+1}}{6x^2}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (d*(a + b*ArcSinh[c*x]))/(3*x^3) - (c^
2*d*(a + b*ArcSinh[c*x]))/x - (5*b*c^3*d*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rubi [A] time = 0.0843184, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {14, 5730, 12, 446, 78, 63, 208}

$$-\frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{c^2x^2+1}}{6x^2}-\frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```


```
[Out] -(b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (d*(a + b*ArcSinh[c*x]))/(3*x^3) - (c^
2*d*(a + b*ArcSinh[c*x]))/x - (5*b*c^3*d*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x} - (bc)\int \frac{d(-1-3c^2x^2)}{3x^3\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x} - \frac{1}{3}(bcd)\int \frac{-1-3c^2x^2}{x^3\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x} - \frac{1}{6}(bcd)\,\text{Subst}\left(\int \frac{-1-3c^2x}{x^2\sqrt{1+c^2x}}\,d$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{12}\left(5bc^3d\right)\text{Sul}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{6}(5bcd)\,\text{Subst}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}}{6x^2} - \frac{d\left(a+b\sinh^{-1}(cx)\right)}{3x^3} - \frac{c^2d\left(a+b\sinh^{-1}(cx)\right)}{x} - \frac{5}{6}bc^3d\tanh^{-1}\big($$

Mathematica [A] time = 0.0336878, size = 93, normalized size = 1.16

$$-\frac{ac^2d}{x} - \frac{ad}{3x^3} - \frac{bcd\sqrt{c^2x^2+1}}{6x^2} - \frac{5}{6}bc^3d\tanh^{-1}\left(\sqrt{c^2x^2+1}\right) - \frac{bc^2d\sinh^{-1}(cx)}{x} - \frac{bd\sinh^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(a*d)/(3*x^3) - (a*c^2*d)/x - (b*c*d*Sqrt[1 + c^2*x^2])/(6*x^2) - (b*d*Arc
Sinh[c*x])/(3*x^3) - (b*c^2*d*ArcSinh[c*x])/x - (5*b*c^3*d*ArcTanh[Sqrt[1 +
 c^2*x^2]])/6
```

Maple [A] time = 0.012, size = 87, normalized size = 1.1

$$c^3\left(da\left(-\frac{1}{cx} - \frac{1}{3\,c^3x^3}\right) + db\left(-\frac{\text{Arcsinh}\,(cx)}{cx} - \frac{\text{Arcsinh}\,(cx)}{3\,c^3x^3} - \frac{5}{6}\text{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right) - \frac{1}{6\,c^2x^2}\sqrt{c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] c^3*(d*a*(-1/c/x-1/3/c^3/x^3)+d*b*(-arcsinh(c*x)/c/x-1/3*arcsinh(c*x)/c^3/x
^3-5/6*arctanh(1/(c^2*x^2+1)^(1/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.08564, size = 128, normalized size = 1.6

$$-\left(c \operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d + \frac{1}{6}\left(\left(c^2 \operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right) - \frac{\sqrt{c^2x^2+1}}{x^2}\right)c - \frac{2 \operatorname{arsinh}(cx)}{x^3}\right)bd - \frac{ac^2d}{x} - \frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -(c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcsinh(c*x)/x)*b*c^2*d + 1/6*((c^2*arc
sinh(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)
*b*d - a*c^2*d/x - 1/3*a*d/x^3
```

Fricas [B] time = 2.67491, size = 396, normalized size = 4.95

$$-\frac{5\,bc^3dx^3 \log\left(-cx + \sqrt{c^2x^2+1} + 1\right) - 5\,bc^3dx^3 \log\left(-cx + \sqrt{c^2x^2+1} - 1\right) + 6\,ac^2dx^2 - 2\left(3\,bc^2 + b\right)dx^3 \log\left(-cx + \sqrt{c^2 \right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(5*b*c^3*d*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(-
c*x + sqrt(c^2*x^2 + 1) - 1) + 6*a*c^2*d*x^2 - 2*(3*b*c^2 + b)*d*x^3*log(-c
*x + sqrt(c^2*x^2 + 1)) + sqrt(c^2*x^2 + 1)*b*c*d*x + 2*a*d + 2*(3*b*c^2*d*
x^2 - (3*b*c^2 + b)*d*x^3 + b*d)*log(c*x + sqrt(c^2*x^2 + 1)))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{a}{x^4}\,dx + \int \frac{ac^2}{x^2}\,dx + \int \frac{b \operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{bc^2 \operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d*(Integral(a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(b*asinh(c*x)/x
**4, x) + Integral(b*c**2*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)(b\operatorname{arsinh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.10 $\int x^4 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=181

$$\frac{1}{9}c^4d^2x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{9/2}}{81c^5}+\frac{10bd^2\left(c^2x^2\right.}{441c^5}$$

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(315*c^5) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(945
*c^5) - (b*d^2*(1 + c^2*x^2)^(5/2))/(525*c^5) + (10*b*d^2*(1 + c^2*x^2)^(7/
2))/(441*c^5) - (b*d^2*(1 + c^2*x^2)^(9/2))/(81*c^5) + (d^2*x^5*(a + b*ArcS
inh[c*x]))/5 + (2*c^2*d^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^2*x^9*(a + b
*ArcSinh[c*x]))/9
```

Rubi [A] time = 0.208181, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {270, 5730, 12, 1251, 897, 1153}

$$\frac{1}{9}c^4d^2x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{9/2}}{81c^5}+\frac{10bd^2\left(c^2x^2\right.}{441c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(315*c^5) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(945
*c^5) - (b*d^2*(1 + c^2*x^2)^(5/2))/(525*c^5) + (10*b*d^2*(1 + c^2*x^2)^(7/
2))/(441*c^5) - (b*d^2*(1 + c^2*x^2)^(9/2))/(81*c^5) + (d^2*x^5*(a + b*ArcS
inh[c*x]))/5 + (2*c^2*d^2*x^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^2*x^9*(a + b
*ArcSinh[c*x]))/9
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
```

```
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{7}c^2d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a+b\sin\right.$$

$$= -\frac{8bd^2\sqrt{1+c^2x^2}}{315c^5}-\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{945c^5}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{525c^5}+\frac{10bd^2\left(1+c^2x^2\right.}{441c^5}$$

Mathematica [A] time = 0.0974155, size = 119, normalized size = 0.66

$$\frac{d^2\left(315ac^5x^5\left(35c^4x^4+90c^2x^2+63\right)-b\sqrt{c^2x^2+1}\left(1225c^8x^8+2650c^6x^6+789c^4x^4-1052c^2x^2+2104\right)+315bc^5x^5\left(3\right.\right.}{99225c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(2
104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c^5
*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSinh[c*x]))/(99225*c^5)
```

Maple [A] time = 0.01, size = 167, normalized size = 0.9

$$\frac{1}{c^5}\left(d^2a\left(\frac{c^9x^9}{9}+\frac{2c^7x^7}{7}+\frac{c^5x^5}{5}\right)+d^2b\left(\frac{\mathrm{Arcsinh}(cx)c^9x^9}{9}+\frac{2\,\mathrm{Arcsinh}(cx)c^7x^7}{7}+\frac{\mathrm{Arcsinh}(cx)c^5x^5}{5}-\frac{c^8x^8}{81}\sqrt{c^2x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^5*(d^2*a*(1/9*c^9*x^9+2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b*(1/9*arcsinh(c*x)*
c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^2
*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^2
+1)^(1/2)+1052/99225*c^2*x^2*(c^2*x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2)
))
```

Maxima [B] time = 1.18542, size = 431, normalized size = 2.38

$$\frac{1}{9}\,ac^4d^2x^9 + \frac{2}{7}\,ac^2d^2x^7 + \frac{1}{2835}\left(315\,x^9\,\mathrm{arsinh}\,(cx) - \left(\frac{35\,\sqrt{c^2x^2+1}x^8}{c^2} - \frac{40\,\sqrt{c^2x^2+1}x^6}{c^4} + \frac{48\,\sqrt{c^2x^2+1}x^4}{c^6} - \frac{64\,\sqrt{c^2x^2+}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 + 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2
+ 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 -
16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqr
t(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/
c^6)*c)*b*d^2
```

Fricas [A] time = 2.34645, size = 387, normalized size = 2.14

$$\frac{11025\,ac^9d^2x^9 + 28350\,ac^7d^2x^7 + 19845\,ac^5d^2x^5 + 315\left(35\,bc^9d^2x^9 + 90\,bc^7d^2x^7 + 63\,bc^5d^2x^5\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - (1}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 + 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 + 1)) - (1225*b*c^8*d^2*x^8 + 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 - 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 24.1854, size = 230, normalized size = 1.27

$$\begin{cases} \frac{ac^4d^2x^9}{9} + \frac{2ac^2d^2x^7}{7} + \frac{ad^2x^5}{5} + \frac{bc^4d^2x^9 \operatorname{asinh}(cx)}{9} - \frac{bc^3d^2x^8\sqrt{c^2x^2+1}}{81} + \frac{2bc^2d^2x^7 \operatorname{asinh}(cx)}{7} - \frac{106bcd^2x^6\sqrt{c^2x^2+1}}{3969} + \frac{bd^2x^5 \operatorname{asinh}(cx)}{5} - \frac{263bd^2x}{33} \\ \frac{ad^2x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 + 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*asinh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 + 1)/81 + 2*b*c
**2*d**2*x**7*asinh(c*x)/7 - 106*b*c*d**2*x**6*sqrt(c**2*x**2 + 1)/3969 + b
*d**2*x**5*asinh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 + 1)/(33075*c) + 1
052*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 + 1)/(99225*c**5), Ne(c, 0)), (a*d**2*x**5/5, True))
```

Giac [A] time = 1.6058, size = 402, normalized size = 2.22

$$\frac{1}{9}ac^4d^2x^9 + \frac{2}{7}ac^2d^2x^7 + \frac{1}{2835}\left(315x^9\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{35\left(c^2x^2+1\right)^{\frac{9}{2}} - 180\left(c^2x^2+1\right)^{\frac{7}{2}} + 378\left(c^2x^2+1\right)^{\frac{5}{2}} - 42}{c^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/9*a*c^4*d^2*x^9 + 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*log(c*x + sqrt(c^2*
x^2 + 1)) - (35*(c^2*x^2 + 1)^(9/2) - 180*(c^2*x^2 + 1)^(7/2) + 378*(c^2*x^
2 + 1)^(5/2) - 420*(c^2*x^2 + 1)^(3/2) + 315*sqrt(c^2*x^2 + 1))/c^9)*b*c^4*
d^2 + 1/5*a*d^2*x^5 + 2/245*(35*x^7*log(c*x + sqrt(c^2*x^2 + 1)) - (5*(c^2*
x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1)^(3/2) - 35*sqrt(
c^2*x^2 + 1))/c^7)*b*c^2*d^2 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) -
(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5
)*b*d^2
```

3.11 $\int x^3 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=180

$$\frac{1}{8} c^4 d^2 x^8 \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{3} c^2 d^2 x^6 \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{4} d^2 x^4 \left(a+b \sinh^{-1}(cx)\right)-\frac{1}{64} b c^3 d^2 x^7 \sqrt{c^2 x^2+1}-\frac{43 b c d^2 x^5 \sqrt{}}{115}$$

```
[Out] (73*b*d^2*x*Sqrt[1 + c^2*x^2])/(3072*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]
)/(4608*c) - (43*b*c*d^2*x^5*Sqrt[1 + c^2*x^2])/1152 - (b*c^3*d^2*x^7*Sqrt[
1 + c^2*x^2])/64 - (73*b*d^2*ArcSinh[c*x])/(3072*c^4) + (d^2*x^4*(a + b*Arc
Sinh[c*x]))/4 + (c^2*d^2*x^6*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^8*(a + b*
ArcSinh[c*x]))/8
```

Rubi [A] time = 0.17493, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 43, 5730, 12, 1267, 459, 321, 215}

$$\frac{1}{8} c^4 d^2 x^8 \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{3} c^2 d^2 x^6 \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{4} d^2 x^4 \left(a+b \sinh^{-1}(cx)\right)-\frac{1}{64} b c^3 d^2 x^7 \sqrt{c^2 x^2+1}-\frac{43 b c d^2 x^5 \sqrt{}}{115}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (73*b*d^2*x*Sqrt[1 + c^2*x^2])/(3072*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]
)/(4608*c) - (43*b*c*d^2*x^5*Sqrt[1 + c^2*x^2])/1152 - (b*c^3*d^2*x^7*Sqrt[
1 + c^2*x^2])/64 - (73*b*d^2*ArcSinh[c*x])/(3072*c^4) + (d^2*x^4*(a + b*Arc
Sinh[c*x]))/4 + (c^2*d^2*x^6*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^8*(a + b*
ArcSinh[c*x]))/8
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8\left(a+b\sinh\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8\left(a+b\sinh\right.$$

$$= -\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^2d^2x^6\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+\frac{1}{4}d^2x^4\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c$$

$$= -\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1+c^2x^2}+\frac{1}{4}d^2x^4\left(a\right.$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1}$$

$$= \frac{73bd^2x\sqrt{1+c^2x^2}}{3072c^3}-\frac{73bd^2x^3\sqrt{1+c^2x^2}}{4608c}-\frac{43bcd^2x^5\sqrt{1+c^2x^2}}{1152}-\frac{1}{64}bc^3d^2x^7\sqrt{1}$$

Mathematica [A] time = 0.0821231, size = 115, normalized size = 0.64

$$\frac{d^2\left(384ac^4x^4\left(3c^4x^4+8c^2x^2+6\right)-bcx\sqrt{c^2x^2+1}\left(144c^6x^6+344c^4x^4+146c^2x^2-219\right)+3b\left(384c^8x^8+1024c^6x^6+768\right.\right.}{9216c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(384*a*c^4*x^4*(6 + 8*c^2*x^2 + 3*c^4*x^4) - b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6) + 3*b*(-73 + 768*c^4*x^4 +
1024*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]))/(9216*c^4)
```

Maple [A] time = 0.009, size = 156, normalized size = 0.9

$$\frac{1}{c^4}\left(d^2a\left(\frac{c^8x^8}{8}+\frac{c^6x^6}{3}+\frac{c^4x^4}{4}\right)+d^2b\left(\frac{\mathrm{Arcsinh}(cx)c^8x^8}{8}+\frac{\mathrm{Arcsinh}(cx)c^6x^6}{3}+\frac{\mathrm{Arcsinh}(cx)c^4x^4}{4}-\frac{c^7x^7}{64}\sqrt{c^2x^2+1}-\frac{43}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^4*(d^2*a*(1/8*c^8*x^8+1/3*c^6*x^6+1/4*c^4*x^4)+d^2*b*(1/8*arcsinh(c*x)*
c^8*x^8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/64*c^7*x^7*(c^2
*x^2+1)^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*c^3*x^3*(c^2*x^2+1)
^(1/2)+73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x)))
```

Maxima [B] time = 1.21305, size = 443, normalized size = 2.46

$$\frac{1}{8}ac^4d^2x^8+\frac{1}{3}ac^2d^2x^6+\frac{1}{3072}\left(384x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^2*x^8 + 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(
sqrt(c^2)*c^8))*c)*b*c^4*d^2 + 1/4*a*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) -
 (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*
x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^2*d^2
+ 1/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2
+ 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d^2
```

Fricas [A] time = 2.15863, size = 373, normalized size = 2.07

$$\frac{1152\,ac^8d^2x^8+3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\left(384\,bc^8d^2x^8+1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2\right)\log\left(cx+\sqrt{c^2x^2}\right.}{9216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*d^2*x^8 + 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(3
84*b*c^8*d^2*x^8 + 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c
```

```
*x + sqrt(c^2*x^2 + 1)) - (144*b*c^7*d^2*x^7 + 344*b*c^5*d^2*x^5 + 146*b*c^
3*d^2*x^3 - 219*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 16.4422, size = 218, normalized size = 1.21

$$\begin{cases} \frac{ac^4d^2x^8}{8} + \frac{ac^2d^2x^6}{3} + \frac{ad^2x^4}{4} + \frac{bc^4d^2x^8 \operatorname{asinh}(cx)}{8} - \frac{bc^3d^2x^7\sqrt{c^2x^2+1}}{64} + \frac{bc^2d^2x^6 \operatorname{asinh}(cx)}{3} - \frac{43bcd^2x^5\sqrt{c^2x^2+1}}{1152} + \frac{bd^2x^4 \operatorname{asinh}(cx)}{4} - \frac{73bd^2x^3\sqrt{c^2x^2}}{4608c} \\ \frac{ad^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**8/8 + a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4
*d**2*x**8*asinh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)/64 + b*c**2*
d**2*x**6*asinh(c*x)/3 - 43*b*c*d**2*x**5*sqrt(c**2*x**2 + 1)/1152 + b*d**2
*x**4*asinh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 + 1)/(4608*c) + 73*b*d**
2*x*sqrt(c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*asinh(c*x)/(3072*c**4), Ne(
c, 0)), (a*d**2*x**4/4, True))
```

Giac [B] time = 1.79885, size = 451, normalized size = 2.51

$$\frac{1}{8}\,ac^4d^2x^8 + \frac{1}{3}\,ac^2d^2x^6 + \frac{1}{3072}\left(384\,x^8\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(\sqrt{c^2x^2+1}\left(2\left(4\,x^2\left(\frac{6\,x^2}{c^2} - \frac{7}{c^4}\right) + \frac{35}{c^6}\right)x^2 - \frac{105}{c^8}\right)x - \frac{105\,\mathrm{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*a*c^4*d^2*x^8 + 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*log(c*x + sqrt(c^2*
x^2 + 1)) - (sqrt(c^2*x^2 + 1)*(2*(4*x^2*(6*x^2/c^2 - 7/c^4) + 35/c^6)*x^2
- 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^8*abs(c)))*c)
*b*c^4*d^2 + 1/4*a*d^2*x^4 + 1/144*(48*x^6*log(c*x + sqrt(c^2*x^2 + 1)) - (
sqrt(c^2*x^2 + 1)*(2*x^2*(4*x^2/c^2 - 5/c^4) + 15/c^6)*x + 15*log(abs(-x*ab
s(c) + sqrt(c^2*x^2 + 1)))/(c^6*abs(c)))*c)*b*c^2*d^2 + 1/32*(8*x^4*log(c*x
 + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*x*(2*x^2/c^2 - 3/c^4) - 3*log(ab
s(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4*abs(c)))*c)*b*d^2
```

3.12 $\int x^2 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=157

$$\frac{1}{7}c^4d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{7/2}}{49c^3}+\frac{bd^2\left(c^2x^2+1\right.}{175c^3}$$

```
[Out] (8*b*d^2*Sqrt[1 + c^2*x^2])/(105*c^3) + (4*b*d^2*(1 + c^2*x^2)^(3/2))/(315*
c^3) + (b*d^2*(1 + c^2*x^2)^(5/2))/(175*c^3) - (b*d^2*(1 + c^2*x^2)^(7/2))/
(49*c^3) + (d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (2*c^2*d^2*x^5*(a + b*ArcSinh
[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcSinh[c*x]))/7
```

Rubi [A] time = 0.168962, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {270, 5730, 12, 1251, 771}

$$\frac{1}{7}c^4d^2x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{7/2}}{49c^3}+\frac{bd^2\left(c^2x^2+1\right.}{175c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (8*b*d^2*Sqrt[1 + c^2*x^2])/(105*c^3) + (4*b*d^2*(1 + c^2*x^2)^(3/2))/(315*
c^3) + (b*d^2*(1 + c^2*x^2)^(5/2))/(175*c^3) - (b*d^2*(1 + c^2*x^2)^(7/2))/
(49*c^3) + (d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (2*c^2*d^2*x^5*(a + b*ArcSinh
[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcSinh[c*x]))/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\sinh\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\sinh\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\sinh\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{5}c^2d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7\left(a+b\sinh\right.$$

$$=\frac{8bd^2\sqrt{1+c^2x^2}}{105c^3}+\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{315c^3}+\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{175c^3}-\frac{bd^2\left(1+c^2x^2\right)^{7/2}}{49c^3}+$$

Mathematica [A] time = 0.0851899, size = 111, normalized size = 0.71

$$\frac{d^2\left(105ac^3x^3\left(15c^4x^4+42c^2x^2+35\right)-b\sqrt{c^2x^2+1}\left(225c^6x^6+612c^4x^4+409c^2x^2-818\right)+105bc^3x^3\left(15c^4x^4+42c^2x^2+\right.\right.}{11025c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(-
818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 + 42*c^2
*x^2 + 15*c^4*x^4)*ArcSinh[c*x]))/(11025*c^3)
```

Maple [A] time = 0.003, size = 148, normalized size = 0.9

$$\frac{1}{c^3}\left(d^2a\left(\frac{c^7x^7}{7}+\frac{2\,c^5x^5}{5}+\frac{c^3x^3}{3}\right)+d^2b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^7x^7}{7}+\frac{2\,\mathrm{Arcsinh}\,(cx)\,c^5x^5}{5}+\frac{\mathrm{Arcsinh}\,(cx)\,c^3x^3}{3}-\frac{c^6x^6}{49}\sqrt{c^2x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^3*(d^2*a*(1/7*c^7*x^7+2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b*(1/7*arcsinh(c*x)*
c^7*x^7+2/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/49*c^6*x^6*(c^2
*x^2+1)^(1/2)-68/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/11025*c^2*x^2*(c^2*x^2+
1)^(1/2)+818/11025*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.18923, size = 352, normalized size = 2.24

$$\frac{1}{7}\,ac^4d^2x^7+\frac{2}{5}\,ac^2d^2x^5+\frac{1}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 + 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^2 + 2/75*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2
```

Fricas [A] time = 2.43742, size = 351, normalized size = 2.24

$$\frac{1575\,ac^7d^2x^7 + 4410\,ac^5d^2x^5 + 3675\,ac^3d^2x^3 + 105\left(15\,bc^7d^2x^7 + 42\,bc^5d^2x^5 + 35\,bc^3d^2x^3\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(225\,b\right.}{11025\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 + 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 + 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 + 1)) - (225*b*c^6*d^2*x^6 + 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 -
818*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 9.19659, size = 202, normalized size = 1.29

$$\begin{cases} \frac{ac^4d^2x^7}{7} + \frac{2ac^2d^2x^5}{5} + \frac{ad^2x^3}{3} + \frac{bc^4d^2x^7\operatorname{asinh}(cx)}{7} - \frac{bc^3d^2x^6\sqrt{c^2x^2+1}}{49} + \frac{2bc^2d^2x^5\operatorname{asinh}(cx)}{5} - \frac{68bcd^2x^4\sqrt{c^2x^2+1}}{1225} + \frac{bd^2x^3\operatorname{asinh}(cx)}{3} - \frac{409bd^2x^2\sqrt{c}}{11025} \\ \frac{ad^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 + 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*asinh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 + 1)/49 + 2*b*c
**2*d**2*x**5*asinh(c*x)/5 - 68*b*c*d**2*x**4*sqrt(c**2*x**2 + 1)/1225 + b*
d**2*x**3*asinh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(11025*c) + 81
8*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (a*d**2*x**3/3, True)
)
```

Giac [A] time = 1.71359, size = 347, normalized size = 2.21

$$\frac{1}{7}\,ac^4d^2x^7 + \frac{2}{5}\,ac^2d^2x^5 + \frac{1}{245}\left(35\,x^7\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{5\left(c^2x^2+1\right)^{\frac{7}{2}} - 21\left(c^2x^2+1\right)^{\frac{5}{2}} + 35\left(c^2x^2+1\right)^{\frac{3}{2}} - 35\sqrt{c^2x^2}}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/7*a*c^4*d^2*x^7 + 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*log(c*x + sqrt(c^2*x^
2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1
)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*b*c^4*d^2 + 2/75*(15*x^5*log(c*x + sqr
t(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt
(c^2*x^2 + 1))/c^5)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c
^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*d^2
```

3.13 $\int x \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=120

$$\frac{d^2 \left(c^2 x^2+1\right)^3 \left(a+b \sinh^{-1}(cx)\right)}{6 c^2}-\frac{b d^2 x \left(c^2 x^2+1\right)^{5/2}}{36 c}-\frac{5 b d^2 x \left(c^2 x^2+1\right)^{3/2}}{144 c}-\frac{5 b d^2 x \sqrt{c^2 x^2+1}}{96 c}-\frac{5 b d^2 \sinh^{-1}(cx)}{96 c^2}$$

```
[Out] (-5*b*d^2*x*Sqrt[1 + c^2*x^2])/(96*c) - (5*b*d^2*x*(1 + c^2*x^2)^(3/2))/(14
4*c) - (b*d^2*x*(1 + c^2*x^2)^(5/2))/(36*c) - (5*b*d^2*ArcSinh[c*x])/(96*c^
2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2)
```

Rubi [A] time = 0.0646363, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5717, 195, 215}

$$\frac{d^2 \left(c^2 x^2+1\right)^3 \left(a+b \sinh^{-1}(cx)\right)}{6 c^2}-\frac{b d^2 x \left(c^2 x^2+1\right)^{5/2}}{36 c}-\frac{5 b d^2 x \left(c^2 x^2+1\right)^{3/2}}{144 c}-\frac{5 b d^2 x \sqrt{c^2 x^2+1}}{96 c}-\frac{5 b d^2 \sinh^{-1}(cx)}{96 c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[1 + c^2*x^2])/(96*c) - (5*b*d^2*x*(1 + c^2*x^2)^(3/2))/(14
4*c) - (b*d^2*x*(1 + c^2*x^2)^(5/2))/(36*c) - (5*b*d^2*ArcSinh[c*x])/(96*c^
2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(6*c^2)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)dx=\frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{\left(bd^2\right)\int\left(1+c^2x^2\right)^{5/2}dx}{6c}$$
$$=-\frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c}+\frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}-\frac{\left(5bd^2\right)\int\left(1+c^2x^2\right.}{36c}$$
$$=-\frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{144c}-\frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c}+\frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{6c^2}$$
$$=-\frac{5bd^2x\sqrt{1+c^2x^2}}{96c}-\frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{144c}-\frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c}+\frac{d^2\left(1+c^2x^2\right)^3}{}$$
$$=-\frac{5bd^2x\sqrt{1+c^2x^2}}{96c}-\frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{144c}-\frac{bd^2x\left(1+c^2x^2\right)^{5/2}}{36c}-\frac{5bd^2\sinh^{-1}(c}{96c^2}$$

Mathematica [A] time = 0.132565, size = 104, normalized size = 0.87

$$\frac{d^2\left(cx\left(48acx\left(c^4x^4+3c^2x^2+3\right)-b\sqrt{c^2x^2+1}\left(8c^4x^4+26c^2x^2+33\right)\right)+3b\left(16c^6x^6+48c^4x^4+48c^2x^2+11\right)\sinh^{-1}(cx\right.}{288c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(c*x*(48*a*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(33 + 2
6*c^2*x^2 + 8*c^4*x^4)) + 3*b*(11 + 48*c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6)*A
rcSinh[c*x]))/(288*c^2)
```

Maple [A] time = 0.005, size = 137, normalized size = 1.1

$$\frac{1}{c^2}\left(d^2a\left(\frac{c^6x^6}{6}+\frac{c^4x^4}{2}+\frac{c^2x^2}{2}\right)+d^2b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^6x^6}{6}+\frac{\mathrm{Arcsinh}\,(cx)\,c^4x^4}{2}+\frac{\mathrm{Arcsinh}\,(cx)\,c^2x^2}{2}-\frac{c^5x^5}{36}\sqrt{c^2x^2+1}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^2*(d^2*a*(1/6*c^6*x^6+1/2*c^4*x^4+1/2*c^2*x^2)+d^2*b*(1/6*arcsinh(c*x)*
c^6*x^6+1/2*arcsinh(c*x)*c^4*x^4+1/2*arcsinh(c*x)*c^2*x^2-1/36*c^5*x^5*(c^2
*x^2+1)^(1/2)-13/144*c^3*x^3*(c^2*x^2+1)^(1/2)-11/96*c*x*(c^2*x^2+1)^(1/2)+
11/96*arcsinh(c*x)))
```

Maxima [B] time = 1.05321, size = 365, normalized size = 3.04

$$\frac{1}{6}\,ac^4d^2x^6+\frac{1}{2}\,ac^2d^2x^4+\frac{1}{288}\left(48\,x^6\operatorname{arsinh}(cx)-\left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\,\sqrt{c^2x^2+1}x^3}{c^4}+\frac{15\,\sqrt{c^2x^2+1}x}{c^6}-\frac{15\,\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{c^2}c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 + 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arcsinh(c*x) - (8*sqr
t(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1
)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^4*d^2 + 1/16*
(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/
c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*c^2*d^2 + 1/2*a*d^2*
x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c^2*x/
sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d^2
```

Fricas [A] time = 2.32668, size = 328, normalized size = 2.73

$$\frac{48\,ac^6d^2x^6+144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\left(16\,bc^6d^2x^6+48\,bc^4d^2x^4+48\,bc^2d^2x^2+11\,bd^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(8\,bc\right.}{288\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 + 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 + 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 + 11*b*d^2)*log(c*x + sqrt
```

```
(c^2*x^2 + 1)) - (8*b*c^5*d^2*x^5 + 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))/c^2
```

Sympy [A] time = 5.81135, size = 190, normalized size = 1.58

$$\begin{cases} \frac{ac^4d^2x^6}{6} + \frac{ac^2d^2x^4}{2} + \frac{ad^2x^2}{2} + \frac{bc^4d^2x^6\operatorname{asinh}(cx)}{6} - \frac{bc^3d^2x^5\sqrt{c^2x^2+1}}{36} + \frac{bc^2d^2x^4\operatorname{asinh}(cx)}{2} - \frac{13bcd^2x^3\sqrt{c^2x^2+1}}{144} + \frac{bd^2x^2\operatorname{asinh}(cx)}{2} - \frac{11bd^2x\sqrt{c^2x}}{96c} \\ \frac{ad^2x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 + a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*asinh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)/36 + b*c**2*
d**2*x**4*asinh(c*x)/2 - 13*b*c*d**2*x**3*sqrt(c**2*x**2 + 1)/144 + b*d**2*
x**2*asinh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 + 1)/(96*c) + 11*b*d**2*asin
h(c*x)/(96*c**2), Ne(c, 0)), (a*d**2*x**2/2, True))
```

Giac [B] time = 1.77561, size = 405, normalized size = 3.38

$$\frac{1}{6}ac^4d^2x^6 + \frac{1}{2}ac^2d^2x^4 + \frac{1}{288}\left(48x^6\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(\sqrt{c^2x^2+1}\left(2x^2\left(\frac{4x^2}{c^2} - \frac{5}{c^4}\right) + \frac{15}{c^6}\right)x + \frac{15\log\left(\left|-x|c| + \sqrt{c^2}\right.\right.}{c^6|c|}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*a*c^4*d^2*x^6 + 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*log(c*x + sqrt(c^2*x^
2 + 1)) - (sqrt(c^2*x^2 + 1)*(2*x^2*(4*x^2/c^2 - 5/c^4) + 15/c^6)*x + 15*lo
g(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^6*abs(c)))*c)*b*c^4*d^2 + 1/16*(8*
x^4*log(c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*x*(2*x^2/c^2 - 3/c^4)
 - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4*abs(c)))*c)*b*c^2*d^2 + 1
/2*a*d^2*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 +
1)*x/c^2 + log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^2*abs(c))))*b*d^2
```

3.14 $\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=128

$$\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{5/2}}{25c}-\frac{4bd^2\left(c^2x^2+1\right)^{3/}}{45c}$$

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(15*c) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(45*c)
- (b*d^2*(1 + c^2*x^2)^(5/2))/(25*c) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c^2*
d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rubi [A] time = 0.101974, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {194, 5679, 12, 1247, 698}

$$\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{bd^2\left(c^2x^2+1\right)^{5/2}}{25c}-\frac{4bd^2\left(c^2x^2+1\right)^{3/}}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-8*b*d^2*Sqrt[1 + c^2*x^2])/(15*c) - (4*b*d^2*(1 + c^2*x^2)^(3/2))/(45*c)
- (b*d^2*(1 + c^2*x^2)^(5/2))/(25*c) + d^2*x*(a + b*ArcSinh[c*x]) + (2*c^2*
d^2*x^3*(a + b*ArcSinh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)dx = d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(c\right.$$
$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(c\right.$$
$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(c\right.$$
$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}c^2d^2x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\sinh^{-1}(c\right.$$
$$= -\frac{8bd^2\sqrt{1+c^2x^2}}{15c}-\frac{4bd^2\left(1+c^2x^2\right)^{3/2}}{45c}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}}{25c}+d^2x\left(a+b\sinh^{-1}(c\right.$$

Mathematica [A] time = 0.120046, size = 95, normalized size = 0.74

$$\frac{d^2\left(15acx\left(3c^4x^4+10c^2x^2+15\right)-b\sqrt{c^2x^2+1}\left(9c^4x^4+38c^2x^2+149\right)+15bcx\left(3c^4x^4+10c^2x^2+15\right)\sinh^{-1}(cx)\right)}{225c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 + c^2*x^2]*(149 + 3
8*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x
]))/(225*c)
```

Maple [A] time = 0.005, size = 119, normalized size = 0.9

$$\frac{1}{c}\left(d^2a\left(\frac{c^5x^5}{5}+\frac{2\,c^3x^3}{3}+cx\right)+d^2b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^5x^5}{5}+\frac{2\,\mathrm{Arcsinh}\,(cx)\,c^3x^3}{3}+\mathrm{Arcsinh}\,(cx)\,cx-\frac{c^4x^4}{25}\sqrt{c^2x^2+1}-\frac{38\,c^2}{225}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(d^2*a*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b*(1/5*arcsinh(c*x)*c^5*x^5+2/
3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-38/2
25*c^2*x^2*(c^2*x^2+1)^(1/2)-149/225*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.19368, size = 262, normalized size = 2.05

$$\frac{1}{5}\,ac^4d^2x^5+\frac{1}{75}\left(15\,x^5\,\mathrm{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^4d^2+\frac{2}{3}\,ac^2d^2x^3+\frac{2}{9}\left(3\,x^3\,\mathrm{arsin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 + 2
/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c
^2*x^2 + 1))*b*d^2/c
```

Fricas [A] time = 2.43673, size = 296, normalized size = 2.31

$$\frac{45\,ac^5d^2x^5+150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5+10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(9\,bc^4d^2x^4+38\,b\right.}{225\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 + 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 + 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (9
*b*c^4*d^2*x^4 + 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] time = 2.86785, size = 165, normalized size = 1.29

$$\begin{cases} \frac{ac^4d^2x^5}{5} + \frac{2ac^2d^2x^3}{3} + ad^2x + \frac{bc^4d^2x^5 \operatorname{asinh}(cx)}{5} - \frac{bc^3d^2x^4\sqrt{c^2x^2+1}}{25} + \frac{2bc^2d^2x^3 \operatorname{asinh}(cx)}{3} - \frac{38bcd^2x^2\sqrt{c^2x^2+1}}{225} + bd^2x \operatorname{asinh}(cx) - \frac{149bd^2}{2 \\ ad^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**5/5 + 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d*
*2*x**5*asinh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)/25 + 2*b*c**2*d
**2*x**3*asinh(c*x)/3 - 38*b*c*d**2*x**2*sqrt(c**2*x**2 + 1)/225 + b*d**2*x
*asinh(c*x) - 149*b*d**2*sqrt(c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**2*x,
 True))
```

Giac [A] time = 1.56878, size = 281, normalized size = 2.2

$$\frac{1}{5}ac^4d^2x^5 + \frac{1}{75}\left(15\,x^5\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{3\left(c^2x^2+1\right)^{\frac{5}{2}} - 10\left(c^2x^2+1\right)^{\frac{3}{2}} + 15\sqrt{c^2x^2+1}}{c^5}\right)bc^4d^2 + \frac{2}{3}ac^2d^2x^3 + \frac{2}{9}\Big(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2
 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*b*c^4*d^2
 + 2/3*a*c^2*d^2*x^3 + 2/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2
+ 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*c^2*d^2 + (x*log(c*x + sqrt(c^2*x^
2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^2 + a*d^2*x
```

3.15 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{2}bd^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+\frac{1}{4}d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{d^2\left(a+b\sin\right.}{2b}$$

```
[Out] (-11*b*c*d^2*x*Sqrt[1 + c^2*x^2])/32 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2))/16 -
 (11*b*d^2*ArcSinh[c*x])/32 + (d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 +
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 + (d^2*(a + b*ArcSinh[c*x])^2)
/(2*b) + d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (b*d^2*Pol
yLog[2, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.202739, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5726, 5659, 3716, 2190, 2279, 2391, 195, 215}

$$\frac{1}{2}bd^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{4}d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\sinh^{-}\right.}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-11*b*c*d^2*x*Sqrt[1 + c^2*x^2])/32 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2))/16 -
 (11*b*d^2*ArcSinh[c*x])/32 + (d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 +
(d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/4 - (d^2*(a + b*ArcSinh[c*x])^2)
/(2*b) + d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (b*d^2*Poly
Log[2, E^(2*ArcSinh[c*x])])/2
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx-\frac{1}{4}$$

$$= -\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a\right.$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}+\frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}d^2\left(1+c^2x^2\right.$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}d^2\left(1+c^2x^2\right.$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}d^2\left(1+c^2x^2\right.$$

$$= -\frac{11}{32}bcd^2x\sqrt{1+c^2x^2}-\frac{1}{16}bcd^2x\left(1+c^2x^2\right)^{3/2}-\frac{11}{32}bd^2\sinh^{-1}(cx)+\frac{1}{2}d^2\left(1+c^2x^2\right.$$

Mathematica [A] time = 0.200318, size = 173, normalized size = 1.01

$$\frac{d^2\left(16b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-16a^2+8abc^4x^4+32abc^2x^2+b\sinh^{-1}(cx)\left(-32a+b\left(8c^4x^4+32c^2x^2+13\right)+32b\log\right.\right.}{32b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^2*(-16*a^2 + 24*a*b + 32*a*b*c^2*x^2 + 8*a*b*c^4*x^4 - 13*b^2*c*x*Sqrt[1
 + c^2*x^2] - 2*b^2*c^3*x^3*Sqrt[1 + c^2*x^2] - 16*b^2*ArcSinh[c*x]^2 + 32*
a*b*Log[1 - E^(2*ArcSinh[c*x])] + b*ArcSinh[c*x]*(-32*a + b*(13 + 32*c^2*x^
2 + 8*c^4*x^4) + 32*b*Log[1 - E^(2*ArcSinh[c*x])]) + 16*b^2*PolyLog[2, E^(2
*ArcSinh[c*x])]))/(32*b)
```

Maple [A] time = 0.116, size = 231, normalized size = 1.3

$$\frac{d^2ac^4x^4}{4}+d^2ac^2x^2+d^2a\ln(cx)+\frac{d^2b\text{Arcsinh}(cx)\,c^4x^4}{4}+d^2b\text{Arcsinh}(cx)\,c^2x^2+\frac{13\,bd^2\text{Arcsinh}(cx)}{32}-\frac{d^2bc^3x^3}{16}\sqrt{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/4*d^2*a*c^4*x^4+d^2*a*c^2*x^2+d^2*a*ln(c*x)+1/4*d^2*b*arcsinh(c*x)*c^4*x^
4+d^2*b*arcsinh(c*x)*c^2*x^2+13/32*b*d^2*arcsinh(c*x)-1/16*d^2*b*c^3*x^3*(c
^2*x^2+1)^(1/2)-13/32*b*c*d^2*x*(c^2*x^2+1)^(1/2)+d^2*b*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))+d^2*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*d^
2*b*arcsinh(c*x)^2+d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+d^2*b*polylog(2,
c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}ac^4d^2x^4 + ac^2d^2x^2 + ad^2\log(x) + \int bc^4d^2x^3\log\left(cx + \sqrt{c^2x^2+1}\right) + 2\,bc^2d^2x\log\left(cx + \sqrt{c^2x^2+1}\right) + \frac{bd^2\log\left(cx + \sqrt{}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^2*x^4 + a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*b*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) +
 b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\left(cx\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a}{x}\,dx + \int 2ac^2x\,dx + \int ac^4x^3\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 2bc^2x\,\mathrm{asinh}\,(cx)\,dx + \int bc^4x^3\,\mathrm{asinh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(2*a*c**2*x, x) + Integral(a*c**4*x**3, x)
 + Integral(b*asinh(c*x)/x, x) + Integral(2*b*c**2*x*asinh(c*x), x) + Integ
ral(b*c**4*x**3*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^2(b\,\mathrm{arsinh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)/x, x)
```

3.16 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx)\right)+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2\left(c^2x^2+1\right)^{3/2}-\frac{5}{3}bcd^2\sqrt{c^2x^2+1}$$

```
[Out] (-5*b*c*d^2*Sqrt[1 + c^2*x^2])/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/9 - (d^2*(
a + b*ArcSinh[c*x]))/x + 2*c^2*d^2*x*(a + b*ArcSinh[c*x]) + (c^4*d^2*x^3*(a
 + b*ArcSinh[c*x]))/3 - b*c*d^2*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rubi [A] time = 0.157219, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {270, 5730, 12, 1251, 897, 1153, 208}

$$\frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx)\right)+2c^2d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2\left(c^2x^2+1\right)^{3/2}-\frac{5}{3}bcd^2\sqrt{c^2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```


```
[Out] (-5*b*c*d^2*Sqrt[1 + c^2*x^2])/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/9 - (d^2*(
a + b*ArcSinh[c*x]))/x + 2*c^2*d^2*x*(a + b*ArcSinh[c*x]) + (c^4*d^2*x^3*(a
 + b*ArcSinh[c*x]))/3 - b*c*d^2*ArcTanh[Sqrt[1 + c^2*x^2]]
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{3}c^4d^2x^3\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2} - \frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2} - \frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+\right.$$

$$= -\frac{5}{3}bcd^2\sqrt{1+c^2x^2} - \frac{1}{9}bcd^2\left(1+c^2x^2\right)^{3/2} - \frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{x} + 2c^2d^2x\left(a+\right.$$

Mathematica [A] time = 0.123593, size = 124, normalized size = 1.03

$$\frac{d^2\left(3ac^4x^4+18ac^2x^2-9a-bc^3x^3\sqrt{c^2x^2+1}-16bcx\sqrt{c^2x^2+1}-9bcx\log\left(\sqrt{c^2x^2+1}+1\right)+3b\left(c^4x^4+6c^2x^2-3\right)\sinh\right.}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a + 18*a*c^2*x^2 + 3*a*c^4*x^4 - 16*b*c*x*Sqrt[1 + c^2*x^2] - b*c^
3*x^3*Sqrt[1 + c^2*x^2] + 3*b*(-3 + 6*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b
*c*x*Log[x] - 9*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(9*x)
```

Maple [A] time = 0.009, size = 114, normalized size = 1.

$$c\left(d^2a\left(\frac{c^3x^3}{3}+2\,cx-\frac{1}{cx}\right)+d^2b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^3x^3}{3}+2\,\mathrm{Arcsinh}\,(cx)\,cx-\frac{\mathrm{Arcsinh}\,(cx)}{cx}-\frac{c^2x^2}{9}\sqrt{c^2x^2+1}-\frac{16}{9}\sqrt{c^2x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] c*(d^2*a*(1/3*c^3*x^3+2*c*x-1/c/x)+d^2*b*(1/3*arcsinh(c*x)*c^3*x^3+2*arcsin
h(c*x)*c*x-arcsinh(c*x)/c/x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-16/9*(c^2*x^2+1)^
(1/2)-arctanh(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.17115, size = 196, normalized size = 1.63

$$\frac{1}{3}\,ac^4d^2x^3 + \frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^4d^2 + 2\,ac^2d^2x + 2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bcd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^4*d^2 + 2*a*c^2*d^2*x + 2*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c*d^2 - (c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcsinh(
c*x)/x)*b*d^2 - a*d^2/x
```

Fricas [B] time = 2.63919, size = 505, normalized size = 4.21

$$\frac{3\,ac^4d^2x^4 + 18\,ac^2d^2x^2 - 9\,bcd^2x\log\left(-cx + \sqrt{c^2x^2+1}+1\right) + 9\,bcd^2x\log\left(-cx + \sqrt{c^2x^2+1}-1\right) - 3\left(bc^4 + 6\,bc^2 - 3\,b\right)d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 + 18*a*c^2*d^2*x^2 - 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x
^2 + 1) + 1) + 9*b*c*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 3*(b*c^4 + 6
*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 + 1)) - 9*a*d^2 + 3*(b*c^4*d^2*
x^4 + 6*b*c^2*d^2*x^2 - (b*c^4 + 6*b*c^2 - 3*b)*d^2*x - 3*b*d^2)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^3*d^2*x^3 + 16*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int 2ac^2\,dx+\int \frac{a}{x^2}\,dx+\int ac^4x^2\,dx+\int 2bc^2\operatorname{asinh}(cx)\,dx+\int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx+\int bc^4x^2\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2, x
) + Integral(2*b*c**2*asinh(c*x), x) + Integral(b*asinh(c*x)/x**2, x) + Int
egral(b*c**4*x**2*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2(b\operatorname{arsinh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)/x^2, x)
```

3.17 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=187

$$-bc^2d^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{c^2d^2\left(a+b\sinh\right.}{b}$$

```
[Out] (b*c^3*d^2*x*Sqrt[1 + c^2*x^2])/4 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/(2*x) + (
b*c^2*d^2*ArcSinh[c*x])/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (d
^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(2*x^2) + (c^2*d^2*(a + b*ArcSinh[
c*x])^2)/b + 2*c^2*d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] -
b*c^2*d^2*PolyLog[2, E^(-2*ArcSinh[c*x])]
```

Rubi [A] time = 0.210942, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5728, 277, 195, 215, 5726, 5659, 3716, 2190, 2279, 2391}

$$bc^2d^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2d^2\left(a+b\sinh\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[1 + c^2*x^2])/4 - (b*c*d^2*(1 + c^2*x^2)^(3/2))/(2*x) + (
b*c^2*d^2*ArcSinh[c*x])/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (d
^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*d^2*(a + b*ArcSinh[
c*x])^2)/b + 2*c^2*d^2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b
*c^2*d^2*PolyLog[2, E^(2*ArcSinh[c*x])]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\left(2c^2d\right)\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x}+c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.}{2x^2}$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2}-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x}+c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{d^2}{}$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2}-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x}+\frac{1}{4}bc^2d^2\sinh^{-1}(cx)+c^2d^2\left(1+c^2x^2\right)\left(a\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2}-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x}+\frac{1}{4}bc^2d^2\sinh^{-1}(cx)+c^2d^2\left(1+c^2x^2\right)\left(a\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2}-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x}+\frac{1}{4}bc^2d^2\sinh^{-1}(cx)+c^2d^2\left(1+c^2x^2\right)\left(a\right.$$

$$= \frac{1}{4}bc^3d^2x\sqrt{1+c^2x^2}-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}}{2x}+\frac{1}{4}bc^2d^2\sinh^{-1}(cx)+c^2d^2\left(1+c^2x^2\right)\left(a\right.$$

Mathematica [A] time = 0.34301, size = 143, normalized size = 0.76

$$\frac{1}{4}d^2\left(4c^2\left(b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+2\log\left(1-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)\right)+2c^4x^2\left(a+b\sinh^{-1}(cx)\right)-\frac{4c^2\left(a+b\,\mathrm{s}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^2*((-2*b*c*Sqrt[1 + c^2*x^2])/x + b*c^2*(-(c*x*Sqrt[1 + c^2*x^2]) + ArcS
inh[c*x]) - (2*(a + b*ArcSinh[c*x]))/x^2 + 2*c^4*x^2*(a + b*ArcSinh[c*x]) -
 (4*c^2*(a + b*ArcSinh[c*x])^2)/b + 4*c^2*(2*(a + b*ArcSinh[c*x])*Log[1 - E
^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])])))/4
```

Maple [A] time = 0.244, size = 262, normalized size = 1.4

$$\frac{c^4d^2ax^2}{2}+2\,c^2d^2a\ln(cx)-\frac{d^2a}{2\,x^2}-c^2d^2b\,(\mathrm{Arcsinh}\,(cx))^2+\frac{c^4d^2b\mathrm{Arcsinh}\,(cx)\,x^2}{2}-\frac{bc^3d^2x}{4}\sqrt{c^2x^2+1}+\frac{bc^2d^2\mathrm{Arcsinh}\,(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] 1/2*c^4*d^2*a*x^2+2*c^2*d^2*a*ln(c*x)-1/2*d^2*a/x^2-c^2*d^2*b*arcsinh(c*x)^
2+1/2*c^4*d^2*b*arcsinh(c*x)*x^2-1/4*b*c^3*d^2*x*(c^2*x^2+1)^(1/2)+1/4*b*c^
2*d^2*arcsinh(c*x)+1/2*d^2*b*c^2-1/2*c*d^2*b/x*(c^2*x^2+1)^(1/2)-1/2*d^2*b*
arcsinh(c*x)/x^2+2*c^2*d^2*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*c^2
*d^2*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*c^2*d^2*b*arcsinh(c*x)*ln(1-c*x-
(c^2*x^2+1)^(1/2))+2*c^2*d^2*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,ac^4d^2x^2+2\,ac^2d^2\log(x)-\frac{1}{2}\,bd^2\left(\frac{\sqrt{c^2x^2+1}c}{x}+\frac{\mathrm{arsinh}\,(cx)}{x^2}\right)-\frac{ad^2}{2\,x^2}+\int bc^4d^2x\log\left(cx+\sqrt{c^2x^2+1}\right)+\frac{2\,bc^2d^2\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 + 2*a*c^2*d^2*log(x) - 1/2*b*d^2*(sqrt(c^2*x^2 + 1)*c/x +
 arcsinh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arsinh}\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{a}{x^3}\,dx + \int\frac{2ac^2}{x}\,dx + \int ac^4x\,dx + \int\frac{b\operatorname{asinh}\left(cx\right)}{x^3}\,dx + \int\frac{2bc^2\operatorname{asinh}\left(cx\right)}{x}\,dx + \int bc^4x\operatorname{asinh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c**2/x, x) + Integral(a*c**4*x, x)
 + Integral(b*asinh(c*x)/x**3, x) + Integral(2*b*c**2*asinh(c*x)/x, x) + In
tegral(b*c**4*x*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2dx^2 + d\right)^2\left(b\operatorname{arsinh}\left(cx\right) + a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)/x^3, x)
```

3.18 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=126

$$c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-bc^3d^2\sqrt{c^2x^2+1}-\frac{bcd^2\sqrt{c^2x^2+1}}{6x^2}-\frac{11}{6}bc$$

```
[Out] -(b*c^3*d^2*Sqrt[1 + c^2*x^2]) - (b*c*d^2*Sqrt[1 + c^2*x^2])/(6*x^2) - (d^2
*(a + b*ArcSinh[c*x]))/(3*x^3) - (2*c^2*d^2*(a + b*ArcSinh[c*x]))/x + c^4*d
^2*x*(a + b*ArcSinh[c*x]) - (11*b*c^3*d^2*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rubi [A] time = 0.158875, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5730, 12, 1251, 897, 1157, 388, 208}

$$c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-bc^3d^2\sqrt{c^2x^2+1}-\frac{bcd^2\sqrt{c^2x^2+1}}{6x^2}-\frac{11}{6}bc^3$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c^3*d^2*Sqrt[1 + c^2*x^2]) - (b*c*d^2*Sqrt[1 + c^2*x^2])/(6*x^2) - (d^2
*(a + b*ArcSinh[c*x]))/(3*x^3) - (2*c^2*d^2*(a + b*ArcSinh[c*x]))/x + c^4*d
^2*x*(a + b*ArcSinh[c*x]) - (11*b*c^3*d^2*ArcTanh[Sqrt[1 + c^2*x^2]])/6
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-$$

$$=-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-$$

$$=-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-$$

$$=-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\sinh^{-1}(cx)\right)-$$

$$=-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh^{-1}(cx)\right)}{x}+c^4d^2x\left(a\right.$$

$$=-bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh\right.}{x}$$

$$=-bc^3d^2\sqrt{1+c^2x^2}-\frac{bcd^2\sqrt{1+c^2x^2}}{6x^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{2c^2d^2\left(a+b\sinh\right.}{x}$$

Mathematica [A] time = 0.125521, size = 133, normalized size = 1.06

$$\frac{d^2\left(6ac^4x^4-12ac^2x^2-2a-6bc^3x^3\sqrt{c^2x^2+1}-bcx\sqrt{c^2x^2+1}+11bc^3x^3\log(x)-11bc^3x^3\log\left(\sqrt{c^2x^2+1}+1\right)+2b\left(3c^4\right.\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a - 12*a*c^2*x^2 + 6*a*c^4*x^4 - b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3
*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-1 - 6*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] + 11
*b*c^3*x^3*Log[x] - 11*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(6*x^3)
```

Maple [A] time = 0.012, size = 114, normalized size = 0.9

$$c^3\left(d^2a\left(cx-2\frac{1}{cx}-\frac{1}{3c^3x^3}\right)+d^2b\left(\mathrm{Arcsinh}\,(cx)\,cx-2\frac{\mathrm{Arcsinh}\,(cx)}{cx}-\frac{\mathrm{Arcsinh}\,(cx)}{3c^3x^3}-\sqrt{c^2x^2+1}-\frac{11}{6}\mathrm{Artanh}\left(\frac{1}{\sqrt{c^2x^2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] c^3*(d^2*a*(c*x-2/c/x-1/3/c^3/x^3)+d^2*b*(arcsinh(c*x)*c*x-2*arcsinh(c*x)/c
/x-1/3*arcsinh(c*x)/c^3/x^3-(c^2*x^2+1)^(1/2)-11/6*arctanh(1/(c^2*x^2+1)^(1
/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.12379, size = 190, normalized size = 1.51

$$ac^4d^2x + \left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bc^3d^2 - 2\left(c\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right) + \frac{\operatorname{arsinh}(cx)}{x}\right)bc^2d^2 + \frac{1}{6}\left(\left(c^2\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right) - \frac{\sqrt{c^2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*c^3*d^2 - 2*(c*arcsi
nh(1/(sqrt(c^2)*abs(x))) + arcsinh(c*x)/x)*b*c^2*d^2 + 1/6*((c^2*arcsinh(1/
(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d^2
- 2*a*c^2*d^2/x - 1/3*a*d^2/x^3
```

Fricas [B] time = 3.05978, size = 525, normalized size = 4.17

$$6\,ac^4d^2x^4 - 11\,bc^3d^2x^3\log\left(-cx + \sqrt{c^2x^2+1} + 1\right) + 11\,bc^3d^2x^3\log\left(-cx + \sqrt{c^2x^2+1} - 1\right) - 12\,ac^2d^2x^2 - 2\left(3\,bc^4 - 6\,bc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) + 1) +
 11*b*c^3*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 12*a*c^2*d^2*x^2 - 2*
(3*b*c^4 - 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^2 + 1)) - 2*a*d^2 + 2
*(3*b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (3*b*c^4 - 6*b*c^2 - b)*d^2*x^3 - b*d
^2)*log(c*x + sqrt(c^2*x^2 + 1)) - (6*b*c^3*d^2*x^3 + b*c*d^2*x)*sqrt(c^2*x
^2 + 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int ac^4\,dx+\int \frac{a}{x^4}\,dx+\int \frac{2ac^2}{x^2}\,dx+\int bc^4\operatorname{asinh}(cx)\,dx+\int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx+\int \frac{2bc^2\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(2*a*c**2/x**2, x
) + Integral(b*c**4*asinh(c*x), x) + Integral(b*asinh(c*x)/x**4, x) + Integ
ral(2*b*c**2*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2(b\operatorname{arsinh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)/x^4, x)
```

3.19 $\int x^4 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=226

$$\frac{1}{11}c^6d^3x^{11}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{ba}{}$$

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(1155*c^5) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(3
465*c^5) - (2*b*d^3*(1 + c^2*x^2)^(5/2))/(1925*c^5) - (b*d^3*(1 + c^2*x^2)^
(7/2))/(1617*c^5) + (4*b*d^3*(1 + c^2*x^2)^(9/2))/(297*c^5) - (b*d^3*(1 + c
^2*x^2)^(11/2))/(121*c^5) + (d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^2*d^3*x
^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcSinh[c*x]))/3 + (c^6*d^
3*x^11*(a + b*ArcSinh[c*x]))/11
```

Rubi [A] time = 0.281484, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {270, 5730, 12, 1799, 1620}

$$\frac{1}{11}c^6d^3x^{11}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{ba}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(1155*c^5) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(3
465*c^5) - (2*b*d^3*(1 + c^2*x^2)^(5/2))/(1925*c^5) - (b*d^3*(1 + c^2*x^2)^
(7/2))/(1617*c^5) + (4*b*d^3*(1 + c^2*x^2)^(9/2))/(297*c^5) - (b*d^3*(1 + c
^2*x^2)^(11/2))/(121*c^5) + (d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^2*d^3*x
^7*(a + b*ArcSinh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcSinh[c*x]))/3 + (c^6*d^
3*x^11*(a + b*ArcSinh[c*x]))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sin\right.$$

$$= \frac{1}{5}d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^2d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9\left(a+b\sin\right.$$

$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{1155c^5}-\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{3465c^5}-\frac{2bd^3\left(1+c^2x^2\right)^{5/2}}{1925c^5}-\frac{bd^3\left(1+c^2x^2\right)}{1617c^5}$$

Mathematica [A] time = 0.124513, size = 143, normalized size = 0.63

$$\frac{d^3\left(3465ac^5x^5\left(105c^6x^6+385c^4x^4+495c^2x^2+231\right)-b\sqrt{c^2x^2+1}\left(33075c^{10}x^{10}+111475c^8x^8+117625c^6x^6+18933c^4x\right.\right.}{4002075c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6) - b*Sq
rt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 117625*c^6*x^6 + 1
11475*c^8*x^8 + 33075*c^10*x^10) + 3465*b*c^5*x^5*(231 + 495*c^2*x^2 + 385*
c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*x]))/(4002075*c^5)
```

Maple [A] time = 0.017, size = 206, normalized size = 0.9

$$\frac{1}{c^5}\left(d^3a\left(\frac{c^{11}x^{11}}{11}+\frac{c^9x^9}{3}+\frac{3\,c^7x^7}{7}+\frac{c^5x^5}{5}\right)+d^3b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^{11}x^{11}}{11}+\frac{\mathrm{Arcsinh}\,(cx)\,c^9x^9}{3}+\frac{3\,\mathrm{Arcsinh}\,(cx)\,c^7x^7}{7}+\frac{\mathrm{Arcs}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^5*(d^3*a*(1/11*c^11*x^11+1/3*c^9*x^9+3/7*c^7*x^7+1/5*c^5*x^5)+d^3*b*(1/
11*arcsinh(c*x)*c^11*x^11+1/3*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7
+1/5*arcsinh(c*x)*c^5*x^5-1/121*c^10*x^10*(c^2*x^2+1)^(1/2)-91/3267*c^8*x^8
*(c^2*x^2+1)^(1/2)-4705/160083*c^6*x^6*(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x
^4*(c^2*x^2+1)^(1/2)+25244/4002075*c^2*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*
(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.08555, size = 628, normalized size = 2.78

$$\frac{1}{11}\,ac^6d^3x^{11}+\frac{1}{3}\,ac^4d^3x^9+\frac{3}{7}\,ac^2d^3x^7+\frac{1}{7623}\left(693\,x^{11}\,\mathrm{arsinh}\,(cx)-\left(\frac{63\,\sqrt{c^2x^2+1}x^{10}}{c^2}-\frac{70\,\sqrt{c^2x^2+1}x^8}{c^4}+\frac{80\,\sqrt{c^2x^2+1}}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 + 3/7*a*c^2*d^3*x^7 + 1/7623*(693*x
^11*arcsinh(c*x) - (63*sqrt(c^2*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^
8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*s
qrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqrt(c^2*x^2 + 1)/c^12)*c)*b*c^6*d^3 + 1/94
5*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 +
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 + 3/245*(35*x^7*a
rcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 +
8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^2*d^3 + 1/75
*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^3
```

Fricas [A] time = 2.70114, size = 509, normalized size = 2.25

$$\frac{363825\,ac^{11}d^3x^{11} + 1334025\,ac^9d^3x^9 + 1715175\,ac^7d^3x^7 + 800415\,ac^5d^3x^5 + 3465\left(105\,bc^{11}d^3x^{11} + 385\,bc^9d^3x^9 + 495\,b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4002075*(363825*a*c^11*d^3*x^11 + 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*d
^3*x^7 + 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 + 385*b*c^9*d^3*x
^9 + 495*b*c^7*d^3*x^7 + 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1)) -
(33075*b*c^10*d^3*x^10 + 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 + 1893
3*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 + 50488*b*d^3)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 62.2965, size = 289, normalized size = 1.28

$$\begin{cases} \frac{ac^6d^3x^{11}}{11} + \frac{ac^4d^3x^9}{3} + \frac{3ac^2d^3x^7}{7} + \frac{ad^3x^5}{5} + \frac{bc^6d^3x^{11}\operatorname{asinh}(cx)}{11} - \frac{bc^5d^3x^{10}\sqrt{c^2x^2+1}}{121} + \frac{bc^4d^3x^9\operatorname{asinh}(cx)}{3} - \frac{91bc^3d^3x^8\sqrt{c^2x^2+1}}{3267} + \frac{3bc^2d^3x^7\operatorname{asinh}\cdots}{7} \cdots \\ \frac{ad^3x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 + 3*a*c**2*d**3*x**7/7
 + a*d**3*x**5/5 + b*c**6*d**3*x**11*asinh(c*x)/11 - b*c**5*d**3*x**10*sqrt
```

```
(c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*asinh(c*x)/3 - 91*b*c**3*d**3*x**8*s
qrt(c**2*x**2 + 1)/3267 + 3*b*c**2*d**3*x**7*asinh(c*x)/7 - 4705*b*c*d**3*x
**6*sqrt(c**2*x**2 + 1)/160083 + b*d**3*x**5*asinh(c*x)/5 - 6311*b*d**3*x**
4*sqrt(c**2*x**2 + 1)/(1334025*c) + 25244*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(
4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 + 1)/(4002075*c**5), Ne(c, 0)),
 (a*d**3*x**5/5, True))
```

Giac [B] time = 1.82439, size = 574, normalized size = 2.54

$$\frac{1}{11}ac^6d^3x^{11} + \frac{1}{3}ac^4d^3x^9 + \frac{3}{7}ac^2d^3x^7 + \frac{1}{7623}\left(693\,x^{11}\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{63\left(c^2x^2+1\right)^{\frac{11}{2}} - 385\left(c^2x^2+1\right)^{\frac{9}{2}} + 990\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 + 3/7*a*c^2*d^3*x^7 + 1/7623*(693*x
^11*log(c*x + sqrt(c^2*x^2 + 1)) - (63*(c^2*x^2 + 1)^(11/2) - 385*(c^2*x^2
+ 1)^(9/2) + 990*(c^2*x^2 + 1)^(7/2) - 1386*(c^2*x^2 + 1)^(5/2) + 1155*(c^2
*x^2 + 1)^(3/2) - 693*sqrt(c^2*x^2 + 1))/c^11)*b*c^6*d^3 + 1/945*(315*x^9*l
og(c*x + sqrt(c^2*x^2 + 1)) - (35*(c^2*x^2 + 1)^(9/2) - 180*(c^2*x^2 + 1)^(
7/2) + 378*(c^2*x^2 + 1)^(5/2) - 420*(c^2*x^2 + 1)^(3/2) + 315*sqrt(c^2*x^2
 + 1))/c^9)*b*c^4*d^3 + 1/5*a*d^3*x^5 + 3/245*(35*x^7*log(c*x + sqrt(c^2*x^
2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1
)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*b*c^2*d^3 + 1/75*(15*x^5*log(c*x + sqr
t(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt
(c^2*x^2 + 1))/c^5)*b*d^3
```

3.20 $\int x^3 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=199

$$\frac{d^3 \left(c^2x^2+1\right)^5 \left(a+b\sinh^{-1}(cx)\right)}{10c^4}-\frac{d^3 \left(c^2x^2+1\right)^4 \left(a+b\sinh^{-1}(cx)\right)}{8c^4}-\frac{bd^3x\left(c^2x^2+1\right)^{9/2}}{100c^3}+\frac{7bd^3x\left(c^2x^2+1\right)^{7/2}}{1600c^3}+\frac{49bd}{}$$

```
[Out] (49*b*d^3*x*Sqrt[1 + c^2*x^2])/(5120*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(3/2)
)/(7680*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(5/2))/(9600*c^3) + (7*b*d^3*x*(1
+ c^2*x^2)^(7/2))/(1600*c^3) - (b*d^3*x*(1 + c^2*x^2)^(9/2))/(100*c^3) + (4
9*b*d^3*ArcSinh[c*x])/(5120*c^4) - (d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]
))/(8*c^4) + (d^3*(1 + c^2*x^2)^5*(a + b*ArcSinh[c*x]))/(10*c^4)
```

Rubi [A] time = 0.168706, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {266, 43, 5730, 12, 388, 195, 215}

$$\frac{d^3 \left(c^2x^2+1\right)^5 \left(a+b\sinh^{-1}(cx)\right)}{10c^4}-\frac{d^3 \left(c^2x^2+1\right)^4 \left(a+b\sinh^{-1}(cx)\right)}{8c^4}-\frac{bd^3x\left(c^2x^2+1\right)^{9/2}}{100c^3}+\frac{7bd^3x\left(c^2x^2+1\right)^{7/2}}{1600c^3}+\frac{49bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (49*b*d^3*x*Sqrt[1 + c^2*x^2])/(5120*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(3/2)
)/(7680*c^3) + (49*b*d^3*x*(1 + c^2*x^2)^(5/2))/(9600*c^3) + (7*b*d^3*x*(1
+ c^2*x^2)^(7/2))/(1600*c^3) - (b*d^3*x*(1 + c^2*x^2)^(9/2))/(100*c^3) + (4
9*b*d^3*ArcSinh[c*x])/(5120*c^4) - (d^3*(1 + c^2*x^2)^4*(a + b*ArcSinh[c*x]
))/(8*c^4) + (d^3*(1 + c^2*x^2)^5*(a + b*ArcSinh[c*x]))/(10*c^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = -\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4} + \frac{d^3\left(1+c^2x^2\right)^5\left(a+b\sinh^{-1}(cx)\right)}{10c^4} - (bc$$

$$= -\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4} + \frac{d^3\left(1+c^2x^2\right)^5\left(a+b\sinh^{-1}(cx)\right)}{10c^4} - (bc$$

$$= -\frac{bd^3x\left(1+c^2x^2\right)^{9/2}}{100c^3} - \frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4} + \frac{d^3\left(1+c^2x^2\right)^5(a-}{10c}$$

$$= \frac{7bd^3x\left(1+c^2x^2\right)^{7/2}}{1600c^3} - \frac{bd^3x\left(1+c^2x^2\right)^{9/2}}{100c^3} - \frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^4}$$

$$= \frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3} + \frac{7bd^3x\left(1+c^2x^2\right)^{7/2}}{1600c^3} - \frac{bd^3x\left(1+c^2x^2\right)^{9/2}}{100c^3} - \frac{d^3(1+c^2}{}$$

$$= \frac{49bd^3x\left(1+c^2x^2\right)^{3/2}}{7680c^3} + \frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3} + \frac{7bd^3x\left(1+c^2x^2\right)^{7/2}}{1600c^3} - \frac{bd^3x(1}{1}$$

$$= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3} + \frac{49bd^3x\left(1+c^2x^2\right)^{3/2}}{7680c^3} + \frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3} + \frac{7bd^3x(1}{16}$$

$$= \frac{49bd^3x\sqrt{1+c^2x^2}}{5120c^3} + \frac{49bd^3x\left(1+c^2x^2\right)^{3/2}}{7680c^3} + \frac{49bd^3x\left(1+c^2x^2\right)^{5/2}}{9600c^3} + \frac{7bd^3x(1}{16}$$

Mathematica [A] time = 0.116578, size = 139, normalized size = 0.7

$$\frac{d^3\left(1920ac^4x^4\left(4c^6x^6+15c^4x^4+20c^2x^2+10\right) - bcx\sqrt{c^2x^2+1}\left(768c^8x^8+2736c^6x^6+3208c^4x^4+790c^2x^2-1185\right)+15}{76800c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(1920*a*c^4*x^4*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - b*c*x*Sqr
t[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8
*x^8) + 15*b*(-79 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x
^10)*ArcSinh[c*x]))/(76800*c^4)
```

Maple [A] time = 0.013, size = 195, normalized size = 1.

$$\frac{1}{c^4}\left(d^3a\left(\frac{c^{10}x^{10}}{10}+\frac{3\,c^8x^8}{8}+\frac{c^6x^6}{2}+\frac{c^4x^4}{4}\right)+d^3b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^{10}x^{10}}{10}+\frac{3\,\mathrm{Arcsinh}\,(cx)\,c^8x^8}{8}+\frac{\mathrm{Arcsinh}\,(cx)\,c^6x^6}{2}+\mathrm{Ar}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^4*(d^3*a*(1/10*c^10*x^10+3/8*c^8*x^8+1/2*c^6*x^6+1/4*c^4*x^4)+d^3*b*(1/
10*arcsinh(c*x)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6
+1/4*arcsinh(c*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(
c^2*x^2+1)^(1/2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^
2+1)^(1/2)+79/5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x)))
```

Maxima [B] time = 1.1828, size = 644, normalized size = 3.24

$$\frac{1}{10}\,ac^6d^3x^{10} + \frac{3}{8}\,ac^4d^3x^8 + \frac{1}{2}\,ac^2d^3x^6 + \frac{1}{12800}\left(1280\,x^{10}\,\mathrm{arsinh}\,(cx) - \left(\frac{128\,\sqrt{c^2x^2+1}x^9}{c^2} - \frac{144\,\sqrt{c^2x^2+1}x^7}{c^4} + \frac{168\,\sqrt{c^2x^2}}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 + 1/2*a*c^2*d^3*x^6 + 1/12800*(1280
*x^10*arcsinh(c*x) - (128*sqrt(c^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)
*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 +
315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^10
))*c)*b*c^6*d^3 + 1/1024*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/
c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqr
t(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^8))*c)*b*c
^4*d^3 + 1/4*a*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x
^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arc
sinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^2*d^3 + 1/32*(8*x^4*arcsinh(c
*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(
c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*d^3
```

Fricas [A] time = 2.51547, size = 474, normalized size = 2.38

$$\frac{7680\,ac^{10}d^3x^{10} + 28800\,ac^8d^3x^8 + 38400\,ac^6d^3x^6 + 19200\,ac^4d^3x^4 + 15\left(512\,bc^{10}d^3x^{10} + 1920\,bc^8d^3x^8 + 2560\,bc^6d^3x^6 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*d^3*x^10 + 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6 +
 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 + 1920*b*c^8*d^3*x^8 + 2560*
b*c^6*d^3*x^6 + 1280*b*c^4*d^3*x^4 - 79*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1))
 - (768*b*c^9*d^3*x^9 + 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 + 790*b*c^3
*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 42.7176, size = 280, normalized size = 1.41

$$\begin{cases} \frac{ac^6d^3x^{10}}{10} + \frac{3ac^4d^3x^8}{8} + \frac{ac^2d^3x^6}{2} + \frac{ad^3x^4}{4} + \frac{bc^6d^3x^{10}\operatorname{asinh}(cx)}{10} - \frac{bc^5d^3x^9\sqrt{c^2x^2+1}}{100} + \frac{3bc^4d^3x^8\operatorname{asinh}(cx)}{8} - \frac{57bc^3d^3x^7\sqrt{c^2x^2+1}}{1600} + \frac{bc^2d^3x^6\operatorname{asinh} \\ \frac{ad^3x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 + a*c**2*d**3*x**6/2
 + a*d**3*x**4/4 + b*c**6*d**3*x**10*asinh(c*x)/10 - b*c**5*d**3*x**9*sqrt(
c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*asinh(c*x)/8 - 57*b*c**3*d**3*x**7*
sqrt(c**2*x**2 + 1)/1600 + b*c**2*d**3*x**6*asinh(c*x)/2 - 401*b*c*d**3*x**
5*sqrt(c**2*x**2 + 1)/9600 + b*d**3*x**4*asinh(c*x)/4 - 79*b*d**3*x**3*sqrt
(c**2*x**2 + 1)/(7680*c) + 79*b*d**3*x*sqrt(c**2*x**2 + 1)/(5120*c**3) - 79
*b*d**3*asinh(c*x)/(5120*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

Giac [B] time = 1.89606, size = 633, normalized size = 3.18

$$\frac{1}{10}\,ac^6d^3x^{10} + \frac{3}{8}\,ac^4d^3x^8 + \frac{1}{2}\,ac^2d^3x^6 + \frac{1}{12800}\left(1280\,x^{10}\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(\sqrt{c^2x^2+1}\left(2\left(4\left(2\,x^2\left(\frac{8\,x^2}{c^2} - \frac{9}{c^4}\right) + \frac{21}{c^6}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 + 1/2*a*c^2*d^3*x^6 + 1/12800*(1280
*x^10*log(c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*(2*(4*(2*x^2*(8*x^2
/c^2 - 9/c^4) + 21/c^6)*x^2 - 105/c^8)*x^2 + 315/c^10)*x + 315*log(abs(-x*a
```

```
bs(c) + sqrt(c^2*x^2 + 1)))/(c^10*abs(c)))*c)*b*c^6*d^3 + 1/1024*(384*x^8*l
og(c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*(2*(4*x^2*(6*x^2/c^2 - 7/c
^4) + 35/c^6)*x^2 - 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1))
)/(c^8*abs(c)))*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 + 1/96*(48*x^6*log(c*x + sqrt(
c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*(2*x^2*(4*x^2/c^2 - 5/c^4) + 15/c^6)*x +
 15*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^6*abs(c)))*c)*b*c^2*d^3 + 1/
32*(8*x^4*log(c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*x*(2*x^2/c^2 -
3/c^4) - 3*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4*abs(c)))*c)*b*d^3
```

3.21 $\int x^2 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=202

$$\frac{1}{9}c^6d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd}{}$$

```
[Out] (16*b*d^3*Sqrt[1 + c^2*x^2])/(315*c^3) + (8*b*d^3*(1 + c^2*x^2)^(3/2))/(945
*c^3) + (2*b*d^3*(1 + c^2*x^2)^(5/2))/(525*c^3) + (b*d^3*(1 + c^2*x^2)^(7/2
))/(441*c^3) - (b*d^3*(1 + c^2*x^2)^(9/2))/(81*c^3) + (d^3*x^3*(a + b*ArcSi
nh[c*x]))/3 + (3*c^2*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^4*d^3*x^7*(a +
b*ArcSinh[c*x]))/7 + (c^6*d^3*x^9*(a + b*ArcSinh[c*x]))/9
```

Rubi [A] time = 0.24858, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {270, 5730, 12, 1799, 1620}

$$\frac{1}{9}c^6d^3x^9\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)-\frac{bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (16*b*d^3*Sqrt[1 + c^2*x^2])/(315*c^3) + (8*b*d^3*(1 + c^2*x^2)^(3/2))/(945
*c^3) + (2*b*d^3*(1 + c^2*x^2)^(5/2))/(525*c^3) + (b*d^3*(1 + c^2*x^2)^(7/2
))/(441*c^3) - (b*d^3*(1 + c^2*x^2)^(9/2))/(81*c^3) + (d^3*x^3*(a + b*ArcSi
nh[c*x]))/3 + (3*c^2*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (3*c^4*d^3*x^7*(a +
b*ArcSinh[c*x]))/7 + (c^6*d^3*x^9*(a + b*ArcSinh[c*x]))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
```

```
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\sinh\right.$$

$$= \frac{16bd^3\sqrt{1+c^2x^2}}{315c^3}+\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{945c^3}+\frac{2bd^3\left(1+c^2x^2\right)^{5/2}}{525c^3}+\frac{bd^3\left(1+c^2x^2\right)^{7/2}}{441c^3}$$

Mathematica [A] time = 0.103453, size = 135, normalized size = 0.67

$$\frac{d^3\left(315ac^3x^3\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)-b\sqrt{c^2x^2+1}\left(1225c^8x^8+4675c^6x^6+6297c^4x^4+2629c^2x^2-5258\right)+\right.}{99225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(315*a*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - b*Sqrt
[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6 + 1225*c^
8*x^8) + 315*b*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSi
nh[c*x]))/(99225*c^3)
```

Maple [A] time = 0.008, size = 187, normalized size = 0.9

$$\frac{1}{c^3}\left(d^3a\left(\frac{c^9x^9}{9}+\frac{3\,c^7x^7}{7}+\frac{3\,c^5x^5}{5}+\frac{c^3x^3}{3}\right)+d^3b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^9x^9}{9}+\frac{3\,\mathrm{Arcsinh}\,(cx)\,c^7x^7}{7}+\frac{3\,\mathrm{Arcsinh}\,(cx)\,c^5x^5}{5}+\mathrm{Arc}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^3*(d^3*a*(1/9*c^9*x^9+3/7*c^7*x^7+3/5*c^5*x^5+1/3*c^3*x^3)+d^3*b*(1/9*a
rcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*a
rcsinh(c*x)*c^3*x^3-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^
2+1)^(1/2)-2099/33075*c^4*x^4*(c^2*x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2
+1)^(1/2)+5258/99225*(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.1779, size = 524, normalized size = 2.59

$$\frac{1}{9}\,ac^6d^3x^9+\frac{3}{7}\,ac^4d^3x^7+\frac{1}{2835}\left(315\,x^9\,\mathrm{arsinh}\,(cx)-\left(\frac{35\,\sqrt{c^2x^2+1}x^8}{c^2}-\frac{40\,\sqrt{c^2x^2+1}x^6}{c^4}+\frac{48\,\sqrt{c^2x^2+1}x^4}{c^6}-\frac{64\,\sqrt{c^2x^2}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 + 1/2835*(315*x^9*arcsinh(c*x) - (35*
sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2
+ 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c
)*b*c^6*d^3 + 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*
x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^
6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 + 1/25*(15*x^5*arcsinh(c*x) - (3
```

```
*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^3
```

Fricas [A] time = 2.53317, size = 448, normalized size = 2.22

$$\frac{11025\,ac^9d^3x^9 + 42525\,ac^7d^3x^7 + 59535\,ac^5d^3x^5 + 33075\,ac^3d^3x^3 + 315\left(35\,bc^9d^3x^9 + 135\,bc^7d^3x^7 + 189\,bc^5d^3x^5 + 105\,b\right.}{99225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^3*x^9 + 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 +
33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 + 135*b*c^7*d^3*x^7 + 189*b*c^5
*d^3*x^5 + 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*d^
3*x^8 + 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 + 2629*b*c^2*d^3*x^2 - 5258
*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 23.7592, size = 265, normalized size = 1.31

$$\begin{cases} \frac{ac^6d^3x^9}{9} + \frac{3ac^4d^3x^7}{7} + \frac{3ac^2d^3x^5}{5} + \frac{ad^3x^3}{3} + \frac{bc^6d^3x^9\operatorname{asinh}(cx)}{9} - \frac{bc^5d^3x^8\sqrt{c^2x^2+1}}{81} + \frac{3bc^4d^3x^7\operatorname{asinh}(cx)}{7} - \frac{187bc^3d^3x^6\sqrt{c^2x^2+1}}{3969} + \frac{3bc^2d^3x^5\operatorname{asinh}(}{5} \\ \frac{ad^3x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 + 3*a*c**2*d**3*x**5/5
 + a*d**3*x**3/3 + b*c**6*d**3*x**9*asinh(c*x)/9 - b*c**5*d**3*x**8*sqrt(c*
*2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*asinh(c*x)/7 - 187*b*c**3*d**3*x**6*sq
rt(c**2*x**2 + 1)/3969 + 3*b*c**2*d**3*x**5*asinh(c*x)/5 - 2099*b*c*d**3*x*
*4*sqrt(c**2*x**2 + 1)/33075 + b*d**3*x**3*asinh(c*x)/3 - 2629*b*d**3*x**2*
sqrt(c**2*x**2 + 1)/(99225*c) + 5258*b*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3
), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Giac [B] time = 1.83501, size = 501, normalized size = 2.48

$$\frac{1}{9}\,ac^{6}d^{3}x^{9} + \frac{3}{7}\,ac^{4}d^{3}x^{7} + \frac{1}{2835}\left(315\,x^{9}\log\left(cx + \sqrt{c^{2}x^{2}+1}\right) - \frac{35\,\left(c^{2}x^{2}+1\right)^{\frac{9}{2}} - 180\,\left(c^{2}x^{2}+1\right)^{\frac{7}{2}} + 378\,\left(c^{2}x^{2}+1\right)^{\frac{5}{2}} - 42}{c^{9}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 + 1/2835*(315*x^9*log(c*x + sqrt(c^2*
x^2 + 1)) - (35*(c^2*x^2 + 1)^(9/2) - 180*(c^2*x^2 + 1)^(7/2) + 378*(c^2*x^
2 + 1)^(5/2) - 420*(c^2*x^2 + 1)^(3/2) + 315*sqrt(c^2*x^2 + 1))/c^9)*b*c^6*
d^3 + 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*log(c*x + sqrt(c^2*x^2 + 1)) - (5*(
c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1)^(3/2) - 35*s
qrt(c^2*x^2 + 1))/c^7)*b*c^4*d^3 + 1/25*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)
) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))
/c^5)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) -
 ((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*d^3
```

3.22 $\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=145

$$\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}}{64c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}}{384c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{c^2x^2+1}}{1024c}-$$

```
[Out] (-35*b*d^3*x*Sqrt[1 + c^2*x^2])/(1024*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2))
/(1536*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2))/(384*c) - (b*d^3*x*(1 + c^2*x^2
)^(7/2))/(64*c) - (35*b*d^3*ArcSinh[c*x])/(1024*c^2) + (d^3*(1 + c^2*x^2)^4
*(a + b*ArcSinh[c*x]))/(8*c^2)
```

Rubi [A] time = 0.0697191, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5717, 195, 215}

$$\frac{d^3\left(c^2x^2+1\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}}{64c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}}{384c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}}{1536c}-\frac{35bd^3x\sqrt{c^2x^2+1}}{1024c}-$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-35*b*d^3*x*Sqrt[1 + c^2*x^2])/(1024*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2))
/(1536*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2))/(384*c) - (b*d^3*x*(1 + c^2*x^2
)^(7/2))/(64*c) - (35*b*d^3*ArcSinh[c*x])/(1024*c^2) + (d^3*(1 + c^2*x^2)^4
*(a + b*ArcSinh[c*x]))/(8*c^2)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)dx=\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\left(bd^3\right)\int\left(1+c^2x^2\right)^{7/2}dx}{8c}$$
$$=-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{64c}+\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\left(7bd^3\right)\int\left(1+c^2x^2\right.}{64c}$$
$$=-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{64c}+\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)}{8c^2}$$
$$=-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}}{1536c}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{64c}+\frac{d^3\left(1+c\right.}{}$$
$$=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}}{1536c}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-\frac{bd^3x\left(1+\right.}{64}$$
$$=-\frac{35bd^3x\sqrt{1+c^2x^2}}{1024c}-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}}{1536c}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}}{384c}-\frac{bd^3x\left(1+\right.}{64}$$

Mathematica [A] time = 0.173852, size = 128, normalized size = 0.88

$$\frac{d^3\left(cx\left(384acx\left(c^6x^6+4c^4x^4+6c^2x^2+4\right)-b\sqrt{c^2x^2+1}\left(48c^6x^6+200c^4x^4+326c^2x^2+279\right)\right)+3b\left(128c^8x^8+512c^6x^6\right.}{3072c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(c*x*(384*a*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*b*(93 + 512*c^2*x
^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]))/(3072*c^2)
```

Maple [A] time = 0.004, size = 176, normalized size = 1.2

$$\frac{1}{c^2}\left(d^3a\left(\frac{c^8x^8}{8}+\frac{c^6x^6}{2}+\frac{3\,c^4x^4}{4}+\frac{c^2x^2}{2}\right)+d^3b\left(\frac{\operatorname{Arcsinh}(cx)\,c^8x^8}{8}+\frac{\operatorname{Arcsinh}(cx)\,c^6x^6}{2}+\frac{3\operatorname{Arcsinh}(cx)\,c^4x^4}{4}+\frac{\operatorname{Arcsinh}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c^2*(d^3*a*(1/8*c^8*x^8+1/2*c^6*x^6+3/4*c^4*x^4+1/2*c^2*x^2)+d^3*b*(1/8*a
rcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^4*x^4+1/2*a
rcsinh(c*x)*c^2*x^2-1/64*c^7*x^7*(c^2*x^2+1)^(1/2)-25/384*c^5*x^5*(c^2*x^2+
1)^(1/2)-163/1536*c^3*x^3*(c^2*x^2+1)^(1/2)-93/1024*c*x*(c^2*x^2+1)^(1/2)+9
3/1024*arcsinh(c*x)))
```

Maxima [B] time = 1.12706, size = 540, normalized size = 3.72

$$\frac{1}{8}\,ac^6d^3x^8+\frac{1}{2}\,ac^4d^3x^6+\frac{1}{3072}\left(384\,x^8\operatorname{arsinh}(cx)-\left(\frac{48\sqrt{c^2x^2+1}x^7}{c^2}-\frac{56\sqrt{c^2x^2+1}x^5}{c^4}+\frac{70\sqrt{c^2x^2+1}x^3}{c^6}-\frac{105\sqrt{c^2x^2}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 + 1/3072*(384*x^8*arcsinh(c*x) - (48*
sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2
+ 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(
sqrt(c^2)*c^8))*c)*b*c^6*d^3 + 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arcsinh(c*x
) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c
^2*x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*b*c^4*d
^3 + 3/32*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x
^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*b*c^2*d^3 +
1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcs
inh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*b*d^3
```

Fricas [A] time = 2.39912, size = 424, normalized size = 2.92

$$\frac{384\,ac^8d^3x^8+1536\,ac^6d^3x^6+2304\,ac^4d^3x^4+1536\,ac^2d^3x^2+3\left(128\,bc^8d^3x^8+512\,bc^6d^3x^6+768\,bc^4d^3x^4+512\,bc^2d^3x^2\right.}{3072\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*d^3*x^8 + 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 + 1536*
a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 + 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x^
4 + 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 + 1)) - (48*b*c^7*
d^3*x^7 + 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 + 279*b*c*d^3*x)*sqrt(c^2*x
^2 + 1))/c^2
```

Sympy [A] time = 16.0277, size = 253, normalized size = 1.74

$$\begin{cases} \frac{ac^6d^3x^8}{8} + \frac{ac^4d^3x^6}{2} + \frac{3ac^2d^3x^4}{4} + \frac{ad^3x^2}{2} + \frac{bc^6d^3x^8 \operatorname{asinh}(cx)}{8} - \frac{bc^5d^3x^7\sqrt{c^2x^2+1}}{64} + \frac{bc^4d^3x^6 \operatorname{asinh}(cx)}{2} - \frac{25bc^3d^3x^5\sqrt{c^2x^2+1}}{384} + \frac{3bc^2d^3x^4 \operatorname{asinh}(c}{4} \\ \frac{ad^3x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 + 3*a*c**2*d**3*x**4/4 +
 a*d**3*x**2/2 + b*c**6*d**3*x**8*asinh(c*x)/8 - b*c**5*d**3*x**7*sqrt(c**2
*x**2 + 1)/64 + b*c**4*d**3*x**6*asinh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c*
*2*x**2 + 1)/384 + 3*b*c**2*d**3*x**4*asinh(c*x)/4 - 163*b*c*d**3*x**3*sqrt
(c**2*x**2 + 1)/1536 + b*d**3*x**2*asinh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x**
2 + 1)/(1024*c) + 93*b*d**3*asinh(c*x)/(1024*c**2), Ne(c, 0)), (a*d**3*x**2
/2, True))
```

Giac [B] time = 2.01012, size = 572, normalized size = 3.94

$$\frac{1}{8}ac^6d^3x^8 + \frac{1}{2}ac^4d^3x^6 + \frac{1}{3072}\left(384\,x^8\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(\sqrt{c^2x^2+1}\left(2\left(4\,x^2\left(\frac{6\,x^2}{c^2} - \frac{7}{c^4}\right) + \frac{35}{c^6}\right)x^2 - \frac{105}{c^8}\right)x - \frac{105}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 + 1/3072*(384*x^8*log(c*x + sqrt(c^2*
x^2 + 1)) - (sqrt(c^2*x^2 + 1)*(2*(4*x^2*(6*x^2/c^2 - 7/c^4) + 35/c^6)*x^2
```

```
- 105/c^8)*x - 105*log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^8*abs(c)))*c)
*b*c^6*d^3 + 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*log(c*x + sqrt(c^2*x^2 + 1))
- (sqrt(c^2*x^2 + 1)*(2*x^2*(4*x^2/c^2 - 5/c^4) + 15/c^6)*x + 15*log(abs(-x
*abs(c) + sqrt(c^2*x^2 + 1)))/(c^6*abs(c)))*c)*b*c^4*d^3 + 3/32*(8*x^4*log(
c*x + sqrt(c^2*x^2 + 1)) - (sqrt(c^2*x^2 + 1)*x*(2*x^2/c^2 - 3/c^4) - 3*log
(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^4*abs(c)))*c)*b*c^2*d^3 + 1/2*a*d^3
*x^2 + 1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2
 + log(abs(-x*abs(c) + sqrt(c^2*x^2 + 1)))/(c^2*abs(c))))*b*d^3
```

3.23 $\int \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=170

$$\frac{1}{7} c^6 d^3 x^7 \left(a+b \sinh^{-1}(cx)\right)+\frac{3}{5} c^4 d^3 x^5 \left(a+b \sinh^{-1}(cx)\right)+c^2 d^3 x^3 \left(a+b \sinh^{-1}(cx)\right)+d^3 x \left(a+b \sinh^{-1}(cx)\right)-\frac{b d^3 \left(c^2\right.}{}$$

```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(35*c) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(105*c
) - (6*b*d^3*(1 + c^2*x^2)^(5/2))/(175*c) - (b*d^3*(1 + c^2*x^2)^(7/2))/(49
*c) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^
4*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```

Rubi [A] time = 0.16143, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {194, 5679, 12, 1799, 1850}

$$\frac{1}{7} c^6 d^3 x^7 \left(a+b \sinh^{-1}(cx)\right)+\frac{3}{5} c^4 d^3 x^5 \left(a+b \sinh^{-1}(cx)\right)+c^2 d^3 x^3 \left(a+b \sinh^{-1}(cx)\right)+d^3 x \left(a+b \sinh^{-1}(cx)\right)-\frac{b d^3 \left(c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-16*b*d^3*Sqrt[1 + c^2*x^2])/(35*c) - (8*b*d^3*(1 + c^2*x^2)^(3/2))/(105*c
) - (6*b*d^3*(1 + c^2*x^2)^(5/2))/(175*c) - (b*d^3*(1 + c^2*x^2)^(7/2))/(49
*c) + d^3*x*(a + b*ArcSinh[c*x]) + c^2*d^3*x^3*(a + b*ArcSinh[c*x]) + (3*c^
4*d^3*x^5*(a + b*ArcSinh[c*x]))/5 + (c^6*d^3*x^7*(a + b*ArcSinh[c*x]))/7
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+c^2d^3x^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{16bd^3\sqrt{1+c^2x^2}}{35c}-\frac{8bd^3\left(1+c^2x^2\right)^{3/2}}{105c}-\frac{6bd^3\left(1+c^2x^2\right)^{5/2}}{175c}-\frac{bd^3\left(1+c^2x^2\right)^{7/2}}{49c}$$

Mathematica [A] time = 0.165447, size = 119, normalized size = 0.7

$$\frac{d^3\left(105acx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)-b\sqrt{c^2x^2+1}\left(75c^6x^6+351c^4x^4+757c^2x^2+2161\right)+105bcx\left(5c^6x^6+21c^4x^4\right.\right.}{3675c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(105*a*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - b*Sqrt[1 + c^2
*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(35 + 35*
c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]))/(3675*c)
```

Maple [A] time = 0.006, size = 156, normalized size = 0.9

$$\frac{1}{c}\left(d^3a\left(\frac{c^7x^7}{7}+\frac{3\,c^5x^5}{5}+c^3x^3+cx\right)+d^3b\left(\frac{\operatorname{Arcsinh}(cx)\,c^7x^7}{7}+\frac{3\operatorname{Arcsinh}(cx)\,c^5x^5}{5}+\operatorname{Arcsinh}(cx)\,c^3x^3+\operatorname{Arcsinh}(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(d^3*a*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b*(1/7*arcsinh(c*x)*c^
7*x^7+3/5*arcsinh(c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/49*c
^6*x^6*(c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-757/3675*c^2*x^
2*(c^2*x^2+1)^(1/2)-2161/3675*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.14275, size = 406, normalized size = 2.39

$$\frac{1}{7}\,ac^6d^3x^7+\frac{3}{5}\,ac^4d^3x^5+\frac{1}{245}\left(35\,x^7\operatorname{arsinh}(cx)-\left(\frac{5\,\sqrt{c^2x^2+1}x^6}{c^2}-\frac{6\,\sqrt{c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{c^2x^2+1}x^2}{c^6}-\frac{16\,\sqrt{c^2x^2+1}}{c^8}\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*arcsinh(c*x) - (5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsinh(c*x
) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 + 1)/c^6)*c)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(
sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d^3*x +
 (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A] time = 2.3234, size = 387, normalized size = 2.28

$$\frac{525\,ac^7d^3x^7+2205\,ac^5d^3x^5+3675\,ac^3d^3x^3+3675\,acd^3x+105\left(5\,bc^7d^3x^7+21\,bc^5d^3x^5+35\,bc^3d^3x^3+35\,bcd^3x\right)\log\left(cx\right.}{3675\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*d^3*x^7 + 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 + 3675*
a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 + 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 + 35
*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*d^3*x^6 + 351*b*c^4*d^
3*x^4 + 757*b*c^2*d^3*x^2 + 2161*b*d^3)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] time = 8.66608, size = 221, normalized size = 1.3

$$\begin{cases}\frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}+ac^2d^3x^3+ad^3x+\frac{bc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{bc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{3bc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{117bc^3d^3x^4\sqrt{c^2x^2+1}}{1225}+bc^2d^3x^3\operatorname{asin} \\ ad^3x\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 + a*c**2*d**3*x**3 + a
*d**3*x + b*c**6*d**3*x**7*asinh(c*x)/7 - b*c**5*d**3*x**6*sqrt(c**2*x**2 +
 1)/49 + 3*b*c**4*d**3*x**5*asinh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*x
**2 + 1)/1225 + b*c**2*d**3*x**3*asinh(c*x) - 757*b*c*d**3*x**2*sqrt(c**2*x
**2 + 1)/3675 + b*d**3*x*asinh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 + 1)/(3675
*c), Ne(c, 0)), (a*d**3*x, True))
```

Giac [B] time = 1.67599, size = 416, normalized size = 2.45

$$\frac{1}{7}\,ac^6d^3x^7+\frac{3}{5}\,ac^4d^3x^5+\frac{1}{245}\left(35\,x^7\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{5\left(c^2x^2+1\right)^{\frac{7}{2}}-21\left(c^2x^2+1\right)^{\frac{5}{2}}+35\left(c^2x^2+1\right)^{\frac{3}{2}}-35\sqrt{c^2x^2}\right.}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 + 1/245*(35*x^7*log(c*x + sqrt(c^2*x^
2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1
)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*b*c^6*d^3 + 1/25*(15*x^5*log(c*x + sqr
t(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt
(c^2*x^2 + 1))/c^5)*b*c^4*d^3 + a*c^2*d^3*x^3 + 1/3*(3*x^3*log(c*x + sqrt(c
^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*c^2*d^3 +
 (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^3 + a*d^3*x
```

3.24 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=221

$$-\frac{1}{2}bd^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^3\left(c^2x^2+\right.$$

```
[Out] (-19*b*c*d^3*x*Sqrt[1 + c^2*x^2])/48 - (7*b*c*d^3*x*(1 + c^2*x^2)^(3/2))/72
 - (b*c*d^3*x*(1 + c^2*x^2)^(5/2))/36 - (19*b*d^3*ArcSinh[c*x])/48 + (d^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[
c*x]))/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/6 + (d^3*(a + b*ArcSi
nh[c*x])^2)/(2*b) + d^3*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] -
 (b*d^3*PolyLog[2, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.28416, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5726, 5659, 3716, 2190, 2279, 2391, 195, 215}

$$\frac{1}{2}bd^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{6}d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}d^3\left(c^2x^2+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-19*b*c*d^3*x*Sqrt[1 + c^2*x^2])/48 - (7*b*c*d^3*x*(1 + c^2*x^2)^(3/2))/72
 - (b*c*d^3*x*(1 + c^2*x^2)^(5/2))/36 - (19*b*d^3*ArcSinh[c*x])/48 + (d^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[
c*x]))/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/6 - (d^3*(a + b*ArcSi
nh[c*x])^2)/(2*b) + d^3*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] +
(b*d^3*PolyLog[2, E^(2*ArcSinh[c*x])])/2
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx-\frac{1}{6}$$

$$=-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{4}d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(\right.$$

$$=-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{2}d^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(c\right.$$

$$=-\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}+\frac{1}{2}d^3\left(1+\right.$$

$$=-\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}-\frac{19}{48}bd^3\,\mathrm{si}$$

$$=-\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}-\frac{19}{48}bd^3\,\mathrm{si}$$

$$=-\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}-\frac{19}{48}bd^3\,\mathrm{si}$$

$$=-\frac{19}{48}bcd^3x\sqrt{1+c^2x^2}-\frac{7}{72}bcd^3x\left(1+c^2x^2\right)^{3/2}-\frac{1}{36}bcd^3x\left(1+c^2x^2\right)^{5/2}-\frac{19}{48}bd^3\,\mathrm{si}$$

Mathematica [A] time = 0.140979, size = 189, normalized size = 0.86

$$\frac{1}{144}d^3\left(72b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+3\sinh^{-1}(cx)\left(-48a+b\left(8c^6x^6+36c^4x^4+72c^2x^2+25\right)+48b\log\left(1-e^{2\sinh^{-1}(cx)}\right)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (d^3*(216*a*c^2*x^2 + 108*a*c^4*x^4 + 24*a*c^6*x^6 - 75*b*c*x*Sqrt[1 + c^2*
x^2] - 22*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 4*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*
b*ArcSinh[c*x]^2 + 144*a*Log[1 - E^(2*ArcSinh[c*x])] + 3*ArcSinh[c*x]*(-48*
a + b*(25 + 72*c^2*x^2 + 36*c^4*x^4 + 8*c^6*x^6) + 48*b*Log[1 - E^(2*ArcSin
h[c*x])]) + 72*b*PolyLog[2, E^(2*ArcSinh[c*x])]))/144
```

Maple [A] time = 0.135, size = 284, normalized size = 1.3

$$\frac{d^3ac^6x^6}{6}+\frac{3\,d^3ac^4x^4}{4}+\frac{3\,d^3ac^2x^2}{2}+d^3a\ln(cx)+\frac{3\,d^3b\mathrm{Arcsinh}\,(cx)\,c^4x^4}{4}+\frac{3\,d^3b\mathrm{Arcsinh}\,(cx)\,c^2x^2}{2}+\frac{d^3b\mathrm{Arcsinh}\,(cx)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/6*d^3*a*c^6*x^6+3/4*d^3*a*c^4*x^4+3/2*d^3*a*c^2*x^2+d^3*a*ln(c*x)+3/4*d^3
*b*arcsinh(c*x)*c^4*x^4+3/2*d^3*b*arcsinh(c*x)*c^2*x^2+1/6*d^3*b*arcsinh(c*
x)*c^6*x^6+25/48*b*d^3*arcsinh(c*x)+d^3*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+
d^3*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-1/36*d^3*b*c^5*x^5*(c^2*x^2+1)^(1/2
)-11/72*d^3*b*c^3*x^3*(c^2*x^2+1)^(1/2)-25/48*b*c*d^3*x*(c^2*x^2+1)^(1/2)-1
/2*d^3*b*arcsinh(c*x)^2+d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+d^3*
b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,ac^6d^3x^6+\frac{3}{4}\,ac^4d^3x^4+\frac{3}{2}\,ac^2d^3x^2+ad^3\log(x)+\int bc^6d^3x^5\log\left(cx+\sqrt{c^2x^2+1}\right)+3\,bc^4d^3x^3\log\left(cx+\sqrt{c^2x^2+1}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 + 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) +
integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x^3*log(
c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^2*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + b*d
^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ac^6d^3x^6+3\,ac^4d^3x^4+3\,ac^2d^3x^2+ad^3+\left(bc^6d^3x^6+3\,bc^4d^3x^4+3\,bc^2d^3x^2+bd^3\right)\mathrm{arsinh}\,(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a}{x}\,dx + \int 3ac^2x\,dx + \int 3ac^4x^3\,dx + \int ac^6x^5\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 3bc^2x\,\mathrm{asinh}\,(cx)\,dx + \int 3bc^4x^3\,\mathrm{asinh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x,x)
```

```
[Out] d**3*(Integral(a/x, x) + Integral(3*a*c**2*x, x) + Integral(3*a*c**4*x**3,
x) + Integral(a*c**6*x**5, x) + Integral(b*asinh(c*x)/x, x) + Integral(3*b*
c**2*x*asinh(c*x), x) + Integral(3*b*c**4*x**3*asinh(c*x), x) + Integral(b*
c**6*x**5*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^3 (b\,\mathrm{arsinh}\,(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)/x, x)
```

3.25 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=160

$$\frac{1}{5}c^6d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{25}bcd^3\left(\right.$$

```
[Out] (-11*b*c*d^3*Sqrt[1 + c^2*x^2])/5 - (b*c*d^3*(1 + c^2*x^2)^(3/2))/5 - (b*c*
d^3*(1 + c^2*x^2)^(5/2))/25 - (d^3*(a + b*ArcSinh[c*x]))/x + 3*c^2*d^3*x*(a
 + b*ArcSinh[c*x]) + c^4*d^3*x^3*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^5*(a + b
*ArcSinh[c*x]))/5 - b*c*d^3*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rubi [A] time = 0.221344, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {270, 5730, 12, 1799, 1620, 63, 208}

$$\frac{1}{5}c^6d^3x^5\left(a+b\sinh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^2d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{1}{25}bcd^3\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*Sqrt[1 + c^2*x^2])/5 - (b*c*d^3*(1 + c^2*x^2)^(3/2))/5 - (b*c*
d^3*(1 + c^2*x^2)^(5/2))/25 - (d^3*(a + b*ArcSinh[c*x]))/x + 3*c^2*d^3*x*(a
 + b*ArcSinh[c*x]) + c^4*d^3*x^3*(a + b*ArcSinh[c*x]) + (c^6*d^3*x^5*(a + b
*ArcSinh[c*x]))/5 - b*c*d^3*ArcTanh[Sqrt[1 + c^2*x^2]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^3 (a+b\sinh^{-1}(cx))}{x^2} dx = -\frac{d^3 (a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x (a+b\sinh^{-1}(cx)) + c^4d^3x^3 (a+b\sinh^{-1}(cx)$$

$$= -\frac{d^3 (a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x (a+b\sinh^{-1}(cx)) + c^4d^3x^3 (a+b\sinh^{-1}(cx)$$

$$= -\frac{d^3 (a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x (a+b\sinh^{-1}(cx)) + c^4d^3x^3 (a+b\sinh^{-1}(cx)$$

$$= -\frac{d^3 (a+b\sinh^{-1}(cx))}{x} + 3c^2d^3x (a+b\sinh^{-1}(cx)) + c^4d^3x^3 (a+b\sinh^{-1}(cx)$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3 (1+c^2x^2)^{3/2} - \frac{1}{25}bcd^3 (1+c^2x^2)^{5/2} - \frac{d^3 (a+b\sin}{x}$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3 (1+c^2x^2)^{3/2} - \frac{1}{25}bcd^3 (1+c^2x^2)^{5/2} - \frac{d^3 (a+b\sin}{x}$$

$$= -\frac{11}{5}bcd^3\sqrt{1+c^2x^2} - \frac{1}{5}bcd^3 (1+c^2x^2)^{3/2} - \frac{1}{25}bcd^3 (1+c^2x^2)^{5/2} - \frac{d^3 (a+b\sin}{x}$$

Mathematica [A] time = 0.165508, size = 163, normalized size = 1.02

$$\frac{d^3 \left(5ac^6x^6 + 25ac^4x^4 + 75ac^2x^2 - 25a - bc^5x^5\sqrt{c^2x^2+1} - 7bc^3x^3\sqrt{c^2x^2+1} - 61bcx\sqrt{c^2x^2+1} - 25bcx\log\left(\sqrt{c^2x^2+1}\right.\right.}{25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-25*a + 75*a*c^2*x^2 + 25*a*c^4*x^4 + 5*a*c^6*x^6 - 61*b*c*x*Sqrt[1 +
 c^2*x^2] - 7*b*c^3*x^3*Sqrt[1 + c^2*x^2] - b*c^5*x^5*Sqrt[1 + c^2*x^2] + 5
*b*(-5 + 15*c^2*x^2 + 5*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 25*b*c*x*Log[x] -
 25*b*c*x*Log[1 + Sqrt[1 + c^2*x^2]]))/(25*x)
```

Maple [A] time = 0.008, size = 151, normalized size = 0.9

$$c\left(d^3a\left(\frac{c^5x^5}{5} + c^3x^3 + 3cx - \frac{1}{cx}\right) + d^3b\left(\frac{\mathrm{Arcsinh}(cx)c^5x^5}{5} + \mathrm{Arcsinh}(cx)c^3x^3 + 3\,\mathrm{Arcsinh}(cx)cx - \frac{\mathrm{Arcsinh}(cx)}{cx} - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] c*(d^3*a*(1/5*c^5*x^5+c^3*x^3+3*c*x-1/c/x)+d^3*b*(1/5*arcsinh(c*x)*c^5*x^5+
arcsinh(c*x)*c^3*x^3+3*arcsinh(c*x)*c*x-arcsinh(c*x)/c/x-1/25*c^4*x^4*(c^2*
x^2+1)^(1/2)-7/25*c^2*x^2*(c^2*x^2+1)^(1/2)-61/25*(c^2*x^2+1)^(1/2)-arctanh
(1/(c^2*x^2+1)^(1/2))))
```

Maxima [A] time = 1.24493, size = 315, normalized size = 1.97

$$\frac{1}{5}\,ac^6d^3x^5 + \frac{1}{75}\left(15\,x^5\,\mathrm{arsinh}\,(cx) - \left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2} - \frac{4\,\sqrt{c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{c^2x^2+1}}{c^6}\right)c\right)bc^6d^3 + ac^4d^3x^3 + \frac{1}{3}\left(3\,x^3\,\mathrm{arsinh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*c^6*d^3*x^5 + 1/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^
2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*c^6*d^3 + a
*c^4*d^3*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*s
qrt(c^2*x^2 + 1)/c^4))*b*c^4*d^3 + 3*a*c^2*d^3*x + 3*(c*x*arcsinh(c*x) - sq
rt(c^2*x^2 + 1))*b*c*d^3 - (c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcsinh(c*x)/
x)*b*d^3 - a*d^3/x
```

Fricas [A] time = 2.77926, size = 616, normalized size = 3.85

$$\frac{5\,ac^6d^3x^6 + 25\,ac^4d^3x^4 + 75\,ac^2d^3x^2 - 25\,bcd^3x\log\left(-cx + \sqrt{c^2x^2+1} + 1\right) + 25\,bcd^3x\log\left(-cx + \sqrt{c^2x^2+1} - 1\right) - 5\left(bc^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/25*(5*a*c^6*d^3*x^6 + 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 25*b*c*d^3*x*
log(-c*x + sqrt(c^2*x^2 + 1) + 1) + 25*b*c*d^3*x*log(-c*x + sqrt(c^2*x^2 +
1) - 1) - 5*(b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x*log(-c*x + sqrt(c^2*x^
2 + 1)) - 25*a*d^3 + 5*(b*c^6*d^3*x^6 + 5*b*c^4*d^3*x^4 + 15*b*c^2*d^3*x^2
```

```
- (b*c^6 + 5*b*c^4 + 15*b*c^2 - 5*b)*d^3*x - 5*b*d^3)*log(c*x + sqrt(c^2*x^
2 + 1)) - (b*c^5*d^3*x^5 + 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sqrt(c^2*x^2 + 1
))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int 3ac^4x^2\,dx + \int ac^6x^4\,dx + \int 3bc^2\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^2}\,dx + \int 3bc^4x^2\operatorname{asinh}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c**4*x**2,
 x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**2, x) + Integral(3*b*c**4*x**2*asinh(c*x), x) + Integral(
b*c**6*x**4*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^3 (b\operatorname{arsinh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)/x^2, x)
```

3.26 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=249

$$-\frac{3}{2}bc^2d^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)-\frac{d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2d^3\left(c^2\right.$$

```
[Out] (-3*b*c^3*d^3*x*Sqrt[1 + c^2*x^2])/32 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2))
/16 - (b*c*d^3*(1 + c^2*x^2)^(5/2))/(2*x) - (3*b*c^2*d^3*ArcSinh[c*x])/32 +
 (3*c^2*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (3*c^2*d^3*(1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/4 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(2
*x^2) + (3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/(2*b) + 3*c^2*d^3*(a + b*ArcSinh
[c*x])*Log[1 - E^(-2*ArcSinh[c*x])] - (3*b*c^2*d^3*PolyLog[2, E^(-2*ArcSinh
[c*x])])/2
```

Rubi [A] time = 0.30335, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5728, 277, 195, 215, 5726, 5659, 3716, 2190, 2279, 2391}

$$\frac{3}{2}bc^2d^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-\frac{d^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{3}{4}c^2d^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2d^3\left(c^2x^2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[1 + c^2*x^2])/32 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2))
/16 - (b*c*d^3*(1 + c^2*x^2)^(5/2))/(2*x) - (3*b*c^2*d^3*ArcSinh[c*x])/32 +
 (3*c^2*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/2 + (3*c^2*d^3*(1 + c^2*x^2
)^2*(a + b*ArcSinh[c*x]))/4 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(2
*x^2) - (3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/(2*b) + 3*c^2*d^3*(a + b*ArcSinh
[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + (3*b*c^2*d^3*PolyLog[2, E^(2*ArcSinh[c
*x])])/2
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
```

```
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{2x^2} + \left(3c^2d\right)\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}$$
$$= -\frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} + \frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right) - \frac{d^3\left(1+c^2x^2\right)^3\left(a\right.}{2x^2}$$
$$= \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} + \frac{3}{2}c^2d^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(c\right.$$
$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} + \frac{3}{2}c^2d^3\left(1\right.$$
$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3$$
$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3$$
$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3$$
$$= -\frac{3}{32}bc^3d^3x\sqrt{1+c^2x^2} + \frac{7}{16}bc^3d^3x\left(1+c^2x^2\right)^{3/2} - \frac{bcd^3\left(1+c^2x^2\right)^{5/2}}{2x} - \frac{3}{32}bc^2d^3$$

Mathematica [A] time = 0.436417, size = 184, normalized size = 0.74

$$\frac{1}{32}d^3\left(48bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right) + 8c^6x^4\left(a+b\sinh^{-1}(cx)\right) + 48c^4x^2\left(a+b\sinh^{-1}(cx)\right) - \frac{48c^2\left(a+b\sinh^{-1}(cx)\right)^2}{b}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (d^3*((-16*b*c*Sqrt[1 + c^2*x^2])/x - 2*b*c^5*x^3*Sqrt[1 + c^2*x^2] - 21*b*
c^2*(c*x*Sqrt[1 + c^2*x^2] - ArcSinh[c*x]) - (16*(a + b*ArcSinh[c*x]))/x^2
+ 48*c^4*x^2*(a + b*ArcSinh[c*x]) + 8*c^6*x^4*(a + b*ArcSinh[c*x]) - (48*c^
2*(a + b*ArcSinh[c*x])^2)/b + 96*c^2*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcS
inh[c*x])] + 48*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])]))/32
```

Maple [A] time = 0.285, size = 313, normalized size = 1.3

$$\frac{c^6d^3ax^4}{4} + \frac{3\,c^4d^3ax^2}{2} + 3\,c^2d^3a\ln(cx) - \frac{d^3a}{2\,x^2} + \frac{d^3bc^2}{2} + \frac{c^6d^3b\text{Arcsinh}(cx)\,x^4}{4} + \frac{3\,c^4d^3b\text{Arcsinh}(cx)\,x^2}{2} - \frac{d^3b\text{Arcsinh}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] 1/4*c^6*d^3*a*x^4+3/2*c^4*d^3*a*x^2+3*c^2*d^3*a*ln(c*x)-1/2*d^3*a/x^2+1/2*d
^3*b*c^2+1/4*c^6*d^3*b*arcsinh(c*x)*x^4+3/2*c^4*d^3*b*arcsinh(c*x)*x^2-1/2*
d^3*b*arcsinh(c*x)/x^2+21/32*b*c^2*d^3*arcsinh(c*x)-1/16*c^5*d^3*b*x^3*(c^2
*x^2+1)^(1/2)-21/32*b*c^3*d^3*x*(c^2*x^2+1)^(1/2)-3/2*c^2*d^3*b*arcsinh(c*x
)^2+3*c^2*d^3*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+3*c^2*d^3*b*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))+3*c^2*d^3*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+3*
c^2*d^3*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/2*c*d^3*b/x*(c^2*x^2+1
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}ac^6d^3x^4 + \frac{3}{2}ac^4d^3x^2 + 3ac^2d^3\log(x) - \frac{1}{2}bd^3\left(\frac{\sqrt{c^2x^2+1}c}{x} + \frac{\operatorname{arsinh}(cx)}{x^2}\right) - \frac{ad^3}{2x^2} + \int bc^6d^3x^3\log\left(cx + \sqrt{c^2x^2+1}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 + 3*a*c^2*d^3*log(x) - 1/2*b*d^3*(sqr
t(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a*d^3/x^2 + integrate(b*c^6*d^
3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 +
 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ac^6d^3x^6 + 3ac^4d^3x^4 + 3ac^2d^3x^2 + ad^3 + \left(bc^6d^3x^6 + 3bc^4d^3x^4 + 3bc^2d^3x^2 + bd^3\right)\operatorname{arsinh}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))/x^3, x
```

)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a}{x^3}\,dx + \int \frac{3ac^2}{x}\,dx + \int 3ac^4x\,dx + \int ac^6x^3\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x^3}\,dx + \int \frac{3bc^2\,\mathrm{asinh}\,(cx)}{x}\,dx + \int 3bc^4x\,\mathrm{asinh}\,(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(3*a*c**4*x,
x) + Integral(a*c**6*x**3, x) + Integral(b*asinh(c*x)/x**3, x) + Integral(3
*b*c**2*asinh(c*x)/x, x) + Integral(3*b*c**4*x*asinh(c*x), x) + Integral(b*
c**6*x**3*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^3 (b\,\mathrm{arsinh}\,(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)/x^3, x)
```

3.27 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=174

$$\frac{1}{3}c^6d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{1}{9}bc^3d^3\left(c^2x\right.$$

```
[Out] (-8*b*c^3*d^3*Sqrt[1 + c^2*x^2])/3 - (b*c*d^3*Sqrt[1 + c^2*x^2])/(6*x^2) -
(b*c^3*d^3*(1 + c^2*x^2)^(3/2))/9 - (d^3*(a + b*ArcSinh[c*x]))/(3*x^3) - (3
*c^2*d^3*(a + b*ArcSinh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcSinh[c*x]) + (c^6*
d^3*x^3*(a + b*ArcSinh[c*x]))/3 - (17*b*c^3*d^3*ArcTanh[Sqrt[1 + c^2*x^2]])
/6
```

Rubi [A] time = 0.255775, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5730, 12, 1799, 1621, 897, 1153, 208}

$$\frac{1}{3}c^6d^3x^3\left(a+b\sinh^{-1}(cx)\right)+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{1}{9}bc^3d^3\left(c^2x\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```


```
[Out] (-8*b*c^3*d^3*Sqrt[1 + c^2*x^2])/3 - (b*c*d^3*Sqrt[1 + c^2*x^2])/(6*x^2) -
(b*c^3*d^3*(1 + c^2*x^2)^(3/2))/9 - (d^3*(a + b*ArcSinh[c*x]))/(3*x^3) - (3
*c^2*d^3*(a + b*ArcSinh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcSinh[c*x]) + (c^6*
d^3*x^3*(a + b*ArcSinh[c*x]))/3 - (17*b*c^3*d^3*ArcTanh[Sqrt[1 + c^2*x^2]])
/6
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
```

```
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a\right.$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a\right.$$

$$= -\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\sinh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a\right.$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2}-\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2}-\frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}-\frac{d^3\left(a+b\sinh^{-1}(c\right.}{3x^3}$$

$$= -\frac{8}{3}bc^3d^3\sqrt{1+c^2x^2}-\frac{bcd^3\sqrt{1+c^2x^2}}{6x^2}-\frac{1}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}-\frac{d^3\left(a+b\sinh^{-1}(c\right.}{3x^3}$$

Mathematica [A] time = 0.163344, size = 171, normalized size = 0.98

$$\frac{d^3\left(6ac^6x^6+54ac^4x^4-54ac^2x^2-6a-2bc^5x^5\sqrt{c^2x^2+1}-50bc^3x^3\sqrt{c^2x^2+1}-3bcx\sqrt{c^2x^2+1}+51bc^3x^3\log(x)-51bc^3x\right.}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-6*a - 54*a*c^2*x^2 + 54*a*c^4*x^4 + 6*a*c^6*x^6 - 3*b*c*x*Sqrt[1 + c
^2*x^2] - 50*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 + c^2*x^2] +
```

```
6*b*(-1 - 9*c^2*x^2 + 9*c^4*x^4 + c^6*x^6)*ArcSinh[c*x] + 51*b*c^3*x^3*Log[
x] - 51*b*c^3*x^3*Log[1 + Sqrt[1 + c^2*x^2]]))/(18*x^3)
```

Maple [A] time = 0.01, size = 155, normalized size = 0.9

$$c^3\left(d^3a\left(\frac{c^3x^3}{3}+3\,cx-3\,\frac{1}{cx}-\frac{1}{3\,c^3x^3}\right)+d^3b\left(\frac{\mathrm{Arcsinh}\,(cx)\,c^3x^3}{3}+3\,\mathrm{Arcsinh}\,(cx)\,cx-3\,\frac{\mathrm{Arcsinh}\,(cx)}{cx}-\frac{\mathrm{Arcsinh}\,(cx)}{3\,c^3x^3}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] c^3*(d^3*a*(1/3*c^3*x^3+3*c*x-3/c/x-1/3/c^3/x^3)+d^3*b*(1/3*arcsinh(c*x)*c^
3*x^3+3*arcsinh(c*x)*c*x-3*arcsinh(c*x)/c/x-1/3*arcsinh(c*x)/c^3/x^3-1/9*c^
2*x^2*(c^2*x^2+1)^(1/2)-25/9*(c^2*x^2+1)^(1/2)-17/6*arctanh(1/(c^2*x^2+1)^(
1/2))-1/6/c^2/x^2*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.13797, size = 286, normalized size = 1.64

$$\frac{1}{3}\,ac^6d^3x^3+\frac{1}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x+3\left(cx\,\mathrm{arsinh}\,(cx)-\sqrt{c^2x^2+1}\right)bc^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^6*d^3*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*c^3*d^3 - 3*(c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcs
inh(c*x)/x)*b*c^2*d^3 + 1/6*((c^2*arcsinh(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*
x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*b*d^3 - 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Fricas [A] time = 3.07185, size = 624, normalized size = 3.59

$$6\,ac^6d^3x^6+54\,ac^4d^3x^4-51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}+1\right)+51\,bc^3d^3x^3\log\left(-cx+\sqrt{c^2x^2+1}-1\right)-54\,ac^2d^3x^2-6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^6*d^3*x^6 + 54*a*c^4*d^3*x^4 - 51*b*c^3*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1) + 1) + 51*b*c^3*d^3*x^3*log(-c*x + sqrt(c^2*x^2 + 1) - 1) - 5
4*a*c^2*d^3*x^2 - 6*(b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3*log(-c*x + sqrt
(c^2*x^2 + 1)) - 6*a*d^3 + 6*(b*c^6*d^3*x^6 + 9*b*c^4*d^3*x^4 - 9*b*c^2*d^3
*x^2 - (b*c^6 + 9*b*c^4 - 9*b*c^2 - b)*d^3*x^3 - b*d^3)*log(c*x + sqrt(c^2*
x^2 + 1)) - (2*b*c^5*d^3*x^5 + 50*b*c^3*d^3*x^3 + 3*b*c*d^3*x)*sqrt(c^2*x^2
 + 1))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \frac{3ac^2}{x^2}\,dx + \int ac^6x^2\,dx + \int 3bc^4\operatorname{asinh}(cx)\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{3bc^2\operatorname{asinh}(cx)}{x^2}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(3*a*c**4, x) + Integral(a/x**4, x) + Integral(3*a*c**2/x**2,
 x) + Integral(a*c**6*x**2, x) + Integral(3*b*c**4*asinh(c*x), x) + Integra
l(b*asinh(c*x)/x**4, x) + Integral(3*b*c**2*asinh(c*x)/x**2, x) + Integral(
b*c**6*x**2*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^3(b\operatorname{arsinh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)/x^4, x)
```

3.28 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=156

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{2\tan^{-1}\left(e^{\dots}\right.}{}$$

```
[Out] (4*b*Sqrt[1 + c^2*x^2])/(3*c^5*d) - (b*(1 + c^2*x^2)^(3/2))/(9*c^5*d) - (x*
(a + b*ArcSinh[c*x]))/(c^4*d) + (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*d) + (2*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d) - (I*b*PolyLog[2, (-I)*
E^ArcSinh[c*x]])/(c^5*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d)
```

Rubi [A] time = 0.240951, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5767, 5693, 4180, 2279, 2391, 261, 266, 43}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^5d}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{2\tan^{-1}\left(e^{\dots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (4*b*Sqrt[1 + c^2*x^2])/(3*c^5*d) - (b*(1 + c^2*x^2)^(3/2))/(9*c^5*d) - (x*
(a + b*ArcSinh[c*x]))/(c^4*d) + (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*d) + (2*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d) - (I*b*PolyLog[2, (-I)*
E^ArcSinh[c*x]])/(c^5*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d)
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} - \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^3}{\sqrt{1+c^2x^2}}\,dx}{3cd}$$

$$= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{c^4} + \frac{b\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{c^3d} - \frac{b\,S}{}$$

$$= \frac{b\sqrt{1+c^2x^2}}{c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{\text{Subst}\left(\int(a+bx)\mathbf{sech}(x)\,d\right.}{c^5d}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b\left(1+c^2x^2\right)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{2\left(a+b\right.}{}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b\left(1+c^2x^2\right)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{2\left(a+b\right.}{}$$

$$= \frac{4b\sqrt{1+c^2x^2}}{3c^5d} - \frac{b\left(1+c^2x^2\right)^{3/2}}{9c^5d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \frac{2\left(a+b\right.}{}$$

Mathematica [A] time = 0.231707, size = 170, normalized size = 1.09

$$\frac{-9ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+9ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+3ac^3x^3-9acx+9a\tan^{-1}(cx)-bc^2x^2\sqrt{c^2x^2+1}+11b\sqrt{c}}{9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (-9*a*c*x + 3*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] - b*c^2*x^2*Sqrt[1 + c^2*x
^2] - 9*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x] + (
9*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*b*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (9*I)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*Pol
yLog[2, I*E^ArcSinh[c*x]])/(9*c^5*d)
```

Maple [A] time = 0.158, size = 266, normalized size = 1.7

$$\frac{ax^3}{3\,c^2d} - \frac{ax}{c^4d} + \frac{a\arctan(cx)}{c^5d} + \frac{b\text{Arcsinh}(cx)\,x^3}{3\,c^2d} - \frac{b\text{Arcsinh}(cx)\,x}{c^4d} + \frac{b\text{Arcsinh}(cx)\arctan(cx)}{c^5d} - \frac{bx^2}{9\,c^3d}\sqrt{c^2x^2+1} + \frac{}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/3/c^2*a/d*x^3-1/c^4*a/d*x+1/c^5*a/d*arctan(c*x)+1/3/c^2*b/d*arcsinh(c*x)*
x^3-1/c^4*b/d*arcsinh(c*x)*x+1/c^5*b/d*arcsinh(c*x)*arctan(c*x)-1/9/c^3*b/d
*x^2*(c^2*x^2+1)^(1/2)+11/9*b*(c^2*x^2+1)^(1/2)/c^5/d+1/c^5*b/d*arctan(c*x)
*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/c^5*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-I/c^5*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^5*
b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a\left(\frac{c^2x^3-3x}{c^4d}+\frac{3\arctan(cx)}{c^5d}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + b*integrate(x^4*l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4\,\text{arsinh}(cx)+ax^4}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{ax^4}{c^2x^2+1}\,dx+\int\frac{bx^4\,\text{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2 + 1), x) + Integral(b*x**4*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d), x)
```

3.29 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=135

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}-$$

```
[Out] -(b*x*Sqrt[1 + c^2*x^2])/(4*c^3*d) + (b*ArcSinh[c*x])/(4*c^4*d) + (x^2*(a +
 b*ArcSinh[c*x]))/(2*c^2*d) + (a + b*ArcSinh[c*x])^2/(2*b*c^4*d) - ((a + b*
ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d) - (b*PolyLog[2, -E^(2*Ar
cSinh[c*x])])/(2*c^4*d)
```

Rubi [A] time = 0.195492, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5767, 5714, 3718, 2190, 2279, 2391, 321, 215}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -(b*x*Sqrt[1 + c^2*x^2])/(4*c^3*d) + (b*ArcSinh[c*x])/(4*c^4*d) + (x^2*(a +
 b*ArcSinh[c*x]))/(2*c^2*d) + (a + b*ArcSinh[c*x])^2/(2*b*c^4*d) - ((a + b*
ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d) - (b*PolyLog[2, -E^(2*Ar
cSinh[c*x])])/(2*c^4*d)
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2}{\sqrt{1+c^2x^2}}\,dx}{2cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} - \frac{\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^4d} + \frac{b}{}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d} - \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d} - \frac{\left(a+b\sinh\right.}{}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d} - \frac{\left(a+b\sinh\right.}{}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{4c^3d} + \frac{b\sinh^{-1}(cx)}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d} - \frac{\left(a+b\sinh\right.}{}$$

Mathematica [A] time = 0.201929, size = 181, normalized size = 1.34

$$-\frac{4b\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+4b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)-2ac^2x^2+2a\log\left(c^2x^2+1\right)+bcx\sqrt{c^2x^2+1}-2bc^2x^2\sinh^{-1}}{4c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -(-2*a*c^2*x^2 + b*c*x*Sqrt[1 + c^2*x^2] - b*ArcSinh[c*x] - 2*b*c^2*x^2*Arc
Sinh[c*x] - 2*b*ArcSinh[c*x]^2 + 4*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + 4*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2
*a*Log[1 + c^2*x^2] + 4*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*P
olyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(4*c^4*d)
```

Maple [A] time = 0.081, size = 161, normalized size = 1.2

$$\frac{ax^2}{2c^2d} - \frac{a\ln\left(c^2x^2+1\right)}{2c^4d} + \frac{b\left(\text{Arcsinh}\,(cx)\right)^2}{2c^4d} + \frac{b\text{Arcsinh}\,(cx)\,x^2}{2c^2d} - \frac{bx}{4c^3d}\sqrt{c^2x^2+1} + \frac{b\text{Arcsinh}\,(cx)}{4c^4d} - \frac{b\text{Arcsinh}\,(cx)}{c^4d}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a/d*x^2-1/2/c^4*a/d*ln(c^2*x^2+1)+1/2/c^4*b/d*arcsinh(c*x)^2+1/2/c^
2*b/d*arcsinh(c*x)*x^2-1/4*b*x*(c^2*x^2+1)^(1/2)/c^3/d+1/4*b*arcsinh(c*x)/c
^4/d-1/c^4*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{x^2}{c^2d} - \frac{\log\left(c^2x^2+1\right)}{c^4d}\right) - \frac{1}{8}b\left(\frac{2\,c^2x^2 - \log\left(c^2x^2+1\right)^2 - 4\left(c^2x^2 - \log\left(c^2x^2+1\right)\right)\log\left(cx+\sqrt{c^2x^2+1}\right) - 2\,\log\left(c^2x^2\right.}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) - 1/8*b*((2*c^2*x^2 - log(c^
2*x^2 + 1)^2 - 4*(c^2*x^2 - log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 2*log(c^2*x^2 + 1))/(c^4*d) - 8*integrate(-1/2*(c^2*x^2 - log(c^2*x^2 + 1
))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqrt(c^2*x^2 + 1)), x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3\,\text{arsinh}\,(cx) + ax^3}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^2x^2+1}\,dx + \int \frac{bx^3\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2 + 1), x) + Integral(b*x**3*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)x^3}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(c^2*d*x^2 + d), x)
```

3.30 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=108

$$\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{c^3d}$$

```
[Out] -((b*Sqrt[1 + c^2*x^2])/(c^3*d)) + (x*(a + b*ArcSinh[c*x]))/(c^2*d) - (2*(a
 + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d) + (I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]])/(c^3*d) - (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rubi [A] time = 0.139653, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5767, 5693, 4180, 2279, 2391, 261}

$$\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^3d}-\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^3d}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```


```
[Out] -((b*Sqrt[1 + c^2*x^2])/(c^3*d)) + (x*(a + b*ArcSinh[c*x]))/(c^2*d) - (2*(a
 + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d) + (I*b*PolyLog[2, (-I)*E
^ArcSinh[c*x]])/(c^3*d) - (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d)
```


Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^3d}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} + \frac{(ib)\,\text{Sub}}{}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} + \frac{(ib)\,\text{Sub}}{}$$

$$= -\frac{b\sqrt{1+c^2x^2}}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d} + \frac{ib\mathrm{Li}_2\left(-\right.}{c}$$

Mathematica [A] time = 0.155034, size = 121, normalized size = 1.12

$$\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) - ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) + acx - a\tan^{-1}(cx) - b\sqrt{c^2x^2+1} + bcx\sinh^{-1}(cx) - ib\sinh^{-1}}{c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] (a*c*x - b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - I*b*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] + I*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d)
```

Maple [A] time = 0.009, size = 215, normalized size = 2.

$$\frac{ax}{c^2d} - \frac{a\arctan(cx)}{c^3d} - \frac{b\arctan(cx)}{c^3d}\ln\left(1+i(1+icx)\frac{1}{\sqrt{c^2x^2+1}}\right) + \frac{b\arctan(cx)}{c^3d}\ln\left(1-i(1+icx)\frac{1}{\sqrt{c^2x^2+1}}\right) + \frac{ib}{c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/c^2*a/d*x-1/c^3*a/d*arctan(c*x)-1/c^3*b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c
^2*x^2+1)^(1/2))+1/c^3*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
```

```
I/c^3*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^3*b/d*dilog(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-1/c^3*b/d*arcsinh(c*x)*arctan(c*x)+1/c^2*b/d*arcsinh
(c*x)*x-b*(c^2*x^2+1)^(1/2)/c^3/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{x}{c^2d} - \frac{\arctan(cx)}{c^3d}\right) + b\int \frac{x^2\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^
2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\,\text{arsinh}(cx) + ax^2}{c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 1), x) + Integral(b*x**2*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^2}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d), x)
```

3.31 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=73

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}$$

```
[Out] -(a + b*ArcSinh[c*x])^2/(2*b*c^2*d) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*Ar
cSinh[c*x])])/(c^2*d) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rubi [A] time = 0.117086, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5714, 3718, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^2/(2*b*c^2*d) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*Ar
cSinh[c*x])])/(c^2*d) + (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx &= \frac{\operatorname{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}-\frac{b\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\right.}{c^2d}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}-\frac{b\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.}{2c^2d}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^2d}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d}+\frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}
\end{aligned}$$

Mathematica [B] time = 0.0687814, size = 167, normalized size = 2.29

$$\frac{b\operatorname{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d}+\frac{b\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{c^2d}+\frac{a\log\left(c^2x^2+1\right)}{2c^2d}-\frac{b\sinh^{-1}(cx)^2}{2c^2d}+\frac{b\sinh^{-1}(cx)\log\left(1\right.}{c^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] -(b*ArcSinh[c*x]^2)/(2*c^2*d) + (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^ArcSi
nh[c*x])/c])/(c^2*d) + (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/
c])/(c^2*d) + (a*Log[1 + c^2*x^2])/(2*c^2*d) + (b*PolyLog[2, -((Sqrt[-c^2]*
E^ArcSinh[c*x])/c)])/(c^2*d) + (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c]
)/(c^2*d)
```

Maple [A] time = 0.033, size = 98, normalized size = 1.3

$$\frac{a\ln\left(c^2x^2+1\right)}{2\,c^2d}-\frac{b\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{2\,c^2d}+\frac{b\mathrm{Arcsinh}\left(cx\right)}{c^2d}\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right)+\frac{b}{2\,c^2d}\mathrm{polylog}\left(2,-\left(cx+\sqrt{c^2x^2+1}\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a/d*ln(c^2*x^2+1)-1/2/c^2*b/d*arcsinh(c*x)^2+1/c^2*b/d*arcsinh(c*x)
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)/c^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\,b\left(\frac{\log\left(c^2x^2+1\right)^2-4\,\log\left(c^2x^2+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2d}+8\int\frac{\log\left(c^2x^2+1\right)}{2\left(c^4dx^3+c^2dx+\left(c^3dx^2+cd\right)\sqrt{c^2x^2+1}\right)}\,dx\right)+\frac{a\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*b*((log(c^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1
)))/(c^2*d) + 8*integrate(1/2*log(c^2*x^2 + 1)/(c^4*d*x^3 + c^2*d*x + (c^3*
d*x^2 + c*d)*sqrt(c^2*x^2 + 1)), x)) + 1/2*a*log(c^2*d*x^2 + d)/(c^2*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx\,\mathrm{arsinh}\left(cx\right)+ax}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arcsinh(c*x) + a*x)/(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{c^2x^2+1}\,dx + \int \frac{bx\,\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x/(c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(c**2*x**2 + 1)
, x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\operatorname{arsinh}(cx) + a)x}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d), x)
```

3.32 $\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=70

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}$$

```
[Out] (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d) - (I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]])/(c*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

Rubi [A] time = 0.0641422, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5693, 4180, 2279, 2391}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{cd}+\frac{2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d) - (I*b*PolyLog[2, (-I
)*E^ArcSinh[c*x]])/(c*d) + (I*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(ib)\,\text{Subst}\left(\int \log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(cx)\right)}{cd} + \frac{(ib)\,\text{S}}{}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{\sinh^{-1}(cx)}\right)}{cd} + \frac{(ib)\,\text{Subst}}{}$$
$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{ib\text{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{ib\text{Li}_2\left(ie^{\sinh^{-1}(cx)}\right)}{cd}$$

Mathematica [A] time = 0.103496, size = 135, normalized size = 1.93

$$-\frac{c\left(bc\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)-bc\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)+a\sqrt{-c^2}\tan^{-1}(cx)-bc\sinh^{-1}(cx)\log\left(\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}+1\right)+\right.}{\left(-c^2\right)^{3/2}d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a*Sqrt[-c^2]*ArcTan[c*x] - b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b
*c*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*c*PolyLog[2, (Sqrt[-c^2]*E
^ArcSinh[c*x])/c]))/((-c^2)^(3/2)*d))
```

Maple [A] time = 0.036, size = 171, normalized size = 2.4

$$\frac{a\arctan(cx)}{cd}+\frac{b\operatorname{Arcsinh}(cx)\arctan(cx)}{cd}-\frac{b\arctan(cx)}{cd}\ln\left(1-i(1+icx)\frac{1}{\sqrt{c^2x^2+1}}\right)+\frac{b\arctan(cx)}{cd}\ln\left(1+i(1+ic\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] 1/c*a/d*arctan(c*x)+1/c*b/d*arcsinh(c*x)*arctan(c*x)-1/c*b/d*arctan(c*x)*ln
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/c*b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))+I/c*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c*b/d*dilog
(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx+\frac{a\arctan(cx)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x) + a*arctan(c*x
)/(c*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{arsinh}(cx)+a}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^2+1}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**2*x**2 + 1), x)
)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d), x)
```

3.33 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=61

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}$$

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rubi [A] time = 0.116704, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5720, 5461, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```



```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```


Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx = \frac{\text{Subst}\left(\int (a+bx)\operatorname{csch}(x)\operatorname{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int (a+bx)\operatorname{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{b\,\text{S}}{}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{2\sinh^{-1}(cx)}\right)}{2d} + \frac{b\,\text{Subst}}{}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\text{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{2d} + \frac{b\text{Li}_2\left(e^{2\sinh^{-1}(cx)}\right)}{2d}$$

Mathematica [B] time = 0.0903523, size = 207, normalized size = 3.39

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d} - \frac{b\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)}{d} + \frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d} - \frac{a\log\left(c^2x^2+1\right)}{2d} - \frac{a\sinh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -((a*ArcSinh[c*x])/d) - (b*ArcSinh[c*x]*Log[1 - (Sqrt[-c^2]*E^ArcSinh[c*x])
/c])/d - (b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (a*Log
[1 - E^(2*ArcSinh[c*x])])/d + (b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])])/
d - (a*Log[1 + c^2*x^2])/(2*d) - (b*PolyLog[2, -((Sqrt[-c^2]*E^ArcSinh[c*x]
)/c)])/d - (b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/d + (b*PolyLog[2,
E^(2*ArcSinh[c*x])])/(2*d)
```

Maple [A] time = 0.042, size = 74, normalized size = 1.2

$$\frac{a\ln(cx)}{d} - \frac{a\ln\left(c^2x^2+1\right)}{2d} + \frac{b}{d}\mathrm{dilog}\left(\left(cx+\sqrt{c^2x^2+1}\right)^{-2}\right) - \frac{b}{4d}\mathrm{dilog}\left(\left(cx+\sqrt{c^2x^2+1}\right)^{-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x)
```

```
[Out] a/d*ln(c*x)-1/2*a/d*ln(c^2*x^2+1)+b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^2)-1/
4*b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{\log\left(c^2x^2+1\right)}{d} - \frac{2\log(x)}{d}\right) + b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c^2*x
^2 + 1))/(c^2*d*x^3 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}(cx)+a}{c^2dx^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^3+x}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^2x^3+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**2*x**3 + x), x)
)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\left(c^2dx^2 + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x), x)
```

3.34 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=101

$$\frac{ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{a+b\sinh^{-1}(cx)}{dx}-\frac{2c\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}$$

```
[Out] -((a + b*ArcSinh[c*x])/(d*x)) - (2*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d + (I*b*c*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/d - (I*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rubi [A] time = 0.149639, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5747, 5693, 4180, 2279, 2391, 266, 63, 208}

$$\frac{ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{d}-\frac{a+b\sinh^{-1}(cx)}{dx}-\frac{2c\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x)) - (2*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d + (I*b*c*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/d - (I*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx} - c^2\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\int \frac{1}{x\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{c\,\mathrm{Subst}\left(\int(a+bx)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{(bc)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx, \right.}{2d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c\right.}{cd}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)}{d} + \frac{(ibc)\,\mathrm{S}}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{bc\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)}{d} + \frac{ibc\mathbf{Li}_2}{}$$

Mathematica [A] time = 0.162546, size = 182, normalized size = 1.8

$$-\frac{-b\sqrt{-c^2}x\mathrm{PolyLog}\left(2, \frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + b\sqrt{-c^2}x\mathrm{PolyLog}\left(2, \frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right) + acx\tan^{-1}(cx) + a + bcx\tanh^{-1}\left(\sqrt{c^2x^2+1}\right) +}{dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x] + a*c*x*ArcTan[c*x] + b*c*x*ArcTanh[Sqrt[1 + c^2*x^2]
] + b*Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - b*
Sqrt[-c^2]*x*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*Sqrt[-
c^2]*x*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + b*Sqrt[-c^2]*x*PolyLog[2
, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(d*x))
```

Maple [A] time = 0.013, size = 202, normalized size = 2.

$$-\frac{a}{dx} - \frac{ca\arctan(cx)}{d} - \frac{b\mathrm{Arcsinh}(cx)}{dx} - \frac{bc\mathrm{Arcsinh}(cx)\arctan(cx)}{d} - \frac{bc}{d}\mathrm{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right) - \frac{bc\arctan(cx)}{d}\ln\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x)
```

```
[Out] -a/d/x-c*a/d*arctan(c*x)-b/d*arcsinh(c*x)/x-c*b/d*arcsinh(c*x)*arctan(c*x)-
c*b/d*arctanh(1/(c^2*x^2+1)^(1/2))-c*b/d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+c*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*c*b/d
*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*c*b/d*dilog(1-I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{c\arctan(cx)}{d}+\frac{1}{dx}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^4+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(c*arctan(c*x)/d + 1/(d*x)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^4 + d*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}(cx)+a}{c^2dx^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^4 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^2x^4+x^2}\,dx+\int\frac{b\,\mathrm{asinh}(cx)}{c^2x^4+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c**2*x**4 + x**
2), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^2), x)
```

3.35 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)}\,dx$

Optimal. Leaf size=113

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{2c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{a+b\sinh}{2dx}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*d*x) - (a + b*ArcSinh[c*x])/(2*d*x^2) + (2*c^2*
(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d + (b*c^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rubi [A] time = 0.198218, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5747, 5720, 5461, 4182, 2279, 2391, 264}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d}+\frac{2c^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{a+b\sinh}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*d*x) - (a + b*ArcSinh[c*x])/(2*d*x^2) + (2*c^2*
(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d + (b*c^2*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2} - c^2\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{1}{x^2\sqrt{1+c^2x^2}}\,dx}{2d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} - \frac{c^2\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} - \frac{\left(2c^2\right)\text{Subst}\left(\int (a+bx)\mathbf{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{\left(bc^2\right)\text{Subs}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{\left(bc^2\right)\text{Subs}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx} - \frac{a+b\sinh^{-1}(cx)}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{bc^2\mathbf{Li}_2\left(-e\right.}{2}$$

Mathematica [B] time = 0.272453, size = 240, normalized size = 2.12

$$\frac{-c^2\left(b\text{PolyLog}\left(2, e^{2\sinh^{-1}(cx)}\right) + 2\log\left(1-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)\right) + 2bc^2\text{PolyLog}\left(2, \frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2bc^2\text{PolyL}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] (-((b*c*Sqrt[1 + c^2*x^2])/x) - b*c^2*ArcSinh[c*x]^2 - (a + b*ArcSinh[c*x])
/x^2 + (c^2*(a + b*ArcSinh[c*x])^2)/b + 2*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSi
nh[c*x])/c] + a*c^2*Log[1 + c^2*x^2] + 2*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + 2*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - c^2*(2*
(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSin
h[c*x])]))/(2*d)
```

Maple [A] time = 0.097, size = 266, normalized size = 2.4

$$-\frac{a}{2\,dx^2} - \frac{c^2a\ln(cx)}{d} + \frac{c^2a\ln\left(c^2x^2+1\right)}{2\,d} - \frac{bc}{2\,dx}\sqrt{c^2x^2+1} + \frac{c^2b}{2\,d} - \frac{b\text{Arcsinh}(cx)}{2\,dx^2} + \frac{c^2b\text{Arcsinh}(cx)}{d}\ln\left(1+\left(cx+\sqrt{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x)
```

```
[Out] -1/2*a/d/x^2-c^2*a/d*ln(c*x)+1/2*c^2*a/d*ln(c^2*x^2+1)-1/2*b*c*(c^2*x^2+1)^
(1/2)/d/x+1/2*c^2*b/d-1/2*b/d*arcsinh(c*x)/x^2+c^2*b/d*arcsinh(c*x)*ln(1+(c
*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d-
c^2*b/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-c^2*b/d*polylog(2,-c*x-(c^
2*x^2+1)^(1/2))-c^2*b/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-c^2*b/d*po
lylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(\frac{c^2\log\left(c^2x^2+1\right)}{d}-\frac{2\,c^2\log\left(x\right)}{d}-\frac{1}{dx^2}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^5+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arsinh}\left(cx\right)+a}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^2x^5+x^3}\,dx+\int\frac{b\,\text{asinh}\left(cx\right)}{c^2x^5+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c**2*x**5 + x**
3), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(c^2dx^2 + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^3), x)
```

3.36 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)}\,dx$

Optimal. Leaf size=156

$$-\frac{ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}+\frac{2c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sin\right.}{d}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*d*x^2) - (a + b*ArcSinh[c*x])/(3*d*x^3) + (c^2*
(a + b*ArcSinh[c*x]))/(d*x) + (2*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d + (7*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d) - (I*b*c^3*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/d + (I*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d
```

Rubi [A] time = 0.246556, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5747, 5693, 4180, 2279, 2391, 266, 63, 208, 51}

$$-\frac{ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{d}+\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx}+\frac{2c^3\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sin\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```



```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*d*x^2) - (a + b*ArcSinh[c*x])/(3*d*x^3) + (c^2*
(a + b*ArcSinh[c*x]))/(d*x) + (2*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[
c*x]])/d + (7*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d) - (I*b*c^3*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/d + (I*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3} - c^2\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{1}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + c^4\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+c^2x}}\right.}{6d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{c^3\,\text{Subst}\left(\int (a+bx)\,\mathbf{sech}(x)\,dx, x, \right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2} - \frac{a+b\sinh^{-1}(cx)}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{2c^3\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh}\right.}{d}$$

Mathematica [A] time = 0.213495, size = 247, normalized size = 1.58

$$\frac{6b\left(-c^2\right)^{3/2}x^3\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 6b\left(-c^2\right)^{3/2}x^3\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right) + 6ac^2x^2 + 6ac^3x^3\tan^{-1}(cx) - 2a - bcx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + 6*b*c^2*
x^2*ArcSinh[c*x] + 6*a*c^3*x^3*ArcTan[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 + c
```

```
^2*x^2]] - 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqr
t[-c^2]] + 6*b*(-c^2)^(3/2)*x^3*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[
c*x])/c] + 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] -
 6*b*(-c^2)^(3/2)*x^3*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*d*x^3)
```

Maple [A] time = 0.016, size = 261, normalized size = 1.7

$$-\frac{a}{3\,d{x}^{3}}+\frac{{c}^{2}a}{dx}+\frac{{c}^{3}a\arctan \left(cx \right)}{d}-\frac{b{\rm Arcsinh} \left(cx \right)}{3\,d{x}^{3}}+\frac{{c}^{2}b{\rm Arcsinh} \left(cx \right)}{dx}+\frac{b{c}^{3}{\rm Arcsinh} \left(cx \right)\arctan \left(cx \right)}{d}-\frac{bc}{6\,d{x}^{2}}\sqrt{{c}^{2}{x}^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x)
```

```
[Out] -1/3*a/d/x^3+c^2*a/d/x+c^3*a/d*arctan(c*x)-1/3*b/d*arcsinh(c*x)/x^3+c^2*b/d
*arcsinh(c*x)/x+c^3*b/d*arcsinh(c*x)*arctan(c*x)-1/6*b*c*(c^2*x^2+1)^(1/2)/
d/x^2+7/6*c^3*b/d*arctanh(1/(c^2*x^2+1)^(1/2))+c^3*b/d*arctan(c*x)*ln(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-c^3*b/d*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-I*c^3*b/d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*c^3*b/d*dilog(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(\frac{3\,c^{3}\arctan\left(cx\right)}{d}+\frac{3\,c^{2}x^{2}-1}{dx^{3}}\right)a+b\int\frac{\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{c^{2}dx^{6}+dx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a + b*integrate(log(c*x
 + sqrt(c^2*x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{b\operatorname{arsinh}\left(cx\right)+a}{c^{2}dx^{6}+dx^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^2*d*x^6 + d*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^6+x^4}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^6+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a/(c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c**2*x**6 + x**
4), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{(c^2dx^2+d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)*x^4), x)
```

3.37 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=171

$$\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\tan^{-1}}{}$$

```
[Out] b/(2*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[1 + c^2*x^2])/(c^5*d^2) + (3*x*(a
 + b*ArcSinh[c*x]))/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
+ c^2*x^2)) - (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) + (
((3*I)/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^2) - (((3*I)/2)*b*PolyL
og[2, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rubi [A] time = 0.240776, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5751, 5767, 5693, 4180, 2279, 2391, 261, 266, 43}

$$\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^5d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{3\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[1 + c^2*x^2])/(c^5*d^2) + (3*x*(a
 + b*ArcSinh[c*x]))/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1
+ c^2*x^2)) - (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) + (
((3*I)/2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^2) - (((3*I)/2)*b*PolyL
og[2, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
```

)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1]

Rule 5767

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_ .)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c ^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2 *d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]

Rule 5693

Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]

Rule 4180

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2 , -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n) ^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&

```
NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= \frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{(3b)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx}{2c^3d^2}+\frac{b\,\text{Subst}\left(\int \frac{x}{\left(1+c^2x\right)^{3/2}}\right.}{4cd^2}$$

$$= -\frac{3b\sqrt{1+c^2x^2}}{2c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\,\text{Subst}\left(\int(a+bx)\text{sec}\right.}{2c^5}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\left(a+b\right.}{}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\left(a+b\right.}{}$$

$$= \frac{b}{2c^5d^2\sqrt{1+c^2x^2}}-\frac{b\sqrt{1+c^2x^2}}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{3\left(a+b\right.}{}$$

Mathematica [A] time = 0.339399, size = 268, normalized size = 1.57

$$\frac{3ib\left(c^2x^2+1\right)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-3ib\left(c^2x^2+1\right)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+2ac^3x^3-3ac^2x^2\tan^{-1}(cx)+3acx-3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (3*a*c*x + 2*a*c^3*x^3 - b*Sqrt[1 + c^2*x^2] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^2
] + 3*b*c*x*ArcSinh[c*x] + 2*b*c^3*x^3*ArcSinh[c*x] - 3*a*ArcTan[c*x] - 3*a
*c^2*x^2*ArcTan[c*x] - (3*I)*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*
I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)*b*ArcSinh[c*x]*
Log[1 + I*E^ArcSinh[c*x]] + (3*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] + (3*I)*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (3*I)*b*(
1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^5*d^2*(1 + c^2*x^2))
```

Maple [A] time = 0.016, size = 285, normalized size = 1.7

$$\frac{ax}{c^4d^2}+\frac{ax}{2\,c^4d^2\left(c^2x^2+1\right)}-\frac{3\,a\arctan(cx)}{2\,c^5d^2}+\frac{b\text{Arcsinh}(cx)\,x}{c^4d^2}+\frac{b\text{Arcsinh}(cx)\,x}{2\,c^4d^2\left(c^2x^2+1\right)}-\frac{3\,b\text{Arcsinh}(cx)\arctan(cx)}{2\,c^5d^2}-\frac{bx^2}{c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^4*a/d^2*x+1/2/c^4*a/d^2*x/(c^2*x^2+1)-3/2/c^5*a/d^2*arctan(c*x)+1/c^4*b
/d^2*arcsinh(c*x)*x+1/2/c^4*b/d^2*arcsinh(c*x)*x/(c^2*x^2+1)-3/2/c^5*b/d^2*
arcsinh(c*x)*arctan(c*x)-1/c^3*b/d^2*x^2/(c^2*x^2+1)^(1/2)-1/2*b/c^5/d^2/(c
^2*x^2+1)^(1/2)-3/2/c^5*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))+3/2/c^5*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I/c^5*
b/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I/c^5*b/d^2*dilog(1-I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{x}{c^6d^2x^2+c^4d^2}+\frac{2\,x}{c^4d^2}-\frac{3\,\arctan(cx)}{c^5d^2}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2))
 + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^
2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arsinh}(cx) + ax^4}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^4}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^4 \operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**4*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^2, x)
```

3.38 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=145

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-$$

```
[Out] -(b*x)/(2*c^3*d^2*Sqrt[1 + c^2*x^2]) + (b*ArcSinh[c*x])/(2*c^4*d^2) - (x^2*
(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2
*b*c^4*d^2) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d^2)
+ (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rubi [A] time = 0.190549, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 5714, 3718, 2190, 2279, 2391, 288, 215}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```


```
[Out] -(b*x)/(2*c^3*d^2*Sqrt[1 + c^2*x^2]) + (b*ArcSinh[c*x])/(2*c^4*d^2) - (x^2*
(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2
*b*c^4*d^2) + ((a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^4*d^2)
+ (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

```
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx=-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}+\frac{\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{c^2d}$$

$$=-\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^4d^2}$$

$$=-\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{2\,\text{Subs}}{}$$

$$=-\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\left(a+b\right.}{}$$

$$=-\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\left(a+b\right.}{}$$

$$=-\frac{bx}{2c^3d^2\sqrt{1+c^2x^2}}+\frac{b\sinh^{-1}(cx)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\left(a+b\right.}{}$$

Mathematica [C] time = 0.218747, size = 241, normalized size = 1.66

$$\frac{2b\left(c^2x^2+1\right)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+2b\left(c^2x^2+1\right)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+ac^2x^2\log\left(c^2x^2+1\right)+a\log\left(c^2x^2+1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a - b*c*x*Sqrt[1 + c^2*x^2] + b*ArcSinh[c*x] - b*ArcSinh[c*x]^2 - b*c^2*x^
2*ArcSinh[c*x]^2 + 2*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*c^2*x^2
*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*b*ArcSinh[c*x]*Log[1 + I*E^ArcS
inh[c*x]] + 2*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + a*Log[1 +
c^2*x^2] + a*c^2*x^2*Log[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)*PolyLog[2, (-I)*E
^ArcSinh[c*x]] + 2*b*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]])/(2*c^4*d^2
```

```
*(1 + c^2*x^2))
```

Maple [A] time = 0.141, size = 206, normalized size = 1.4

$$\frac{a}{2\,c^4d^2\left(c^2x^2+1\right)}+\frac{a\ln\left(c^2x^2+1\right)}{2\,c^4d^2}-\frac{b\left(\operatorname{Arcsinh}\left(cx\right)\right)^2}{2\,c^4d^2}-\frac{bx}{2\,c^3d^2}\frac{1}{\sqrt{c^2x^2+1}}+\frac{bx^2}{2\,c^2d^2\left(c^2x^2+1\right)}+\frac{b\operatorname{Arcsinh}\left(cx\right)}{2\,c^4d^2\left(c^2x^2+1\right)}+\frac{}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/2/c^4*a/d^2/(c^2*x^2+1)+1/2/c^4*a/d^2*ln(c^2*x^2+1)-1/2/c^4*b/d^2*arcsinh
(c*x)^2-1/2*b*x/c^3/d^2/(c^2*x^2+1)^(1/2)+1/2/c^2*b/d^2*x^2/(c^2*x^2+1)+1/2
/c^4*b/d^2*arcsinh(c*x)/(c^2*x^2+1)+1/2/c^4*b/d^2/(c^2*x^2+1)+1/c^4*b/d^2*a
rcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c^2*x^2+
1)^(1/2))^2)/c^4/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\,b\left(\frac{\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)^2-4\left(\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-2}{c^6d^2x^2+c^4d^2}+8\int\frac{(}{2\left(c^8d^2x^5+2\,c^6d^2x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - 4*((c^2*x^2 + 1)*log(c^2*x^2 +
1) + 1)*log(c*x + sqrt(c^2*x^2 + 1)) - 2)/(c^6*d^2*x^2 + c^4*d^2) + 8*integ
rate(1/2*((c^2*x^2 + 1)*log(c^2*x^2 + 1) + 1)/(c^8*d^2*x^5 + 2*c^6*d^2*x^3
+ c^4*d^2*x + (c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x
)) + 1/2*a*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^3\operatorname{arsinh}\left(cx\right)+ax^3}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arcsinh(c*x) + a*x^3)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^3 \operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**3*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^3}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(c^2*d*x^2 + d)^2, x)
```

3.39 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=127

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}$$

```
[Out] -b/(2*c^3*d^2*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 +
 c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d^2) - ((I/
2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + ((I/2)*b*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^3*d^2)
```

Rubi [A] time = 0.133786, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5751, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2c^3d^2}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```


```
[Out] -b/(2*c^3*d^2*Sqrt[1 + c^2*x^2]) - (x*(a + b*ArcSinh[c*x]))/(2*c^2*d^2*(1 +
 c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^3*d^2) - ((I/
2)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + ((I/2)*b*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^3*d^2)
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

```
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{2c^2d}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int(a+bx)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{2c^3d^2}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2} - \frac{(ib)\,\text{Sub}}{}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2} - \frac{(ib)\,\text{Sub}}{}$$

$$= -\frac{b}{2c^3d^2\sqrt{1+c^2x^2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2} - \frac{ib\mathbf{Li}_2\left(-\right.}{2}$$

Mathematica [A] time = 0.264338, size = 221, normalized size = 1.74

$$-\frac{ib\left(c^2x^2+1\right)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) - ib\left(c^2x^2+1\right)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) - ac^2x^2\tan^{-1}(cx) + acx - a\tan^{-1}(cx) + }{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -(a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] - a*ArcTan[c*x] - a*c^2*
x^2*ArcTan[c*x] - I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*c^2*x^2*
ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSi
nh[c*x]] + I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + I*b*(1 + c^
2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - I*b*(1 + c^2*x^2)*PolyLog[2, I*E^A
rcSinh[c*x]])/(2*c^3*d^2*(1 + c^2*x^2))
```

Maple [A] time = 0.012, size = 240, normalized size = 1.9

$$-\frac{ax}{2\,c^2d^2\left(c^2x^2+1\right)} + \frac{a\arctan(cx)}{2\,c^3d^2} - \frac{b\text{Arcsinh}(cx)\,x}{2\,c^2d^2\left(c^2x^2+1\right)} + \frac{b\text{Arcsinh}(cx)\arctan(cx)}{2\,c^3d^2} + \frac{b\arctan(cx)}{2\,c^3d^2}\ln\left(1+i(1+icx)\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2/c^2*a/d^2*x/(c^2*x^2+1)+1/2/c^3*a/d^2*arctan(c*x)-1/2/c^2*b/d^2*arcsin
h(c*x)*x/(c^2*x^2+1)+1/2/c^3*b/d^2*arcsinh(c*x)*arctan(c*x)+1/2/c^3*b/d^2*a
rctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/c^3*b/d^2*arctan(c*x)*ln
(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I/c^3*b/d^2*dilog(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+1/2*I/c^3*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*b/
c^3/d^2/(c^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{x}{c^4d^2x^2+c^2d^2}-\frac{\arctan\left(cx\right)}{c^3d^2}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + b*integrate(x^
2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arsinh}\left(cx\right)+ax^2}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**2*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^2, x)
```

3.40 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=55

$$\frac{bx}{2cd^2\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

```
[Out] (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*d^2*(1 + c^
2*x^2))
```

Rubi [A] time = 0.0496438, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5717, 191}

$$\frac{bx}{2cd^2\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(2*c^2*d^2*(1 + c^
2*x^2))
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2cd^2}$$
$$= \frac{bx}{2cd^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{2c^2d^2\left(1+c^2x^2\right)}$$

Mathematica [A] time = 0.0674566, size = 74, normalized size = 1.35

$$-\frac{a}{2c^2d^2\left(c^2x^2+1\right)} + \frac{bx}{2cd^2\sqrt{c^2x^2+1}} - \frac{b\sinh^{-1}(cx)}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] -a/(2*c^2*d^2*(1 + c^2*x^2)) + (b*x)/(2*c*d^2*Sqrt[1 + c^2*x^2]) - (b*ArcSi
nh[c*x])/(2*c^2*d^2*(1 + c^2*x^2))
```

Maple [A] time = 0.006, size = 61, normalized size = 1.1

$$\frac{1}{c^2}\left(-\frac{a}{2\,d^2\left(c^2x^2+1\right)} + \frac{b}{d^2}\left(-\frac{\mathrm{Arcsinh}\,(cx)}{2\,c^2x^2+2} + \frac{cx}{2}\frac{1}{\sqrt{c^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2+1)+b/d^2*(-1/2/(c^2*x^2+1)*arcsinh(c*x)+1/2*c*x/
(c^2*x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\,b\left(\frac{2\,\log\left(cx+\sqrt{c^2x^2+1}\right)+1}{c^4d^2x^2+c^2d^2} - 4\int \frac{1}{2\left(c^6d^2x^5+2\,c^4d^2x^3+c^2d^2x+\left(c^5d^2x^4+2\,c^3d^2x^2+cd^2\right)\sqrt{c^2x^2+1}\right)}\,dx\right) - \frac{}{2\left(c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b*((2*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^4*d^2*x^2 + c^2*d^2) - 4*in
tegrate(1/2/(c^6*d^2*x^5 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3
*d^2*x^2 + c*d^2)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a/(c^4*d^2*x^2 + c^2*d^2)
```

Fricas [A] time = 2.39258, size = 135, normalized size = 2.45

$$\frac{ac^2x^2 + \sqrt{c^2x^2+1}bcx - b\log\left(cx + \sqrt{c^2x^2+1}\right)}{2\left(c^4d^2x^2 + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x - b*log(c*x + sqrt(c^2*x^2 + 1)))/
(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x*asinh(c*x)/(
c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)x}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^2, x)
```

3.41 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx}{cd^2}$$

```
[Out] b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^
2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d^2) - ((I/2)*b*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/(c*d^2) + ((I/2)*b*PolyLog[2, I*E^ArcSinh[c*x]
])/(c*d^2)
```

Rubi [A] time = 0.0978521, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2cd^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a+b\sinh^{-1}(cx}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] b/(2*c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^
2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*d^2) - ((I/2)*b*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/(c*d^2) + ((I/2)*b*PolyLog[2, I*E^ArcSinh[c*x]
])/(c*d^2)
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{2cd^2}$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{(ib)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{(ib)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b}{2cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{ib\text{Li}_2\left(-ie^{\sinh}\right.}{2cd^2}$$

Mathematica [A] time = 0.105299, size = 216, normalized size = 1.74

$$\frac{-ib\left(c^2x^2+1\right)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+ib\left(c^2x^2+1\right)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+ac^2x^2\tan^{-1}(cx)+acx+a\tan^{-1}(cx)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a*c*x + b*Sqrt[1 + c^2*x^2] + b*c*x*ArcSinh[c*x] + a*ArcTan[c*x] + a*c^2*x
^2*ArcTan[c*x] + I*b*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*b*c^2*x^2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*b*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] - I*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*b*(1 + c^2
*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + I*b*(1 + c^2*x^2)*PolyLog[2, I*E^Ar
cSinh[c*x]])/(2*d^2*(c + c^3*x^2))
```

Maple [A] time = 0.01, size = 234, normalized size = 1.9

$$\frac{ax}{2\,d^2\left(c^2x^2+1\right)} + \frac{a\arctan(cx)}{2\,cd^2} + \frac{b\text{Arcsinh}(cx)\,x}{2\,d^2\left(c^2x^2+1\right)} + \frac{b\text{Arcsinh}(cx)\arctan(cx)}{2\,cd^2} + \frac{b}{2\,cd^2}\frac{1}{\sqrt{c^2x^2+1}} + \frac{b\arctan(cx)}{2\,cd^2}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/2*a/d^2*x/(c^2*x^2+1)+1/2/c*a/d^2*arctan(c*x)+1/2*b/d^2*arcsinh(c*x)*x/(c
^2*x^2+1)+1/2/c*b/d^2*arcsinh(c*x)*arctan(c*x)+1/2*b/c/d^2/(c^2*x^2+1)^(1/2
)+1/2/c*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/c*b/d^2*a
rctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I/c*b/d^2*dilog(1+I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))+1/2*I/c*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{x}{c^2d^2x^2+d^2}+\frac{\arctan\left(cx\right)}{cd^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + b*integrate(log(c*x +
 sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\left(cx\right)+a}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^4x^4+2c^2x^2+1}\,dx+\int\frac{b\,\mathrm{asinh}\left(cx\right)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*asinh(c*x)/(c**4
*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^2, x)
```

3.42 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{a+b\sinh^{-1}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}$$

```
[Out] -(b*c*x)/(2*d^2*Sqrt[1 + c^2*x^2]) + (a + b*ArcSinh[c*x])/(2*d^2*(1 + c^2*x
^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 - (b*PolyLo
g[2, -E^(2*ArcSinh[c*x])])/(2*d^2) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*
d^2)
```

Rubi [A] time = 0.176965, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5755, 5720, 5461, 4182, 2279, 2391, 191}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^2}+\frac{a+b\sinh^{-1}(cx)}{2d^2\left(c^2x^2+1\right)}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(b*c*x)/(2*d^2*Sqrt[1 + c^2*x^2]) + (a + b*ArcSinh[c*x])/(2*d^2*(1 + c^2*x
^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 - (b*PolyLo
g[2, -E^(2*ArcSinh[c*x])])/(2*d^2) + (b*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*
d^2)
```

Rule *5755*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] && !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
```

)

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} + \frac{2\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d^2}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2} - \frac{b\,\text{Subst}\left(\int \text{lo}\right.}{}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2} - \frac{b\,\text{Subst}\left(\int \frac{\text{lo}}{}\right.}{}$$

$$= -\frac{bcx}{2d^2\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d^2} - \frac{b\mathbf{Li}_2\left(-e^{2\sinh}\right.}{2d^2}$$

Mathematica [B] time = 0.444966, size = 234, normalized size = 2.13

$$-\frac{2b\text{PolyLog}\left(2, \frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 2b\text{PolyLog}\left(2, \frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right) - b\text{PolyLog}\left(2, e^{2\sinh^{-1}(cx)}\right) + \frac{a^2}{b} - \frac{a}{c^2x^2+1} + a\log\left(c^2x^2+1\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(a^2/b - a/(1 + c^2*x^2) + (b*c*x)/Sqrt[1 + c^2*x^2] + 2*a*ArcSinh[c*x] -
(b*ArcSinh[c*x])/(1 + c^2*x^2) + 2*b*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]
)/Sqrt[-c^2]] + 2*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2
*a*Log[1 - E^(2*ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x]
)] + a*Log[1 + c^2*x^2] + 2*b*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 2
*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - b*PolyLog[2, E^(2*ArcSinh[c*
x])])/(2*d^2)
```

Maple [B] time = 0.085, size = 283, normalized size = 2.6

$$\frac{a\ln(cx)}{d^2}+\frac{a}{2d^2(c^2x^2+1)}-\frac{a\ln(c^2x^2+1)}{2d^2}-\frac{bcx}{2d^2}\frac{1}{\sqrt{c^2x^2+1}}+\frac{bc^2x^2}{2d^2(c^2x^2+1)}+\frac{b\mathrm{Arcsinh}(cx)}{2d^2(c^2x^2+1)}+\frac{b}{2d^2(c^2x^2+1)}-\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x)
```

```
[Out] a/d^2*ln(c*x)+1/2*a/d^2/(c^2*x^2+1)-1/2*a/d^2*ln(c^2*x^2+1)-1/2*b*c*x/d^2/(
c^2*x^2+1)^(1/2)+1/2*b/d^2*c^2*x^2/(c^2*x^2+1)+1/2*b/d^2*arcsinh(c*x)/(c^2*
x^2+1)+1/2*b/d^2/(c^2*x^2+1)-b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+b/d^2*arcsinh(c*x)*ln(
1+c*x+(c^2*x^2+1)^(1/2))+b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b/d^2*arcs
inh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{1}{c^2d^2x^2+d^2}-\frac{\log(c^2x^2+1)}{d^2}+\frac{2\log(x)}{d^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^5+2c^2d^2x^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + b*int
egrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}(cx)+a}{c^4d^2x^5+2c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^5+2c^2x^3+x}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b*asinh(c*x)/(c**4
*x**5 + 2*c**2*x**3 + x), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\left(c^2dx^2 + d\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x), x)
```

3.43 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=168

$$\frac{3ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{d^2x\left(c^2x^2+1\right)}-\frac{3c\tan^{-1}}{}$$

```
[Out] -(b*c)/(2*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(d^2*x*(1 + c^2*x^2
)) - (3*c^2*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (3*c*(a + b*Arc
Sinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d
^2 + (((3*I)/2)*b*c*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - (((3*I)/2)*b*c*P
olyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rubi [A] time = 0.187181, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$\frac{3ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2}-\frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{d^2x\left(c^2x^2+1\right)}-\frac{3c\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(d^2*x*(1 + c^2*x^2
)) - (3*c^2*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (3*c*(a + b*Arc
Sinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 - (b*c*ArcTanh[Sqrt[1 + c^2*x^2]])/d
^2 + (((3*I)/2)*b*c*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - (((3*I)/2)*b*c*P
olyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)^{3/2}}\,dx, x, x^2\right)}{2d^2} + \frac{\left(3bc^3\right)\int \frac{}{(1}}{2a}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{(3c)\,\text{Subst}\left(\int (a+bx)\text{sech}(x)\,a\right.}{2d^2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\text{si}}\right.}{d^2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\text{si}}\right.}{d^2}$$

$$= -\frac{bc}{2d^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\text{si}}\right.}{d^2}$$

Mathematica [C] time = 0.59534, size = 253, normalized size = 1.51

$$-\frac{\frac{bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},c^2x^2+1\right)}{\sqrt{c^2x^2+1}} - 3b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 3b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right) - \frac{a}{c^2x^3+x} + 3ac\,\text{t}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -((3*a)/x - a/(x + c^2*x^3) + (3*b*ArcSinh[c*x])/x - (b*ArcSinh[c*x])/(x +
c^2*x^3) + 3*a*c*ArcTan[c*x] + 3*b*c*ArcTanh[Sqrt[1 + c^2*x^2]] + (b*c*Hype
rgeometric2F1[-1/2, 1, 1/2, 1 + c^2*x^2])/Sqrt[1 + c^2*x^2] + 3*b*Sqrt[-c^2
]*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 3*b*Sqrt[-c^2]*ArcS
inh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 3*b*Sqrt[-c^2]*PolyLog[2,
 (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 3*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]*E^A
rcSinh[c*x])/c])/(2*d^2)
```

Maple [A] time = 0.018, size = 267, normalized size = 1.6

$$-\frac{a}{d^2x}-\frac{ac^2x}{2\,d^2\left(c^2x^2+1\right)}-\frac{3\,ca\arctan\left(cx\right)}{2\,d^2}-\frac{b\mathrm{Arcsinh}\left(cx\right)}{d^2x}-\frac{b\mathrm{Arcsinh}\left(cx\right)c^2x}{2\,d^2\left(c^2x^2+1\right)}-\frac{3\,bc\mathrm{Arcsinh}\left(cx\right)\arctan\left(cx\right)}{2\,d^2}-\frac{3\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2*a/d^2*c^2*x/(c^2*x^2+1)-3/2*c*a/d^2*arctan(c*x)-b/d^2*arcsinh(
c*x)/x-1/2*b/d^2*arcsinh(c*x)*c^2*x/(c^2*x^2+1)-3/2*c*b/d^2*arcsinh(c*x)*ar
ctan(c*x)-3/2*c*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*c
*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*c*b/d^2*dilog(
1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*c*b/d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))-1/2*b*c/d^2/(c^2*x^2+1)^(1/2)-c*b/d^2*arctanh(1/(c^2*x^2+1)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{3\,c^2x^2+2}{c^2d^2x^3+d^2x}+\frac{3\,c\arctan\left(cx\right)}{d^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + b*in
tegrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\left(cx\right)+a}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b*asinh(c*x)/(c
**4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^2), x)
```

3.44 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{4c^2\tanh^{-1}\left(\right.}{}$$

```
[Out] -(b*c)/(2*d^2*x*Sqrt[1 + c^2*x^2]) - (c^2*(a + b*ArcSinh[c*x]))/(d^2*(1 + c
^2*x^2)) - (a + b*ArcSinh[c*x])/(2*d^2*x^2*(1 + c^2*x^2)) + (4*c^2*(a + b*A
rcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 + (b*c^2*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2
```

Rubi [A] time = 0.259758, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5747, 5755, 5720, 5461, 4182, 2279, 2391, 191, 271}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d^2}-\frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(c^2x^2+1\right)}-\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(c^2x^2+1\right)}+\frac{4c^2\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```


```
[Out] -(b*c)/(2*d^2*x*Sqrt[1 + c^2*x^2]) - (c^2*(a + b*ArcSinh[c*x]))/(d^2*(1 + c
^2*x^2)) - (a + b*ArcSinh[c*x])/(2*d^2*x^2*(1 + c^2*x^2)) + (4*c^2*(a + b*A
rcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 + (b*c^2*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int(a+bx)\mathbf{csch}(x)\right.}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(4c^2\right)\text{Subst}\left(\int(a+bx)\mathbf{csch}(2\right.}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)}{d^2\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d^2}$$

Mathematica [B] time = 0.481177, size = 326, normalized size = 2.23

$$\frac{4bc^2\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+4bc^2\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)-2bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+\frac{2a^2c^2}{b}+\frac{a}{c^2x^4+x^2}+2ac^2\,\text{l}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((2*a^2*c^2)/b - (2*a)/x^2 + (b*c)/(x*Sqrt[1 + c^2*x^2]) + (2*b*c^3*x)/Sqrt
[1 + c^2*x^2] - (2*b*c*Sqrt[1 + c^2*x^2])/x + a/(x^2 + c^2*x^4) + 4*a*c^2*A
rcSinh[c*x] - (2*b*ArcSinh[c*x])/x^2 + (b*ArcSinh[c*x])/(x^2 + c^2*x^4) + 4
*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*ArcSin
h[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 4*a*c^2*Log[1 - E^(2*ArcSin
h[c*x])] - 4*b*c^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a*c^2*Log[1
 + c^2*x^2] + 4*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b*c^2*P
olyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b*c^2*PolyLog[2, E^(2*ArcSinh[
c*x])])/(2*d^2)
```

Maple [A] time = 0.112, size = 311, normalized size = 2.1

$$-\frac{a}{2\,d^2x^2}-2\,\frac{c^2a\ln(cx)}{d^2}-\frac{c^2a}{2\,d^2\left(c^2x^2+1\right)}+\frac{c^2a\ln\left(c^2x^2+1\right)}{d^2}-\frac{c^2b\mathrm{Arcsinh}(cx)}{d^2\left(c^2x^2+1\right)}-\frac{bc}{2\,d^2x}\frac{1}{\sqrt{c^2x^2+1}}-\frac{b\mathrm{Arcsinh}(cx)}{2\,d^2x^2\left(c^2x^2+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2/x^2-2*c^2*a/d^2*ln(c*x)-1/2*c^2*a/d^2/(c^2*x^2+1)+c^2*a/d^2*ln(c
^2*x^2+1)-c^2*b/d^2*arcsinh(c*x)/(c^2*x^2+1)-1/2*b*c/d^2/x/(c^2*x^2+1)^(1/2
)-1/2*b/d^2/x^2/(c^2*x^2+1)*arcsinh(c*x)+2*c^2*b/d^2*arcsinh(c*x)*ln(1+(c*x
+(c^2*x^2+1)^(1/2))^2)+b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2-2*c^
2*b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*c^2*b/d^2*polylog(2,-c*x
-(c^2*x^2+1)^(1/2))-2*c^2*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*
c^2*b/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{2\,c^2\log\left(c^2x^2+1\right)}{d^2}-\frac{4\,c^2\log(x)}{d^2}-\frac{2\,c^2x^2+1}{c^2d^2x^4+d^2x^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c^2
*d^2*x^4 + d^2*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^
7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\mathrm{arsinh}\,(cx)+a}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^7+2c^2x^5+x^3}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b*asinh(c*x)/(c
**4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^3), x)
```

3.45 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=239

$$-\frac{5ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(c^2x^2+1\right)}-\frac{a}{3}$$

```
[Out] (b*c^3)/(3*d^2*Sqrt[1 + c^2*x^2]) - (b*c)/(6*d^2*x^2*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])
)/(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*
x^2)) + (5*c^3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/d^2 + (13*b*c^3
*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d^2) - (((5*I)/2)*b*c^3*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/d^2 + (((5*I)/2)*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rubi [A] time = 0.309059, antiderivative size = 264, normalized size of antiderivative = 1.1, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$-\frac{5ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{2d^2}+\frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(c^2x^2+1\right)}+\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(c^2x^2+1\right)}-\frac{a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] (5*b*c^3)/(6*d^2*Sqrt[1 + c^2*x^2]) + (b*c)/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) -
 (b*c*Sqrt[1 + c^2*x^2])/(2*d^2*x^2) - (a + b*ArcSinh[c*x])/(3*d^2*x^3*(1 +
 c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x]))/(3*d^2*x*(1 + c^2*x^2)) + (5*c^4*
x*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) + (5*c^3*(a + b*ArcSinh[c*x])
*ArcTan[E^ArcSinh[c*x]])/d^2 + (13*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d^2
) - (((5*I)/2)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + (((5*I)/2)*b*c^
3*PolyLog[2, I*E^ArcSinh[c*x]])/d^2
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
```

```
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^2}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} - \frac{1}{3}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^2}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)} + \left(5c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\,\mathrm{Subst}\left(\int -\right.}{6}$$

$$= \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)} + \frac{5c^4x\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)} + \frac{(bc}{}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)}$$

$$= \frac{5bc^3}{6d^2\sqrt{1+c^2x^2}} + \frac{bc}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{bc\sqrt{1+c^2x^2}}{2d^2x^2} - \frac{a+b\sinh^{-1}(cx)}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^2x\left(1+c^2x^2\right)}$$

Mathematica [C] time = 0.634604, size = 311, normalized size = 1.3

$$\frac{\frac{bc^3\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},c^2x^2+1\right)}{\sqrt{c^2x^2+1}} + 5b\left(-c^2\right)^{3/2}\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 5b\left(-c^2\right)^{3/2}\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right) + \frac{a}{c^2x^5+x}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-5*a)/(3*x^3) + (5*a*c^2)/x - (5*b*c*Sqrt[1 + c^2*x^2])/(6*x^2) + a/(x^3
+ c^2*x^5) - (5*b*ArcSinh[c*x])/(3*x^3) + (5*b*c^2*ArcSinh[c*x])/x + (b*Arc
Sinh[c*x])/(x^3 + c^2*x^5) + 5*a*c^3*ArcTan[c*x] + (35*b*c^3*ArcTanh[Sqrt[1
 + c^2*x^2]])/6 + (b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^2])/Sqrt
[1 + c^2*x^2] - 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sq
```

```
rt[-c^2]] + 5*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x
])/c] + 5*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 5*b*(-
c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(2*d^2)
```

Maple [A] time = 0.021, size = 332, normalized size = 1.4

$$-\frac{a}{3\,d^2x^3}+2\,\frac{c^2a}{d^2x}+\frac{c^4ax}{2\,d^2\left(c^2x^2+1\right)}+\frac{5\,c^3a\arctan(cx)}{2\,d^2}-\frac{b\text{Arcsinh}(cx)}{3\,d^2x^3}+2\,\frac{c^2b\text{Arcsinh}(cx)}{d^2x}+\frac{c^4b\text{Arcsinh}(cx)\,x}{2\,d^2\left(c^2x^2+1\right)}+\frac{5\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3+2*c^2*a/d^2/x+1/2*c^4*a/d^2*x/(c^2*x^2+1)+5/2*c^3*a/d^2*arct
an(c*x)-1/3*b/d^2*arcsinh(c*x)/x^3+2*c^2*b/d^2*arcsinh(c*x)/x+1/2*c^4*b/d^2
*arcsinh(c*x)*x/(c^2*x^2+1)+5/2*c^3*b/d^2*arcsinh(c*x)*arctan(c*x)+1/3*b*c^
3/d^2/(c^2*x^2+1)^(1/2)+13/6*c^3*b/d^2*arctanh(1/(c^2*x^2+1)^(1/2))-1/6*b*c
/d^2/x^2/(c^2*x^2+1)^(1/2)+5/2*c^3*b/d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))-5/2*c^3*b/d^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
-5/2*I*c^3*b/d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+5/2*I*c^3*b/d^2*dil
og(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\left(\frac{15\,c^3\arctan(cx)}{d^2}+\frac{15\,c^4x^4+10\,c^2x^2-2}{c^2d^2x^5+d^2x^3}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^8+2\,c^2d^2x^6+d^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8 + 2*c^2
*d^2*x^6 + d^2*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{c^4 d^2 x^8 + 2 c^2 d^2 x^6 + d^2 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b \operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b*asinh(c*x)/(c
**4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2 dx^2 + d\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^2*x^4), x)
```

3.46 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=186

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(c^2x^2+1\right)}+\frac{3\tan^{-1}}{}$$

```
[Out] b/(12*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (5*b)/(8*c^5*d^3*Sqrt[1 + c^2*x^2]) -
(x^3*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a + b*ArcSin
h[c*x]))/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(4*c^5*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*
d^3) + (((3*I)/8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rubi [A] time = 0.232901, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5751, 5693, 4180, 2279, 2391, 261, 266, 43}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8c^5d^3}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(c^2x^2+1\right)}+\frac{3\tan^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (5*b)/(8*c^5*d^3*Sqrt[1 + c^2*x^2]) -
(x^3*(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a + b*ArcSin
h[c*x]))/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(4*c^5*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*
d^3) + (((3*I)/8)*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
```

```
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4cd^3}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+\frac{(3b)\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{8c^3d^3}+\frac{b\,\mathrm{Subst}\left(\int\frac{x}{\left(1+c^2x\right)\right.}\right.}{8cd^3}$$

$$= -\frac{3b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+\frac{3\,\mathrm{Subst}\left(\int(a+bx)\mathrm{se}\right.}{8c}$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+$$

$$= \frac{b}{12c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b}{8c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8c^4d^3\left(1+c^2x^2\right)}+$$

Mathematica [A] time = 0.64223, size = 341, normalized size = 1.83

$$-\frac{9ib\left(c^2x^2+1\right)^2\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-9ib\left(c^2x^2+1\right)^2\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+15ac^3x^3-9ac^4x^4\tan^{-1}(cx)-18a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] -(9*a*c*x + 15*a*c^3*x^3 + 13*b*Sqrt[1 + c^2*x^2] + 15*b*c^2*x^2*Sqrt[1 + c
^2*x^2] + 9*b*c*x*ArcSinh[c*x] + 15*b*c^3*x^3*ArcSinh[c*x] - 9*a*ArcTan[c*x
```

```
] - 18*a*c^2*x^2*ArcTan[c*x] - 9*a*c^4*x^4*ArcTan[c*x] - (9*I)*b*ArcSinh[c*
x]*Log[1 - I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^Ar
cSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (9*I
)*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*
Log[1 + I*E^ArcSinh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSin
h[c*x]] + (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9*I)*b
*(1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c^5*d^3*(1 + c^2*x^2)^2)
```

Maple [A] time = 0.016, size = 313, normalized size = 1.7

$$-\frac{5\,a x^3}{8\,c^2 d^3\left(c^2x^2+1\right)^2}-\frac{3\,a x}{8\,c^4 d^3\left(c^2x^2+1\right)^2}+\frac{3\,a\arctan(cx)}{8\,c^5 d^3}-\frac{5\,b\text{Arcsinh}\,(cx)\,x^3}{8\,c^2 d^3\left(c^2x^2+1\right)^2}-\frac{3\,b\text{Arcsinh}\,(cx)\,x}{8\,c^4 d^3\left(c^2x^2+1\right)^2}+\frac{3\,b\text{Arcsinh}\,(cx}{8\,c^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] -5/8/c^2*a/d^3*x^3/(c^2*x^2+1)^2-3/8/c^4*a/d^3*x/(c^2*x^2+1)^2+3/8/c^5*a/d^
3*arctan(c*x)-5/8/c^2*b/d^3*arcsinh(c*x)*x^3/(c^2*x^2+1)^2-3/8/c^4*b/d^3*ar
csinh(c*x)*x/(c^2*x^2+1)^2+3/8/c^5*b/d^3*arcsinh(c*x)*arctan(c*x)+3/8/c^5*b
/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8/c^5*b/d^3*arctan(c
*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I/c^5*b/d^3*dilog(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))+3/8*I/c^5*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-
5/8/c^3*b/d^3*x^2/(c^2*x^2+1)^(3/2)-13/24*b/c^5/d^3/(c^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\,a\left(\frac{5\,c^2x^3+3\,x}{c^8d^3x^4+2\,c^6d^3x^2+c^4d^3}-\frac{3\,\arctan(cx)}{c^5d^3}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arcta
n(c*x)/(c^5*d^3)) + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x
^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{arsinh}(cx) + ax^4}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^4 \operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^3, x)
```

3.47 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=97

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3}{12cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{c^2x^2+1}}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}$$

```
[Out] (b*x^3)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(4*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (b*ArcSinh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c
^2*x^2)^2)
```

Rubi [A] time = 0.0861107, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5723, 288, 215}

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3}{12cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{c^2x^2+1}}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(4*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (b*ArcSinh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c
^2*x^2)^2)
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx &= \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{(bc)\int\frac{x^4}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3}\\ &= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{b\int\frac{x^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{4cd^3}\\ &= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1+c^2x^2}}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}-\frac{b\int\frac{1}{\sqrt{1+c^2x^2}}\,dx}{4c^3d^3}\\ &= \frac{bx^3}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{b\sinh^{-1}(cx)}{4c^4d^3}+\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.14508, size = 79, normalized size = 0.81

$$\frac{-3a\left(2c^2x^2+1\right)+bcx\sqrt{c^2x^2+1}\left(4c^2x^2+3\right)-3\left(2bc^2x^2+b\right)\sinh^{-1}(cx)}{12c^4d^3\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*(1 + 2*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2) - 3*(b + 2*
b*c^2*x^2)*ArcSinh[c*x])/(12*c^4*d^3*(1 + c^2*x^2)^2)
```

Maple [A] time = 0.016, size = 108, normalized size = 1.1

$$\frac{1}{c^4}\left(\frac{a}{d^3}\left(\frac{1}{4\left(c^2x^2+1\right)^2}-\frac{1}{2c^2x^2+2}\right)+\frac{b}{d^3}\left(\frac{\operatorname{Arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}-\frac{\operatorname{Arcsinh}(cx)}{2c^2x^2+2}-\frac{cx}{12}\left(c^2x^2+1\right)^{-\frac{3}{2}}+\frac{cx}{3}\frac{1}{\sqrt{c^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^4*(a/d^3*(1/4/(c^2*x^2+1)^2-1/2/(c^2*x^2+1))+b/d^3*(1/4/(c^2*x^2+1)^2*a
rcsinh(c*x)-1/2/(c^2*x^2+1)*arcsinh(c*x)-1/12/(c^2*x^2+1)^(3/2)*c*x+1/3*c*x
/(c^2*x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16}b\left(\frac{4c^2x^2+4\left(2c^2x^2+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+3}{c^8d^3x^4+2c^6d^3x^2+c^4d^3}-16\int\frac{2c^2x^2+1}{4\left(c^{10}d^3x^7+3c^8d^3x^5+3c^6d^3x^3+c^4d^3x+\left(c^9d^3x^6+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1)) + 3)/(
c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 16*integrate(1/4*(2*c^2*x^2 + 1)/(
c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^3*x + (c^9*d^3*x^6 + 3
*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*c^
2*x^2 + 1)*a/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

Fricas [A] time = 2.33659, size = 209, normalized size = 2.15

$$\frac{3ac^4x^4-3\left(2bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(4bc^3x^3+3bcx\right)\sqrt{c^2x^2+1}}{12\left(c^8d^3x^4+2c^6d^3x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - 3*(2*b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + (4*b
*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d
^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^3 \operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^3}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(c^2*d*x^2 + d)^3, x)
```

3.48 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=184

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(c^2x^2+1\right)}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{\tan^{-1}\left(e^{\sinh\ldots}\right.}{}$$

```
[Out] -b/(12*c^3*d^3*(1 + c^2*x^2)^(3/2)) + b/(8*c^3*d^3*Sqrt[1 + c^2*x^2]) - (x*
(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSinh[c*x])
)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/(4*c^3*d^3) - ((I/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/8)
*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3)
```

Rubi [A] time = 0.175743, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5751, 5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8c^3d^3}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(c^2x^2+1\right)}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{\tan^{-1}\left(e^{\sinh\ldots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```


```
[Out] -b/(12*c^3*d^3*(1 + c^2*x^2)^(3/2)) + b/(8*c^3*d^3*Sqrt[1 + c^2*x^2]) - (x*
(a + b*ArcSinh[c*x]))/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSinh[c*x])
)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])
/(4*c^3*d^3) - ((I/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/8)
*b*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3)
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
```

```
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int \frac{x}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4cd^3}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$=-\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}-\frac{b\int \frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{8cd^3}+$$

$$=-\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}+$$

$$=-\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}+$$

$$=-\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}+$$

$$=-\frac{b}{12c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{8c^2d^3\left(1+c^2x^2\right)}+$$

Mathematica [A] time = 0.24785, size = 340, normalized size = 1.85

$$\frac{-3ib\left(c^2x^2+1\right)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+3ib\left(c^2x^2+1\right)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+3ac^3x^3+3ac^4x^4\tan^{-1}(cx)+6ac}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a*c*x + 3*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 3*b*c^2*x^2*Sqrt[1 + c^2*x^
2] - 3*b*c*x*ArcSinh[c*x] + 3*b*c^3*x^3*ArcSinh[c*x] + 3*a*ArcTan[c*x] + 6*
a*c^2*x^2*ArcTan[c*x] + 3*a*c^4*x^4*ArcTan[c*x] + (3*I)*b*ArcSinh[c*x]*Log[
1 - I*E^ArcSinh[c*x]] + (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*
x]] + (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*b*ArcS
inh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (6*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 + I
```

```
*E^ArcSinh[c*x]] - (3*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] -
 (3*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*b*(1 + c^2
*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c^3*d^3*(1 + c^2*x^2)^2)
```

Maple [A] time = 0.013, size = 307, normalized size = 1.7

$$\frac{ax^3}{8\,d^3\left(c^2x^2+1\right)^2}-\frac{ax}{8\,c^2d^3\left(c^2x^2+1\right)^2}+\frac{a\arctan(cx)}{8\,c^3d^3}+\frac{b\mathrm{Arcsinh}(cx)\,x^3}{8\,d^3\left(c^2x^2+1\right)^2}-\frac{b\mathrm{Arcsinh}(cx)\,x}{8\,c^2d^3\left(c^2x^2+1\right)^2}+\frac{b\mathrm{Arcsinh}(cx)\arctan(c}{8\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/8*a/d^3*x^3/(c^2*x^2+1)^2-1/8/c^2*a/d^3*x/(c^2*x^2+1)^2+1/8/c^3*a/d^3*arc
tan(c*x)+1/8*b/d^3*arcsinh(c*x)*x^3/(c^2*x^2+1)^2-1/8/c^2*b/d^3*arcsinh(c*x
)*x/(c^2*x^2+1)^2+1/8/c^3*b/d^3*arcsinh(c*x)*arctan(c*x)+1/8/c^3*b/d^3*arct
an(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8/c^3*b/d^3*arctan(c*x)*ln(1-
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/8*I/c^3*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))+1/8*I/c^3*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/8/c*b/d
^3*x^2/(c^2*x^2+1)^(3/2)+1/24*b/c^3/d^3/(c^2*x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,a\left(\frac{c^2x^3-x}{c^6d^3x^4+2\,c^4d^3x^2+c^2d^3}+\frac{\arctan(cx)}{c^3d^3}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x)/
(c^3*d^3)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arsinh}(cx)+ax^2}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^3, x)
```

3.49 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=80

$$-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{bx}{6cd^3\sqrt{c^2x^2+1}}+\frac{bx}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

```
[Out] (b*x)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(6*c*d^3*Sqrt[1 + c^2*x^2]) -
(a + b*ArcSinh[c*x])/(4*c^2*d^3*(1 + c^2*x^2)^2)
```

Rubi [A] time = 0.053746, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5717, 192, 191}

$$-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{bx}{6cd^3\sqrt{c^2x^2+1}}+\frac{bx}{12cd^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (b*x)/(6*c*d^3*Sqrt[1 + c^2*x^2]) -
(a + b*ArcSinh[c*x])/(4*c^2*d^3*(1 + c^2*x^2)^2)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
&& NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4cd^3}\\
&= \frac{bx}{12cd^3\left(1+c^2x^2\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{6cd^3}\\
&= \frac{bx}{12cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx}{6cd^3\sqrt{1+c^2x^2}}-\frac{a+b\sinh^{-1}(cx)}{4c^2d^3\left(1+c^2x^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0699482, size = 56, normalized size = 0.7

$$\frac{-3a+bcx\sqrt{c^2x^2+1}\left(2c^2x^2+3\right)-3b\sinh^{-1}(cx)}{12d^3\left(c^3x^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2) - 3*b*ArcSinh[c*x])/(12*d^3
*(c + c^3*x^2)^2)
```

Maple [A] time = 0.006, size = 76, normalized size = 1.

$$\frac{1}{c^2}\left(-\frac{a}{4\,d^3\left(c^2x^2+1\right)^2}+\frac{b}{d^3}\left(-\frac{\operatorname{Arcsinh}(cx)}{4\left(c^2x^2+1\right)^2}+\frac{cx}{12}\left(c^2x^2+1\right)^{-\frac{3}{2}}+\frac{cx}{6}\frac{1}{\sqrt{c^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^2*(-1/4*a/d^3/(c^2*x^2+1)^2+b/d^3*(-1/4/(c^2*x^2+1)^2*arcsinh(c*x)+1/12
/(c^2*x^2+1)^(3/2)*c*x+1/6*c*x/(c^2*x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16}\,b\left(\frac{4\,\log\left(cx+\sqrt{c^2x^2+1}\right)+1}{c^6d^3x^4+2\,c^4d^3x^2+c^2d^3}-16\int\frac{1}{4\left(c^8d^3x^7+3\,c^6d^3x^5+3\,c^4d^3x^3+c^2d^3x+\left(c^7d^3x^6+3\,c^5d^3x^4+3\,c^3d^3x^2+cd^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*b*((4*log(c*x + sqrt(c^2*x^2 + 1)) + 1)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2
+ c^2*d^3) - 16*integrate(1/4/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3
+ c^2*d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^
2*x^2 + 1)), x)) - 1/4*a/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Fricas [A] time = 2.34944, size = 207, normalized size = 2.59

$$\frac{3\,ac^4x^4+6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(2\,bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}}{12\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 + 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 + 1)) + (2*b*c
^3*x^3 + 3*b*c*x)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{ax}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx+\int\frac{bx\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*
x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x}{\left(c^2dx^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^3, x)
```

3.50 $\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=178

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{3\tan^{-1}\left(\right.}{}$$

```
[Out] b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (3*b)/(8*c*d^3*Sqrt[1 + c^2*x^2]) + (x*(
a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]))/(
8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*c
*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/8)*
b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3)
```

Rubi [A] time = 0.140758, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8cd^3}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}+\frac{3\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c*d^3*(1 + c^2*x^2)^(3/2)) + (3*b)/(8*c*d^3*Sqrt[1 + c^2*x^2]) + (x*(
a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]))/(
8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*c
*d^3) - (((3*I)/8)*b*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/8)*
b*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3)
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

```
&& GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^3}\,dx = \frac{x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2} - \frac{(bc)\int \frac{x}{(1+c^2x^2)^{5/2}}\,dx}{4d^3} + \frac{3\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^2}\,dx}{4d}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)} - \frac{(3bc)\int \frac{x}{(1+c^2x^2)^{3/2}}\,dx}{8d^3} + \frac{3}{\ldots}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)} + \frac{3\,\mathrm{Subst}\ldots}{\ldots}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)} + \frac{3(a+b\ldots}{\ldots}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)} + \frac{3(a+b\ldots}{\ldots}$$

$$= \frac{b}{12cd^3(1+c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1+c^2x^2}} + \frac{x(a+b\sinh^{-1}(cx))}{4d^3(1+c^2x^2)^2} + \frac{3x(a+b\sinh^{-1}(cx))}{8d^3(1+c^2x^2)} + \frac{3(a+b\ldots}{\ldots}$$

Mathematica [A] time = 0.152428, size = 341, normalized size = 1.92

$$\frac{-9ib\left(c^2x^2+1\right)^2 \mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) + 9ib\left(c^2x^2+1\right)^2 \mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) + 9ac^3x^3 + 9ac^4x^4\tan^{-1}(cx) + 18ac\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^3,x]
```

```
[Out] (15*a*c*x + 9*a*c^3*x^3 + 11*b*Sqrt[1 + c^2*x^2] + 9*b*c^2*x^2*Sqrt[1 + c^2
*x^2] + 15*b*c*x*ArcSinh[c*x] + 9*b*c^3*x^3*ArcSinh[c*x] + 9*a*ArcTan[c*x]
+ 18*a*c^2*x^2*ArcTan[c*x] + 9*a*c^4*x^4*ArcTan[c*x] + (9*I)*b*ArcSinh[c*x]
*Log[1 - I*E^ArcSinh[c*x]] + (18*I)*b*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcS
inh[c*x]] + (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (9*I)*
b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - (18*I)*b*c^2*x^2*ArcSinh[c*x]*Lo
g[1 + I*E^ArcSinh[c*x]] - (9*I)*b*c^4*x^4*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[
c*x]] - (9*I)*b*(1 + c^2*x^2)^2*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (9*I)*b*(
1 + c^2*x^2)^2*PolyLog[2, I*E^ArcSinh[c*x]])/(24*c*d^3*(1 + c^2*x^2)^2)
```

Maple [A] time = 0.01, size = 295, normalized size = 1.7

$$\frac{ax}{4\,d^3\left(c^2x^2+1\right)^2}+\frac{3\,ax}{8\,d^3\left(c^2x^2+1\right)}+\frac{3\,a\arctan\left(cx\right)}{8\,cd^3}+\frac{b\mathrm{Arcsinh}\left(cx\right)x}{4\,d^3\left(c^2x^2+1\right)^2}+\frac{3\,b\mathrm{Arcsinh}\left(cx\right)x}{8\,d^3\left(c^2x^2+1\right)}+\frac{3\,b\mathrm{Arcsinh}\left(cx\right)\arctan}{8\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] 1/4*a/d^3*x/(c^2*x^2+1)^2+3/8*a/d^3*x/(c^2*x^2+1)+3/8/c*a/d^3*arctan(c*x)+1
/4*b/d^3*arcsinh(c*x)*x/(c^2*x^2+1)^2+3/8*b/d^3*arcsinh(c*x)*x/(c^2*x^2+1)+
3/8/c*b/d^3*arcsinh(c*x)*arctan(c*x)+3/8*c*b/d^3*x^2/(c^2*x^2+1)^(3/2)+11/2
4*b/c/d^3/(c^2*x^2+1)^(3/2)+3/8/c*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))-3/8/c*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/
8*I/c*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/8*I/c*b/d^3*dilog(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,a\left(\frac{3\,c^2x^3+5\,x}{c^4d^3x^4+2\,c^2d^3x^2+d^3}+\frac{3\,\arctan\left(cx\right)}{cd^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c*x
)/(c*d^3)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\left(cx\right)+a}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
+ d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b*as
inh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^3, x)
```

3.51 $\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{a+b\sinh^{-1}(cx)}{2d^3\left(c^2x^2+1\right)}+\frac{a+b\sinh^{-1}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(c\ldots}\right.}{}$$

```
[Out] -(b*c*x)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[1 + c^2*x^2])
 + (a + b*ArcSinh[c*x])/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])/(2*d
^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^
3 - (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*d^3) + (b*PolyLog[2, E^(2*ArcSin
h[c*x])])/(2*d^3)
```

Rubi [A] time = 0.25158, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5755, 5720, 5461, 4182, 2279, 2391, 191, 192}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^3}+\frac{a+b\sinh^{-1}(cx)}{2d^3\left(c^2x^2+1\right)}+\frac{a+b\sinh^{-1}(cx)}{4d^3\left(c^2x^2+1\right)^2}-\frac{2\tanh^{-1}\left(e^{2\sinh^{-1}(c\ldots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```



```
[Out] -(b*c*x)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[1 + c^2*x^2])
 + (a + b*ArcSinh[c*x])/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])/(2*d
^3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^
3 - (b*PolyLog[2, -E^(2*ArcSinh[c*x])])/(2*d^3) + (b*PolyLog[2, E^(2*ArcSin
h[c*x])])/(2*d^3)
```


Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
```

```
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
&& NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx = \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^2}\,dx}{d}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{1}{\left(1+c^2x^2\right)^{3/2}}\,dx}{6d^3} - \frac{(bc)\int \frac{}{(1+c}}{2d}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} + \frac{\mathrm{Subst}\left(\int (a+b\right.}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} + \frac{2\,\mathrm{Subst}\left(\int (a+\right.}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}\right.}{}$$

$$= -\frac{bcx}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1+c^2x^2}} + \frac{a+b\sinh^{-1}(cx)}{4d^3\left(1+c^2x^2\right)^2} + \frac{a+b\sinh^{-1}(cx)}{2d^3\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}\right.}{}$$

Mathematica [A] time = 0.632703, size = 289, normalized size = 1.82

$$\frac{-4b\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 4b\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right) + 2b\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right) - \frac{2a^2}{b} + \frac{2a}{c^2x^2+1} + \frac{a}{\left(c^2x^2+1\right)^2} - 2a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-2*a^2)/b + a/(1 + c^2*x^2)^2 - (b*c*x)/(3*(1 + c^2*x^2)^(3/2)) + (2*a)/(
1 + c^2*x^2) - (8*b*c*x)/(3*Sqrt[1 + c^2*x^2]) - 4*a*ArcSinh[c*x] + (b*ArcS
inh[c*x])/(1 + c^2*x^2)^2 + (2*b*ArcSinh[c*x])/(1 + c^2*x^2) - 4*b*ArcSinh[
```

```
c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*ArcSinh[c*x]*Log[1 + (Sqr
t[-c^2]*E^ArcSinh[c*x])/c] + 4*a*Log[1 - E^(2*ArcSinh[c*x])] + 4*b*ArcSinh[
c*x]*Log[1 - E^(2*ArcSinh[c*x])] - 2*a*Log[1 + c^2*x^2] - 4*b*PolyLog[2, (c
*E^ArcSinh[c*x])/Sqrt[-c^2]] - 4*b*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
] + 2*b*PolyLog[2, E^(2*ArcSinh[c*x])])/(4*d^3)
```

Maple [B] time = 0.156, size = 451, normalized size = 2.8

$$\frac{a\ln(cx)}{d^3}+\frac{a}{4d^3\left(c^2x^2+1\right)^2}+\frac{a}{2d^3\left(c^2x^2+1\right)}-\frac{a\ln\left(c^2x^2+1\right)}{2d^3}-\frac{2bc^3x^3}{3d^3\left(c^4x^4+2c^2x^2+1\right)}\sqrt{c^2x^2+1}+\frac{2bc^4x^4}{3d^3\left(c^4x^4+2c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x)
```

```
[Out] a/d^3*ln(c*x)+1/4*a/d^3/(c^2*x^2+1)^2+1/2*a/d^3/(c^2*x^2+1)-1/2*a/d^3*ln(c^
2*x^2+1)-2/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^3*x^3*(c^2*x^2+1)^(1/2)+2/3*b/d^
3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4+1/2*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*c^2*x^2-3/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^2+1)^(1/2)+4/3*b/d^3/(
c^4*x^4+2*c^2*x^2+1)*c^2*x^2+3/4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+2
/3*b/d^3/(c^4*x^4+2*c^2*x^2+1)-b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/
2))^2)-1/2*b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+b/d^3*arcsinh(c*x)*l
n(1+c*x+(c^2*x^2+1)^(1/2))+b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b/d^3*ar
csinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a\left(\frac{2c^2x^2+3}{c^4d^3x^4+2c^2d^3x^2+d^3}-\frac{2\log\left(c^2x^2+1\right)}{d^3}+\frac{4\log(x)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^7+3c^4d^3x^5+3c^2d^3x^3+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^2
+ 1)/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
```

```
3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\operatorname{arsinh}(cx)+a}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
+ d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int \frac{b\operatorname{asinh}(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b*as
inh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x), x)
```

3.52 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=222

$$\frac{15ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-$$

```
[Out] -(b*c)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c*x])
)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2
*x^2)) - (15*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (b*c*
ArcTanh[Sqrt[1 + c^2*x^2]])/d^3 + (((15*I)/8)*b*c*PolyLog[2, (-I)*E^ArcSinh
[c*x]])/d^3 - (((15*I)/8)*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rubi [A] time = 0.238339, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$\frac{15ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8d^3}-\frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}-\frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```


```
[Out] -(b*c)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c*x])
)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2
*x^2)) - (15*c*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (b*c*
ArcTanh[Sqrt[1 + c^2*x^2]])/d^3 + (((15*I)/8)*b*c*PolyLog[2, (-I)*E^ArcSinh
[c*x]])/d^3 - (((15*I)/8)*b*c*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)^{5/2}}\,dx,x,x^2\right)}{2d^3} + \frac{\left(5bc^3\right)\int}{4}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15c^2x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15c^2x\left(a\right.}{8d^3}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15c^2x\left(a\right.}{8d^3}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15c^2x\left(a\right.}{8d^3}$$

$$= -\frac{bc}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc}{8d^3\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{15c^2x\left(a\right.}{8d^3}$$

Mathematica [C] time = 1.21853, size = 298, normalized size = 1.34

$$-\frac{\frac{2bc\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},c^2x^2+1\right)}{\left(c^2x^2+1\right)^{3/2}} + \frac{15bc\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},c^2x^2+1\right)}{\sqrt{c^2x^2+1}} - 45b\sqrt{-c^2}\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) + 45b\sqrt{-c^2}\text{I}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -((45*(a + b*ArcSinh[c*x]))/x - (6*(a + b*ArcSinh[c*x]))/(x*(1 + c^2*x^2)^2
) - (15*(a + b*ArcSinh[c*x]))/(x + c^2*x^3) + 45*a*c*ArcTan[c*x] + 45*b*c*A
rcTanh[Sqrt[1 + c^2*x^2]] + (2*b*c*Hypergeometric2F1[-3/2, 1, -1/2, 1 + c^2
*x^2])/(1 + c^2*x^2)^(3/2) + (15*b*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^
```

```
2*x^2])/Sqrt[1 + c^2*x^2] + 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (c*E^ArcSi
nh[c*x])/Sqrt[-c^2]] - 45*b*Sqrt[-c^2]*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^A
rcSinh[c*x])/c] - 45*b*Sqrt[-c^2]*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]]
 + 45*b*Sqrt[-c^2]*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(24*d^3)
```

Maple [A] time = 0.016, size = 357, normalized size = 1.6

$$-\frac{a}{d^3x}-\frac{7\,ac^4x^3}{8\,d^3\left(c^2x^2+1\right)^2}-\frac{9\,ac^2x}{8\,d^3\left(c^2x^2+1\right)^2}-\frac{15\,ca\arctan(cx)}{8\,d^3}-\frac{b\mathrm{Arcsinh}(cx)}{d^3x}-\frac{7\,b\mathrm{Arcsinh}(cx)\,c^4x^3}{8\,d^3\left(c^2x^2+1\right)^2}-\frac{9\,b\mathrm{Arcsinh}(c}{8\,d^3\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] -a/d^3/x-7/8*a/d^3*c^4*x^3/(c^2*x^2+1)^2-9/8*a/d^3*c^2*x/(c^2*x^2+1)^2-15/8
*c*a/d^3*arctan(c*x)-b/d^3*arcsinh(c*x)/x-7/8*b/d^3*arcsinh(c*x)*c^4*x^3/(c
^2*x^2+1)^2-9/8*b/d^3*arcsinh(c*x)*c^2*x/(c^2*x^2+1)^2-15/8*c*b/d^3*arcsinh
(c*x)*arctan(c*x)-15/8*c*b/d^3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+15/8*c*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+15/8*I*c*b
/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-15/8*I*c*b/d^3*dilog(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-15/8*b/d^3*c^3*x^2/(c^2*x^2+1)^(3/2)-47/24*b*c/d^3/(
c^2*x^2+1)^(3/2)+b*c/d^3/(c^2*x^2+1)^(1/2)-c*b/d^3*arctanh(1/(c^2*x^2+1)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}a\left(\frac{15\,c^4x^4+25\,c^2x^2+8}{c^4d^3x^5+2\,c^2d^3x^3+d^3x}+\frac{15\,c\arctan(cx)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*x)
 + 15*c*arctan(c*x)/d^3) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^
3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
+ d^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx+\int \frac{b\,\text{asinh}\,(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integral(b
*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^2), x)
```

3.53 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=232

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{a+}{2d^3x}$$

```
[Out] -(b*c)/(2*d^3*x*(1 + c^2*x^2)^(3/2)) - (5*b*c^3*x)/(12*d^3*(1 + c^2*x^2)^(3
/2)) + (2*b*c^3*x)/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a + b*ArcSinh[c*x]))
/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])/(2*d^3*x^2*(1 + c^2*x^2)^2)
 - (3*c^2*(a + b*ArcSinh[c*x]))/(2*d^3*(1 + c^2*x^2)) + (6*c^2*(a + b*ArcSi
nh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rubi [A] time = 0.343745, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5747, 5755, 5720, 5461, 4182, 2279, 2391, 191, 192, 271}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{2d^3}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^3\left(c^2x^2+1\right)}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(c^2x^2+1\right)^2}-\frac{a+}{2d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(b*c)/(2*d^3*x*(1 + c^2*x^2)^(3/2)) - (5*b*c^3*x)/(12*d^3*(1 + c^2*x^2)^(3
/2)) + (2*b*c^3*x)/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a + b*ArcSinh[c*x]))
/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])/(2*d^3*x^2*(1 + c^2*x^2)^2)
 - (3*c^2*(a + b*ArcSinh[c*x]))/(2*d^3*(1 + c^2*x^2)) + (6*c^2*(a + b*ArcSi
nh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
```

```
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} + \frac{\left(3bc^3\right)\int \frac{1}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3} - \cdots$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sinh^{-1}(cx)}{2d^3x^2\left(1+c^2x^2\right)^2} - \frac{3c^2\left(\cdots\right.}{2\cdots}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sin\cdots}{2d^3x^2\left(1\cdots\right.}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sin\cdots}{2d^3x^2\left(1\cdots\right.}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sin\cdots}{2d^3x^2\left(1\cdots\right.}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sin\cdots}{2d^3x^2\left(1\cdots\right.}$$

$$= -\frac{bc}{2d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x}{12d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc^3x}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{a+b\sin\cdots}{2d^3x^2\left(1\cdots\right.}$$

Mathematica [A] time = 0.846377, size = 353, normalized size = 1.52

$$\frac{-18c^2\left(b\,\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+2\log\left(1-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)\right)+36bc^2\,\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)+36bc^2\mathrm{P}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-18*b*c*Sqrt[1 + c^2*x^2])/x + (9*b*c*(1 + 2*c^2*x^2))/(x*Sqrt[1 + c^2*x^
2]) + (b*c*(3 + 12*c^2*x^2 + 8*c^4*x^4))/(x*(1 + c^2*x^2)^(3/2)) - 18*b*c^2
*ArcSinh[c*x]^2 - (18*(a + b*ArcSinh[c*x]))/x^2 + (3*(a + b*ArcSinh[c*x]))/
```

```
(x + c^2*x^3)^2 + (9*(a + b*ArcSinh[c*x]))/(x^2 + c^2*x^4) + (18*c^2*(a + b
*ArcSinh[c*x])^2)/b + 36*b*c^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] + 36*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 18
*a*c^2*Log[1 + c^2*x^2] + 36*b*c^2*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]
] + 36*b*c^2*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 18*c^2*(2*(a + b*A
rcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])] + b*PolyLog[2, E^(2*ArcSinh[c*x])]
))/(12*d^3)
```

Maple [B] time = 0.194, size = 575, normalized size = 2.5

$$-\frac{a}{2\,d^3x^2}-3\,\frac{c^2a\ln(cx)}{d^3}-\frac{c^2a}{4\,d^3\left(c^2x^2+1\right)^2}-\frac{c^2a}{d^3\left(c^2x^2+1\right)}+\frac{3\,c^2a\ln\left(c^2x^2+1\right)}{2\,d^3}+\frac{2\,c^5bx^3}{3\,d^3\left(c^4x^4+2\,c^2x^2+1\right)}\sqrt{c^2x^2+1}-\frac{}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/2*a/d^3/x^2-3*c^2*a/d^3*ln(c*x)-1/4*c^2*a/d^3/(c^2*x^2+1)^2-c^2*a/d^3/(c
^2*x^2+1)+3/2*c^2*a/d^3*ln(c^2*x^2+1)+2/3*c^5*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x
^3*(c^2*x^2+1)^(1/2)-2/3*c^6*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^4-3/2*c^4*b/d^3/
(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^2+1/4*c^3*b/d^3/(c^4*x^4+2*c^2*x^2+1)*
x*(c^2*x^2+1)^(1/2)-4/3*c^4*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^2-9/4*c^2*b/d^3/(
c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)-1/2*c*b/d^3/(c^4*x^4+2*c^2*x^2+1)/x*(c^2*
x^2+1)^(1/2)-2/3*c^2*b/d^3/(c^4*x^4+2*c^2*x^2+1)-1/2*b/d^3/(c^4*x^4+2*c^2*x
^2+1)/x^2*arcsinh(c*x)+3*c^2*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)+3/2*b*c^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3-3*c^2*b/d^3*arcsin
h(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-3*c^2*b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-3*c^2*b/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-3*c^2*b/d^3*poly
log(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\,a\left(\frac{6\,c^4x^4+9\,c^2x^2+2}{c^4d^3x^6+2\,c^2d^3x^4+d^3x^2}-\frac{6\,c^2\log\left(c^2x^2+1\right)}{d^3}+\frac{12\,c^2\log(x)}{d^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^2)
 - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + b*integrate(log(c*x +
sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{c^6d^3x^9 + 3\,c^4d^3x^7 + 3\,c^2d^3x^5 + d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
+ d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^3 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^3), x)
```

3.54 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^3}\,dx$

Optimal. Leaf size=295

$$-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{35c^4x\left(a+b\sinh^{-1}(cx\right.}{12d^3\left(c^2x^2+1\right)^2}$$

```
[Out] -(b*c^3)/(12*d^3*(1 + c^2*x^2)^(3/2)) - (b*c)/(6*d^3*x^2*(1 + c^2*x^2)^(3/2
)) + (29*b*c^3)/(24*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d^3*x^
3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x]))/(3*d^3*x*(1 + c^2*x^2)^2)
 + (35*c^4*x*(a + b*ArcSinh[c*x]))/(12*d^3*(1 + c^2*x^2)^2) + (35*c^4*x*(a
+ b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*ArcSinh[c*x])*Arc
Tan[E^ArcSinh[c*x]])/(4*d^3) + (19*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/(6*d^3
) - (((35*I)/8)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/8)*b*
c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rubi [A] time = 0.367184, antiderivative size = 345, normalized size of antiderivative = 1.17, number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5747, 5690, 5693, 4180, 2279, 2391, 261, 266, 51, 63, 208}

$$-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{8d^3}+\frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(c^2x^2+1\right)}+\frac{35c^4x\left(a+b\sinh^{-1}(cx\right.}{12d^3\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] (7*b*c^3)/(36*d^3*(1 + c^2*x^2)^(3/2)) + (b*c)/(9*d^3*x^2*(1 + c^2*x^2)^(3/
2)) + (49*b*c^3)/(24*d^3*Sqrt[1 + c^2*x^2]) + (5*b*c)/(9*d^3*x^2*Sqrt[1 + c
^2*x^2]) - (5*b*c*Sqrt[1 + c^2*x^2])/(6*d^3*x^2) - (a + b*ArcSinh[c*x])/(3*
d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x]))/(3*d^3*x*(1 + c^2*x
^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x]))/(12*d^3*(1 + c^2*x^2)^2) + (35*c^4
*x*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*ArcSinh[c*x
])*ArcTan[E^ArcSinh[c*x]])/(4*d^3) + (19*b*c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/
(6*d^3) - (((35*I)/8)*b*c^3*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)
/8)*b*c^3*PolyLog[2, I*E^ArcSinh[c*x]])/d^3
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^3}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} - \frac{1}{3}\left(7c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)^{5/2}}\,dx}{3d^3}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(1+c^2x^2\right)^2} + \frac{1}{3}\left(35c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)}{3d^3x\left(1+c^2x^2\right)^2} + \frac{35c^4x\left(a+b\sinh^{-1}(cx)\right)}{12d^3\left(1+c^2x^2\right)^2}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{a+b\sinh^{-1}(cx)}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\right.}{3d^3x\left(1\right.}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1+c^2x^2}}{6d^3x^2}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1+c^2x^2}}{6d^3x^2}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1+c^2x^2}}{6d^3x^2}$$

$$= \frac{7bc^3}{36d^3\left(1+c^2x^2\right)^{3/2}} + \frac{bc}{9d^3x^2\left(1+c^2x^2\right)^{3/2}} + \frac{49bc^3}{24d^3\sqrt{1+c^2x^2}} + \frac{5bc}{9d^3x^2\sqrt{1+c^2x^2}} - \frac{5bc\sqrt{1+c^2x^2}}{6d^3x^2}$$

Mathematica [C] time = 1.04604, size = 380, normalized size = 1.29

$$\frac{\frac{4bc^3\mathrm{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},c^2x^2+1\right)}{\left(c^2x^2+1\right)^{3/2}} + \frac{42bc^3\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},c^2x^2+1\right)}{\sqrt{c^2x^2+1}} + 210b\left(-c^2\right)^{3/2}\mathrm{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right) - 210b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((-70*a)/x^3 + (210*a*c^2)/x + (12*a)/(x^3*(1 + c^2*x^2)^2) - (35*b*c*Sqrt[
1 + c^2*x^2])/x^2 + (42*a)/(x^3 + c^2*x^5) - (70*b*ArcSinh[c*x])/x^3 + (210
*b*c^2*ArcSinh[c*x])/x + (12*b*ArcSinh[c*x])/(x^3*(1 + c^2*x^2)^2) + (42*b*
ArcSinh[c*x])/(x^3 + c^2*x^5) + 210*a*c^3*ArcTan[c*x] + 245*b*c^3*ArcTanh[S
qrt[1 + c^2*x^2]] + (4*b*c^3*Hypergeometric2F1[-3/2, 2, -1/2, 1 + c^2*x^2])
/(1 + c^2*x^2)^(3/2) + (42*b*c^3*Hypergeometric2F1[-1/2, 2, 1/2, 1 + c^2*x^
2])/Sqrt[1 + c^2*x^2] - 210*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] + 210*b*(-c^2)^(3/2)*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E
^ArcSinh[c*x])/c] + 210*b*(-c^2)^(3/2)*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] - 210*b*(-c^2)^(3/2)*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(48*d
^3)
```

Maple [A] time = 0.02, size = 425, normalized size = 1.4

$$-\frac{a}{3\,d^3x^3}+3\,\frac{c^2a}{d^3x}+\frac{11\,c^6ax^3}{8\,d^3\left(c^2x^2+1\right)^2}+\frac{13\,c^4ax}{8\,d^3\left(c^2x^2+1\right)^2}+\frac{35\,c^3a\arctan\left(cx\right)}{8\,d^3}-\frac{b\mathrm{Arcsinh}\left(cx\right)}{3\,d^3x^3}+3\,\frac{c^2b\mathrm{Arcsinh}\left(cx\right)}{d^3x}+\frac{11\,c}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3+3*c^2*a/d^3/x+11/8*c^6*a/d^3*x^3/(c^2*x^2+1)^2+13/8*c^4*a/d^
3*x/(c^2*x^2+1)^2+35/8*c^3*a/d^3*arctan(c*x)-1/3*b/d^3*arcsinh(c*x)/x^3+3*c
^2*b/d^3*arcsinh(c*x)/x+11/8*c^6*b/d^3*arcsinh(c*x)*x^3/(c^2*x^2+1)^2+13/8*
c^4*b/d^3*arcsinh(c*x)*x/(c^2*x^2+1)^2+35/8*c^3*b/d^3*arcsinh(c*x)*arctan(c
*x)+35/8*c^5*b/d^3*x^2/(c^2*x^2+1)^(3/2)+103/24*b*c^3/d^3/(c^2*x^2+1)^(3/2)
-19/6*b*c^3/d^3/(c^2*x^2+1)^(1/2)+19/6*c^3*b/d^3*arctanh(1/(c^2*x^2+1)^(1/2
))-1/6*b*c/d^3/x^2/(c^2*x^2+1)^(3/2)+35/8*c^3*b/d^3*arctan(c*x)*ln(1+I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))-35/8*c^3*b/d^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+35/8*I*c^3*b/d^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-35/8*I
*c^3*b/d^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\,a\left(\frac{105\,c^3\arctan\left(cx\right)}{d^3}+\frac{105\,c^6x^6+175\,c^4x^4+56\,c^2x^2-8}{c^4d^3x^7+2\,c^2d^3x^5+d^3x^3}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2 -
 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsinh}(cx) + a}{c^6d^3x^{10} + 3\,c^4d^3x^8 + 3\,c^2d^3x^6 + d^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6
 + d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^3 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^3*x^4), x)
```

3.55 $\int x^3\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=109

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}+\frac{2\sqrt{\pi}bx}{15c^3}-\frac{1}{25}\sqrt{\pi}bcx^5-\frac{\sqrt{\pi}bx^3}{45c}$$

```
[Out] (2*b*Sqrt[Pi]*x)/(15*c^3) - (b*Sqrt[Pi]*x^3)/(45*c) - (b*c*Sqrt[Pi]*x^5)/25
 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + ((Pi + c^2*P
i*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*Pi^2)
```

Rubi [A] time = 0.120906, antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43, 5732, 12}

$$\frac{\sqrt{\pi}\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-\frac{\sqrt{\pi}\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{2\sqrt{\pi}bx}{15c^3}-\frac{1}{25}\sqrt{\pi}bcx^5-\frac{\sqrt{\pi}bx^3}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Sqrt[Pi]*x)/(15*c^3) - (b*Sqrt[Pi]*x^3)/(45*c) - (b*c*Sqrt[Pi]*x^5)/25
 - (Sqrt[Pi]*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4) + (Sqrt[Pi]*
(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx = -\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-$$

$$= -\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-$$

$$= \frac{2b\sqrt{\pi}x}{15c^3}-\frac{b\sqrt{\pi}x^3}{45c}-\frac{1}{25}bc\sqrt{\pi}x^5-\frac{\sqrt{\pi}\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}+\frac{\sqrt{\pi}(1}{}$$

Mathematica [A] time = 0.194575, size = 106, normalized size = 0.97

$$\frac{\sqrt{\pi}\left(15a\sqrt{c^2x^2+1}\left(3c^4x^4+c^2x^2-2\right)+b\left(-9c^5x^5-5c^3x^3+30cx\right)+15b\sqrt{c^2x^2+1}\left(3c^4x^4+c^2x^2-2\right)\sinh^{-1}(cx)\right)}{225c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(15*a*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4) + b*(30*c*x -
5*c^3*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*
ArcSinh[c*x]))/(225*c^4)
```

Maple [A] time = 0.077, size = 164, normalized size = 1.5

$$a\left(\frac{x^2}{5\pi c^2}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{2}{15\pi c^4}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}\right)+\frac{b\sqrt{\pi}}{225c^4}\left(45\,\mathrm{Arcsinh}\,(cx)\,c^6x^6+60\,\mathrm{Arcsinh}\,(cx)\,c^4x^4-9\,c^5x^5\sqrt{c^2x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*(1/5*x^2*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-2/15/Pi/c^4*(Pi*c^2*x^2+Pi)^(3/2))+
1/225*b/c^4*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6+60*arcsinh(
c*x)*c^4*x^4-9*c^5*x^5*(c^2*x^2+1)^(1/2)-15*arcsinh(c*x)*c^2*x^2-5*c^3*x^3*
(c^2*x^2+1)^(1/2)-30*arcsinh(c*x)+30*c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.20101, size = 181, normalized size = 1.66

$$\frac{1}{15}\,b\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)\operatorname{arsinh}(cx)+\frac{1}{15}\,a\left(\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}{\pi c^4}\right)-\frac{\left(9\sqrt{\pi}c^4x^5+5\cdots\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*b*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(3/2)/
(pi*c^4))*arcsinh(c*x) + 1/15*a*(3*(pi + pi*c^2*x^2)^(3/2)*x^2/(pi*c^2) - 2
*(pi + pi*c^2*x^2)^(3/2)/(pi*c^4)) - 1/225*(9*sqrt(pi)*c^4*x^5 + 5*sqrt(pi)
*c^2*x^3 - 30*sqrt(pi)*x)*b/c^3
```

Fricas [A] time = 2.40754, size = 351, normalized size = 3.22

$$\frac{15\sqrt{\pi+\pi c^2x^2}\left(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-\left(\cdots\right.\right.}{225\left(c^6x^2+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 + 60*
a*c^4*x^4 - 15*a*c^2*x^2 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*
```

```
x^2 + 1) - 30*a))/(c^6*x^2 + c^4)
```

Sympy [A] time = 21.3508, size = 221, normalized size = 2.03

$$\begin{cases} \frac{\sqrt{\pi}ax^4\sqrt{c^2x^2+1}}{5} + \frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{15c^2} - \frac{2\sqrt{\pi}a\sqrt{c^2x^2+1}}{15c^4} - \frac{\sqrt{\pi}bcx^5}{25} + \frac{\sqrt{\pi}bx^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5} - \frac{\sqrt{\pi}bx^3}{45c} + \frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^2} + \frac{2\sqrt{\pi}bx}{15c^3} - 2\sqrt{} \\ \frac{\sqrt{\pi}ax^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**4*sqrt(c**2*x**2 + 1)/5 + sqrt(pi)*a*x**2*sqrt(c**
2*x**2 + 1)/(15*c**2) - 2*sqrt(pi)*a*sqrt(c**2*x**2 + 1)/(15*c**4) - sqrt(p
i)*b*c*x**5/25 + sqrt(pi)*b*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5 - sqrt(pi
)*b*x**3/(45*c) + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**2)
+ 2*sqrt(pi)*b*x/(15*c**3) - 2*sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
5*c**4), Ne(c, 0)), (sqrt(pi)*a*x**4/4, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.56 $\int x^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=119

$$\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi}x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3}-\frac{1}{16}\sqrt{\pi}bcx^4-\frac{\sqrt{\pi}b}{16c}$$

```
[Out] -(b*Sqrt[Pi]*x^2)/(16*c) - (b*c*Sqrt[Pi]*x^4)/16 + (Sqrt[Pi]*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*Arc
Sinh[c*x]))/4 - (Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3)
```

Rubi [A] time = 0.197875, antiderivative size = 181, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5742, 5758, 5675, 30}

$$\frac{1}{4}x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{\pi c^2x}}{16\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x^2*Sqrt[Pi + c^2*Pi*x^2])/(16*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[Pi
+ c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*Arc
Sinh[c*x]))/(8*c^2) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/4 -
(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi+c^2\pi x^2}\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx}{4\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{\right.}}{}$$

$$=-\frac{bcx^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{\pi+c^2\pi x^2}\left(a+\right.$$

$$=-\frac{bx^2\sqrt{\pi+c^2\pi x^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x$$

Mathematica [A] time = 0.168271, size = 79, normalized size = 0.66

$$\frac{\sqrt{\pi}\left(\sinh^{-1}(cx)\left(4b\sinh\left(4\sinh^{-1}(cx)\right)-16a\right)+16acx\sqrt{c^2x^2+1}\left(2c^2x^2+1\right)-8b\sinh^{-1}(cx)^2-b\cosh\left(4\sinh^{-1}(cx)\right)\right)}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(16*a*c*x*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2) - 8*b*ArcSinh[c*x]^2
- b*Cosh[4*ArcSinh[c*x]] + ArcSinh[c*x]*(-16*a + 4*b*Sinh[4*ArcSinh[c*x]]))
```

```
)/(128*c^3)
```

Maple [A] time = 0.055, size = 170, normalized size = 1.4

$$\frac{ax}{4\pi c^2}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{ax}{8c^2}\sqrt{\pi c^2x^2+\pi}-\frac{a\pi}{8c^2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+\frac{b\sqrt{\pi}\mathrm{Arcsinh}\,(cx)\,x^3}{4}\sqrt{c^2x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/4*a*x*(Pi*c^2*x^2+Pi)^(3/2)/Pi/c^2-1/8*a/c^2*x*(Pi*c^2*x^2+Pi)^(1/2)-1/8*
a/c^2*Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1
/4*b*Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/16*b*c*x^4*Pi^(1/2)+1/8*
b*Pi^(1/2)/c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-1/16*b*x^2*Pi^(1/2)/c-1/16*
b*Pi^(1/2)/c^3*arcsinh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(bx^2\,\mathrm{arsinh}\,(cx)+ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\pi}\left(\int ax^2\sqrt{c^2x^2+1}\,dx+\int bx^2\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] sqrt(pi)*(Integral(a*x**2*sqrt(c**2*x**2 + 1), x) + Integral(b*x**2*sqrt(c*
*2*x**2 + 1)*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\pi+\pi c^2x^2}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)*x^2, x)
```

3.57 $\int x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{1}{9}\sqrt{\pi}bcx^3-\frac{\sqrt{\pi}bx}{3c}$$

```
[Out] -(b*Sqrt[Pi]*x)/(3*c) - (b*c*Sqrt[Pi]*x^3)/9 + ((Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/(3*c^2*Pi)
```

Rubi [A] time = 0.0678192, antiderivative size = 105, normalized size of antiderivative = 1.72, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5717}

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{bcx^3\sqrt{\pi c^2x^2+\pi}}{9\sqrt{c^2x^2+1}}-\frac{bx\sqrt{\pi c^2x^2+\pi}}{3c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x*Sqrt[Pi + c^2*Pi*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[Pi + c
^2*Pi*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/(3*c^2*Pi)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3c^2\pi} - \frac{\left(b\sqrt{\pi + c^2\pi x^2}\right)\int\left(1 + c^2x^2\right)\,dx}{3c\sqrt{1 + c^2x^2}}$$

$$= -\frac{bx\sqrt{\pi + c^2\pi x^2}}{3c\sqrt{1 + c^2x^2}} - \frac{bcx^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3c^2\pi}$$

Mathematica [A] time = 0.116606, size = 63, normalized size = 1.03

$$\frac{\sqrt{\pi}\left(3a\left(c^2x^2+1\right)^{3/2} - bcx\left(c^2x^2+3\right) + 3b\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)\right)}{9c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(3*a*(1 + c^2*x^2)^(3/2) - b*c*x*(3 + c^2*x^2) + 3*b*(1 + c^2*x^2
)^(3/2)*ArcSinh[c*x]))/(9*c^2)
```

Maple [B] time = 0.047, size = 108, normalized size = 1.8

$$\frac{a}{3\,\pi\,c^2}\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}} + \frac{b\sqrt{\pi}}{9\,c^2}\left(3\,\mathrm{Arcsinh}\,(cx)\,c^4x^4 + 6\,\mathrm{Arcsinh}\,(cx)\,c^2x^2 - c^3x^3\sqrt{c^2x^2+1} + 3\,\mathrm{Arcsinh}\,(cx) - 3\,cx\sqrt{c^2x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/3*a/Pi/c^2*(Pi*c^2*x^2+Pi)^(3/2)+1/9*b/c^2*Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*
arcsinh(c*x)*c^4*x^4+6*arcsinh(c*x)*c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)+3*arc
sinh(c*x)-3*c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.22158, size = 99, normalized size = 1.62

$$\frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} b\,\mathrm{arsinh}\,(cx)}{3\,\pi c^2} - \frac{\left(\pi^{\frac{3}{2}}c^2x^3 + 3\,\pi^{\frac{3}{2}}x\right)b}{9\,\pi c} + \frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} a}{3\,\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*c^2) - 1/9*(pi^(3/2)*c^2*x^3
 + 3*pi^(3/2)*x)*b/(pi*c) + 1/3*(pi + pi*c^2*x^2)^(3/2)*a/(pi*c^2)
```

Fricas [B] time = 2.54178, size = 278, normalized size = 4.56

$$\frac{3\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4+2\,bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3\,ac^4x^4+6\,ac^2x^2-\left(bc^3x^3+3\,bcx\right)\sqrt{c^2x^2+1}+\right.}{9\left(c^4x^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^
3 + 3*b*c*x)*sqrt(c^2*x^2 + 1) + 3*a))/(c^4*x^2 + c^2)
```

Sympy [A] time = 2.65681, size = 141, normalized size = 2.31

$$\begin{cases} \frac{\sqrt{\pi}ax^2\sqrt{c^2x^2+1}}{3}+\frac{\sqrt{\pi}a\sqrt{c^2x^2+1}}{3c^2}-\frac{\sqrt{\pi}bcx^3}{9}+\frac{\sqrt{\pi}bx^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3}-\frac{\sqrt{\pi}bx}{3c}+\frac{\sqrt{\pi}b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2} & \text{for}\ c\neq 0 \\ \frac{\sqrt{\pi}ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((sqrt(pi)*a*x**2*sqrt(c**2*x**2 + 1)/3 + sqrt(pi)*a*sqrt(c**2*x**
2 + 1)/(3*c**2) - sqrt(pi)*b*c*x**3/9 + sqrt(pi)*b*x**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/3 - sqrt(pi)*b*x/(3*c) + sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(3*c**2), Ne(c, 0)), (sqrt(pi)*a*x**2/2, True))
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.58 $\int \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc}-\frac{1}{4}\sqrt{\pi}bcx^2$$

```
[Out] -(b*c*Sqrt[Pi]*x^2)/4 + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 +
(Sqrt[Pi]*(a + b*ArcSinh[c*x])^2)/(4*b*c)
```

Rubi [A] time = 0.0593713, antiderivative size = 111, normalized size of antiderivative = 1.66, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5682, 5675, 30}

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{\pi c^2x^2+\pi}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[Pi + c^2*Pi*x^2])/(4*Sqrt[1 + c^2*x^2]) + (x*Sqrt[Pi + c^2*P
i*x^2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
)^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)dx = \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi + c^2\pi x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1 + c^2x^2}} - \frac{\left(bc\sqrt{\pi + c^2\right.}{2\sqrt{1}}$$

$$= -\frac{bcx^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1 + c^2x^2}} + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sin\right.}{4bc\sqrt{1 + c^2x^2}}$$

Mathematica [A] time = 0.1366, size = 69, normalized size = 1.03

$$\frac{\sqrt{\pi}\left(2\sinh^{-1}(cx)\left(2a + b\sinh\left(2\sinh^{-1}(cx)\right)\right) + 4acx\sqrt{c^2x^2 + 1} + 2b\sinh^{-1}(cx)^2 - b\cosh\left(2\sinh^{-1}(cx)\right)\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[Pi]*(4*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSin
h[c*x]] + 2*ArcSinh[c*x]*(2*a + b*Sinh[2*ArcSinh[c*x]])))/(8*c)
```

Maple [B] time = 0.041, size = 112, normalized size = 1.7

$$\frac{ax}{2}\sqrt{\pi c^2x^2 + \pi} + \frac{a\pi}{2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}} + \sqrt{\pi c^2x^2 + \pi}\right)\frac{1}{\sqrt{\pi c^2}} + \frac{b\sqrt{\pi}\mathrm{Arcsinh}(cx)\,x}{2}\sqrt{c^2x^2 + 1} - \frac{bcx^2\sqrt{\pi}}{4} + \frac{b\sqrt{\pi}\,(\mathrm{Arcsin}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/2*a*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x
^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*
x-1/4*b*c*x^2*Pi^(1/2)+1/4*b*Pi^(1/2)/c*arcsinh(c*x)^2-1/4*b*Pi^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\pi}\left(\int a\sqrt{c^2x^2+1}\,dx + \int b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 +
1)*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a), x)
```

3.59 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=89

$$-\sqrt{\pi}b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\sqrt{\pi}b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-2\sqrt{\pi}\tanh^{-1}\left(e^{\sinh^{-1}(}\right.$$

```
[Out] -(b*c*Sqrt[Pi]*x) + Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) - 2*Sqrt[Pi]
*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Sqrt[Pi]*PolyLog[2, -E^Ar
cSinh[c*x]] + b*Sqrt[Pi]*PolyLog[2, E^ArcSinh[c*x]]
```

Rubi [A] time = 0.19175, antiderivative size = 177, normalized size of antiderivative = 1.99, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{b\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[Pi + c^2*Pi*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[Pi + c^2*Pi*x^2]*
(a + b*ArcSinh[c*x]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTan
h[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E
^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx = \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi + c^2\pi x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1 + c^2x^2}} - \frac{\left(bc\sqrt{\pi + c^2\pi\right.}{\sqrt{1 +}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{\sqrt{\pi + c^2\pi x^2}\,\mathrm{Subst}\left(\int(a\right.}{\sqrt{}}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh\right.}{\sqrt{1 +}}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh\right.}{\sqrt{1 +}}$$

$$= -\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh\right.}{\sqrt{1 +}}$$

Mathematica [A] time = 0.188321, size = 131, normalized size = 1.47

$$\sqrt{\pi}\left(b\left(\mathrm{PolyLog}\left(2, -e^{-\sinh^{-1}(cx)}\right) - \mathrm{PolyLog}\left(2, e^{-\sinh^{-1}(cx)}\right) + \sqrt{c^2x^2 + 1}\sinh^{-1}(cx) - cx + \sinh^{-1}(cx)\log\left(1 - e^{-\sinh^{-1}}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] Sqrt[Pi]*(a*Sqrt[1 + c^2*x^2] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])
] + b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-A
rcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))
```

Maple [A] time = 0.148, size = 171, normalized size = 1.9

$$-\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi\,c^2x^2 + \pi}}\right)\sqrt{\pi}a + a\sqrt{\pi\,c^2x^2 + \pi} + \mathrm{Arcsinh}\,(cx)\,\sqrt{\pi}\ln\left(1 - cx - \sqrt{c^2x^2 + 1}\right)b - \mathrm{Arcsinh}\,(cx)\,\sqrt{\pi}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x,x)
```

```
[Out] -arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))*Pi^(1/2)*a+a*(Pi*c^2*x^2+Pi)^(1/2)
+arcsinh(c*x)*Pi^(1/2)*ln(1-c*x-(c^2*x^2+1)^(1/2))*b-arcsinh(c*x)*Pi^(1/2)*
ln(1+c*x+(c^2*x^2+1)^(1/2))*b+arcsinh(c*x)*Pi^(1/2)*(c^2*x^2+1)^(1/2)*b-b*c
*x*Pi^(1/2)+b*polylog(2,c*x+(c^2*x^2+1)^(1/2))*Pi^(1/2)-b*polylog(2,-c*x-(c
^2*x^2+1)^(1/2))*Pi^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(\sqrt{\pi}\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)-\sqrt{\pi+\pi c^2x^2}\right)a+b\int\frac{\sqrt{\pi+\pi c^2x^2}\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(pi)*arcsinh(1/(sqrt(c^2)*abs(x))) - sqrt(pi + pi*c^2*x^2))*a + b*int
egrate(sqrt(pi + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\operatorname{arsinh}(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int\frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2
+ 1)*asinh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x, x)
```

3.60 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{\sqrt{\pi}c\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+\sqrt{\pi}bc\log(x)$$

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[Pi]*(a + b*ArcS
inh[c*x])^2)/(2*b) + b*c*Sqrt[Pi]*Log[x]
```

Rubi [A] time = 0.107924, antiderivative size = 105, normalized size of antiderivative = 1.72, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5737, 29, 5675}

$$\frac{c\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc\sqrt{\pi c^2x^2+\pi}\log(x)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[Pi + c^2*Pi*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[Pi + c^2*Pi*x
^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int \frac{1}{x}\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(c^2\sqrt{\pi+c^2\pi x^2}\right)}{\sqrt{1+}}$$

$$= -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{c\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{1+c^2x^2}} + \frac{bc\sqrt{\pi}}{\sqrt{}}$$

Mathematica [A] time = 0.163245, size = 75, normalized size = 1.23

$$\frac{\sqrt{\pi}\left(2\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\right)-2a\sqrt{c^2x^2+1}+2bcx\log(cx)+bcx\sinh^{-1}(cx)^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*Sqrt[1 + c^2*x^2] + 2*(a*c*x - b*Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*c*x*ArcSinh[c*x]^2 + 2*b*c*x*Log[c*x]))/(2*x)
```

Maple [B] time = 0.114, size = 155, normalized size = 2.5

$$-\frac{a}{\pi x}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}} + ac^2x\sqrt{\pi c^2x^2+\pi} + ac^2\pi\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}} + \frac{bc\sqrt{\pi}\,(\mathrm{Arcsinh}\,(cx))^2}{2} - bc\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^2,x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^2*Pi*ln(Pi*
x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c*Pi^(1/2)
*arcsinh(c*x)^2-b*c*Pi^(1/2)*arcsinh(c*x)-b*Pi^(1/2)*arcsinh(c*x)/x*(c^2*x^
```

```
2+1)^(1/2)+b*c*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\frac{\pi c^2 \operatorname{arsinh}\left(\frac{c^2 x}{\sqrt{c^2}}\right)}{\sqrt{\pi c^2}} - \frac{\sqrt{\pi + \pi c^2 x^2}}{x}\right)a + b\int \frac{\sqrt{\pi + \pi c^2 x^2}\log\left(cx + \sqrt{c^2 x^2 + 1}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] (pi*c^2*arcsinh(c^2*x/sqrt(c^2))/sqrt(pi*c^2) - sqrt(pi + pi*c^2*x^2)/x)*a
+ b*integrate(sqrt(pi + pi*c^2*x^2)*log(c*x + sqrt(c^2*x^2 + 1))/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^2, x)
```

Sympy [B] time = 3.19936, size = 110, normalized size = 1.8

$$-\frac{\sqrt{\pi} a c^2 x}{\sqrt{c^2 x^2 + 1}} + \sqrt{\pi} a c \operatorname{asinh}(cx) - \frac{\sqrt{\pi} a}{x\sqrt{c^2 x^2 + 1}} + \sqrt{\pi} b c \log(x) + \frac{\sqrt{\pi} b c \operatorname{asinh}^2(cx)}{2} - \frac{\sqrt{\pi} b \sqrt{c^2 x^2 + 1} \operatorname{asinh}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**2,x)
```

```
[Out] -sqrt(pi)*a*c**2*x/sqrt(c**2*x**2 + 1) + sqrt(pi)*a*c*asinh(c*x) - sqrt(pi)
*a/(x*sqrt(c**2*x**2 + 1)) + sqrt(pi)*b*c*log(x) + sqrt(pi)*b*c*asinh(c*x)*
*2/2 - sqrt(pi)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.61 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{1}{2}\sqrt{\pi}bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{1}{2}\sqrt{\pi}bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\sqrt{\pi}c^2\tanh^{-1}\left(\right.$$

```
[Out] -(b*c*Sqrt[Pi])/(2*x) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*x^2
) - c^2*Sqrt[Pi]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - (b*c^2*Sqrt
[Pi]*PolyLog[2, -E^ArcSinh[c*x]])/2 + (b*c^2*Sqrt[Pi]*PolyLog[2, E^ArcSinh[
c*x]])/2
```

Rubi [A] time = 0.195438, antiderivative size = 201, normalized size of antiderivative = 1.78, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5737, 30, 5760, 4182, 2279, 2391}

$$-\frac{bc^2\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Sqrt[Pi + c^2*Pi*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSin
h[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[Pi + c^2*P
i*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[Pi
+ c^2*Pi*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{\left(bc\sqrt{\pi + c^2\pi x^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{1 + c^2x^2}} + \frac{\left(c^2\sqrt{\pi + c^2\pi x^2}\right)}{2\sqrt{1 +}}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{\left(c^2\sqrt{\pi + c^2\pi x^2}\right)\text{Subst}\left(\int(\right.}{2\sqrt{}}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-}\right.}{\sqrt{1 +}}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-}\right.}{\sqrt{1 +}}$$

$$= -\frac{bc\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-}\right.}{\sqrt{1 +}}$$

Mathematica [A] time = 3.25718, size = 185, normalized size = 1.64

$$\frac{1}{8}\sqrt{\pi}\left(bc^2\left(4\text{PolyLog}\left(2, -e^{-\sinh^{-1}(cx)}\right) - 4\text{PolyLog}\left(2, e^{-\sinh^{-1}(cx)}\right) + 4\sinh^{-1}(cx)\log\left(1 - e^{-\sinh^{-1}(cx)}\right) - 4\sinh^{-1}(cx)\,\text{lo}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Sqrt[Pi]*((-4*a*Sqrt[1 + c^2*x^2])/x^2 + 4*a*c^2*Log[x] - 4*a*c^2*Log[Pi*(
1 + Sqrt[1 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csc
h[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*Pol
yLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[A
rcSinh[c*x]/2])))/8
```

Maple [A] time = 0.195, size = 243, normalized size = 2.2

$$-\frac{a}{2\pi x^2}\left(\pi c^2x^2 + \pi\right)^{\frac{3}{2}} - \frac{a\sqrt{\pi}c^2}{2}\text{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi c^2x^2 + \pi}}\right) + \frac{ac^2}{2}\sqrt{\pi c^2x^2 + \pi} - \frac{b\sqrt{\pi}\text{Arcsinh}(cx)\,c^2}{2}\frac{1}{\sqrt{c^2x^2 + 1}} - \frac{b\sqrt{\pi}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^3,x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(3/2)-1/2*a*Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2
*x^2+Pi)^(1/2))*c^2+1/2*a*(Pi*c^2*x^2+Pi)^(1/2)*c^2-1/2*b*Pi^(1/2)/(c^2*x^2
+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(1/2)/x-1/2*b*Pi^(1/2)/(c^2*x^2+1)^(1
/2)/x^2*arcsinh(c*x)-1/2*b*c^2*Pi^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))-1/2*b*c^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*Pi^(1/2)+1/2*b*c^2*Pi^(1/
2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+1/2*b*c^2*polylog(2,c*x+(c^2*x^
2+1)^(1/2))*Pi^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\sqrt{\pi}c^{2}\operatorname{arsinh}\left(\frac{1}{\sqrt{c^{2}}|x|}\right)-\sqrt{\pi+\pi c^{2}x^{2}}c^{2}+\frac{\left(\pi+\pi c^{2}x^{2}\right)^{\frac{3}{2}}}{\pi x^{2}}\right)a+b\int\frac{\sqrt{\pi+\pi c^{2}x^{2}}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(sqrt(pi)*c^2*arcsinh(1/(sqrt(c^2)*abs(x))) - sqrt(pi + pi*c^2*x^2)*c^
2 + (pi + pi*c^2*x^2)^(3/2)/(pi*x^2))*a + b*integrate(sqrt(pi + pi*c^2*x^2)
*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\pi+\pi c^{2}x^{2}}(b\operatorname{arsinh}(cx)+a)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\pi}\left(\int \frac{a\sqrt{c^2x^2+1}}{x^3}\,dx+\int \frac{b\sqrt{c^2x^2+1}\,\operatorname{asinh}(cx)}{x^3}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**3,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\pi+\pi c^2x^2}(b\operatorname{arsinh}(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.62 $\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{1}{3}\sqrt{\pi}bc^3\log(x)-\frac{\sqrt{\pi}bc}{6x^2}$$

```
[Out] -(b*c*Sqrt[Pi])/(6*x^2) - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*Pi*x^3) + (b*c^3*Sqrt[Pi]*Log[x])/3
```

Rubi [A] time = 0.0895466, antiderivative size = 106, normalized size of antiderivative = 1.71, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 14}

$$-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{bc\sqrt{\pi c^2x^2+\pi}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\sqrt{\pi c^2x^2+\pi}\log(x)}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[Pi + c^2*Pi*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - ((Pi + c^2*Pi*x^2)
^(3/2)*(a + b*ArcSinh[c*x]))/(3*Pi*x^3) + (b*c^3*Sqrt[Pi + c^2*Pi*x^2]*Log[
x])/(3*Sqrt[1 + c^2*x^2])
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int\frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}$$
$$= -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{\left(bc\sqrt{\pi+c^2\pi x^2}\right)\int\left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx}{3\sqrt{1+c^2x^2}}$$
$$= -\frac{bc\sqrt{\pi+c^2\pi x^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{bc^3\sqrt{\pi+c^2\pi x^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.129709, size = 78, normalized size = 1.26

$$\frac{1}{3}\sqrt{\pi}bc^3\log(x)-\frac{\sqrt{\pi}\left(2a\left(c^2x^2+1\right)^{3/2}+3bc^3x^3+2b\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)+bcx\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(Sqrt[Pi]*(b*c*x + 3*b*c^3*x^3 + 2*a*(1 + c^2*x^2)^(3/2) + 2*b*(1 + c^2*x^
2)^(3/2)*ArcSinh[c*x]))/(6*x^3) + (b*c^3*Sqrt[Pi]*Log[x])/3
```

Maple [B] time = 0.2, size = 501, normalized size = 8.1

$$-\frac{a}{3\pi x^3}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{2bc^3\sqrt{\pi}\mathrm{Arcsinh}(cx)}{3}+\frac{b\sqrt{\pi}x^4\mathrm{Arcsinh}(cx)c^7}{3c^4x^4+3c^2x^2+1}-\frac{b\sqrt{\pi}x^3\mathrm{Arcsinh}(cx)c^6}{3c^4x^4+3c^2x^2+1}\sqrt{c^2x^2+1}+\frac{b\sqrt{}}{18c^4x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(Pi*c^2*x^2+Pi)^(1/2)/x^4,x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(3/2)-2/3*b*c^3*Pi^(1/2)*arcsinh(c*x)+b*Pi^(1
/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-b*Pi^(1/2)/(3*c^4*x^4+3*c^
2*x^2+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6+1/6*b*Pi^(1/2)/(3*c^4*x^4+3
```

```
*c^2*x^2+1)*x^4*c^7-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*(c^2*x^2+1)*
c^5+b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-2*b*Pi^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-1/3*b*Pi^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*(c^2*x^2+1)*c^3+1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+
1)*arcsinh(c*x)*c^3-4/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x*arcsinh(c*x)*(
c^2*x^2+1)^(1/2)*c^2-1/6*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2*(c^2*x^2+1)
*c-1/3*b*Pi^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)+1/3*b*c^3*Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [B] time = 1.19868, size = 193, normalized size = 3.11

$$\frac{\left(\pi^2 c^4 \sqrt{\frac{1}{\pi c^4}} \log\left(x^2 + \frac{1}{c^2}\right) - \pi^{\frac{3}{2}} (-1)^{2\pi + 2\pi c^2 x^2} c^2 \log\left(2\pi c^2 + \frac{2\pi}{x^2}\right) - \frac{\pi\sqrt{\pi + \pi c^4 x^4 + 2\pi c^2 x^2}}{x^2}\right) bc}{6\pi} - \frac{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} b \operatorname{arsinh}(cx)}{3\pi x^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(pi^2*c^4*sqrt(1/(pi*c^4))*log(x^2 + 1/c^2) - pi^(3/2)*(-1)^(2*pi + 2*p
i*c^2*x^2)*c^2*log(2*pi*c^2 + 2*pi/x^2) - pi*sqrt(pi + pi*c^4*x^4 + 2*pi*c^
2*x^2)/x^2)*b*c/pi - 1/3*(pi + pi*c^2*x^2)^(3/2)*b*arcsinh(c*x)/(pi*x^3) -
1/3*(pi + pi*c^2*x^2)^(3/2)*a/(pi*x^3)
```

Fricas [B] time = 2.84194, size = 486, normalized size = 7.84

$$-\frac{2\sqrt{\pi + \pi c^2 x^2}\left(bc^4 x^4 + 2bc^2 x^2 + b\right) \log\left(cx + \sqrt{c^2 x^2 + 1}\right) - \sqrt{\pi}\left(bc^5 x^5 + bc^3 x^3\right) \log\left(\frac{\pi + \pi c^2 x^6 + \pi c^2 x^2 + \pi x^4 + \sqrt{\pi}\sqrt{\pi + \pi c^2 x^2}\sqrt{c^2 x^2}}{c^2 x^4 + x^2}\right.}{6\left(c^2 x^5 + x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*log(c*x + sqrt(
c^2*x^2 + 1)) - sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi*
c^2*x^2 + pi*x^4 + sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(2*a*c^4*x^4 + 4*a*c^2*x^2 - (
```

```
b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) + 2*a))/(c^2*x^5 + x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\pi}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^4}\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\mathrm{asinh}\,(cx)}{x^4}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(pi*c**2*x**2+pi)**(1/2)/x**4,x)
```

```
[Out] sqrt(pi)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(b*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/x**4, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}\,(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(pi*c^2*x^2+pi)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.63 $\int x^3 \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=125

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^4}-\frac{1}{49}\pi^{3/2}bc^3x^7+\frac{2\pi^{3/2}bx}{35c^3}-\frac{8}{175}\pi^{3/2}bcx^5-\frac{\pi^{3/2}}{105}$$

```
[Out] (2*b*Pi^(3/2)*x)/(35*c^3) - (b*Pi^(3/2)*x^3)/(105*c) - (8*b*c*Pi^(3/2)*x^5)
/175 - (b*c^3*Pi^(3/2)*x^7)/49 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*
x]))/(5*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi^
2)
```

Rubi [A] time = 0.143125, antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 12, 373}

$$\frac{\pi^{3/2}\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}-\frac{\pi^{3/2}\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-\frac{1}{49}\pi^{3/2}bc^3x^7+\frac{2\pi^{3/2}bx}{35c^3}-\frac{8}{175}\pi^{3/2}bcx^5-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(3/2)*x)/(35*c^3) - (b*Pi^(3/2)*x^3)/(105*c) - (8*b*c*Pi^(3/2)*x^5)
/175 - (b*c^3*Pi^(3/2)*x^7)/49 - (Pi^(3/2)*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x]))/(5*c^4) + (Pi^(3/2)*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*
c^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^3 \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx = -\frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4} + \frac{\pi^{3/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}$$

$$= -\frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4} + \frac{\pi^{3/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}$$

$$= -\frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4} + \frac{\pi^{3/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}$$

$$= \frac{2b\pi^{3/2}x}{35c^3} - \frac{b\pi^{3/2}x^3}{105c} - \frac{8}{175}bc\pi^{3/2}x^5 - \frac{1}{49}bc^3\pi^{3/2}x^7 - \frac{\pi^{3/2}\left(1+c^2x^2\right)^{5/2}\left(a+b\right.}{5c^4}$$

Mathematica [A] time = 0.1717, size = 100, normalized size = 0.8

$$\frac{\pi^{3/2}\left(105a\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^{5/2} - bcx\left(75c^6x^6+168c^4x^4+35c^2x^2-210\right) + 105b\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)\right)}{3675c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(105*a*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x^2) - b*c*x*(-210 + 35*c^
2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c^2*x^2)^(5/2)*(-2 + 5*c^2*x
^2)*ArcSinh[c*x]))/(3675*c^4)
```

Maple [A] time = 0.092, size = 195, normalized size = 1.6

$$a\left(\frac{x^2}{7\pi c^2}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}-\frac{2}{35\pi c^4}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}\right)+\frac{b\pi^{\frac{3}{2}}}{3675c^4}\left(525\operatorname{Arcsinh}(cx)c^8x^8+1365\operatorname{Arcsinh}(cx)c^6x^6-75c^7x^7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/7*x^2*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-2/35/Pi/c^4*(Pi*c^2*x^2+Pi)^(5/2))+
1/3675*b/c^4*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(525*arcsinh(c*x)*c^8*x^8+1365*arcs
inh(c*x)*c^6*x^6-75*c^7*x^7*(c^2*x^2+1)^(1/2)+945*arcsinh(c*x)*c^4*x^4-168*
c^5*x^5*(c^2*x^2+1)^(1/2)-105*arcsinh(c*x)*c^2*x^2-35*c^3*x^3*(c^2*x^2+1)^(
1/2)-210*arcsinh(c*x)+210*c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.20039, size = 196, normalized size = 1.57

$$\frac{1}{35}\left(\frac{5\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi c^4}\right)b\operatorname{arsinh}(cx)+\frac{1}{35}\left(\frac{5\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi c^4}\right)a-\frac{\left(75\pi^{\frac{3}{2}}c^6x^7+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(5/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/35*(5*(pi + pi*c^2*x^2)^(5/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(5/2)/(pi*c^4))*a - 1/3675*(75*pi^(3/2)*c^6*x^7 + 168*pi^(
3/2)*c^4*x^5 + 35*pi^(3/2)*c^2*x^3 - 210*pi^(3/2)*x)*b/c^3
```

Fricas [A] time = 2.39489, size = 485, normalized size = 3.88

$$\frac{105 \sqrt{\pi + \pi c^2 x^2}\left(5 \pi b c^8 x^8 + 13 \pi b c^6 x^6 + 9 \pi b c^4 x^4 - \pi b c^2 x^2 - 2 \pi b\right) \log\left(c x + \sqrt{c^2 x^2 + 1}\right) + \sqrt{\pi + \pi c^2 x^2}\left(525 \pi a c^8 x^8 + 13\right.}{3675 \left(c^6 x^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3675*(105*sqrt(pi + pi*c^2*x^2)*(5*pi*b*c^8*x^8 + 13*pi*b*c^6*x^6 + 9*pi*
b*c^4*x^4 - pi*b*c^2*x^2 - 2*pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi +
 pi*c^2*x^2)*(525*pi*a*c^8*x^8 + 1365*pi*a*c^6*x^6 + 945*pi*a*c^4*x^4 - 105
*pi*a*c^2*x^2 - 210*pi*a - (75*pi*b*c^7*x^7 + 168*pi*b*c^5*x^5 + 35*pi*b*c^
3*x^3 - 210*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.64 $\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=165

$$\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}\pi x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\pi^{3/2}x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}-\frac{\pi}{}$$

```
[Out] -(b*Pi^(3/2)*x^2)/(32*c) - (7*b*c*Pi^(3/2)*x^4)/96 - (b*c^3*Pi^(3/2)*x^6)/3
6 + (Pi^(3/2)*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (Pi*x^3*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(Pi + c^2*Pi*x^2)^(3/2
)*(a + b*ArcSinh[c*x]))/6 - (Pi^(3/2)*(a + b*ArcSinh[c*x])^2)/(32*b*c^3)
```

Rubi [A] time = 0.321175, antiderivative size = 254, normalized size of antiderivative = 1.54, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5744, 5742, 5758, 5675, 30, 14}

$$\frac{1}{6}x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}\pi x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}-\frac{\pi}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (7*b*c*Pi*x^4*
Sqrt[Pi + c^2*Pi*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^6*Sqrt[Pi + c^2
*Pi*x^2])/(36*Sqrt[1 + c^2*x^2]) + (Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (Pi*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8
+ (x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (Pi*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}\pi\int x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh\right.$$

$$=\frac{1}{8}\pi x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.$$

$$=-\frac{7bc\pi x^4\sqrt{\pi+c^2\pi x^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^6\sqrt{\pi+c^2\pi x^2}}{36\sqrt{1+c^2x^2}}+\frac{\pi x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh\right.}{16c^2}$$

$$=-\frac{b\pi x^2\sqrt{\pi+c^2\pi x^2}}{32c\sqrt{1+c^2x^2}}-\frac{7bc\pi x^4\sqrt{\pi+c^2\pi x^2}}{96\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^6\sqrt{\pi+c^2\pi x^2}}{36\sqrt{1+c^2x^2}}+\frac{\pi x\sqrt{\pi}}{}$$

Mathematica [A] time = 0.347191, size = 154, normalized size = 0.93

$$\frac{\pi^{3/2}\left(-12\sinh^{-1}(cx)\left(12a+3b\sinh\left(2\sinh^{-1}(cx)\right)-3b\sinh\left(4\sinh^{-1}(cx)\right)-b\sinh\left(6\sinh^{-1}(cx)\right)\right)+384ac^5x^5\sqrt{c^2x^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(144*a*c*x*Sqrt[1 + c^2*x^2] + 672*a*c^3*x^3*Sqrt[1 + c^2*x^2] +
384*a*c^5*x^5*Sqrt[1 + c^2*x^2] - 72*b*ArcSinh[c*x]^2 + 18*b*Cosh[2*ArcSinh
[c*x]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 12*ArcSinh[c
*x]*(12*a + 3*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4*ArcSinh[c*x]] - b*Sinh[6*
ArcSinh[c*x]])))/(2304*c^3)
```

Maple [A] time = 0.058, size = 240, normalized size = 1.5

$$\frac{ax}{6\pi c^2}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}-\frac{ax}{24c^2}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{a\pi x}{16c^2}\sqrt{\pi c^2x^2+\pi}-\frac{a\pi^2}{16c^2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+\frac{b\pi^{\frac{3}{2}}c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*a*x*(Pi*c^2*x^2+Pi)^(5/2)/Pi/c^2-1/24*a/c^2*x*(Pi*c^2*x^2+Pi)^(3/2)-1/1
6*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)-1/16*a/c^2*Pi^2*ln(Pi*x*c^2/(Pi*c^2)^(1/
```

```
2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6*b*Pi^(3/2)*c^2*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*x^5-1/36*b*c^3*Pi^(3/2)*x^6+7/24*b*Pi^(3/2)*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*x^3-7/96*b*c*Pi^(3/2)*x^4+1/16*b*Pi^(3/2)/c^2*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*x-1/32*b*Pi^(3/2)*x^2/c-1/32*b*Pi^(3/2)/c^3*arcsinh(c*x)^
2+1/72*b*Pi^(3/2)/c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\pi + \pi c^2x^2}\left(\pi ac^2x^4 + \pi ax^2 + \left(\pi bc^2x^4 + \pi bx^2\right)\text{arsinh}\,(cx)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^4 + pi*a*x^2 + (pi*b*c^2*x^4 + p
i*b*x^2)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

3.65 $\int x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=77

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}-\frac{1}{25}\pi^{3/2}bc^3x^5-\frac{2}{15}\pi^{3/2}bcx^3-\frac{\pi^{3/2}bx}{5c}$$

```
[Out] -(b*Pi^(3/2)*x)/(5*c) - (2*b*c*Pi^(3/2)*x^3)/15 - (b*c^3*Pi^(3/2)*x^5)/25 +
 ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rubi [A] time = 0.087301, antiderivative size = 146, normalized size of antiderivative = 1.9, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}-\frac{\pi bc^3x^5\sqrt{\pi c^2x^2+\pi}}{25\sqrt{c^2x^2+1}}-\frac{2\pi bcx^3\sqrt{\pi c^2x^2+\pi}}{15\sqrt{c^2x^2+1}}-\frac{\pi bx\sqrt{\pi c^2x^2+\pi}}{5c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*Pi*x*Sqrt[Pi + c^2*Pi*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (2*b*c*Pi*x^3*Sqr
t[Pi + c^2*Pi*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^5*Sqrt[Pi + c^2*Pi
*x^2])/(25*Sqrt[1 + c^2*x^2]) + ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x
]))/(5*c^2*Pi)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi}-\frac{\left(b\pi\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+c^2x^2\right)^2\,dx}{5c\sqrt{1+c^2x^2}}$$

$$=\frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi}-\frac{\left(b\pi\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+2c^2x^2+c^4x^4\right)}{5c\sqrt{1+c^2x^2}}$$

$$=-\frac{b\pi x\sqrt{\pi+c^2\pi x^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bc\pi x^3\sqrt{\pi+c^2\pi x^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3\pi x^5\sqrt{\pi+c^2\pi x^2}}{25\sqrt{1+c^2x^2}}+\frac{\left(\pi+c^2\pi\right.}{}$$

Mathematica [A] time = 0.124941, size = 72, normalized size = 0.94

$$\frac{\pi^{3/2}\left(15a\left(c^2x^2+1\right)^{5/2}-bcx\left(3c^4x^4+10c^2x^2+15\right)+15b\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)\right)}{75c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(15*a*(1 + c^2*x^2)^(5/2) - b*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) +
 15*b*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]))/(75*c^2)
```

Maple [B] time = 0.057, size = 139, normalized size = 1.8

$$\frac{a}{5\,\pi\,c^2}\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}+\frac{b\pi^{\frac{3}{2}}}{75\,c^2}\left(15\,\mathrm{Arcsinh}\,(cx)\,c^6x^6+45\,\mathrm{Arcsinh}\,(cx)\,c^4x^4-3\,c^5x^5\sqrt{c^2x^2+1}+45\,\mathrm{Arcsinh}\,(cx)\,c^2x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/5*a/Pi/c^2*(Pi*c^2*x^2+Pi)^(5/2)+1/75*b/c^2*Pi^(3/2)/(c^2*x^2+1)^(1/2)*(1
5*arcsinh(c*x)*c^6*x^6+45*arcsinh(c*x)*c^4*x^4-3*c^5*x^5*(c^2*x^2+1)^(1/2)+
45*arcsinh(c*x)*c^2*x^2-10*c^3*x^3*(c^2*x^2+1)^(1/2)+15*arcsinh(c*x)-15*c*x
*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.16566, size = 115, normalized size = 1.49

$$\frac{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} b \operatorname{arsinh}(cx)}{5\,\pi c^2} + \frac{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} a}{5\,\pi c^2} - \frac{\left(3\,\pi^{\frac{5}{2}} c^4 x^5 + 10\,\pi^{\frac{5}{2}} c^2 x^3 + 15\,\pi^{\frac{5}{2}} x\right) b}{75\,\pi c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(pi + pi*c^2*x^2)^(5/2)*b*arcsinh(c*x)/(pi*c^2) + 1/5*(pi + pi*c^2*x^2)
^(5/2)*a/(pi*c^2) - 1/75*(3*pi^(5/2)*c^4*x^5 + 10*pi^(5/2)*c^2*x^3 + 15*pi^
(5/2)*x)*b/(pi*c)
```

Fricas [B] time = 2.37377, size = 393, normalized size = 5.1

$$\frac{15\sqrt{\pi + \pi c^2 x^2}\left(\pi b c^6 x^6 + 3\,\pi b c^4 x^4 + 3\,\pi b c^2 x^2 + \pi b\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \sqrt{\pi + \pi c^2 x^2}\left(15\,\pi a c^6 x^6 + 45\,\pi a c^4 x^4 + 45\,\pi a\right.}{75\left(c^4 x^2 + c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*sqrt(pi + pi*c^2*x^2)*(pi*b*c^6*x^6 + 3*pi*b*c^4*x^4 + 3*pi*b*c^2*
x^2 + pi*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(15*pi*a*c
^6*x^6 + 45*pi*a*c^4*x^4 + 45*pi*a*c^2*x^2 + 15*pi*a - (3*pi*b*c^5*x^5 + 10
*pi*b*c^3*x^3 + 15*pi*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

Sympy [A] time = 126.584, size = 221, normalized size = 2.87

$$\begin{cases} \frac{\pi^{\frac{3}{2}} a c^2 x^4 \sqrt{c^2x^2+1}}{5} + \frac{2\pi^{\frac{3}{2}} a x^2 \sqrt{c^2x^2+1}}{5} + \frac{\pi^{\frac{3}{2}} a \sqrt{c^2x^2+1}}{5c^2} - \frac{\pi^{\frac{3}{2}} b c^3 x^5}{25} + \frac{\pi^{\frac{3}{2}} b c^2 x^4 \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{5} - \frac{2\pi^{\frac{3}{2}} b c x^3}{15} + \frac{2\pi^{\frac{3}{2}} b x^2 \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{5} - \frac{\pi^{\frac{3}{2}} b x}{5c} \\ \frac{\pi^{\frac{3}{2}} a x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**4*sqrt(c**2*x**2 + 1)/5 + 2*pi**(3/2)*a*x**2
*sqrt(c**2*x**2 + 1)/5 + pi**(3/2)*a*sqrt(c**2*x**2 + 1)/(5*c**2) - pi**(3/
2)*b*c**3*x**5/25 + pi**(3/2)*b*c**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/5
- 2*pi**(3/2)*b*c*x**3/15 + 2*pi**(3/2)*b*x**2*sqrt(c**2*x**2 + 1)*asinh(c*
x)/5 - pi**(3/2)*b*x/(5*c) + pi**(3/2)*b*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5*
c**2), Ne(c, 0)), (pi**(3/2)*a*x**2/2, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.66 $\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b \sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=111

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc}-\frac{1}{16}\pi^{3/2}bc^3x^4-$$

```
[Out] (-5*b*c*Pi^(3/2)*x^2)/16 - (b*c^3*Pi^(3/2)*x^4)/16 + (3*Pi*x*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh
[c*x]))/4 + (3*Pi^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c)
```

Rubi [A] time = 0.112484, antiderivative size = 180, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5682, 5675, 30, 14}

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{\pi bc^3}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*c*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^
4*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*Pi*x*Sqrt[Pi + c^2*Pi*
x^2]*(a + b*ArcSinh[c*x]))/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/4 + (3*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1
 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right) dx = \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{4}(3\pi)\int \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(c\right.$$

$$= \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) +$$

$$= -\frac{5bc\pi x^2\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^4\sqrt{\pi + c^2\pi x^2}}{16\sqrt{1 + c^2x^2}} + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-}\right.$$

Mathematica [A] time = 0.224868, size = 111, normalized size = 1.

$$\frac{\pi^{3/2}\left(4\sinh^{-1}(cx)\left(12a + 8b\sinh\left(2\sinh^{-1}(cx)\right) + b\sinh\left(4\sinh^{-1}(cx)\right)\right) + 32ac^3x^3\sqrt{c^2x^2+1} + 80acx\sqrt{c^2x^2+1} + 24b\right.}{128c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(3/2)*(80*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24
*b*ArcSinh[c*x]^2 - 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*
ArcSinh[c*x]*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]])))/(
128*c)
```

Maple [A] time = 0.048, size = 170, normalized size = 1.5

$$\frac{ax}{4}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}+\frac{3a\pi x}{8}\sqrt{\pi c^2x^2+\pi}+\frac{3a\pi^2}{8}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+\frac{b\pi^{\frac{3}{2}}c^2\mathrm{Arcsinh}\,(cx)\,x^3}{4}\sqrt{c^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/4*x*(Pi*c^2*x^2+Pi)^(3/2)*a+3/8*a*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a*Pi^2*l
n(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b*Pi^(3
/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/16*b*c^3*Pi^(3/2)*x^4+5/8*b*Pi
^(3/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-5/16*b*c*Pi^(3/2)*x^2+3/16*b*Pi^(3/
2)/c*arcsinh(c*x)^2-1/4*b*Pi^(3/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(\pi ac^2x^2+\pi a+\left(\pi bc^2x^2+\pi b\right)\mathrm{arsinh}\,(cx)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x)), x)
```

Sympy [A] time = 52.3446, size = 185, normalized size = 1.67

$$\begin{cases} \frac{\pi^{\frac{3}{2}} ac^2x^3\sqrt{c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} ax\sqrt{c^2x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a \operatorname{asinh}(cx)}{8c} - \frac{\pi^{\frac{3}{2}} bc^3x^4}{16} + \frac{\pi^{\frac{3}{2}} bc^2x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4} - \frac{5\pi^{\frac{3}{2}} bcx^2}{16} + \frac{5\pi^{\frac{3}{2}} bx\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8} + 3\pi \\ \pi^{\frac{3}{2}} ax \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((pi**(3/2)*a*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x*sq
rt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a*asinh(c*x)/(8*c) - pi**(3/2)*b*c**3*x**
4/16 + pi**(3/2)*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/4 - 5*pi**(3/2)
*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/8 + 3*pi**(3/
2)*b*asinh(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*a*x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\pi + \pi c^2x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a), x)
```

3.67 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=134

$$-\pi^{3/2}b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\pi^{3/2}b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\pi\sqrt{\pi c^2x^2+\pi}\left(a\right.$$

```
[Out] (-4*b*c*Pi^(3/2)*x)/3 - (b*c^3*Pi^(3/2)*x^3)/9 + Pi*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - 2*
Pi^(3/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Pi^(3/2)*PolyLog[
2, -E^ArcSinh[c*x]] + b*Pi^(3/2)*PolyLog[2, E^ArcSinh[c*x]]
```

Rubi [A] time = 0.304125, antiderivative size = 249, normalized size of antiderivative = 1.86, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{\pi b\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{\pi b\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{3}\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-4*b*c*Pi*x*Sqrt[Pi + c^2*Pi*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x^3*S
qrt[Pi + c^2*Pi*x^2])/(9*Sqrt[1 + c^2*x^2]) + Pi*Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*Pi
*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1
 + c^2*x^2] - (b*Pi*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt
[1 + c^2*x^2] + (b*Pi*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqr
t[1 + c^2*x^2]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{3}\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \pi\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx$$

$$= -\frac{bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)$$

$$= -\frac{4bc\pi x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)$$

Mathematica [A] time = 0.353417, size = 180, normalized size = 1.34

$$\frac{1}{9}\pi^{3/2}\left(9b\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - \text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + \sqrt{c^2x^2+1}\sinh^{-1}(cx) - cx + \sinh^{-1}(cx)\log\left(1 - e^{-\sin\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(3/2)*(3*a*Sqrt[1 + c^2*x^2]*(4 + c^2*x^2) - b*(3*c*x + c^3*x^3 - 3*(1
+ c^2*x^2)^(3/2)*ArcSinh[c*x]) + 9*a*Log[x] - 9*a*Log[Pi*(1 + Sqrt[1 + c^2*
x^2])] + 9*b*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1
- E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2,
 -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])])))/9
```

Maple [A] time = 0.172, size = 227, normalized size = 1.7

$$\frac{a}{3}\left(\pi c^2x^2 + \pi\right)^{\frac{3}{2}} - a\pi^{\frac{3}{2}}\text{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi c^2x^2 + \pi}}\right) + a\pi\sqrt{\pi c^2x^2 + \pi} - \frac{bc^3\pi^{\frac{3}{2}}x^3}{9} + \frac{4b\text{Arcsinh}(cx)\pi^{3/2}}{3}\sqrt{c^2x^2+1} - \frac{4bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/3*(Pi*c^2*x^2+Pi)^(3/2)*a-a*Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/
2))+a*Pi*(Pi*c^2*x^2+Pi)^(1/2)-1/9*b*c^3*Pi^(3/2)*x^3+4/3*b*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*Pi^(3/2)-4/3*b*c*Pi^(3/2)*x+b*Pi^(3/2)*arcsinh(c*x)*ln(1-c*
x-(c^2*x^2+1)^(1/2))-b*Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*
Pi^(3/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b*Pi^(3/2)*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))+1/3*b*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*Pi^(3/2)*x^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\left(3\,\pi^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)-3\,\pi\sqrt{\pi+\pi c^2x^2}-\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}\right)a+b\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*pi^(3/2)*arcsinh(1/(sqrt(c^2)*abs(x))) - 3*pi*sqrt(pi + pi*c^2*x^2)
 - (pi + pi*c^2*x^2)^(3/2))*a + b*integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x
 + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi ac^2x^2+\pi a+\left(\pi bc^2x^2+\pi b\right)\operatorname{arsinh}\left(cx\right)\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\pi^{\frac{3}{2}}\left(\int\frac{a\sqrt{c^2x^2+1}}{x}\,dx+\int ac^2x\sqrt{c^2x^2+1}\,dx+\int\frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x}\,dx+\int bc^2x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x, x) + Integral(a*c**2*x*sqrt(c*
*2*x**2 + 1), x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x) + Integr
al(b*c**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)/x, x)
```

3.68 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=108

$$\frac{3}{2}\pi c^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{3\pi^{3/2}c\left(a+b\sinh^{-1}(cx)\right)^2}{4b}-\frac{1}{4}\pi^{3/2}bc^3x^2+$$

```
[Out] -(b*c^3*Pi^(3/2)*x^2)/4 + (3*c^2*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[
c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (3*c*Pi^(3/2)
*(a + b*ArcSinh[c*x])^2)/(4*b) + b*c*Pi^(3/2)*Log[x]
```

Rubi [A] time = 0.167999, antiderivative size = 177, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5682, 5675, 30, 14}

$$\frac{3}{2}\pi c^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{3\pi c\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\pi bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c^3*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2])/(4*Sqrt[1 + c^2*x^2]) + (3*c^2*Pi*x*S
qrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/x + (3*c*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/(4*b*Sqrt[1 + c^2*x^2]) + (b*c*Pi*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/Sqrt[1 +
c^2*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\left(3c^2\pi\right)\int\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx\right)$$

$$=\frac{3}{2}c^2\pi x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+$$

$$=-\frac{bc^3\pi x^2\sqrt{\pi+c^2\pi x^2}}{4\sqrt{1+c^2x^2}}+\frac{3}{2}c^2\pi x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{3/}}{}$$

Mathematica [A] time = 0.28834, size = 122, normalized size = 1.13

$$\frac{\pi^{3/2}\left(2\sinh^{-1}(cx)\left(6acx-4b\sqrt{c^2x^2+1}+bcx\sinh\left(2\sinh^{-1}(cx)\right)\right)+4ac^2x^2\sqrt{c^2x^2+1}-8a\sqrt{c^2x^2+1}+8bcx\log(cx)+6b\right.}{8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Pi^(3/2)*(-8*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 6*b*c*x
*ArcSinh[c*x]^2 - b*c*x*Cosh[2*ArcSinh[c*x]] + 8*b*c*x*Log[c*x] + 2*ArcSinh
[c*x]*(6*a*c*x - 4*b*Sqrt[1 + c^2*x^2] + b*c*x*Sinh[2*ArcSinh[c*x]])))/(8*x
)
```

Maple [B] time = 0.148, size = 222, normalized size = 2.1

$$-\frac{a}{\pi x}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}+ac^2x\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}+\frac{3\,ac^2\pi\,x}{2}\sqrt{\pi c^2x^2+\pi}+\frac{3\,ac^2\pi^2}{2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/2*a*c^2*Pi*x*
(Pi*c^2*x^2+Pi)^(1/2)+3/2*a*c^2*Pi^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
+Pi)^(1/2))/(Pi*c^2)^(1/2)+3/4*b*c*Pi^(3/2)*arcsinh(c*x)^2+1/2*b*arcsinh(c*
x)*Pi^(3/2)*(c^2*x^2+1)^(1/2)*x*c^2-1/4*b*c^3*Pi^(3/2)*x^2-b*c*Pi^(3/2)*arc
sinh(c*x)-1/8*b*Pi^(3/2)*c-b*Pi^(3/2)*arcsinh(c*x)/x*(c^2*x^2+1)^(1/2)+b*c*
Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}\left(\pi ac^2x^2 + \pi a + \left(\pi bc^2x^2 + \pi b\right)\text{arsinh}\,(cx)\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\pi^{\frac{3}{2}}\left(\int ac^2\sqrt{c^2x^2+1}\,dx + \int \frac{a\sqrt{c^2x^2+1}}{x^2}\,dx + \int bc^2\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)\,dx + \int \frac{b\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] pi**(3/2)*(Integral(a*c**2*sqrt(c**2*x**2 + 1), x) + Integral(a*sqrt(c**2*x
**2 + 1)/x**2, x) + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x), x) + In
tegral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.69 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=155

$$-\frac{3}{2}\pi^{3/2}bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{3}{2}\pi^{3/2}bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{3}{2}\pi c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2\right.}{}$$

```
[Out] -(b*c*Pi^(3/2))/(2*x) - b*c^3*Pi^(3/2)*x + (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*
(a + b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2
*x^2) - 3*c^2*Pi^(3/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - (3*b*
c^2*Pi^(3/2)*PolyLog[2, -E^ArcSinh[c*x]])/2 + (3*b*c^2*Pi^(3/2)*PolyLog[2,
E^ArcSinh[c*x]])/2
```

Rubi [A] time = 0.302217, antiderivative size = 270, normalized size of antiderivative = 1.74, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5742, 5760, 4182, 2279, 2391, 8, 14}

$$-\frac{3\pi bc^2\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3\pi bc^2\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}\pi c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\,\text{si}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Pi*Sqrt[Pi + c^2*Pi*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (b*c^3*Pi*x*Sqrt[
Pi + c^2*Pi*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/2 - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2)
 - (3*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2] - (3*b*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^A
rcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (3*b*c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
```

```
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{1}{2}\left(3c^2\pi\right)\int \frac{\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}\right.}{x}$$

$$= \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + $$

$$= -\frac{bc\pi\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx\right.$$

$$= -\frac{bc\pi\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx\right.$$

$$= -\frac{bc\pi\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx\right.$$

$$= -\frac{bc\pi\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{bc^3\pi x\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \frac{3}{2}c^2\pi\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx\right.$$

Mathematica [A] time = 1.71188, size = 292, normalized size = 1.88

$$\frac{\pi^{3/2}\left(12bc^2x^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 12bc^2x^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + 8ac^2x^2\sqrt{c^2x^2+1} - 4a\sqrt{c^2x^2+1} + 12ac^2x\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(3/2)*(-8*b*c^3*x^3 - 4*a*Sqrt[1 + c^2*x^2] + 8*a*c^2*x^2*Sqrt[1 + c^2*
x^2] + 8*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*c^3*x^3*Csch[ArcSinh[
c*x]/2]^2 - b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 12*b*c^2*x^2*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*ArcSinh[c*x]*Log[1 + E
^(-ArcSinh[c*x])] + 12*a*c^2*x^2*Log[x] - 12*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 +
 c^2*x^2])] + 12*b*c^2*x^2*PolyLog[2, -E^(-ArcSinh[c*x])] - 12*b*c^2*x^2*Po
```

```
lyLog[2, E^(-ArcSinh[c*x])] + 4*b*c*x*Sinh[ArcSinh[c*x]/2]^2 - 4*b*ArcSinh[
c*x]*Sinh[ArcSinh[c*x]/2]^2))/(8*x^2)
```

Maple [A] time = 0.371, size = 295, normalized size = 1.9

$$-\frac{a}{2\,\pi\,x^2}\left(\pi\,c^2x^2+\pi\right)^{\frac{5}{2}}+\frac{ac^2}{2}\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{3\,ac^2\pi^{3/2}}{2}\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}\right)+\frac{3\,ac^2\pi}{2}\sqrt{\pi\,c^2x^2+\pi}+b\mathrm{Arcsinh}\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(5/2)+1/2*a*c^2*(Pi*c^2*x^2+Pi)^(3/2)-3/2*a*c
^2*Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+3/2*a*c^2*Pi*(Pi*c^2*x^
2+Pi)^(1/2)+b*arcsinh(c*x)*Pi^(3/2)*(c^2*x^2+1)^(1/2)*c^2-b*c^3*Pi^(3/2)*x-
1/2*b*Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c*Pi^(3/2)/x-1/2*b*
Pi^(3/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)-3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)
*ln(1+c*x+(c^2*x^2+1)^(1/2))-3/2*b*c^2*Pi^(3/2)*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))+3/2*b*c^2*Pi^(3/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+3/2*b*c^
2*Pi^(3/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(3\,\pi^{\frac{3}{2}}c^2\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)-3\,\pi\sqrt{\pi+\pi c^2x^2}c^2-\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2+\frac{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}{\pi x^2}\right)a+b\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c^2}\right.}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(3*pi^(3/2)*c^2*arcsinh(1/(sqrt(c^2)*abs(x))) - 3*pi*sqrt(pi + pi*c^2*
x^2)*c^2 - (pi + pi*c^2*x^2)^(3/2)*c^2 + (pi + pi*c^2*x^2)^(5/2)/(pi*x^2))*
a + b*integrate((pi + pi*c^2*x^2)^(3/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi ac^2x^2+\pi a+\left(\pi bc^2x^2+\pi b\right)\text{arsinh}\left(cx\right)\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\pi^{\frac{3}{2}}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^3}\,dx+\int\frac{ac^2\sqrt{c^2x^2+1}}{x}\,dx+\int\frac{b\sqrt{c^2x^2+1}\,\text{asinh}\left(cx\right)}{x^3}\,dx+\int\frac{bc^2\sqrt{c^2x^2+1}\,\text{asinh}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**3, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**3, x) +
Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.70 $\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=115

$$-\frac{\pi c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{\pi^{3/2}c^3\left(a+b\sinh^{-1}(cx)\right)^2}{2b}+\frac{4}{3}\pi^{3/2}bc^3\log(x)$$

```
[Out] -(b*c*Pi^(3/2))/(6*x^2) - (c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/x - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (c^3*Pi^(3/
2)*(a + b*ArcSinh[c*x])^2)/(2*b) + (4*b*c^3*Pi^(3/2)*Log[x])/3
```

Rubi [A] time = 0.218711, antiderivative size = 184, normalized size of antiderivative = 1.6, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5737, 29, 5675, 14}

$$\frac{\pi c^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\pi c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{\pi bc\sqrt{\pi c}}{6x^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Pi*Sqrt[Pi + c^2*Pi*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (c^2*Pi*Sqrt[Pi
 + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/x - ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(3*x^3) + (c^3*Pi*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/(2*b*Sqrt[1 + c^2*x^2]) + (4*b*c^3*Pi*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/(3*Sq
rt[1 + c^2*x^2])
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\left(c^2\pi\right)\int \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx\right.}{x^2}$$

$$= -\frac{c^2\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+$$

$$= -\frac{bc\pi\sqrt{\pi+c^2\pi x^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{c^2\pi\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(\pi+c^2\pi x^2\right)^{3/2}\left(a\right.}{3x^3}$$

Mathematica [A] time = 0.2272, size = 125, normalized size = 1.09

$$\frac{\pi^{3/2}\left(\sinh^{-1}(cx)\left(6ac^3x^3-2b\sqrt{c^2x^2+1}\left(4c^2x^2+1\right)\right)-8ac^2x^2\sqrt{c^2x^2+1}-2a\sqrt{c^2x^2+1}+8bc^3x^3\log(cx)+3bc^3x^3\sinh\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(3/2)*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2] - 8*a*c^2*x^2*Sqrt[1 + c^2*x^2]
 + (6*a*c^3*x^3 - 2*b*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2))*ArcSinh[c*x] + 3*b
*c^3*x^3*ArcSinh[c*x]^2 + 8*b*c^3*x^3*Log[c*x]))/(6*x^3)
```

Maple [B] time = 0.221, size = 622, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(5/2)-2/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(5/2)+2/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(3/2)+a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+a*c^4*Pi^2
*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b*c^3
*Pi^(3/2)*arcsinh(c*x)^2-8/3*b*c^3*Pi^(3/2)*arcsinh(c*x)+32*b*Pi^(3/2)/(24*
c^4*x^4+9*c^2*x^2+1)*x^4*arcsinh(c*x)*c^7-32*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x
^2+1)*x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6+8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*x^4*c^7-8/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*(c^2*x^2+1)*c
^5+12*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-20*b*Pi^(3/2
)/(24*c^4*x^4+9*c^2*x^2+1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4-4/3*b*Pi^(3
/2)/(24*c^4*x^4+9*c^2*x^2+1)*(c^2*x^2+1)*c^3+4/3*b*Pi^(3/2)/(24*c^4*x^4+9*c
^2*x^2+1)*arcsinh(c*x)*c^3-13/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*c^2-1/6*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^2*(
c^2*x^2+1)*c-1/3*b*Pi^(3/2)/(24*c^4*x^4+9*c^2*x^2+1)/x^3*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)+4/3*b*c^3*Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi ac^2x^2+\pi a+\left(\pi bc^2x^2+\pi b\right)\operatorname{arsinh}(cx)\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a*c^2*x^2 + pi*a + (pi*b*c^2*x^2 + pi*b)
*arcsinh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\pi^{\frac{3}{2}}\left(\int\frac{a\sqrt{c^2x^2+1}}{x^4}\,dx+\int\frac{ac^2\sqrt{c^2x^2+1}}{x^2}\,dx+\int\frac{b\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^4}\,dx+\int\frac{bc^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] pi**(3/2)*(Integral(a*sqrt(c**2*x**2 + 1)/x**4, x) + Integral(a*c**2*sqrt(c
**2*x**2 + 1)/x**2, x) + Integral(b*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**4, x)
 + Integral(b*c**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.71 $\int x^3 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=141

$$\frac{\left(\pi c^2x^2+\pi\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9\pi^2c^4}-\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^4}-\frac{1}{81}\pi^{5/2}bc^5x^9-\frac{19}{441}\pi^{5/2}bc^3x^7+\frac{2\pi^{5/2}bx}{63c^3}-\frac{1}{21}\pi$$

```
[Out] (2*b*Pi^(5/2)*x)/(63*c^3) - (b*Pi^(5/2)*x^3)/(189*c) - (b*c*Pi^(5/2)*x^5)/2
1 - (19*b*c^3*Pi^(5/2)*x^7)/441 - (b*c^5*Pi^(5/2)*x^9)/81 - ((Pi + c^2*Pi*x
^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*Pi) + ((Pi + c^2*Pi*x^2)^(9/2)*(a +
b*ArcSinh[c*x]))/(9*c^4*Pi^2)
```

Rubi [A] time = 0.152781, antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 12, 373}

$$\frac{\pi^{5/2}\left(c^2x^2+1\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4}-\frac{\pi^{5/2}\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}-\frac{1}{81}\pi^{5/2}bc^5x^9-\frac{19}{441}\pi^{5/2}bc^3x^7+\frac{2\pi^{5/2}bx}{63c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*b*Pi^(5/2)*x)/(63*c^3) - (b*Pi^(5/2)*x^3)/(189*c) - (b*c*Pi^(5/2)*x^5)/2
1 - (19*b*c^3*Pi^(5/2)*x^7)/441 - (b*c^5*Pi^(5/2)*x^9)/81 - (Pi^(5/2)*(1 +
c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4) + (Pi^(5/2)*(1 + c^2*x^2)^(9/2
)*(a + b*ArcSinh[c*x]))/(9*c^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx &= -\frac{\pi^{5/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}\left(1+c^2x^2\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right.}{9c^4}\\ &= -\frac{\pi^{5/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}\left(1+c^2x^2\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right.}{9c^4}\\ &= -\frac{\pi^{5/2}\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4}+\frac{\pi^{5/2}\left(1+c^2x^2\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right.}{9c^4}\\ &= \frac{2b\pi^{5/2}x}{63c^3}-\frac{b\pi^{5/2}x^3}{189c}-\frac{1}{21}bc\pi^{5/2}x^5-\frac{19}{441}bc^3\pi^{5/2}x^7-\frac{1}{81}bc^5\pi^{5/2}x^9-\frac{\pi^{5/2}\left(1+\right.}{}\end{aligned}$$

Mathematica [A] time = 0.198417, size = 108, normalized size = 0.77

$$\frac{\pi^{5/2}\left(63a\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^{7/2}-bcx\left(49c^8x^8+171c^6x^6+189c^4x^4+21c^2x^2-126\right)+63b\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^{7/2}\sin\right.}{3969c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(63*a*(1 + c^2*x^2)^(7/2)*(-2 + 7*c^2*x^2) - b*c*x*(-126 + 21*c^2
*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8) + 63*b*(1 + c^2*x^2)^(7/2)*(
-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4)
```

Maple [A] time = 0.1, size = 226, normalized size = 1.6

$$a\left(\frac{x^2}{9\,\pi\,c^2}\left(\pi\,c^2x^2+\pi\right)^{\frac{7}{2}}-\frac{2}{63\,\pi\,c^4}\left(\pi\,c^2x^2+\pi\right)^{\frac{7}{2}}\right)+\frac{b\pi^{\frac{5}{2}}}{3969\,c^4}\left(441\,\mathrm{Arcsinh}\,(cx)\,c^{10}x^{10}+1638\,\mathrm{Arcsinh}\,(cx)\,c^8x^8-49\,c^9x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/9*x^2*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-2/63/Pi/c^4*(Pi*c^2*x^2+Pi)^(7/2))+
1/3969*b/c^4*Pi^(5/2)/(c^2*x^2+1)^(1/2)*(441*arcsinh(c*x)*c^10*x^10+1638*ar
csinh(c*x)*c^8*x^8-49*c^9*x^9*(c^2*x^2+1)^(1/2)+2142*arcsinh(c*x)*c^6*x^6-1
71*c^7*x^7*(c^2*x^2+1)^(1/2)+1008*arcsinh(c*x)*c^4*x^4-189*c^5*x^5*(c^2*x^2
+1)^(1/2)-63*arcsinh(c*x)*c^2*x^2-21*c^3*x^3*(c^2*x^2+1)^(1/2)-126*arcsinh(
c*x)+126*c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.20911, size = 211, normalized size = 1.5

$$\frac{1}{63}\left(\frac{7\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}}{\pi c^4}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{63}\left(\frac{7\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}x^2}{\pi c^2}-\frac{2\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}}{\pi c^4}\right)a-\frac{\left(49\,\pi^{\frac{5}{2}}c^8x^9+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(pi + pi*c^2*x^2)^(7/2)/(p
i*c^4))*b*arcsinh(c*x) + 1/63*(7*(pi + pi*c^2*x^2)^(7/2)*x^2/(pi*c^2) - 2*(
pi + pi*c^2*x^2)^(7/2)/(pi*c^4))*a - 1/3969*(49*pi^(5/2)*c^8*x^9 + 171*pi^(
5/2)*c^6*x^7 + 189*pi^(5/2)*c^4*x^5 + 21*pi^(5/2)*c^2*x^3 - 126*pi^(5/2)*x)
*b/c^3
```

Fricas [B] time = 2.47961, size = 613, normalized size = 4.35

$$\frac{63\sqrt{\pi+\pi c^2x^2}\left(7\,\pi^2bc^{10}x^{10}+26\,\pi^2bc^8x^8+34\,\pi^2bc^6x^6+16\,\pi^2bc^4x^4-\pi^2bc^2x^2-2\,\pi^2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] 1/3969*(63*sqrt(pi + pi*c^2*x^2)*(7*pi^2*b*c^10*x^10 + 26*pi^2*b*c^8*x^8 +
34*pi^2*b*c^6*x^6 + 16*pi^2*b*c^4*x^4 - pi^2*b*c^2*x^2 - 2*pi^2*b)*log(c*x
+ sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(441*pi^2*a*c^10*x^10 + 1638*p
i^2*a*c^8*x^8 + 2142*pi^2*a*c^6*x^6 + 1008*pi^2*a*c^4*x^4 - 63*pi^2*a*c^2*x
^2 - 126*pi^2*a - (49*pi^2*b*c^9*x^9 + 171*pi^2*b*c^7*x^7 + 189*pi^2*b*c^5*
x^5 + 21*pi^2*b*c^3*x^3 - 126*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^6*x^2 + c^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.72 $\int x^2 \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=213

$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{64}\pi^2x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(c\right.$$

```
[Out] (-5*b*Pi^(5/2)*x^2)/(256*c) - (59*b*c*Pi^(5/2)*x^4)/768 - (17*b*c^3*Pi^(5/2
)*x^6)/288 - (b*c^5*Pi^(5/2)*x^8)/64 + (5*Pi^(5/2)*x*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(128*c^2) + (5*Pi^2*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/64 + (5*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48
+ (x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*Pi^(5/2)*(a + b
*ArcSinh[c*x])^2)/(256*b*c^3)
```

Rubi [A] time = 0.457729, antiderivative size = 337, normalized size of antiderivative = 1.58, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5758, 5675, 30, 14, 266, 43}

$$\frac{1}{8}x^3\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{64}\pi^2x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*b*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*P
i^2*x^4*Sqrt[Pi + c^2*Pi*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*Pi^2*x^6
*Sqrt[Pi + c^2*Pi*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^8*Sqrt[Pi +
 c^2*Pi*x^2])/(64*Sqrt[1 + c^2*x^2]) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]))/(128*c^2) + (5*Pi^2*x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/64 + (5*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48
+ (x^3*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*Pi^2*Sqrt[Pi +
c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
```

```
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}(5\pi)\int x^2\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\right.$$

$$=\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}x^3\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh\right.$$

$$=\frac{5}{64}\pi^2x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}\pi x^3\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sin\right.$$

$$=-\frac{59bc\pi^2x^4\sqrt{\pi+c^2\pi x^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3\pi^2x^6\sqrt{\pi+c^2\pi x^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5\pi^2x^8\sqrt{\pi+c^2\pi x^2}}{64\sqrt{1+c^2x^2}}$$

$$=-\frac{5b\pi^2x^2\sqrt{\pi+c^2\pi x^2}}{256c\sqrt{1+c^2x^2}}-\frac{59bc\pi^2x^4\sqrt{\pi+c^2\pi x^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3\pi^2x^6\sqrt{\pi+c^2\pi x^2}}{288\sqrt{1+c^2x^2}}-$$

Mathematica [A] time = 0.565704, size = 196, normalized size = 0.92

$$\frac{\pi^{5/2}\left(-24\sinh^{-1}(cx)\left(120a+48b\sinh\left(2\sinh^{-1}(cx)\right)-24b\sinh\left(4\sinh^{-1}(cx)\right)-16b\sinh\left(6\sinh^{-1}(cx)\right)-3b\sinh\left(8\mathrm{s}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(2880*a*c*x*Sqrt[1 + c^2*x^2] + 22656*a*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2] - 1
440*b*ArcSinh[c*x]^2 + 576*b*Cosh[2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*
```

```
x]] - 64*b*Cosh[6*ArcSinh[c*x]] - 9*b*Cosh[8*ArcSinh[c*x]] - 24*ArcSinh[c*x
]*(120*a + 48*b*Sinh[2*ArcSinh[c*x]] - 24*b*Sinh[4*ArcSinh[c*x]] - 16*b*Sin
h[6*ArcSinh[c*x]] - 3*b*Sinh[8*ArcSinh[c*x]])))/(73728*c^3)
```

Maple [A] time = 0.093, size = 301, normalized size = 1.4

$$\frac{ax}{8\pi c^2}\left(\pi c^2x^2+\pi\right)^{\frac{7}{2}}-\frac{ax}{48c^2}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}-\frac{5a\pi x}{192c^2}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{5a\pi^2x}{128c^2}\sqrt{\pi c^2x^2+\pi}-\frac{5a\pi^3}{128c^2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/8*a*x*(Pi*c^2*x^2+Pi)^(7/2)/Pi/c^2-1/48*a/c^2*x*(Pi*c^2*x^2+Pi)^(5/2)-5/1
92*a/c^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)-5/128*a/c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2
)-5/128*a/c^2*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^
2)^(1/2)+1/8*b*Pi^(5/2)*c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^7-1/64*b*c^5*P
i^(5/2)*x^8+17/48*b*Pi^(5/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5-17/288*
b*c^3*Pi^(5/2)*x^6+59/192*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-59/
768*b*c*Pi^(5/2)*x^4+5/128*b*Pi^(5/2)/c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-
5/256*b*Pi^(5/2)*x^2/c-5/256*b*Pi^(5/2)/c^3*arcsinh(c*x)^2+1/72*b*Pi^(5/2)/
c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(\pi^2ac^4x^6+2\,\pi^2ac^2x^4+\pi^2ax^2+\left(\pi^2bc^4x^6+2\,\pi^2bc^2x^4+\pi^2bx^2\right)\mathrm{arsinh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^6 + 2*pi^2*a*c^2*x^4 + pi^2*a*
x^2 + (pi^2*b*c^4*x^6 + 2*pi^2*b*c^2*x^4 + pi^2*b*x^2)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a) x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

3.73 $\int x\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=93

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^2}-\frac{1}{49}\pi^{5/2}bc^5x^7-\frac{3}{35}\pi^{5/2}bc^3x^5-\frac{1}{7}\pi^{5/2}bcx^3-\frac{\pi^{5/2}bx}{7c}$$

```
[Out] -(b*Pi^(5/2)*x)/(7*c) - (b*c*Pi^(5/2)*x^3)/7 - (3*b*c^3*Pi^(5/2)*x^5)/35 -
(b*c^5*Pi^(5/2)*x^7)/49 + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^2*Pi)
```

Rubi [B] time = 0.0868338, antiderivative size = 193, normalized size of antiderivative = 2.08, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(\pi c^2x^2+\pi\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7\pi c^2}-\frac{\pi^2bc^5x^7\sqrt{\pi c^2x^2+\pi}}{49\sqrt{c^2x^2+1}}-\frac{3\pi^2bc^3x^5\sqrt{\pi c^2x^2+\pi}}{35\sqrt{c^2x^2+1}}-\frac{\pi^2bcx^3\sqrt{\pi c^2x^2+\pi}}{7\sqrt{c^2x^2+1}}-\frac{\pi^2bx\sqrt{\pi c^2x^2}}{7c\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (b*c*Pi^2*x^3*S
qrt[Pi + c^2*Pi*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*Pi^2*x^5*Sqrt[Pi + c
^2*Pi*x^2])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^7*Sqrt[Pi + c^2*Pi*x^2])
/(49*Sqrt[1 + c^2*x^2]) + ((Pi + c^2*Pi*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7
*c^2*Pi)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\left(\pi+c^2\pi x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2\pi}-\frac{\left(b\pi^2\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+c^2x^2\right)^3\,dx}{7c\sqrt{1+c^2x^2}}$$

$$=\frac{\left(\pi+c^2\pi x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2\pi}-\frac{\left(b\pi^2\sqrt{\pi+c^2\pi x^2}\right)\int\left(1+3c^2x^2+3c^4x\right.}{7c\sqrt{1+c^2x^2}}$$

$$=-\frac{b\pi^2x\sqrt{\pi+c^2\pi x^2}}{7c\sqrt{1+c^2x^2}}-\frac{bc\pi^2x^3\sqrt{\pi+c^2\pi x^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3\pi^2x^5\sqrt{\pi+c^2\pi x^2}}{35\sqrt{1+c^2x^2}}-\frac{bc^5\pi^2}{49}$$

Mathematica [A] time = 0.144706, size = 80, normalized size = 0.86

$$\frac{\pi^{5/2}\left(35a\left(c^2x^2+1\right)^{7/2}-bcx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)+35b\left(c^2x^2+1\right)^{7/2}\sinh^{-1}(cx)\right)}{245c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(35*a*(1 + c^2*x^2)^(7/2) - b*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 +
 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^(7/2)*ArcSinh[c*x]))/(245*c^2)
```

Maple [B] time = 0.063, size = 170, normalized size = 1.8

$$\frac{a}{7\,\pi\,c^2}\left(\pi\,c^2x^2+\pi\right)^{\frac{7}{2}}+\frac{b\pi^{\frac{5}{2}}}{245\,c^2}\left(35\,\mathrm{Arcsinh}\,(cx)\,c^8x^8+140\,\mathrm{Arcsinh}\,(cx)\,c^6x^6-5\,c^7x^7\sqrt{c^2x^2+1}+210\,\mathrm{Arcsinh}\,(cx)\,c^4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/7*a/Pi/c^2*(Pi*c^2*x^2+Pi)^(7/2)+1/245*b/c^2*Pi^(5/2)/(c^2*x^2+1)^(1/2)*(
35*arcsinh(c*x)*c^8*x^8+140*arcsinh(c*x)*c^6*x^6-5*c^7*x^7*(c^2*x^2+1)^(1/2
)+210*arcsinh(c*x)*c^4*x^4-21*c^5*x^5*(c^2*x^2+1)^(1/2)+140*arcsinh(c*x)*c^
2*x^2-35*c^3*x^3*(c^2*x^2+1)^(1/2)+35*arcsinh(c*x)-35*c*x*(c^2*x^2+1)^(1/2)
)
```

Maxima [A] time = 1.20053, size = 130, normalized size = 1.4

$$\frac{\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}b\operatorname{arsinh}(cx)}{7\,\pi c^2}+\frac{\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}a}{7\,\pi c^2}-\frac{\left(5\,\pi^{\frac{7}{2}}c^6x^7+21\,\pi^{\frac{7}{2}}c^4x^5+35\,\pi^{\frac{7}{2}}c^2x^3+35\,\pi^{\frac{7}{2}}x\right)b}{245\,\pi c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*(pi + pi*c^2*x^2)^(7/2)*b*arcsinh(c*x)/(pi*c^2) + 1/7*(pi + pi*c^2*x^2)
^(7/2)*a/(pi*c^2) - 1/245*(5*pi^(7/2)*c^6*x^7 + 21*pi^(7/2)*c^4*x^5 + 35*pi
^(7/2)*c^2*x^3 + 35*pi^(7/2)*x)*b/(pi*c)
```

Fricas [B] time = 2.60435, size = 508, normalized size = 5.46

$$\frac{35\sqrt{\pi+\pi c^2x^2}\left(\pi^2bc^8x^8+4\,\pi^2bc^6x^6+6\,\pi^2bc^4x^4+4\,\pi^2bc^2x^2+\pi^2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(35\,\pi^2ac^8x^8+\right.}{245\left(c^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*sqrt(pi + pi*c^2*x^2)*(pi^2*b*c^8*x^8 + 4*pi^2*b*c^6*x^6 + 6*pi^2
*b*c^4*x^4 + 4*pi^2*b*c^2*x^2 + pi^2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt
(pi + pi*c^2*x^2)*(35*pi^2*a*c^8*x^8 + 140*pi^2*a*c^6*x^6 + 210*pi^2*a*c^4*
x^4 + 140*pi^2*a*c^2*x^2 + 35*pi^2*a - (5*pi^2*b*c^7*x^7 + 21*pi^2*b*c^5*x^
5 + 35*pi^2*b*c^3*x^3 + 35*pi^2*b*c*x)*sqrt(c^2*x^2 + 1)))/(c^4*x^2 + c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.74 $\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=165

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] (-25*b*c*Pi^(5/2)*x^2)/96 - (5*b*c^3*Pi^(5/2)*x^4)/96 - (b*Pi^(5/2)*(1 + c^
2*x^2)^3)/(36*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/16
 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(Pi + c^2*
Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)
/(32*b*c)
```

Rubi [A] time = 0.164508, antiderivative size = 254, normalized size of antiderivative = 1.54, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5684, 5682, 5675, 30, 14, 261}

$$\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}\pi^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*
Pi^2*x^4*Sqrt[Pi + c^2*Pi*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*Pi^2*(1 + c^2*x
^2)^(5/2)*Sqrt[Pi + c^2*Pi*x^2])/(36*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]))/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*Pi^2*Sqrt
[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right) dx = \frac{1}{6}x\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{6}(5\pi)\int \left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(\right.$$

$$= -\frac{b\pi^2\left(1 + c^2x^2\right)^{5/2}\sqrt{\pi + c^2\pi x^2}}{36c} + \frac{5}{24}\pi x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{6}$$

$$= -\frac{b\pi^2\left(1 + c^2x^2\right)^{5/2}\sqrt{\pi + c^2\pi x^2}}{36c} + \frac{5}{16}\pi^2 x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) + \frac{5}{24}$$

$$= -\frac{25bc\pi^2x^2\sqrt{\pi + c^2\pi x^2}}{96\sqrt{1 + c^2x^2}} - \frac{5bc^3\pi^2x^4\sqrt{\pi + c^2\pi x^2}}{96\sqrt{1 + c^2x^2}} - \frac{b\pi^2\left(1 + c^2x^2\right)^{5/2}\sqrt{\pi + c^2\pi}}{36c}$$

Mathematica [A] time = 0.379247, size = 153, normalized size = 0.93

$$\frac{\pi^{5/2}\left(12\sinh^{-1}(cx)\left(60a + 45b\sinh\left(2\sinh^{-1}(cx)\right) + 9b\sinh\left(4\sinh^{-1}(cx)\right) + b\sinh\left(6\sinh^{-1}(cx)\right)\right) + 384ac^5x^5\sqrt{c^2x^2 +}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Pi^(5/2)*(1584*a*c*x*Sqrt[1 + c^2*x^2] + 1248*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 384*a*c^5*x^5*Sqrt[1 + c^2*x^2] + 360*b*ArcSinh[c*x]^2 - 270*b*Cosh[2*Arc
Sinh[c*x]] - 27*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] + 12*ArcS
inh[c*x]*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*S
inh[6*ArcSinh[c*x]])))/(2304*c)
```

Maple [A] time = 0.052, size = 228, normalized size = 1.4

$$\frac{ax}{6}\left(\pi c^2x^2 + \pi\right)^{\frac{5}{2}} + \frac{5a\pi x}{24}\left(\pi c^2x^2 + \pi\right)^{\frac{3}{2}} + \frac{5a\pi^2 x}{16}\sqrt{\pi c^2x^2 + \pi} + \frac{5a\pi^3}{16}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}} + \sqrt{\pi c^2x^2 + \pi}\right)\frac{1}{\sqrt{\pi c^2}} + \frac{b\pi^{\frac{5}{2}}c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*x*(Pi*c^2*x^2+Pi)^(5/2)*a+5/24*a*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a*Pi^2
*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2
```

```
+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6*b*Pi^(5/2)*c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2
)*x^5-1/36*b*Pi^(5/2)*c^5*x^6+13/24*b*Pi^(5/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*x^3-13/96*b*c^3*Pi^(5/2)*x^4+11/16*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*x-11/32*b*c*Pi^(5/2)*x^2+5/32*b*Pi^(5/2)/c*arcsinh(c*x)^2-17/72*b*
Pi^(5/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\pi + \pi c^2 x^2}\left(\pi^2 a c^4 x^4 + 2\,\pi^2 a c^2 x^2 + \pi^2 a + \left(\pi^2 b c^4 x^4 + 2\,\pi^2 b c^2 x^2 + \pi^2 b\right)\text{arsinh}\,(cx)\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a), x)
```

3.75 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=179

$$-\pi^{5/2}b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\pi^{5/2}b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}\pi\left(\pi c^2x^2+\pi\right.$$

```
[Out] (-23*b*c*Pi^(5/2)*x)/15 - (11*b*c^3*Pi^(5/2)*x^3)/45 - (b*c^5*Pi^(5/2)*x^5)
/25 + Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]) + (Pi*(Pi + c^2*Pi*x^
2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/5 - 2*Pi^(5/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - b*Pi^(
5/2)*PolyLog[2, -E^ArcSinh[c*x]] + b*Pi^(5/2)*PolyLog[2, E^ArcSinh[c*x]]
```

Rubi [A] time = 0.429625, antiderivative size = 329, normalized size of antiderivative = 1.84, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8, 194}

$$-\frac{\pi^2b\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{\pi^2b\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{5}\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-23*b*c*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*P
i^2*x^3*Sqrt[Pi + c^2*Pi*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^5*Sqr
t[Pi + c^2*Pi*x^2])/(25*Sqrt[1 + c^2*x^2]) + Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x]) + (Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + (
(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*Pi^2*Sqrt[Pi + c^2*Pi*
x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*P
i^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
(b*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
```

```
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{5}\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right) + \pi\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{x}$$

$$= \frac{1}{3}\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) + \frac{1}{5}\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)$$

$$= -\frac{8bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi x^2}}{25\sqrt{1 + c^2x^2}} + \pi^2$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi x^2}}{25\sqrt{1 + c^2x^2}} + \pi$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi x^2}}{25\sqrt{1 + c^2x^2}} + \pi$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi x^2}}{25\sqrt{1 + c^2x^2}} + \pi$$

$$= -\frac{23bc\pi^2x\sqrt{\pi + c^2\pi x^2}}{15\sqrt{1 + c^2x^2}} - \frac{11bc^3\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{45\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^5\sqrt{\pi + c^2\pi x^2}}{25\sqrt{1 + c^2x^2}} + \pi$$

Mathematica [A] time = 0.349934, size = 257, normalized size = 1.44

$$\frac{1}{225}\pi^{5/2}\left(225b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 225b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + 45ac^4x^4\sqrt{c^2x^2+1} + 165ac^2x^2\sqrt{c^2x^2+1} + 3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (Pi^(5/2)*(-345*b*c*x - 55*b*c^3*x^3 - 9*b*c^5*x^5 + 345*a*Sqrt[1 + c^2*x^2
] + 165*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 45*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 345*
b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 165*b*c^2*x^2*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + 45*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 225*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 225*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2
```

```
25*a*Log[x] - 225*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 225*b*PolyLog[2, -E^(
-ArcSinh[c*x])] - 225*b*PolyLog[2, E^(-ArcSinh[c*x])]))/225
```

Maple [A] time = 0.235, size = 284, normalized size = 1.6

$$\frac{a}{5}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}+\frac{a\pi}{3}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-a\pi^{\frac{5}{2}}\operatorname{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}}\right)+a\pi^2\sqrt{\pi c^2x^2+\pi}+b\pi^{\frac{5}{2}}\operatorname{polylog}\left(2,cx+\sqrt{c^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/5*(Pi*c^2*x^2+Pi)^(5/2)*a+1/3*a*Pi*(Pi*c^2*x^2+Pi)^(3/2)-a*Pi^(5/2)*arcta
nh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+a*Pi^2*(Pi*c^2*x^2+Pi)^(1/2)+b*Pi^(5/2)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))-b*Pi^(5/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))-b*Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+b*Pi^(5/2)*arcsinh(c
*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/25*b*c^5*Pi^(5/2)*x^5-11/45*b*c^3*Pi^(5/2
)*x^3+23/15*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)-23/15*b*c*Pi^(5/2)*x+
1/5*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x^4*c^4+11/15*b*arcsinh(c*x)*
Pi^(5/2)*(c^2*x^2+1)^(1/2)*x^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{15}\left(15\,\pi^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)-15\,\pi^2\sqrt{\pi+\pi c^2x^2}-5\,\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}-3\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}\right)a+b\int\frac{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}\log\left(cx+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*pi^(5/2)*arcsinh(1/(sqrt(c^2)*abs(x))) - 15*pi^2*sqrt(pi + pi*c^2
*x^2) - 5*pi*(pi + pi*c^2*x^2)^(3/2) - 3*(pi + pi*c^2*x^2)^(5/2))*a + b*int
egrate((pi + pi*c^2*x^2)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi^2ac^4x^4+2\,\pi^2ac^2x^2+\pi^2a+\left(\pi^2bc^4x^4+2\,\pi^2bc^2x^2+\pi^2b\right)\text{arsinh}\left(cx\right)\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)/x, x)
```

3.76 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=157

$$\frac{5}{4}\pi c^2 x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{15}{8}\pi^2c^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+$$

```
[Out] (-9*b*c^3*Pi^(5/2)*x^2)/16 - (b*c^5*Pi^(5/2)*x^4)/16 + (15*c^2*Pi^2*x*Sqrt[
Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*Pi*x*(Pi + c^2*Pi*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/4 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])
)/x + (15*c*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*b) + b*c*Pi^(5/2)*Log[x]
```

Rubi [A] time = 0.235875, antiderivative size = 257, normalized size of antiderivative = 1.64, number of steps used = 10, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5684, 5682, 5675, 30, 14, 266, 43}

$$\frac{5}{4}\pi c^2 x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{15}{8}\pi^2c^2x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{15\pi^2c\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx\right.}{16b\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (-9*b*c^3*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*P
i^2*x^4*Sqrt[Pi + c^2*Pi*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*Pi^2*x*Sqrt
[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*Pi*x*(Pi + c^2*Pi*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/4 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]
))/x + (15*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[
1 + c^2*x^2]) + (b*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
```

```
] && LtQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx=-\frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\left(5c^2\pi\right)\int\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.$$

$$=\frac{5}{4}c^2\pi x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$=\frac{15}{8}c^2\pi^2x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{4}c^2\pi x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

$$=-\frac{9bc^3\pi^2x^2\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^5\pi^2x^4\sqrt{\pi+c^2\pi x^2}}{16\sqrt{1+c^2x^2}}+\frac{15}{8}c^2\pi^2x\sqrt{\pi+c^2\pi x^2}\left(a+b\right.$$

Mathematica [A] time = 0.394832, size = 168, normalized size = 1.07

$$\pi^{5/2}\left(4\sinh^{-1}(cx)\left(60acx-32b\sqrt{c^2x^2+1}+16bcx\sinh\left(2\sinh^{-1}(cx)\right)+bcx\sinh\left(4\sinh^{-1}(cx)\right)\right)+32ac^4x^4\sqrt{c^2x^2+1}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (Pi^(5/2)*(-128*a*Sqrt[1 + c^2*x^2] + 144*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 32*
a*c^4*x^4*Sqrt[1 + c^2*x^2] + 120*b*c*x*ArcSinh[c*x]^2 - 32*b*c*x*Cosh[2*Ar
cSinh[c*x]] - b*c*x*Cosh[4*ArcSinh[c*x]] + 128*b*c*x*Log[c*x] + 4*ArcSinh[c
*x]*(60*a*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*
c*x*Sinh[4*ArcSinh[c*x]])))/(128*x)
```

Maple [B] time = 0.191, size = 283, normalized size = 1.8

$$-\frac{a}{\pi x}\left(\pi c^2x^2+\pi\right)^{\frac{7}{2}}+ac^2x\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}+\frac{5ac^2\pi x}{4}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}+\frac{15ac^2\pi^2x}{8}\sqrt{\pi c^2x^2+\pi}+\frac{15ac^2\pi^3}{8}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+a*c^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/4*a*c^2*Pi*x*
(Pi*c^2*x^2+Pi)^(3/2)+15/8*a*c^2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+15/8*a*c^2*Pi
^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)-1/16*b*
c^5*Pi^(5/2)*x^4-9/16*b*c^3*Pi^(5/2)*x^2+b*c*Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(
1/2))^2-1)-b*c*Pi^(5/2)*arcsinh(c*x)-33/128*b*Pi^(5/2)*c+15/16*b*c*Pi^(5/2)
*arcsinh(c*x)^2+1/4*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x^3*c^4+9/8*b
*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x*c^2-b*Pi^(5/2)*arcsinh(c*x)/x*(c
^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi^2ac^4x^4+2\,\pi^2ac^2x^2+\pi^2a+\left(\pi^2bc^4x^4+2\,\pi^2bc^2x^2+\pi^2b\right)\mathrm{arsinh}\,(cx)\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.77 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=205

$$-\frac{5}{2}\pi^{5/2}bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{5}{2}\pi^{5/2}bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{5}{6}\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{2}\pi^2$$

```
[Out] -(b*c*Pi^(5/2))/(2*x) - (7*b*c^3*Pi^(5/2)*x)/3 - (b*c^5*Pi^(5/2)*x^3)/9 + (
5*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 + (5*c^2*Pi*(Pi +
c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b
*ArcSinh[c*x]))/(2*x^2) - 5*c^2*Pi^(5/2)*(a + b*ArcSinh[c*x])*ArcTanh[E^Arc
Sinh[c*x]] - (5*b*c^2*Pi^(5/2)*PolyLog[2, -E^ArcSinh[c*x]])/2 + (5*b*c^2*Pi
^(5/2)*PolyLog[2, E^ArcSinh[c*x]])/2
```

Rubi [A] time = 0.430901, antiderivative size = 355, normalized size of antiderivative = 1.73, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5739, 5744, 5742, 5760, 4182, 2279, 2391, 8, 270}

$$-\frac{5\pi^2bc^2\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5\pi^2bc^2\sqrt{\pi c^2x^2+\pi}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{6}\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*Pi^2*x
*Sqrt[Pi + c^2*Pi*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x^3*Sqrt[Pi + c
^2*Pi*x^2])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/2 + (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])
)/6 - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*Pi^2*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1
+ c^2*x^2] - (5*b*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, -E^ArcSinh[c*x]
])/(2*Sqrt[1 + c^2*x^2]) + (5*b*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*PolyLog[2, E
^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2} + \frac{1}{2}\left(5c^2\pi\right)\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh\right.}{x}$$

$$= \frac{5}{6}c^2\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right) - \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} + \frac{bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{6\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{5}{2}c^2\pi^2\sqrt{}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{5}{2}c^2\pi^2\sqrt{}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{5}{2}c^2\pi^2\sqrt{}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{5}{2}c^2\pi^2\sqrt{}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{2x\sqrt{1 + c^2x^2}} - \frac{7bc^3\pi^2x\sqrt{\pi + c^2\pi x^2}}{3\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2x^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{5}{2}c^2\pi^2\sqrt{}$$

Mathematica [A] time = 1.89382, size = 349, normalized size = 1.7

$$\frac{\pi^{5/2}\left(180bc^2x^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-180bc^2x^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+24ac^4x^4\sqrt{c^2x^2+1}+168ac^2x^2\sqrt{c^2x^2+1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] (Pi^(5/2)*(-168*b*c^3*x^3 - 8*b*c^5*x^5 - 36*a*Sqrt[1 + c^2*x^2] + 168*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 168*b*c^2*x^2*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] + 24*b*c^4*x^4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
- 9*b*c^3*x^3*Csch[ArcSinh[c*x]/2]^2 - 9*b*c^2*x^2*ArcSinh[c*x]*Csch[ArcSin
h[c*x]/2]^2 + 180*b*c^2*x^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 180*b
*c^2*x^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 180*a*c^2*x^2*Log[x] - 1
80*a*c^2*x^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 180*b*c^2*x^2*PolyLog[2, -E^
(-ArcSinh[c*x])] - 180*b*c^2*x^2*PolyLog[2, E^(-ArcSinh[c*x])] + 36*b*c*x*S
inh[ArcSinh[c*x]/2]^2 - 36*b*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2]^2))/(72*x^2)
```

Maple [A] time = 0.421, size = 356, normalized size = 1.7

$$-\frac{a}{2\pi x^2}\left(\pi c^2x^2+\pi\right)^{\frac{7}{2}}+\frac{ac^2}{2}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}+\frac{5ac^2\pi}{6}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}-\frac{5ac^2\pi^{5/2}}{2}\text{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}}\right)+\frac{5ac^2\pi^2}{2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(7/2)+1/2*a*c^2*(Pi*c^2*x^2+Pi)^(5/2)+5/6*a*c
^2*Pi*(Pi*c^2*x^2+Pi)^(3/2)-5/2*a*c^2*Pi^(5/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2
+Pi)^(1/2))+5/2*a*c^2*Pi^2*(Pi*c^2*x^2+Pi)^(1/2)-5/2*b*c^2*Pi^(5/2)*arcsinh
(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-5/2*b*c^2*Pi^(5/2)*polylog(2,-c*x-(c^2*x^
2+1)^(1/2))+5/2*b*c^2*Pi^(5/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-1/9*b*c^5*P
i^(5/2)*x^3-7/3*b*c^3*Pi^(5/2)*x+5/2*b*c^2*Pi^(5/2)*arcsinh(c*x)*ln(1-c*x-(
c^2*x^2+1)^(1/2))-1/2*b*Pi^(5/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2+1/3*b*a
rcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x^2*c^4-1/2*b*c*Pi^(5/2)/x-1/2*b*Pi^
(5/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)+7/3*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x
^2+1)^(1/2)*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\left(15\,\pi^{\frac{5}{2}}c^2\,\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)-15\,\pi^2\sqrt{\pi+\pi c^2x^2}c^2-5\,\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2-3\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}c^2+\frac{3\left(\pi+\pi c^2x^2\right)^{\frac{7}{2}}}{\pi x^2}\right)a+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(15*pi^(5/2)*c^2*arcsinh(1/(sqrt(c^2)*abs(x))) - 15*pi^2*sqrt(pi + pi*
c^2*x^2)*c^2 - 5*pi*(pi + pi*c^2*x^2)^(3/2)*c^2 - 3*(pi + pi*c^2*x^2)^(5/2)
*c^2 + 3*(pi + pi*c^2*x^2)^(7/2)/(pi*x^2))*a + b*integrate((pi + pi*c^2*x^2
)^(5/2)*log(c*x + sqrt(c^2*x^2 + 1))/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi^2ac^4x^4+2\,\pi^2ac^2x^2+\pi^2a+\left(\pi^2bc^4x^4+2\,\pi^2bc^2x^2+\pi^2b\right)\operatorname{arsinh}\left(cx\right)\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.78 $\int \frac{\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=166

$$\frac{5}{2}\pi^2c^4x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)-\frac{5\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-\frac{\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+$$

```
[Out] -(b*c*Pi^(5/2))/(6*x^2) - (b*c^5*Pi^(5/2)*x^2)/4 + (5*c^4*Pi^2*x*Sqrt[Pi +
c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a
+ b*ArcSinh[c*x]))/(3*x) - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(
3*x^3) + (5*c^3*Pi^(5/2)*(a + b*ArcSinh[c*x])^2)/(4*b) + (7*b*c^3*Pi^(5/2)*
Log[x])/3
```

Rubi [A] time = 0.293949, antiderivative size = 266, normalized size of antiderivative = 1.6, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5739, 5682, 5675, 30, 14, 266, 43}

$$\frac{5}{2}\pi^2c^4x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)+\frac{5\pi^2c^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{5\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*Pi^2*x
^2*Sqrt[Pi + c^2*Pi*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*Pi^2*x*Sqrt[Pi + c
^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)*(a +
 b*ArcSinh[c*x]))/(3*x) - ((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3
*x^3) + (5*c^3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (7*b*c^3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*Log[x])/(3*Sqrt[1 + c^
2*x^2])
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
```

```
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{3x^3} + \frac{1}{3}\left(5c^2\pi\right)\int \frac{\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh\right.}{x^2}$$

$$= -\frac{5c^2\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)}{3x} - \frac{\left(\pi + c^2\pi x^2\right)^{5/2}\left(a + b\sinh^{-1}(cx)\right)}{3x^3}$$

$$= \frac{5}{2}c^4\pi^2 x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right) - \frac{5c^2\pi\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(c\right.}{3x}$$

$$= -\frac{bc\pi^2\sqrt{\pi + c^2\pi x^2}}{6x^2\sqrt{1 + c^2x^2}} - \frac{bc^5\pi^2 x^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1 + c^2x^2}} + \frac{5}{2}c^4\pi^2 x\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh\right.$$

Mathematica [A] time = 0.399065, size = 179, normalized size = 1.08

$$\frac{\pi^{5/2}\left(\sinh^{-1}(cx)\left(60ac^3x^3 - 8b\sqrt{c^2x^2+1}\left(7c^2x^2+1\right) + 6bc^3x^3\sinh\left(2\sinh^{-1}(cx)\right)\right) + 12ac^4x^4\sqrt{c^2x^2+1} - 56ac^2x^2\sqrt{c^2x\right.}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (Pi^(5/2)*(-4*b*c*x - 8*a*Sqrt[1 + c^2*x^2] - 56*a*c^2*x^2*Sqrt[1 + c^2*x^2
] + 12*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 30*b*c^3*x^3*ArcSinh[c*x]^2 - 3*b*c^3*
x^3*Cosh[2*ArcSinh[c*x]] + 56*b*c^3*x^3*Log[c*x] + ArcSinh[c*x]*(60*a*c^3*x
^3 - 8*b*Sqrt[1 + c^2*x^2]*(1 + 7*c^2*x^2) + 6*b*c^3*x^3*Sinh[2*ArcSinh[c*x
]])))/(24*x^3)
```

Maple [B] time = 0.24, size = 692, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(7/2)-4/3*a*c^2/Pi/x*(Pi*c^2*x^2+Pi)^(7/2)+4/
3*a*c^4*x*(Pi*c^2*x^2+Pi)^(5/2)+5/3*a*c^4*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/2*a*
```

```
c^4*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^4*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+
(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+7/3*b*c^3*Pi^(5/2)*ln((c*x+(c^2*x^2+1
)^(1/2))^2-1)-1/4*b*c^5*Pi^(5/2)*x^2-14/3*b*c^3*Pi^(5/2)*arcsinh(c*x)+5/4*b
*c^3*Pi^(5/2)*arcsinh(c*x)^2-1/8*b*Pi^(5/2)*c^3+49/6*b*Pi^(5/2)/(63*c^4*x^4
+15*c^2*x^2+1)*x^4*c^7-147*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^3*arcsinh
(c*x)*(c^2*x^2+1)^(1/2)*c^6-56*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)*c^4-22/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^2+147*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*
x^4*arcsinh(c*x)*c^7-49/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*(c^2*x^2
+1)*c^5+35*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*x^2*arcsinh(c*x)*c^5-7/3*b*
Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)*(c^2*x^2+1)*c^3+7/3*b*Pi^(5/2)/(63*c^4*x
^4+15*c^2*x^2+1)*arcsinh(c*x)*c^3-1/6*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/
x^2*(c^2*x^2+1)*c-1/3*b*Pi^(5/2)/(63*c^4*x^4+15*c^2*x^2+1)/x^3*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)+1/2*b*arcsinh(c*x)*Pi^(5/2)*(c^2*x^2+1)^(1/2)*x*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(\pi^2ac^4x^4+2\,\pi^2ac^2x^2+\pi^2a+\left(\pi^2bc^4x^4+2\,\pi^2bc^2x^2+\pi^2b\right)\mathrm{arsinh}\,(cx)\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a*c^4*x^4 + 2*pi^2*a*c^2*x^2 + pi^2*a
+ (pi^2*b*c^4*x^4 + 2*pi^2*b*c^2*x^2 + pi^2*b)*arcsinh(c*x))/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.79 $\int \sqrt{1+x^2} \sinh^{-1}(x) \, dx$

Optimal. Leaf size=32

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

```
[Out] -x^2/4 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rubi [A] time = 0.0302964, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5682, 5675, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -x^2/4 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+x^2}\sinh^{-1}(x)\,dx = \frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x) + \frac{1}{4}\sinh^{-1}(x)^2$$

Mathematica [A] time = 0.0125163, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-x^2 + 2\sqrt{x^2+1}x\sinh^{-1}(x) + \sinh^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Maple [A] time = 0.024, size = 26, normalized size = 0.8

$$\frac{x\text{Arcsinh}\,(x)}{2}\sqrt{x^2+1} + \frac{(\text{Arcsinh}\,(x))^2}{4} - \frac{x^2}{4} - \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Maxima [A] time = 1.65773, size = 38, normalized size = 1.19

$$-\frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{x^2+1}x + \text{arsinh}\,(x)\right)\text{arsinh}\,(x) - \frac{1}{4}\,\text{arsinh}\,(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

Fricas [A] time = 2.34976, size = 115, normalized size = 3.59

$$\frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2+\frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+1}\operatorname{asinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+1}\operatorname{arsinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

3.80 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=149

$$\frac{x^4\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}-\frac{4x^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^4}+\frac{8\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^6}+\frac{4bx^3}{45\sqrt{\pi}c^3}-$$

```
[Out] (-8*b*x)/(15*c^5*Sqrt[Pi]) + (4*b*x^3)/(45*c^3*Sqrt[Pi]) - (b*x^5)/(25*c*Sq
rt[Pi]) + (8*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*Pi) - (4*x
^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*Pi) + (x^4*Sqrt[Pi +
 c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rubi [A] time = 0.257035, antiderivative size = 215, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^4\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{5\pi c^2}-\frac{4x^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^4}+\frac{8\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{15\pi c^6}-\frac{bx^5\sqrt{c^2x}}{25c\sqrt{\pi c^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[Pi + c^2*Pi*x^2]) + (4*b*x^3*Sqrt[1
 + c^2*x^2])/(45*c^3*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25
*c*Sqrt[Pi + c^2*Pi*x^2]) + (8*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/
(15*c^6*Pi) - (4*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*Pi
) + (x^4*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*Pi)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^4\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2\pi} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{5c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^4\,dx}{5c\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right.}{5c^2\pi}$$

$$= \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} + \frac{8\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6\pi} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}}{15}$$

$$= -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{\pi+c^2\pi x^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} + \frac{8\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx\right.}{15c^6\pi}$$

Mathematica [A] time = 0.184538, size = 108, normalized size = 0.72

$$\frac{15a\sqrt{c^2x^2+1}\left(3c^4x^4-4c^2x^2+8\right)+b\left(-9c^5x^5+20c^3x^3-120cx\right)+15b\sqrt{c^2x^2+1}\left(3c^4x^4-4c^2x^2+8\right)\sinh^{-1}(cx)}{225\sqrt{\pi}c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (15*a*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4) + b*(-120*c*x + 20*c^3*
x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4)*ArcSi
nh[c*x])/(225*c^6*Sqrt[Pi])
```

Maple [A] time = 0.092, size = 193, normalized size = 1.3

$$a\left(\frac{x^4}{5\,\pi\,c^2}\sqrt{\pi\,c^2x^2+\pi}-\frac{4}{5\,c^2}\left(\frac{x^2}{3\,\pi\,c^2}\sqrt{\pi\,c^2x^2+\pi}-\frac{2}{3\,\pi\,c^4}\sqrt{\pi\,c^2x^2+\pi}\right)\right)+\frac{b}{225\,c^6\sqrt{\pi}}\left(45\,\mathrm{Arcsinh}\,(cx)\,c^6x^6-15\,\mathrm{Arcsi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*(1/5*x^4/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-4/5/c^2*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2
+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2)))+1/225*b/c^6/Pi^(1/2)/(c^2*x^2
+1)^(1/2)*(45*arcsinh(c*x)*c^6*x^6-15*arcsinh(c*x)*c^4*x^4-9*c^5*x^5*(c^2*x
^2+1)^(1/2)+60*arcsinh(c*x)*c^2*x^2+20*c^3*x^3*(c^2*x^2+1)^(1/2)+120*arcsin
h(c*x)-120*c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.1725, size = 235, normalized size = 1.58

$$\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^4}+\frac{8\sqrt{\pi+\pi c^2x^2}}{\pi c^6}\right)b\,\mathrm{arsinh}\,(cx)+\frac{1}{15}\left(\frac{3\sqrt{\pi+\pi c^2x^2}x^4}{\pi c^2}-\frac{4\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^4}+\frac{8}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*sqrt(pi + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(p
i*c^4) + 8*sqrt(pi + pi*c^2*x^2)/(pi*c^6))*b*arcsinh(c*x) + 1/15*(3*sqrt(pi
 + pi*c^2*x^2)*x^4/(pi*c^2) - 4*sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^4) + 8*sqrt
(pi + pi*c^2*x^2)/(pi*c^6))*a - 1/225*(9*c^4*x^5 - 20*c^2*x^3 + 120*x)*b/(s
qrt(pi)*c^5)
```

Fricas [A] time = 2.60597, size = 363, normalized size = 2.44

$$\frac{15\sqrt{\pi+\pi c^2x^2}\left(3\,bc^6x^6-bc^4x^4+4\,bc^2x^2+8\,b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(45\,ac^6x^6-15\,ac^4x^4+60\,ac^2x^2-(9\right.}{225\left(\pi c^8x^2+\pi c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/225*(15*sqrt(pi + pi*c^2*x^2)*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*
b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(45*a*c^6*x^6 - 15*
a*c^4*x^4 + 60*a*c^2*x^2 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^
2*x^2 + 1) + 120*a))/(pi*c^8*x^2 + pi*c^6)
```

Sympy [A] time = 26.581, size = 182, normalized size = 1.22

$$\frac{a\left(\begin{cases}\frac{x^4\sqrt{c^2x^2+1}}{5c^2}-\frac{4x^2\sqrt{c^2x^2+1}}{15c^4}+\frac{8\sqrt{c^2x^2+1}}{15c^6} & \text{for}\ c\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^5}{25c}+\frac{x^4\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{5c^2}+\frac{4x^3}{45c^3}-\frac{4x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{15c^4}-\frac{8x}{15c^5}+\right.\\ 0\end{cases}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**4*sqrt(c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(c**2*x**2 + 1)
/(15*c**4) + 8*sqrt(c**2*x**2 + 1)/(15*c**6), Ne(c, 0)), (x**6/6, True))/sq
rt(pi) + b*Piecewise((-x**5/(25*c) + x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(5
*c**2) + 4*x**3/(45*c**3) - 4*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**4)
 - 8*x/(15*c**5) + 8*sqrt(c**2*x**2 + 1)*asinh(c*x)/(15*c**6), Ne(c, 0)), (
0, True))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^5}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)*x^5/sqrt(pi + pi*c^2*x^2), x)
```

3.81 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{4\pi c^2}-\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{8\pi c^4}+\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{16\sqrt{\pi}bc^5}+\frac{3bx^2}{16\sqrt{\pi}c^3}-\frac{bx^4}{16\sqrt{\pi}c}$$

```
[Out] (3*b*x^2)/(16*c^3*Sqrt[Pi]) - (b*x^4)/(16*c*Sqrt[Pi]) - (3*x*Sqrt[Pi + c^2*
Pi*x^2]*(a + b*ArcSinh[c*x]))/(8*c^4*Pi) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/(4*c^2*Pi) + (3*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[Pi]
)
```

Rubi [A] time = 0.226495, antiderivative size = 170, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5758, 5675, 30}

$$\frac{x^3\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{4\pi c^2}-\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{8\pi c^4}+\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{16\sqrt{\pi}bc^5}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{\pi c^2x^2+\pi}}+\frac{3bx}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^4*Sqrt[1
+ c^2*x^2])/(16*c*Sqrt[Pi + c^2*Pi*x^2]) - (3*x*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/(8*c^4*Pi) + (x^3*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x
]))/(4*c^2*Pi) + (3*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[Pi])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{4c^2\pi} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3\,dx}{4c\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx\right)}{4c^2\pi}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x}}{}$$

Mathematica [A] time = 0.237461, size = 111, normalized size = 0.88

$$\frac{4\sinh^{-1}(cx)\left(12a-8b\sinh\left(2\sinh^{-1}(cx)\right)+b\sinh\left(4\sinh^{-1}(cx)\right)\right)+32ac^3x^3\sqrt{c^2x^2+1}-48acx\sqrt{c^2x^2+1}+24b\sinh^{-}}{128\sqrt{\pi}c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-48*a*c*x*Sqrt[1 + c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 + c^2*x^2] + 24*b*ArcSin
h[c*x]^2 + 16*b*Cosh[2*ArcSinh[c*x]] - b*Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c
*x]*(12*a - 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))/(128*c^5*Sq
rt[Pi])
```

Maple [A] time = 0.085, size = 188, normalized size = 1.5

$$\frac{ax^3}{4\,\pi\,c^2}\sqrt{\pi\,c^2x^2+\pi} - \frac{3\,ax}{8\,c^4\pi}\sqrt{\pi\,c^2x^2+\pi} + \frac{3\,a}{8\,c^4}\ln\left(\pi\,c^2x\frac{1}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi\,c^2}} + \frac{b\mathrm{Arcsinh}\,(cx)\,x^3}{4\,c^2\sqrt{\pi}}\sqrt{c^2x^2+1} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/4*a*x^3/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-3/8*a/c^4*x/Pi*(Pi*c^2*x^2+Pi)^(1/2)
+3/8*a/c^4*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
+1/4*b/c^2/Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/16*b*x^4/c/Pi^(1/2
)-3/8*b/c^4/Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x+3/16*b*x^2/c^3/Pi^(1/
2)+3/16*b/c^5/Pi^(1/2)*arcsinh(c*x)^2+1/4*b/c^5/Pi^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^4 \operatorname{arsinh}(cx) + ax^4}{\sqrt{\pi + \pi c^2 x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A] time = 14.4618, size = 185, normalized size = 1.47

$$\frac{ax^5}{4\sqrt{\pi}\sqrt{c^2x^2+1}} - \frac{ax^3}{8\sqrt{\pi}c^2\sqrt{c^2x^2+1}} - \frac{3ax}{8\sqrt{\pi}c^4\sqrt{c^2x^2+1}} + \frac{3a\operatorname{asinh}(cx)}{8\sqrt{\pi}c^5} + \frac{b\left(\begin{cases} -\frac{x^4}{16c} + \frac{x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{4c^2} + \frac{3x^2}{16c^3} - \frac{3x\sqrt{c^2x^2+}}{8} \\ 0 \end{cases}\right.}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(pi)*sqrt(c**2*x**2 + 1)) - a*x**3/(8*sqrt(pi)*c**2*sqrt(c**2
*x**2 + 1)) - 3*a*x/(8*sqrt(pi)*c**4*sqrt(c**2*x**2 + 1)) + 3*a*asinh(c*x)/
(8*sqrt(pi)*c**5) + b*Piecewise((-x**4/(16*c) + x**3*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(4*c**2) + 3*x**2/(16*c**3) - 3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(
8*c**4) + 3*asinh(c*x)**2/(16*c**5), Ne(c, 0)), (0, True))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{\sqrt{\pi + \pi c^2 x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(pi + pi*c^2*x^2), x)
```

3.82 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=98

$$\frac{x^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}+\frac{2bx}{3\sqrt{\pi}c^3}-\frac{bx^3}{9\sqrt{\pi}c}$$

```
[Out] (2*b*x)/(3*c^3*Sqrt[Pi]) - (b*x^3)/(9*c*Sqrt[Pi]) - (2*Sqrt[Pi + c^2*Pi*x^2
]*(a + b*ArcSinh[c*x]))/(3*c^4*Pi) + (x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/(3*c^2*Pi)
```

Rubi [A] time = 0.157292, antiderivative size = 142, normalized size of antiderivative = 1.45, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2}-\frac{2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^4}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{\pi c^2x^2+\pi}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^3*Sqrt[1 + c
^2*x^2])/(9*c*Sqrt[Pi + c^2*Pi*x^2]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcS
inh[c*x]))/(3*c^4*Pi) + (x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3
*c^2*Pi)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^2\,dx}{3c\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi}$$

$$= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}\,(a}{3c^2}$$

Mathematica [A] time = 0.135346, size = 82, normalized size = 0.84

$$\frac{3a\sqrt{c^2x^2+1}\left(c^2x^2-2\right)+b\left(6cx-c^3x^3\right)+3b\sqrt{c^2x^2+1}\left(c^2x^2-2\right)\sinh^{-1}(cx)}{9\sqrt{\pi}c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (3*a*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2] + b*(6*c*x - c^3*x^3) + 3*b*(-2 + c^2
*x^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(9*c^4*Sqrt[Pi])
```

Maple [A] time = 0.083, size = 133, normalized size = 1.4

$$a\left(\frac{x^2}{3\,\pi\,c^2}\sqrt{\pi\,c^2x^2+\pi}-\frac{2}{3\,\pi\,c^4}\sqrt{\pi\,c^2x^2+\pi}\right)+\frac{b}{9\,c^4\sqrt{\pi}}\left(3\,\mathrm{Arcsinh}\,(cx)\,c^4x^4-3\,\mathrm{Arcsinh}\,(cx)\,c^2x^2-c^3x^3\sqrt{c^2x^2+1}-6\,\mathrm{A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*(1/3*x^2/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-2/3/Pi/c^4*(Pi*c^2*x^2+Pi)^(1/2))+1
/9*b/c^4/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(3*arcsinh(c*x)*c^4*x^4-3*arcsinh(c*x)*
c^2*x^2-c^3*x^3*(c^2*x^2+1)^(1/2)-6*arcsinh(c*x)+6*c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.14162, size = 158, normalized size = 1.61

$$\frac{1}{3}\,b\left(\frac{\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^2}-\frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)\mathrm{arsinh}\,(cx)+\frac{1}{3}\,a\left(\frac{\sqrt{\pi+\pi c^2x^2}x^2}{\pi c^2}-\frac{2\,\sqrt{\pi+\pi c^2x^2}}{\pi c^4}\right)-\frac{\left(c^2x^3-6\,x\right)b}{9\,\sqrt{\pi}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + pi*c^2*x^2)/(pi*c^4
))*arcsinh(c*x) + 1/3*a*(sqrt(pi + pi*c^2*x^2)*x^2/(pi*c^2) - 2*sqrt(pi + p
i*c^2*x^2)/(pi*c^4)) - 1/9*(c^2*x^3 - 6*x)*b/(sqrt(pi)*c^3)
```

Fricas [A] time = 2.59313, size = 286, normalized size = 2.92

$$\frac{3\,\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4-bc^2x^2-2\,b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(3\,ac^4x^4-3\,ac^2x^2-\left(bc^3x^3-6\,bcx\right)\sqrt{c^2x^2+1}-6\right.}{9\left(\pi c^6x^2+\pi c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/9*(3*sqrt(pi + pi*c^2*x^2)*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*log(c*x + sqrt(c
^2*x^2 + 1)) + sqrt(pi + pi*c^2*x^2)*(3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^
3 - 6*b*c*x)*sqrt(c^2*x^2 + 1) - 6*a))/(pi*c^6*x^2 + pi*c^4)
```

Sympy [A] time = 5.43048, size = 122, normalized size = 1.24

$$\frac{a\left(\begin{cases}\frac{x^2\sqrt{c^2x^2+1}}{3c^2}-\frac{2\sqrt{c^2x^2+1}}{3c^4} & \text{for } c\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x^3}{9c}+\frac{x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^2}+\frac{2x}{3c^3}-\frac{2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^4} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2*sqrt(c**2*x**2 + 1)/(3*c**2) - 2*sqrt(c**2*x**2 + 1)/(3*c
**4), Ne(c, 0)), (x**4/4, True))/sqrt(pi) + b*Piecewise((-x**3/(9*c) + x**2
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**2) + 2*x/(3*c**3) - 2*sqrt(c**2*x**2
+ 1)*asinh(c*x)/(3*c**4), Ne(c, 0)), (0, True))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/sqrt(pi + pi*c^2*x^2), x)
```

3.83 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=75

$$\frac{x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi c^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4\sqrt{\pi}bc^3}-\frac{bx^2}{4\sqrt{\pi}c}$$

```
[Out] -(b*x^2)/(4*c*Sqrt[Pi]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2
*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*Sqrt[Pi])
```

Rubi [A] time = 0.121491, antiderivative size = 97, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5758, 5675, 30}

$$\frac{x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi c^2}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4\sqrt{\pi}bc^3}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c*Sqrt[Pi + c^2*Pi*x^2]) + (x*Sqrt[Pi + c^2*P
i*x^2]*(a + b*ArcSinh[c*x]))/(2*c^2*Pi) - (a + b*ArcSinh[c*x])^2/(4*b*c^3*S
qrt[Pi])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{\pi+c^2\pi x^2}}$$
$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{\pi+c^2\pi x^2}} + \frac{x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2\pi} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{\pi}}$$

Mathematica [A] time = 0.175136, size = 69, normalized size = 0.92

$$\frac{\sinh^{-1}(cx)\left(2b\sinh\left(2\sinh^{-1}(cx)\right)-4a\right)+4acx\sqrt{c^2x^2+1}-2b\sinh^{-1}(cx)^2-b\cosh\left(2\sinh^{-1}(cx)\right)}{8\sqrt{\pi}c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (4*a*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x]^2 - b*Cosh[2*ArcSinh[c*x]] +
ArcSinh[c*x]*(-4*a + 2*b*Sinh[2*ArcSinh[c*x]]))/(8*c^3*Sqrt[Pi])
```

Maple [A] time = 0.055, size = 125, normalized size = 1.7

$$\frac{ax}{2\,\pi\,c^2}\sqrt{\pi\,c^2x^2+\pi} - \frac{a}{2\,c^2}\ln\left(\pi\,c^2x\frac{1}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi\,c^2}} + \frac{b\mathrm{Arcsinh}\,(cx)\,x}{2\,c^2\sqrt{\pi}}\sqrt{c^2x^2+1} - \frac{bx^2}{4\,c\sqrt{\pi}} - \frac{b\,(\mathrm{Arcsinh}\,(}{4\,c^3\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] 1/2*a*x/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)-1/2*a/c^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(
Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c^2/Pi^(1/2)*arcsinh(c*x)*(c^2*x
```

```
^2+1)^(1/2)*x-1/4*b*x^2/c/Pi^(1/2)-1/4*b/c^3/Pi^(1/2)*arcsinh(c*x)^2-1/4*b/
c^3/Pi^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\,\text{arsinh}\,(cx)+ax^2}{\sqrt{\pi+\pi c^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A] time = 4.64037, size = 92, normalized size = 1.23

$$\frac{ax\sqrt{c^2x^2+1}}{2\sqrt{\pi}c^2}-\frac{a\,\text{asinh}\,(cx)}{2\sqrt{\pi}c^3}+\frac{b\left(\begin{cases}-\frac{x^2}{4c}+\frac{x\sqrt{c^2x^2+1}\,\text{asinh}\,(cx)}{2c^2}-\frac{\text{asinh}^2\,(cx)}{4c^3} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*x*sqrt(c**2*x**2 + 1)/(2*sqrt(pi)*c**2) - a*asinh(c*x)/(2*sqrt(pi)*c**3)
+ b*Piecewise((-x**2/(4*c) + x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c**2) - as
inh(c*x)**2/(4*c**3), Ne(c, 0)), (0, True))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^2}{\sqrt{\pi + \pi c^2 x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(pi + pi*c^2*x^2), x)
```

3.84 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

__Optimal__. Leaf size=42

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2}-\frac{bx}{\sqrt{\pi}c}$$

```
[Out] -((b*x)/(c*Sqrt[Pi])) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^2*P
i)
```

__Rubi__ **[A]** time = 0.0646801, antiderivative size = 64, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 8}

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[Pi + c^2*Pi*x^2])) + (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(c^2*Pi)
```

__Rule 5717__

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

__Rule 8__

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

__Rubi steps__

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{\pi+c^2\pi x^2}}$$
$$= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi}$$

Mathematica [A] time = 0.0792243, size = 49, normalized size = 1.17

$$\frac{a\sqrt{c^2x^2+1}+b\sqrt{c^2x^2+1}\sinh^{-1}(cx)-bcx}{\sqrt{\pi}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c^2*Sq
rt[Pi])
```

Maple [A] time = 0.042, size = 72, normalized size = 1.7

$$\frac{a}{\pi c^2}\sqrt{\pi c^2x^2+\pi}+\frac{b}{c^2\sqrt{\pi}}\left(\mathrm{Arcsinh}\,(cx)\,c^2x^2+\mathrm{Arcsinh}\,(cx)-cx\sqrt{c^2x^2+1}\right)\frac{1}{\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a/Pi/c^2*(Pi*c^2*x^2+Pi)^(1/2)+b/c^2/Pi^(1/2)/(c^2*x^2+1)^(1/2)*(arcsinh(c*
x)*c^2*x^2+arcsinh(c*x)-c*x*(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.2497, size = 74, normalized size = 1.76

$$-\frac{bx}{\sqrt{\pi}c}+\frac{\sqrt{\pi+\pi c^2x^2}b\,\mathrm{arsinh}\,(cx)}{\pi c^2}+\frac{\sqrt{\pi+\pi c^2x^2}a}{\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*x/(sqrt(pi)*c) + sqrt(pi + pi*c^2*x^2)*b*arcsinh(c*x)/(pi*c^2) + sqrt(pi
 + pi*c^2*x^2)*a/(pi*c^2)
```

Fricas [B] time = 2.39019, size = 213, normalized size = 5.07

$$\frac{\sqrt{\pi+\pi c^2x^2}\left(bc^2x^2+b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{\pi+\pi c^2x^2}\left(ac^2x^2-\sqrt{c^2x^2+1}bcx+a\right)}{\pi c^4x^2+\pi c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 + b)*log(c*x + sqrt(c^2*x^2 + 1)) + sqrt(
pi + pi*c^2*x^2)*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a))/(pi*c^4*x^2 + p
i*c^2)
```

Sympy [A] time = 2.2557, size = 60, normalized size = 1.43

$$\frac{a\left(\begin{cases}\frac{x^2}{2} & \text{for}\ c^2=0\\ \frac{\sqrt{c^2x^2+1}}{c^2} & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}+\frac{b\left(\begin{cases}-\frac{x}{c}+\frac{\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c^2} & \text{for}\ c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] a*Piecewise((x**2/2, Eq(c**2, 0)), (sqrt(c**2*x**2 + 1)/c**2, True))/sqrt(p
i) + b*Piecewise((-x/c + sqrt(c**2*x**2 + 1)*asinh(c*x)/c**2, Ne(c, 0)), (0
, True))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)x}{\sqrt{\pi+\pi c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(pi + pi*c^2*x^2), x)
```

3.85 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}bc}$$

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Rubi [A] time = 0.029716, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5675}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]

[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx=\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{\pi}}$$

Mathematica [A] time = 0.0170171, size = 25, normalized size = 1.

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{\pi}bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(2*b*c*Sqrt[Pi])
```

Maple [B] time = 0.037, size = 53, normalized size = 2.1

$$a\ln\left(\pi\,c^2x\frac{1}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi\,c^2}}+\frac{b\left(\mathrm{Arcsinh}\,(cx)\right)^2}{2\,c\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c/
Pi^(1/2)*arcsinh(c*x)^2
```

Maxima [B] time = 1.15035, size = 103, normalized size = 4.12

$$\frac{b\,\mathrm{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)\mathrm{arsinh}\,(cx)}{\sqrt{\pi c^2}}-\frac{b\sqrt{c^2}\,\mathrm{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)^2}{2\,\sqrt{\pi c^2}c}+\frac{a\,\mathrm{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\sqrt{\pi c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] b*arcsinh(c^2*x/sqrt(c^2))*arcsinh(c*x)/sqrt(pi*c^2) - 1/2*b*sqrt(c^2)*arcs
inh(c^2*x/sqrt(c^2))^2/(sqrt(pi*c^2)*c) + a*arcsinh(c^2*x/sqrt(c^2))/sqrt(p
i*c^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{\pi+\pi c^2x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

Sympy [A] time = 2.32607, size = 85, normalized size = 3.4

$$\begin{cases} a \left(\begin{cases} \frac{\sqrt{-\frac{1}{c^2}} \operatorname{asin}\left(x\sqrt{-c^2}\right)}{\sqrt{\pi}} & \text{for } \pi c^2 < 0 \\ \frac{\sqrt{\frac{1}{c^2}} \operatorname{asinh}\left(x\sqrt{c^2}\right)}{\sqrt{\pi}} & \text{for } \pi c^2 > 0 \end{cases}\right) & \text{for } b = 0 \\ \frac{ax}{\sqrt{\pi}} & \text{for } c = 0 \\ \frac{(a+b\operatorname{asinh}(cx))^2}{2\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a*Piecewise((sqrt(-1/c**2)*asin(x*sqrt(-c**2))/sqrt(pi), pi*c**2
 < 0), (sqrt(c**(-2))*asinh(x*sqrt(c**2))/sqrt(pi), pi*c**2 > 0)), Eq(b, 0)
), (a*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**2/(2*sqrt(pi)*b*c), True)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2 x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(pi + pi*c^2*x^2), x)
```

3.86 $\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] - (b*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[Pi] + (b*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[Pi]
```

Rubi [A] time = 0.119054, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5760, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] - (b*PolyLog[2,
-E^ArcSinh[c*x]])/Sqrt[Pi] + (b*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[Pi]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{\pi}}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{\pi}} + \frac{b\,\text{Subs}}{}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} + \frac{b\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} - \frac{b\mathbf{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}} + \frac{b\mathbf{Li}_2\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 0.154927, size = 96, normalized size = 1.71

$$\frac{b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right)+a\log(x)+b\sinh^{-1}(cx)\log\left(1-e^{-}\right.}{\sqrt{\pi}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c*x]*Log[1 + E^(-Arc
Sinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + b*PolyLog[2, -
E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Sqrt[Pi]
```

Maple [A] time = 0.047, size = 72, normalized size = 1.3

$$-\frac{a}{\sqrt{\pi}}\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}\right)+\frac{b}{2\sqrt{\pi}}\left(4\,\mathrm{dilog}\left(\left(cx+\sqrt{c^2x^2+1}\right)^{-1}\right)-\mathrm{dilog}\left(\left(cx+\sqrt{c^2x^2+1}\right)^{-2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -a/Pi^(1/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+1/2*b*(4*dilog(1/(c*x+(
c^2*x^2+1)^(1/2)))-dilog(1/(c*x+(c^2*x^2+1)^(1/2))^2))/Pi^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{\pi+\pi c^2x^2}x}\,dx-\frac{a\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*x), x) - a*
arcsinh(1/(sqrt(c^2)*abs(x)))/sqrt(pi)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}\,(cx)+a)}{\pi c^2x^3+\pi x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^3 + pi*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{x\sqrt{c^2x^2+1}}\,dx + \int \frac{b\,\mathrm{asinh}\,(cx)}{x\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x*sqrt(c**
2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\sqrt{\pi + \pi c^2x^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x), x)
```

3.87 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}$$

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x)) + (b*c*Log[x])/Sqrt[
Pi]
```

Rubi [A] time = 0.0882457, antiderivative size = 63, normalized size of antiderivative = 1.54, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 29}

$$\frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{\pi c^2x^2+\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(Pi*x)) + (b*c*Sqrt[1 + c^2*
x^2]*Log[x])/Sqrt[Pi + c^2*Pi*x^2]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{1}{x}\,dx}{\sqrt{\pi+c^2\pi x^2}}$$
$$= -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{\pi x}+\frac{bc\sqrt{1+c^2x^2}\log(x)}{\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.108567, size = 42, normalized size = 1.02

$$\frac{bc\log(x)}{\sqrt{\pi}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -((Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[Pi]*x)) + (b*c*Log[x])/Sqr
t[Pi]
```

Maple [B] time = 0.066, size = 84, normalized size = 2.1

$$-\frac{a}{\pi x}\sqrt{\pi c^2x^2+\pi}-\frac{bc\text{Arcsinh}\,(cx)}{\sqrt{\pi}}-\frac{b\text{Arcsinh}\,(cx)}{\sqrt{\pi}x}\sqrt{c^2x^2+1}+\frac{bc}{\sqrt{\pi}}\ln\left(\left(cx+\sqrt{c^2x^2+1}\right)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -a/Pi/x*(Pi*c^2*x^2+Pi)^(1/2)-b*c/Pi^(1/2)*arcsinh(c*x)-b/Pi^(1/2)*arcsinh(
c*x)/x*(c^2*x^2+1)^(1/2)+b*c/Pi^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)
```

Maxima [B] time = 1.17426, size = 150, normalized size = 3.66

$$\frac{\left(\pi c^2\sqrt{\frac{1}{\pi c^4}}\log\left(x^2+\frac{1}{c^2}\right)-\sqrt{\pi}\,(-1)^{2\,\pi+2\,\pi c^2x^2}\log\left(2\,\pi c^2+\frac{2\,\pi}{x^2}\right)\right)bc}{2\,\pi}-\frac{\sqrt{\pi+\pi c^2x^2}b\,\text{arsinh}\,(cx)}{\pi x}-\frac{\sqrt{\pi+\pi c^2x^2}a}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(pi*c^2*sqrt(1/(pi*c^4))*log(x^2 + 1/c^2) - sqrt(pi)*(-1)^(2*pi + 2*pi*
c^2*x^2)*log(2*pi*c^2 + 2*pi/x^2))*b*c/pi - sqrt(pi + pi*c^2*x^2)*b*arcsinh
(c*x)/(pi*x) - sqrt(pi + pi*c^2*x^2)*a/(pi*x)
```

Fricas [B] time = 2.8408, size = 319, normalized size = 7.78

$$\frac{\sqrt{\pi}bcx\log\left(\frac{\pi+\pi c^2x^6+\pi c^2x^2+\pi x^4+\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}(x^4-1)}{c^2x^4+x^2}\right)-2\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi+\pi c^2x^2}a}{2\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*b*c*x*log((pi + pi*c^2*x^6 + pi*c^2*x^2 + pi*x^4 + sqrt(pi)*s
qrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 - 1))/(c^2*x^4 + x^2)) - 2*sqrt
(pi + pi*c^2*x^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*
a)/(pi*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{x^2\sqrt{c^2x^2+1}}\,dx+\int \frac{b\,\mathrm{asinh}\,(cx)}{x^2\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**2*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**2*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2 x^2} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^2), x)
```

3.88 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=115

$$\frac{bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2}+\frac{c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a}{\sqrt{\pi}}$$

```
[Out] -(b*c)/(2*Sqrt[Pi]*x) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*Pi*
x^2) + (c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[Pi] + (b*c^2
*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[Pi]) - (b*c^2*PolyLog[2, E^ArcSinh[c*
x]])/(2*Sqrt[Pi])
```

Rubi [A] time = 0.213172, antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5747, 5760, 4182, 2279, 2391, 30}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{\pi}}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2}+\frac{c^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)(a}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*x*Sqrt[Pi + c^2*Pi*x^2]) - (Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(2*Pi*x^2) + (c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^A
rcSinh[c*x]])/Sqrt[Pi] + (b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[Pi]) -
 (b*c^2*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[Pi])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} - \frac{1}{2}c^2\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} - \frac{c^2\,\mathrm{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \sinh^{-}\right.}{2\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{\pi+c^2\pi x^2}} - \frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\pi x^2} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{\pi}}$$

Mathematica [A] time = 2.71724, size = 185, normalized size = 1.61

$$\frac{bc^2\left(-4\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+4\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[1 + c^2*x^2])/x^2 - 4*a*c^2*Log[x] + 4*a*c^2*Log[Pi*(1 + Sqrt[1
 + c^2*x^2])] + b*c^2*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[
c*x]/2]^2 - 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[
1 + E^(-ArcSinh[c*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^
(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x
]/2]))/(8*Sqrt[Pi])
```

Maple [A] time = 0.079, size = 225, normalized size = 2.

$$-\frac{a}{2\,\pi\,x^2}\sqrt{\pi\,c^2x^2+\pi}+\frac{ac^2}{2\sqrt{\pi}}\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}\right)-\frac{b\mathrm{Arcsinh}\,(cx)\,c^2}{2\sqrt{\pi}}\frac{1}{\sqrt{c^2x^2+1}}-\frac{bc}{2\,x\sqrt{\pi}}-\frac{b\mathrm{Arcsinh}\,(cx)}{2\sqrt{\pi}x^2}\frac{1}{\sqrt{c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -1/2*a/Pi/x^2*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a/Pi^(1/2)*c^2*arctanh(Pi^(1/2)/(Pi
*c^2*x^2+Pi)^(1/2))-1/2*b/Pi^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b
*c/x/Pi^(1/2)-1/2*b/Pi^(1/2)/(c^2*x^2+1)^(1/2)/x^2*arcsinh(c*x)+1/2*b*c^2/P
i^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+1/2*b*c^2*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))/Pi^(1/2)-1/2*b*c^2/Pi^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))-1/2*b*c^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))/Pi^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(\frac{c^2 \operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\sqrt{\pi}}-\frac{\sqrt{\pi+\pi c^2x^2}}{\pi x^2}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\sqrt{\pi+\pi c^2x^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(c^2*arcsinh(1/(sqrt(c^2)*abs(x)))/sqrt(pi) - sqrt(pi + pi*c^2*x^2)/(pi
*x^2))*a + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(sqrt(pi + pi*c^2*x^2)*
x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\operatorname{arsinh}(cx)+a)}{\pi c^2x^5+\pi x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi*c^2*x^5 + pi*x^3),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^3\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**3*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**3*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{\pi+\pi c^2x^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^3), x)
```

3.89 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{2c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}-\frac{bc}{6\sqrt{\pi}x^2}$$

```
[Out] -(b*c)/(6*Sqrt[Pi]*x^2) - (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3*P
i*x^3) + (2*c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(3*Pi*x) - (2*b
*c^3*Log[x])/(3*Sqrt[Pi])
```

Rubi [A] time = 0.181917, antiderivative size = 141, normalized size of antiderivative = 1.45, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5747, 5723, 29, 30}

$$\frac{2c^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x}-\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{\pi c^2x^2+\pi}}-\frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*x^2*Sqrt[Pi + c^2*Pi*x^2]) - (Sqrt[Pi + c^2*Pi*
x^2]*(a + b*ArcSinh[c*x]))/(3*Pi*x^3) + (2*c^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b
*ArcSinh[c*x]))/(3*Pi*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[Pi +
c^2*Pi*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{\pi+c^2\pi x^2}}\,dx = -\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}-\frac{1}{3}\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{\pi+c^2\pi x^2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3}\,dx}{3\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{\pi+c^2\pi x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x}-$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{\pi+c^2\pi x^2}}-\frac{\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x^3}+\frac{2c^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi x}-$$

Mathematica [A] time = 0.15839, size = 99, normalized size = 1.02

$$\frac{2a\sqrt{c^2x^2+1}\left(2c^2x^2-1\right)+bcx\left(6c^2x^2-1\right)+2b\sqrt{c^2x^2+1}\left(2c^2x^2-1\right)\sinh^{-1}(cx)}{6\sqrt{\pi}x^3}-\frac{2bc^3\log(x)}{3\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[Pi + c^2*Pi*x^2]),x]
```

```
[Out] (2*a*Sqrt[1 + c^2*x^2]*(-1 + 2*c^2*x^2) + b*c*x*(-1 + 6*c^2*x^2) + 2*b*Sqrt
[1 + c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSinh[c*x])/(6*Sqrt[Pi]*x^3) - (2*b*c^3*Lo
g[x])/(3*Sqrt[Pi])
```

Maple [B] time = 0.09, size = 372, normalized size = 3.8

$$-\frac{a}{3\,\pi\,x^{3}}\sqrt{\pi\,c^{2}x^{2}+\pi}+\frac{2\,ac^{2}}{3\,\pi\,x}\sqrt{\pi\,c^{2}x^{2}+\pi}+\frac{4\,bc^{3}\mathrm{Arcsinh}\,(cx)}{3\,\sqrt{\pi}}-\frac{2\,bx^{4}c^{7}}{3\,\sqrt{\pi}\left(3\,c^{2}x^{2}-1\right)}+\frac{2\,bx^{2}\left(c^{2}x^{2}+1\right)c^{5}}{3\,\sqrt{\pi}\left(3\,c^{2}x^{2}-1\right)}-2\,\frac{bx^{2}\mathrm{Arcsinh}}{\sqrt{\pi}\left(3\,c^{2}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] -1/3*a/Pi/x^3*(Pi*c^2*x^2+Pi)^(1/2)+2/3*a/Pi*c^2/x*(Pi*c^2*x^2+Pi)^(1/2)+4/
3*b*c^3/Pi^(1/2)*arcsinh(c*x)-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*x^4*c^7+2/3*b/Pi
^(1/2)/(3*c^2*x^2-1)*x^2*(c^2*x^2+1)*c^5-2*b/Pi^(1/2)/(3*c^2*x^2-1)*x^2*arc
sinh(c*x)*c^5+2*b/Pi^(1/2)/(3*c^2*x^2-1)*x*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c
^4-2/3*b/Pi^(1/2)/(3*c^2*x^2-1)*(c^2*x^2+1)*c^3+2/3*b/Pi^(1/2)/(3*c^2*x^2-1
)*arcsinh(c*x)*c^3-5/3*b/Pi^(1/2)/(3*c^2*x^2-1)/x*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*c^2+1/6*b/Pi^(1/2)/(3*c^2*x^2-1)/x^2*(c^2*x^2+1)*c+1/3*b/Pi^(1/2)/(3*
c^2*x^2-1)/x^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/3*b*c^3/Pi^(1/2)*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)
```

Maxima [A] time = 1.19238, size = 163, normalized size = 1.68

$$-\frac{1}{6}\left(\frac{4\,c^{2}\log\left(x\right)}{\sqrt{\pi}}+\frac{1}{\sqrt{\pi}x^{2}}\right)bc+\frac{1}{3}\,b\left(\frac{2\,\sqrt{\pi+\pi c^{2}x^{2}}c^{2}}{\pi x}-\frac{\sqrt{\pi+\pi c^{2}x^{2}}}{\pi x^{3}}\right)\mathrm{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,\sqrt{\pi+\pi c^{2}x^{2}}c^{2}}{\pi x}-\frac{\sqrt{\pi+\pi c^{2}x^{2}}}{\pi x^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(4*c^2*log(x)/sqrt(pi) + 1/(sqrt(pi)*x^2))*b*c + 1/3*b*(2*sqrt(pi + pi
*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))*arcsinh(c*x) + 1/3*a
*(2*sqrt(pi + pi*c^2*x^2)*c^2/(pi*x) - sqrt(pi + pi*c^2*x^2)/(pi*x^3))
```

Fricas [B] time = 3.03319, size = 495, normalized size = 5.1

$$\frac{2\,\sqrt{\pi+\pi c^{2}x^{2}}\left(2\,bc^{4}x^{4}+bc^{2}x^{2}-b\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+2\,\sqrt{\pi}\left(bc^{5}x^{5}+bc^{3}x^{3}\right)\log\left(\frac{\pi+\pi c^{2}x^{6}+\pi c^{2}x^{2}+\pi x^{4}-\sqrt{\pi}\sqrt{\pi+\pi c^{2}x^{2}}\sqrt{c^{2}x^{2}+1}}{c^{2}x^{4}+x^{2}}\right.}{6\left(\pi c^{2}x^{5}+\pi x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*sqrt(pi + pi*c^2*x^2)*(2*b*c^4*x^4 + b*c^2*x^2 - b)*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*sqrt(pi)*(b*c^5*x^5 + b*c^3*x^3)*log((pi + pi*c^2*x^6 + pi
*c^2*x^2 + pi*x^4 - sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*(x^4 -
 1))/(c^2*x^4 + x^2)) + sqrt(pi + pi*c^2*x^2)*(4*a*c^4*x^4 + 2*a*c^2*x^2 +
(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 + 1) - 2*a))/(pi*c^2*x^5 + pi*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{x^4\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{x^4\sqrt{c^2x^2+1}}\,dx}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] (Integral(a/(x**4*sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(x**4*sq
rt(c**2*x**2 + 1)), x))/sqrt(pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx) + a}{\sqrt{\pi + \pi c^2x^2}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(pi + pi*c^2*x^2)*x^4), x)
```

3.90 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi^3c^6}-\frac{2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^6}-\frac{a+b\sinh^{-1}(cx)}{\pi c^6\sqrt{\pi c^2x^2+\pi}}-\frac{bx^3}{9\pi^{3/2}c^3}+\frac{5bx}{3\pi^{3/2}c^5}+\frac{b\tan^{-1}(}{\pi^{3/2}c^6}$$

```
[Out] (5*b*x)/(3*c^5*Pi^(3/2)) - (b*x^3)/(9*c^3*Pi^(3/2)) - (a + b*ArcSinh[c*x])/
(c^6*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*
x]))/(c^6*Pi^2) + ((Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*Pi^
3) + (b*ArcTan[c*x])/(c^6*Pi^(3/2))
```

Rubi [A] time = 0.172302, antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 1153, 205}

$$\frac{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}c^6}-\frac{2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}c^6}-\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}c^6\sqrt{c^2x^2+1}}-\frac{bx^3}{9\pi^{3/2}c^3}+\frac{5bx}{3\pi^{3/2}c^5}+\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (5*b*x)/(3*c^5*Pi^(3/2)) - (b*x^3)/(9*c^3*Pi^(3/2)) - (a + b*ArcSinh[c*x])/
(c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
))/(c^6*Pi^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*Pi^(3
/2)) + (b*ArcTan[c*x])/(c^6*Pi^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^6\pi^{3/2}} - $$

$$= -\frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^6\pi^{3/2}} - $$

$$= \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a\right.}{3c^6}$$

$$= \frac{5bx}{3c^5\pi^{3/2}} - \frac{bx^3}{9c^3\pi^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^6\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a\right.}{3c^6}$$

Mathematica [A] time = 0.19999, size = 131, normalized size = 0.96

$$\frac{3ac^4x^4-12ac^2x^2-24a-bc^3x^3\sqrt{c^2x^2+1}+15bcx\sqrt{c^2x^2+1}+9b\sqrt{c^2x^2+1}\tan^{-1}(cx)+3b\left(c^4x^4-4c^2x^2-8\right)\sinh^{-1}(c}{9\pi^{3/2}c^6\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-24*a - 12*a*c^2*x^2 + 3*a*c^4*x^4 + 15*b*c*x*Sqrt[1 + c^2*x^2] - b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x] + 9*b*Sqr
t[1 + c^2*x^2]*ArcTan[c*x])/(9*c^6*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.237, size = 224, normalized size = 1.6

$$\frac{ax^4}{3\,\pi\,c^2}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}-\frac{4\,ax^2}{3\,c^4\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}-\frac{8\,a}{3\,c^6\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}-\frac{bx^3}{9\,c^3\pi^{3/2}}+\frac{5\,bx}{3\,c^5\pi^{3/2}}-\frac{ib}{\pi^{\frac{3}{2}}c^6}\ln\left(cx+\sqrt{c^2x^2+1}-i\right)-\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] 1/3*a*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-4/3*a/c^4*x^2/Pi/(Pi*c^2*x^2+Pi)^(1/
2)-8/3*a/c^6/Pi/(Pi*c^2*x^2+Pi)^(1/2)-1/9*b*x^3/c^3/Pi^(3/2)+5/3*b*x/c^5/Pi
^(3/2)-I*b/c^6/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-5/3*b/Pi^(3/2)/c^6*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)+I*b/c^6/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-b/P
i^(3/2)/(c^2*x^2+1)^(1/2)/c^6*arcsinh(c*x)+1/3*b/Pi^(3/2)/c^4*arcsinh(c*x)*
(c^2*x^2+1)^(1/2)*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a\left(\frac{x^4}{\pi\sqrt{\pi+\pi c^2x^2}c^2}-\frac{4\,x^2}{\pi\sqrt{\pi+\pi c^2x^2}c^4}-\frac{8}{\pi\sqrt{\pi+\pi c^2x^2}c^6}\right)+\frac{1}{3}\,b\left(\frac{\left(\sqrt{\pi}c^4x^4-4\,\sqrt{\pi}c^2x^2-8\,\sqrt{\pi}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^2\sqrt{c^2x^2+1}c^6}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*a*(x^4/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - 4*x^2/(pi*sqrt(pi + pi*c^2*x^2)
*c^4) - 8/(pi*sqrt(pi + pi*c^2*x^2)*c^6)) + 1/3*b*((sqrt(pi)*c^4*x^4 - 4*sq
rt(pi)*c^2*x^2 - 8*sqrt(pi))*log(c*x + sqrt(c^2*x^2 + 1))/(pi^2*sqrt(c^2*x^
```

```
2 + 1)*c^6) - integrate((sqrt(pi)*c^4*x^4 - 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi)
)/(sqrt(c^2*x^2 + 1)*x), x)/(pi^2*c^6) + 3*integrate(1/3*(sqrt(pi)*c^4*x^4
- 4*sqrt(pi)*c^2*x^2 - 8*sqrt(pi))/(pi^2*c^9*x^4 + pi^2*c^7*x^2 + (pi^2*c^8
*x^3 + pi^2*c^6*x)*sqrt(c^2*x^2 + 1)), x))
```

Fricas [A] time = 2.96779, size = 454, normalized size = 3.31

$$-\frac{9\sqrt{\pi}\left(bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)-6\sqrt{\pi+\pi c^2x^2}\left(bc^4x^4-4bc^2x^2-8b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{\pi}}{18\left(\pi^2c^8x^2+\pi^2c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/18*(9*sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*
sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) - 6*sqrt(pi + pi*c^2*x^2)*(b*c^4*x
^4 - 4*b*c^2*x^2 - 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x
^2)*(3*a*c^4*x^4 - 12*a*c^2*x^2 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1)
- 24*a))/(pi^2*c^8*x^2 + pi^2*c^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^5}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^5\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**5/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**5*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^5}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^5/(pi + pi*c^2*x^2)^(3/2), x)
```

3.91 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2c^4}-\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{3/2}bc^5}-\frac{bx^2}{4\pi^{3/2}c^3}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^5}$$

```
[Out] -(b*x^2)/(4*c^3*Pi^(3/2)) - (x^3*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^
2*Pi*x^2]) + (3*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(2*c^4*Pi^2)
- (3*(a + b*ArcSinh[c*x])^2)/(4*b*c^5*Pi^(3/2)) - (b*Log[1 + c^2*x^2])/(2*c
^5*Pi^(3/2))
```

Rubi [A] time = 0.257467, antiderivative size = 181, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5758, 5675, 30, 266, 43}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{3x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2c^4}-\frac{3\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{3/2}bc^5}-\frac{bx^2\sqrt{c^2x^2+1}}{4\pi c^3\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\,l}{2\pi c^5\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c^3*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (x^3*(a + b*A
rcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2]) + (3*x*Sqrt[Pi + c^2*Pi*x^2]*(
a + b*ArcSinh[c*x]))/(2*c^4*Pi^2) - (3*(a + b*ArcSinh[c*x])^2)/(4*b*c^5*Pi^
(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^5*Pi*Sqrt[Pi + c^2*Pi*
x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

```
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}} + \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2} - \frac{3\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{2c^4\pi} - \frac{\left(3b\right.}{2c}$$

$$= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3\pi\sqrt{\pi+c^2\pi x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}} + \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2} - \frac{3\left(a+\right.}{}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3\pi\sqrt{\pi+c^2\pi x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}} + \frac{3x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4\pi^2} - \frac{3\left(a+\right.}{}$$

Mathematica [A] time = 0.347651, size = 147, normalized size = 1.12

$$\frac{\sinh^{-1}(cx)\left(-12a\sqrt{c^2x^2+1}+9bcx+b\sinh\left(3\sinh^{-1}(cx)\right)\right)+4ac^3x^3+12acx-4b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)-6b\sqrt{c^2x^2}}{8\pi^{3/2}c^5\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (12*a*c*x + 4*a*c^3*x^3 - 6*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[1 +
 c^2*x^2]*Cosh[2*ArcSinh[c*x]] - 4*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + A
rcSinh[c*x]*(9*b*c*x - 12*a*Sqrt[1 + c^2*x^2] + b*Sinh[3*ArcSinh[c*x]]))/(8
*c^5*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.178, size = 269, normalized size = 2.1

$$\frac{ax^3}{2\pi c^2}\frac{1}{\sqrt{\pi c^2x^2+\pi}} + \frac{3ax}{2c^4\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}} - \frac{3a}{2c^4\pi}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}} - \frac{3b\left(\mathrm{Arcsinh}\,(cx)\right)^2}{4\pi^{3/2}c^5} + \frac{b\mathrm{Arc}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] 1/2*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+3/2*a/c^4*x/Pi/(Pi*c^2*x^2+Pi)^(1/2)
-3/2*a/c^4/Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1
/2)-3/4*b/c^5/Pi^(3/2)*arcsinh(c*x)^2+1/2*b/Pi^(3/2)/c^4*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)*x-1/4*b*x^2/c^3/Pi^(3/2)+2*b/c^5/Pi^(3/2)*arcsinh(c*x)-1/8*b/P
i^(3/2)/c^5-b/Pi^(3/2)*arcsinh(c*x)/c^3/(c^2*x^2+1)*x^2+b/Pi^(3/2)*arcsinh(
c*x)/c^4/(c^2*x^2+1)^(1/2)*x-b/Pi^(3/2)*arcsinh(c*x)/c^5/(c^2*x^2+1)-b/c^5/
Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{x^3}{\pi\sqrt{\pi+\pi c^2x^2}c^2}+\frac{3\,x}{\pi\sqrt{\pi+\pi c^2x^2}c^4}-\frac{3\,\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\pi\sqrt{\pi c^2}c^4}\right)+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*a*(x^3/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 3*x/(pi*sqrt(pi + pi*c^2*x^2)*c
^4) - 3*arcsinh(c^2*x/sqrt(c^2))/(pi*sqrt(pi*c^2)*c^4)) + b*integrate(x^4*l
og(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^4\operatorname{arsinh}(cx)+ax^4\right)}{\pi^2c^4x^4+2\,\pi^2c^2x^2+\pi^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^4}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^4\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**4/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**4*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)x^4}{(\pi + \pi c^2x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(pi + pi*c^2*x^2)^(3/2), x)
```

3.92 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^4}+\frac{a+b\sinh^{-1}(cx)}{\pi c^4\sqrt{\pi c^2x^2+\pi}}-\frac{bx}{\pi^{3/2}c^3}-\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^4}$$

```
[Out] -((b*x)/(c^3*Pi^(3/2))) + (a + b*ArcSinh[c*x])/(c^4*Pi*Sqrt[Pi + c^2*Pi*x^2
]) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(c^4*Pi^2) - (b*ArcTan[c*
x])/(c^4*Pi^(3/2))
```

Rubi [A] time = 0.142452, antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5732, 388, 205}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}c^4}+\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}c^4\sqrt{c^2x^2+1}}-\frac{bx}{\pi^{3/2}c^3}-\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((b*x)/(c^3*Pi^(3/2))) + (a + b*ArcSinh[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x
^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^4*Pi^(3/2)) - (b*ArcTan[
c*x])/(c^4*Pi^(3/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{(bc)\int\frac{2+c^2x^2}{c^4+c^6x^2}\,dx}{\pi^{3/2}}\\
&= -\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{(bc)\int\frac{1}{c^4+c^6x^2}\,dx}{\pi^{3/2}}\\
&= -\frac{bx}{c^3\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{c^4\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^{3/2}}-\frac{b\tan^{-1}(cx)}{c^4\pi^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.159627, size = 87, normalized size = 1.01

$$\frac{ac^2x^2+2a-bcx\sqrt{c^2x^2+1}-b\sqrt{c^2x^2+1}\tan^{-1}(cx)+b\left(c^2x^2+2\right)\sinh^{-1}(cx)}{\pi^{3/2}c^4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (2*a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] + b*(2 + c^2*x^2)*ArcSinh[c*x] -
 b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.181, size = 158, normalized size = 1.8

$$\frac{ax^2}{\pi c^2}\frac{1}{\sqrt{\pi c^2x^2+\pi}}+2\frac{a}{\pi c^4\sqrt{\pi c^2x^2+\pi}}+\frac{b\mathrm{Arcsinh}(cx)}{\pi^{\frac{3}{2}}c^4}\sqrt{c^2x^2+1}-\frac{bx}{c^3\pi^{\frac{3}{2}}}+\frac{b\mathrm{Arcsinh}(cx)}{\pi^{\frac{3}{2}}c^4}\frac{1}{\sqrt{c^2x^2+1}}+\frac{ib}{\pi^{\frac{3}{2}}c^4}\ln\Big(cx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a*x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+2*a/Pi/c^4/(Pi*c^2*x^2+Pi)^(1/2)+b/Pi^(3
/2)/c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-b*x/c^3/Pi^(3/2)+b/Pi^(3/2)/(c^2*x^2
+1)^(1/2)/c^4*arcsinh(c*x)+I*b/c^4/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-I*b
/c^4/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

Maxima [A] time = 1.7572, size = 161, normalized size = 1.87

$$-bc\left(\frac{x}{\pi^{\frac{3}{2}}c^4}+\frac{\arctan(cx)}{\pi^{\frac{3}{2}}c^5}\right)+b\left(\frac{x^2}{\pi\sqrt{\pi+\pi c^2x^2}c^2}+\frac{2}{\pi\sqrt{\pi+\pi c^2x^2}c^4}\right)\mathrm{arsinh}(cx)+a\left(\frac{x^2}{\pi\sqrt{\pi+\pi c^2x^2}c^2}+\frac{2}{\pi\sqrt{\pi+\pi c^2x^2}c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -b*c*(x/(pi^(3/2)*c^4) + arctan(c*x)/(pi^(3/2)*c^5)) + b*(x^2/(pi*sqrt(pi +
 pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))*arcsinh(c*x) + a*(x^2
/(pi*sqrt(pi + pi*c^2*x^2)*c^2) + 2/(pi*sqrt(pi + pi*c^2*x^2)*c^4))
```

Fricas [B] time = 2.98134, size = 382, normalized size = 4.44

$$\frac{\sqrt{\pi}\left(bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}\left(bc^2x^2+2b\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi+\pi c^2x^2}\Big(ac^2x}{2\left(\pi^2c^6x^2+\pi^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqrt
(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*(b*c^2*x^2 +
 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*(a*c^2*x^2 - s
qrt(c^2*x^2 + 1)*b*c*x + 2*a))/(pi^2*c^6*x^2 + pi^2*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^3\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**3*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)x^3}{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(pi + pi*c^2*x^2)^(3/2), x)
```

3.93 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{3/2}bc^3}+\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c^3}$$

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSinh
[c*x])^2/(2*b*c^3*Pi^(3/2)) + (b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

Rubi [A] time = 0.139521, antiderivative size = 105, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5751, 5675, 260}

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{3/2}bc^3}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2\pi c^3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (a + b*ArcSinh
[c*x])^2/(2*b*c^3*Pi^(3/2)) + (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^3
*Pi*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^2\pi}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{x}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3\pi^{3/2}}+\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3\pi\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.279468, size = 78, normalized size = 0.98

$$\frac{\sinh^{-1}(cx)\left(2a-\frac{2bcx}{\sqrt{c^2x^2+1}}\right)-\frac{2acx}{\sqrt{c^2x^2+1}}+b\log\left(c^2x^2+1\right)+b\sinh^{-1}(cx)^2}{2\pi^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] ((-2*a*c*x)/Sqrt[1 + c^2*x^2] + (2*a - (2*b*c*x)/Sqrt[1 + c^2*x^2])*ArcSinh
[c*x] + b*ArcSinh[c*x]^2 + b*Log[1 + c^2*x^2])/(2*c^3*Pi^(3/2))
```

Maple [B] time = 0.105, size = 196, normalized size = 2.5

$$-\frac{ax}{\pi c^2}\frac{1}{\sqrt{\pi c^2x^2+\pi}}+\frac{a}{\pi c^2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+\frac{b\left(\mathrm{Arcsinh}\,(cx)\right)^2}{2\,c^3\pi^{3/2}}-2\,\frac{b\mathrm{Arcsinh}\,(cx)}{c^3\pi^{3/2}}+\frac{b\mathrm{Arcsinh}}{\pi^{\frac{3}{2}}c\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -a*x/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)+a/Pi/c^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b/c^3/Pi^(3/2)*arcsinh(c*x)^2-2*b/c^3/
Pi^(3/2)*arcsinh(c*x)+b/Pi^(3/2)*arcsinh(c*x)/c/(c^2*x^2+1)*x^2-b/Pi^(3/2)*
arcsinh(c*x)/c^2/(c^2*x^2+1)^(1/2)*x+b/Pi^(3/2)*arcsinh(c*x)/c^3/(c^2*x^2+1
)+b/c^3/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{x}{\pi\sqrt{\pi+\pi c^2x^2}c^2}-\frac{\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\pi\sqrt{\pi c^2}c^2}\right)+b\int\frac{x^2\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] -a*(x/(pi*sqrt(pi + pi*c^2*x^2)*c^2) - arcsinh(c^2*x/sqrt(c^2))/(pi*sqrt(pi
*c^2)*c^2)) + b*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2
)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right)}{\pi^2c^4x^4+2\,\pi^2c^2x^2+\pi^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^2*c^4*x^4 +
 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx^2\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x)
+ Integral(b*x**2*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)x^2}{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(3/2), x)
```

3.94 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{b\tan^{-1}(cx)}{\pi^{3/2}c^2}-\frac{a+b\sinh^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (b*ArcTan[c*x])/(c
^2*Pi^(3/2))
```

Rubi [A] time = 0.0723462, antiderivative size = 70, normalized size of antiderivative = 1.56, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 203}

$$\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{\pi c^2\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])) + (b*Sqrt[1 + c^2*x^
2]*ArcTan[c*x])/(c^2*Pi*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{c\pi\sqrt{\pi+c^2\pi x^2}}$$
$$= -\frac{a+b\sinh^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}+\frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{c^2\pi\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.107267, size = 52, normalized size = 1.16

$$\frac{-a+b\sqrt{c^2x^2+1}\tan^{-1}(cx)-b\sinh^{-1}(cx)}{\pi^{3/2}c^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-a - b*ArcSinh[c*x] + b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^2*Pi^(3/2)*Sqrt[
1 + c^2*x^2])
```

Maple [C] time = 0.086, size = 103, normalized size = 2.3

$$-\frac{a}{\pi c^2}\frac{1}{\sqrt{\pi c^2x^2+\pi}}-\frac{b\mathrm{Arcsinh}\,(cx)}{\pi^{\frac{3}{2}}c^2}\frac{1}{\sqrt{c^2x^2+1}}+\frac{ib}{\pi^{\frac{3}{2}}c^2}\ln\left(cx+\sqrt{c^2x^2+1}+i\right)-\frac{ib}{\pi^{\frac{3}{2}}c^2}\ln\left(cx+\sqrt{c^2x^2+1}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -a/Pi/c^2/(Pi*c^2*x^2+Pi)^(1/2)-b/Pi^(3/2)/(c^2*x^2+1)^(1/2)/c^2*arcsinh(c*
x)+I*b/c^2/Pi^(3/2)*ln(c*x+(c^2*x^2+1)^(1/2)+I)-I*b/c^2/Pi^(3/2)*ln(c*x+(c^
2*x^2+1)^(1/2)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\frac{-\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\pi^{\frac{3}{2}}c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{\frac{3}{2}}\sqrt{c^2x^2+1}c^2}-\int\frac{1}{\pi^{\frac{3}{2}}c^5x^4+\pi^{\frac{3}{2}}c^3x^2+\left(\pi^{\frac{3}{2}}c^4x^3+\pi^{\frac{3}{2}}c^2x\right)\sqrt{c^2x^2+1}}\,dx\right)-\frac{a}{\pi\sqrt{\pi+\pi c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(pi^(3/2)*c^2) - log(c*x + sqrt(c^
2*x^2 + 1))/(pi^(3/2)*sqrt(c^2*x^2 + 1)*c^2) - integrate(1/(pi^(3/2)*c^5*x^
4 + pi^(3/2)*c^3*x^2 + (pi^(3/2)*c^4*x^3 + pi^(3/2)*c^2*x)*sqrt(c^2*x^2 + 1
)), x)) - a/(pi*sqrt(pi + pi*c^2*x^2)*c^2)
```

Fricas [B] time = 2.88157, size = 305, normalized size = 6.78

$$-\frac{\sqrt{\pi}\left(bc^2x^2+b\right)\arctan\left(-\frac{2\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+2\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)+2\sqrt{\pi+\pi c^2x^2}a}{2\left(\pi^2c^4x^2+\pi^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*(b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi + pi*c^2*x^2)*sqr
t(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 2*sqrt(pi + pi*c^2*x^2)*b*log(c*x +
 sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2*x^2)*a)/(pi^2*c^4*x^2 + pi^2*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a*x/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*x*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x}{(\pi + \pi c^2 x^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(3/2), x)
```

3.95 $\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\log\left(c^2x^2+1\right)}{2\pi^{3/2}c}$$

```
[Out] (x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Log[1 + c^2*x^2])/
(2*c*Pi^(3/2))
```

Rubi [A] time = 0.0388463, antiderivative size = 76, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5687, 260}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2\pi c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Sqrt[1 + c^2*x^2]*
Log[1 + c^2*x^2])/(2*c*Pi*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}$$
$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c\pi\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.092849, size = 66, normalized size = 1.29

$$\frac{2acx-b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)+2bcx\sinh^{-1}(cx)}{2\pi^{3/2}c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*
c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.066, size = 132, normalized size = 2.6

$$\frac{ax}{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}}+2\frac{b\mathrm{Arcsinh}(cx)}{c\pi^{3/2}}-\frac{b\mathrm{Arcsinh}(cx)cx^2}{\pi^{\frac{3}{2}}\left(c^2x^2+1\right)}+\frac{b\mathrm{Arcsinh}(cx)x}{\pi^{\frac{3}{2}}}\frac{1}{\sqrt{c^2x^2+1}}-\frac{b\mathrm{Arcsinh}(cx)}{c\pi^{\frac{3}{2}}\left(c^2x^2+1\right)}-\frac{b}{c\pi^{\frac{3}{2}}}\ln\Bigg(1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+2*b/c/Pi^(3/2)*arcsinh(c*x)-b/Pi^(3/2)*arcsinh
(c*x)*c/(c^2*x^2+1)*x^2+b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x-b/Pi^(3
/2)*arcsinh(c*x)/c/(c^2*x^2+1)-b/c/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [A] time = 1.12228, size = 88, normalized size = 1.73

$$-\frac{bc\sqrt{\frac{1}{\pi c^4}}\log\left(x^2+\frac{1}{c^2}\right)}{2\pi}+\frac{bx\,\mathrm{arsinh}(cx)}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{ax}{\pi\sqrt{\pi+\pi c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*sqrt(1/(pi*c^4))*log(x^2 + 1/c^2)/pi + b*x*arcsinh(c*x)/(pi*sqrt(p
i + pi*c^2*x^2)) + a*x/(pi*sqrt(pi + pi*c^2*x^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{\pi^2 c^4 x^4 + 2\,\pi^2 c^2 x^2 + \pi^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^4 + 2*pi^2*
c^2*x^2 + pi^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) + Int
egral(b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x
))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(3/2), x)
```

3.96 $\int \frac{a+b\sinh^{-1}(cx)}{x(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}-$$

```
[Out] (a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*ArcTan[c*x])/Pi^(3/2)
- (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(3/2) - (b*PolyLog[2,
 -E^ArcSinh[c*x]])/Pi^(3/2) + (b*PolyLog[2, E^ArcSinh[c*x]])/Pi^(3/2)
```

Rubi [A] time = 0.221959, antiderivative size = 119, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5755, 5760, 4182, 2279, 2391, 203}

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcT
an[c*x])/(Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^Arc
Sinh[c*x]])/Pi^(3/2) - (b*PolyLog[2, -E^ArcSinh[c*x]])/Pi^(3/2) + (b*PolyLo
g[2, E^ArcSinh[c*x]])/Pi^(3/2)
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\pi^{3/2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}} - \frac{b\,\text{Subst}}{}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}} - \frac{b\,\text{Subst}}{}$$

$$= \frac{a+b\sinh^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{3/2}} - \frac{b\mathbf{Li}_2\left(-e\right.}{\pi}$$

Mathematica [A] time = 0.318419, size = 143, normalized size = 1.52

$$\frac{b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + \frac{a}{\sqrt{c^2x^2+1}} - a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + a\log(x) + \frac{b\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}} +}{\pi^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (a/Sqrt[1 + c^2*x^2] + (b*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 2*b*ArcTan[Tanh
[ArcSinh[c*x]/2]] + b*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - b*ArcSinh[c
*x]*Log[1 + E^(-ArcSinh[c*x])] + a*Log[x] - a*Log[Pi*(1 + Sqrt[1 + c^2*x^2]
)] + b*PolyLog[2, -E^(-ArcSinh[c*x])] - b*PolyLog[2, E^(-ArcSinh[c*x])])/Pi
^(3/2)
```

Maple [A] time = 0.153, size = 156, normalized size = 1.7

$$\frac{a}{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}} - \frac{a}{\pi^{\frac{3}{2}}}\text{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}}\right) + \frac{b\text{Arcsinh}(cx)}{\pi^{\frac{3}{2}}}\frac{1}{\sqrt{c^2x^2+1}} - 2\frac{b\arctan\left(cx+\sqrt{c^2x^2+1}\right)}{\pi^{3/2}} - \frac{b}{\pi^{\frac{3}{2}}}\text{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a/Pi/(Pi*c^2*x^2+Pi)^(1/2)-a/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2
))+b/Pi^(3/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-2*b/Pi^(3/2)*arctan(c*x+(c^2*x
^2+1)^(1/2))-b/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(3/2)*dilog(1+c*x
+(c^2*x^2+1)^(1/2))-b/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\pi^{\frac{3}{2}}} - \frac{1}{\pi\sqrt{\pi + \pi c^2x^2}}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(1/(sqrt(c^2)*abs(x)))/pi^(3/2) - 1/(pi*sqrt(pi + pi*c^2*x^2)))
+ b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\pi + \pi c^2x^2}(b\operatorname{arsinh}(cx) + a)}{\pi^2c^4x^5 + 2\pi^2c^2x^3 + \pi^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^5 + 2*pi^2*
c^2*x^3 + pi^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^3\sqrt{c^2x^2+1}+x\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)), x) + I
ntegral(b*asinh(c*x)/(c**2*x**3*sqrt(c**2*x**2 + 1) + x*sqrt(c**2*x**2 + 1)
), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(\pi + \pi c^2 x^2)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x), x)
```

3.97 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{\pi x\sqrt{\pi c^2x^2+\pi}}+\frac{bc\log\left(c^2x^2+1\right)}{2\pi^{3/2}}+\frac{bc\log(x)}{\pi^{3/2}}$$

```
[Out] -((a + b*ArcSinh[c*x])/(Pi*x*Sqrt[Pi + c^2*Pi*x^2])) - (2*c^2*x*(a + b*ArcS
inh[c*x]))/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*Log[x])/Pi^(3/2) + (b*c*Log[1
+ c^2*x^2])/(2*Pi^(3/2))
```

Rubi [A] time = 0.138361, antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {271, 191, 5732, 446, 72}

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{c^2x^2+1}}+\frac{bc\log\left(c^2x^2+1\right)}{2\pi^{3/2}}+\frac{bc\log(x)}{\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(Pi^(3/2)*x*Sqrt[1 + c^2*x^2])) - (2*c^2*x*(a + b*Ar
cSinh[c*x]))/(Pi^(3/2)*Sqrt[1 + c^2*x^2]) + (b*c*Log[x])/Pi^(3/2) + (b*c*Lo
g[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\int\frac{-1-2c^2x^2}{x\left(1+c^2x^2\right)}\,dx}{\pi^{3/2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\,\mathrm{Subst}\left(\int\frac{-1-2c^2x}{x\left(1+c^2x\right)}\,dx,x,x^2\right)}{2\pi^{3/2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}-\frac{(bc)\,\mathrm{Subst}\left(\int\left(-\frac{1}{x}-\frac{c^2}{1+c^2x}\right)dx,x,x^2\right)}{2\pi^{3/2}}\\ &= -\frac{a+b\sinh^{-1}(cx)}{\pi^{3/2}x\sqrt{1+c^2x^2}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}\sqrt{1+c^2x^2}}+\frac{bc\log(x)}{\pi^{3/2}}+\frac{bc\log\left(1+c^2x^2\right)}{2\pi^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.160248, size = 69, normalized size = 0.74

$$\frac{b\left(\frac{1}{2}c\log\left(c^2x^2+1\right)+c\log(x)\right)}{\pi^{3/2}}-\frac{\left(2c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}x\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(((1 + 2*c^2*x^2)*(a + b*ArcSinh[c*x]))/(Pi^(3/2)*x*Sqrt[1 + c^2*x^2])) +
(b*(c*Log[x] + (c*Log[1 + c^2*x^2])/2))/Pi^(3/2)
```

Maple [B] time = 0.097, size = 180, normalized size = 1.9

$$-\frac{a}{\pi x}\frac{1}{\sqrt{\pi c^2x^2+\pi}}-2\frac{ac^2x}{\pi\sqrt{\pi c^2x^2+\pi}}-4\frac{bc\mathrm{Arcsinh}(cx)}{\pi^{3/2}}+2\frac{b\mathrm{Arcsinh}(cx)x^2c^3}{\pi^{3/2}\left(c^2x^2+1\right)}-2\frac{b\mathrm{Arcsinh}(cx)xc^2}{\pi^{3/2}\sqrt{c^2x^2+1}}+2\frac{bc\mathrm{Arcsin}}{\pi^{3/2}\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -a/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)-2*a/Pi*c^2*x/(Pi*c^2*x^2+Pi)^(1/2)-4*b*c/Pi^(
3/2)*arcsinh(c*x)+2*b/Pi^(3/2)*arcsinh(c*x)*x^2/(c^2*x^2+1)*c^3-2*b/Pi^(3/2
)*arcsinh(c*x)*x/(c^2*x^2+1)^(1/2)*c^2+2*b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1
)*c-b/Pi^(3/2)*arcsinh(c*x)/x/(c^2*x^2+1)^(1/2)+b*c/Pi^(3/2)*ln((c*x+(c^2*x
^2+1)^(1/2))^4-1)
```

Maxima [A] time = 1.21045, size = 161, normalized size = 1.73

$$\frac{1}{2}bc\left(\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{3}{2}}}+\frac{2\log(x)}{\pi^{\frac{3}{2}}}\right)-\left(\frac{2c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x}\right)b\,\mathrm{arsinh}(cx)-\left(\frac{2c^2x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*b*c*(log(c^2*x^2 + 1)/pi^(3/2) + 2*log(x)/pi^(3/2)) - (2*c^2*x/(pi*sqrt
(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*b*arcsinh(c*x) - (2*c^
2*x/(pi*sqrt(pi + pi*c^2*x^2)) + 1/(pi*sqrt(pi + pi*c^2*x^2)*x))*a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}(cx)+a)}{\pi^2c^4x^6+2\pi^2c^2x^4+\pi^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^6 + 2*pi^2*
c^2*x^4 + pi^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx + \int \frac{b\operatorname{asinh}(cx)}{c^2x^4\sqrt{c^2x^2+1}+x^2\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**4*sqrt(c**2*x**2 + 1) + x**2*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2x^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^2), x)
```

3.98 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=162

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{3c^2\tanh^{-}}{}$$

```
[Out] -(b*c)/(2*Pi^(3/2)*x) - (3*c^2*(a + b*ArcSinh[c*x]))/(2*Pi*Sqrt[Pi + c^2*Pi
*x^2]) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c^2*Arc
Tan[c*x])/Pi^(3/2) + (3*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/P
i^(3/2) + (3*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(3/2)) - (3*b*c^2*Pol
yLog[2, E^ArcSinh[c*x]])/(2*Pi^(3/2))
```

Rubi [A] time = 0.350337, antiderivative size = 212, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5747, 5755, 5760, 4182, 2279, 2391, 203, 325}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\pi^{3/2}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi c^2x^2+\pi}}+\frac{3c^2\tanh^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*Pi*x*Sqrt[Pi + c^2*Pi*x^2]) - (3*c^2*(a + b*Arc
Sinh[c*x]))/(2*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*S
qrt[Pi + c^2*Pi*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(Pi*Sqrt[Pi +
 c^2*Pi*x^2]) + (3*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(3/
2) + (3*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(3/2)) - (3*b*c^2*PolyLog[
2, E^ArcSinh[c*x]])/(2*Pi^(3/2))
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{2\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} - \frac{\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{2\pi} -$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} -$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} +$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} +$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi\sqrt{\pi+c^2\pi x^2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\sqrt{\pi+c^2\pi x^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{\pi\sqrt{\pi+c^2\pi x^2}} +$$

Mathematica [A] time = 3.94736, size = 269, normalized size = 1.66

$$-12bc^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+12bc^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\frac{8ac^2}{\sqrt{c^2x^2+1}}-\frac{4a\sqrt{c^2x^2+1}}{x^2}+12ac^2\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] ((-8*a*c^2)/Sqrt[1 + c^2*x^2] - (4*a*Sqrt[1 + c^2*x^2])/x^2 - (8*b*c^2*ArcS
inh[c*x])/Sqrt[1 + c^2*x^2] + 16*b*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 2*b*c
^2*Coth[ArcSinh[c*x]/2] - b*c^2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*b*
c^2*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*b*c^2*ArcSinh[c*x]*Log[1 +
 E^(-ArcSinh[c*x])] - 12*a*c^2*Log[x] + 12*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x
^2])] - 12*b*c^2*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*b*c^2*PolyLog[2, E^(-A
rcSinh[c*x])] - b*c^2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*b*c^2*Tanh[Ar
cSinh[c*x]/2])/(8*Pi^(3/2))
```

Maple [A] time = 0.177, size = 234, normalized size = 1.4

$$-\frac{a}{2\,\pi\,x^2}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}-\frac{3\,ac^2}{2\,\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}+\frac{3\,ac^2}{2\,\pi^{3/2}}\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}\right)-\frac{3\,b\mathrm{Arcsinh}\,(cx)\,c^2}{2\,\pi^{3/2}}\frac{1}{\sqrt{c^2x^2+1}}-\frac{bc}{2\,\pi^{3/2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -1/2*a/Pi/x^2/(Pi*c^2*x^2+Pi)^(1/2)-3/2*a*c^2/Pi/(Pi*c^2*x^2+Pi)^(1/2)+3/2*
a*c^2/Pi^(3/2)*arctanh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))-3/2*b/Pi^(3/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*c^2-1/2*b*c/Pi^(3/2)/x-1/2*b/Pi^(3/2)/(c^2*x^2+1)
^(1/2)/x^2*arcsinh(c*x)+2*b*c^2/Pi^(3/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+3/2*
b*c^2/Pi^(3/2)*dilog(c*x+(c^2*x^2+1)^(1/2))+3/2*b*c^2/Pi^(3/2)*dilog(1+c*x+
(c^2*x^2+1)^(1/2))+3/2*b*c^2/Pi^(3/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(\frac{3\,c^2\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\pi^{\frac{3}{2}}}-\frac{3\,c^2}{\pi\sqrt{\pi+\pi c^2x^2}}-\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x^2}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(3*c^2*arcsinh(1/(sqrt(c^2)*abs(x)))/pi^(3/2) - 3*c^2/(pi*sqrt(pi + pi*
c^2*x^2)) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^2))*a + b*integrate(log(c*x + sqr
```

```
t(c^2*x^2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{\pi^2 c^4 x^7 + 2\,\pi^2 c^2 x^5 + \pi^2 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^7 + 2*pi^2*
c^2*x^5 + pi^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx + \int \frac{b \operatorname{asinh}(cx)}{c^2x^5\sqrt{c^2x^2+1}+x^3\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**2 + 1)), x)
+ Integral(b*asinh(c*x)/(c**2*x**5*sqrt(c**2*x**2 + 1) + x**3*sqrt(c**2*x**
2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^3), x)
```

3.99 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(\pi+c^2\pi x^2)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\sqrt{\pi c^2x^2+\pi}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi x\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\sqrt{\pi c^2x^2+\pi}}-\frac{bc^3\log\left(c^2x^2+1\right)}{2\pi^{3/2}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc}{6\pi^{3/2}x^2}$$

```
[Out] -(b*c)/(6*Pi^(3/2)*x^2) - (a + b*ArcSinh[c*x])/(3*Pi*x^3*Sqrt[Pi + c^2*Pi*x
^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]) + (8*c^4
*x*(a + b*ArcSinh[c*x]))/(3*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (5*b*c^3*Log[x])/(3
*Pi^(3/2)) - (b*c^3*Log[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rubi [A] time = 0.175009, antiderivative size = 156, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {271, 191, 5732, 12, 1251, 893}

$$\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{c^2x^2+1}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{c^2x^2+1}}-\frac{bc^3\log\left(c^2x^2+1\right)}{2\pi^{3/2}}-\frac{5bc^3\log(x)}{3\pi^{3/2}}-\frac{bc}{6\pi^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] -(b*c)/(6*Pi^(3/2)*x^2) - (a + b*ArcSinh[c*x])/(3*Pi^(3/2)*x^3*Sqrt[1 + c^2
*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*Pi^(3/2)*x*Sqrt[1 + c^2*x^2]) + (8
*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^(3/2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*Log[
x])/(3*Pi^(3/2)) - (b*c^3*Log[1 + c^2*x^2])/(2*Pi^(3/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-1+4c^2x^2+8c^4}{3x^3\left(1+c^2x^2\right)}}{\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-1+4c^2x^2+8c^4}{x^3\left(1+c^2x^2\right)}}{3\pi^{3/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\text{Subst}\left(\int \frac{-1+}{x^2}\right.}{6\pi}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\text{Subst}\left(\int\left(-\frac{1}{x}\right.\right.}{}$$

$$= -\frac{bc}{6\pi^{3/2}x^2} - \frac{a+b\sinh^{-1}(cx)}{3\pi^{3/2}x^3\sqrt{1+c^2x^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}x\sqrt{1+c^2x^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{3/2}\sqrt{1+c^2x^2}} - \frac{5bc^3\log}{3\pi^{3/2}}$$

Mathematica [A] time = 0.217, size = 127, normalized size = 0.83

$$\frac{2a\left(8c^4x^4+4c^2x^2-1\right)-bcx\sqrt{c^2x^2+1}+2b\left(8c^4x^4+4c^2x^2-1\right)\sinh^{-1}(cx)}{6\pi^{3/2}x^3\sqrt{c^2x^2+1}} + \frac{-\frac{3}{2}bc^3\log\left(c^2x^2+1\right)-5bc^3\log(x)}{3\pi^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) + 2*a*(-1 + 4*c^2*x^2 + 8*c^4*x^4) + 2*b*(-1 +
4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x])/(6*Pi^(3/2)*x^3*Sqrt[1 + c^2*x^2]) + (
-5*b*c^3*Log[x] - (3*b*c^3*Log[1 + c^2*x^2])/2)/(3*Pi^(3/2))
```

Maple [B] time = 0.19, size = 601, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] -1/3*a/Pi/x^3/(Pi*c^2*x^2+Pi)^(1/2)+4/3*a*c^2/Pi/x/(Pi*c^2*x^2+Pi)^(1/2)+8/
3*a*c^4/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)+16/3*b*c^3/Pi^(3/2)*arcsinh(c*x)-32/3*b/
Pi^(3/2)/(8*c^2*x^2-1)*x^8/(c^2*x^2+1)*c^11+32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x
^6*c^9-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^6/(c^2*x^2+1)*c^9+32/3*b/Pi^(3/2)/(8
*c^2*x^2-1)*x^4*c^7-64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*arcsinh(c
*x)*c^7+64/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^
6-32/3*b/Pi^(3/2)/(8*c^2*x^2-1)*x^4/(c^2*x^2+1)*c^7-56/3*b/Pi^(3/2)/(8*c^2*
x^2-1)*x^2/(c^2*x^2+1)*arcsinh(c*x)*c^5+8*b/Pi^(3/2)/(8*c^2*x^2-1)*x/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^4-4/3*b/Pi^(3/2)/(8*c^2*x^2-1)*c^3+8/3*b/Pi^(3/2
)/(8*c^2*x^2-1)/(c^2*x^2+1)*arcsinh(c*x)*c^3-4*b/Pi^(3/2)/(8*c^2*x^2-1)/x/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^2+1/6*b/Pi^(3/2)/(8*c^2*x^2-1)/x^2*c+1/3*b/
Pi^(3/2)/(8*c^2*x^2-1)/x^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-b*c^3/Pi^(3/2)*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)-5/3*b*c^3/Pi^(3/2)*ln((c*x+(c^2*x^2+1)^(1/2))
^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(\frac{8\,c^4x}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{4\,c^2}{\pi\sqrt{\pi+\pi c^2x^2}x}-\frac{1}{\pi\sqrt{\pi+\pi c^2x^2}x^3}\right)a+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(8*c^4*x/(pi*sqrt(pi + pi*c^2*x^2)) + 4*c^2/(pi*sqrt(pi + pi*c^2*x^2)*x
) - 1/(pi*sqrt(pi + pi*c^2*x^2)*x^3))*a + b*integrate(log(c*x + sqrt(c^2*x^
2 + 1))/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\,\text{arsinh}\,(cx)+a)}{\pi^2c^4x^8+2\,\pi^2c^2x^6+\pi^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^2*c^4*x^8 + 2*pi^2*
c^2*x^6 + pi^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(3/2)*x^4), x)
```

3.100 $\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{5x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^3c^6}-\frac{5\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{5/2}bc^7}-\frac{bx^2}{4\pi^{5/2}c^5}-$$

```
[Out] -(b*x^2)/(4*c^5*Pi^(5/2)) - b/(6*c^7*Pi^(5/2)*(1 + c^2*x^2)) - (x^5*(a + b*
ArcSinh[c*x]))/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c
*x]))/(3*c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (5*x*Sqrt[Pi + c^2*Pi*x^2]*(a +
b*ArcSinh[c*x]))/(2*c^6*Pi^3) - (5*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*Pi^(5/2
)) - (7*b*Log[1 + c^2*x^2])/(6*c^7*Pi^(5/2))
```

Rubi [A] time = 0.425205, antiderivative size = 256, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5758, 5675, 30, 266, 43}

$$-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{5x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\pi^3c^6}-\frac{5\left(a+b\sinh^{-1}(cx)\right)^2}{4\pi^{5/2}bc^7}-\frac{bx^2\sqrt{c^2}}{4\pi^2c^5\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^7*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (b*x^2*Sqrt[1 + c
^2*x^2])/(4*c^5*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3
*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*Pi^2
*Sqrt[Pi + c^2*Pi*x^2]) + (5*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/
(2*c^6*Pi^3) - (5*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*Pi^(5/2)) - (7*b*Sqrt[1
+ c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
```

```
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6 \left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{5\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}dx}{3c^2\pi} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^5}{\left(1+c^2x^2\right)^2}dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{\pi+c^2\pi x^2}}dx}{c^4\pi^2} + \frac{\left(5b\sqrt{1+c^2x^2}\right.}{3c^3\pi^2\sqrt{\pi}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{5x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^6\pi^3} - \frac{5\int}{}$$

$$= -\frac{b}{6c^7\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{13bx^2\sqrt{1+c^2x^2}}{12c^5\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5x^3\left(a+b\,\mathrm{s}\right.}{3c^4\pi^2\sqrt{\pi}}$$

$$= -\frac{b}{6c^7\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{5x^3\left(a+b\,\mathrm{si}\right.}{3c^4\pi^2\sqrt{\pi}}$$

Mathematica [A] time = 0.453433, size = 202, normalized size = 1.05

$$\frac{4\sinh^{-1}(cx)\left(bcx\left(3c^4x^4+20c^2x^2+15\right)-15a\left(c^2x^2+1\right)^{3/2}\right)+12ac^5x^5+80ac^3x^3+60acx-6bc^4x^4\sqrt{c^2x^2+1}-9bc^2x^2\sqrt{}}{24\pi^{5/2}c^7\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (60*a*c*x + 80*a*c^3*x^3 + 12*a*c^5*x^5 - 7*b*Sqrt[1 + c^2*x^2] - 9*b*c^2*x
^2*Sqrt[1 + c^2*x^2] - 6*b*c^4*x^4*Sqrt[1 + c^2*x^2] + 4*(-15*a*(1 + c^2*x^
2)^(3/2) + b*c*x*(15 + 20*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] - 30*b*(1 + c^
2*x^2)^(3/2)*ArcSinh[c*x]^2 - 28*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2
4*c^7*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] time = 0.301, size = 970, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] 5/2*a/c^6/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-1/4*b*x^2/c^5/Pi^(5/2)+1/2*b/Pi^(5/2
)/c^6*(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x-49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*
x^2+49)/(c^2*x^2+1)^2*c*x^8-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^
2*x^2+1)^2/c*x^6-49*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^
3*x^4-98/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*x^2-343
/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7*arcsinh(c*x)+14
7*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^7
+49/6*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c*x^6+14*b/Pi^(5/2
)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)/c^3*x^4+6*b/Pi^(5/2)/(63*c^4*x^4+
111*c^2*x^2+49)/(c^2*x^2+1)/c^5*x^2-1/8*b/Pi^(5/2)/c^7-5/2*a/c^6/Pi^2*ln(Pi
*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*a*x^5/Pi/c^
2/(Pi*c^2*x^2+Pi)^(3/2)+5/6*a/c^4*x^3/Pi/(Pi*c^2*x^2+Pi)^(3/2)-49/6*b/Pi^(5
/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^7+14/3*b/c^7/Pi^(5/2)*arcsi
nh(c*x)-5/4*b/c^7/Pi^(5/2)*arcsinh(c*x)^2-7/3*b/c^7/Pi^(5/2)*ln(1+(c*x+(c^2
*x^2+1)^(1/2))^2)+385*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2*x^2+1)^(3
/2)/c^2*arcsinh(c*x)*x^5+1009/3*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49)/(c^2
*x^2+1)^(3/2)/c^4*arcsinh(c*x)*x^3+98*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x^2+49
)/(c^2*x^2+1)^(3/2)/c^6*arcsinh(c*x)*x-147*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*x
^2+49)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^8-553*b/Pi^(5/2)/(63*c^4*x^4+111*c^2*
x^2+49)/(c^2*x^2+1)^2/c*arcsinh(c*x)*x^6-2338/3*b/Pi^(5/2)/(63*c^4*x^4+111*
c^2*x^2+49)/(c^2*x^2+1)^2/c^3*arcsinh(c*x)*x^4-1463/3*b/Pi^(5/2)/(63*c^4*x^
4+111*c^2*x^2+49)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}a\left(\frac{3x^5}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}+\frac{5x\left(\frac{3x^2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}+\frac{2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^4}\right)}{c^2}+\frac{5x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^6}-\frac{15\,\mathrm{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\pi^2\sqrt{\pi c^2}c^6}\right)+b\int\frac{x^6\log\left(cx+\cdots\right.}{\left(\pi+\pi\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(3*x^5/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 5*x*(3*x^2/(pi*(pi + pi*c^2
*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))/c^2 + 5*x/(pi^2*sqrt
```

```
(pi + pi*c^2*x^2)*c^6) - 15*arcsinh(c^2*x/sqrt(c^2))/(pi^2*sqrt(pi*c^2)*c^6
)) + b*integrate(x^6*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^6\operatorname{arsinh}(cx)+ax^6\right)}{\pi^3c^6x^6+3\,\pi^3c^4x^4+3\,\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^6*arcsinh(c*x) + a*x^6)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)x^6}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^6/(pi + pi*c^2*x^2)^(5/2), x)
```

3.101 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{\pi^3c^6}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^6\sqrt{\pi c^2x^2+\pi}}-\frac{a+b\sinh^{-1}(cx)}{3\pi c^6\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bx}{6\pi^{5/2}c^5\left(c^2x^2+1\right)}-\frac{bx}{\pi^{5/2}c^5}-\frac{11b\tan^{-1}(cx)}{6\pi^{5/2}c^6}$$

```
[Out] -((b*x)/(c^5*Pi^(5/2))) + (b*x)/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*c^6*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/(c
^6*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
))/(c^6*Pi^3) - (11*b*ArcTan[c*x])/(6*c^6*Pi^(5/2))
```

Rubi [A] time = 0.181457, antiderivative size = 149, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {266, 43, 5732, 12, 1157, 388, 203}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}c^6}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}c^6\sqrt{c^2x^2+1}}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}c^6\left(c^2x^2+1\right)^{3/2}}+\frac{bx}{6\pi^{5/2}c^5\left(c^2x^2+1\right)}-\frac{bx}{\pi^{5/2}c^5}-\frac{11b\tan^{-1}(cx)}{6\pi^{5/2}c^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -((b*x)/(c^5*Pi^(5/2))) + (b*x)/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*Arc
Sinh[c*x])/(3*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/
(c^6*Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(c^6*Pi^(5/2)) - (11*b*ArcTan[c*x])/(6*c^6*Pi^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}}-\frac{(bc)\int \frac{8+}{3}}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}}-\frac{b\int \frac{8+12c}{(1}}{3c}$$

$$= \frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{c^6\pi^{5/2}}$$

$$= -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}}{}$$

$$= -\frac{bx}{c^5\pi^{5/2}}+\frac{bx}{6c^5\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3c^6\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6\pi^{5/2}\sqrt{1+c^2x^2}}+\frac{\sqrt{1+c^2x^2}}{}$$

Mathematica [A] time = 0.21416, size = 132, normalized size = 0.9

$$\frac{6ac^4x^4+24ac^2x^2+16a-6bc^3x^3\sqrt{c^2x^2+1}-5bcx\sqrt{c^2x^2+1}-11b\left(c^2x^2+1\right)^{3/2}\tan^{-1}(cx)+2b\left(3c^4x^4+12c^2x^2+8\right)\sin}{6\pi^{5/2}c^6\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (16*a + 24*a*c^2*x^2 + 6*a*c^4*x^4 - 5*b*c*x*Sqrt[1 + c^2*x^2] - 6*b*c^3*x^
3*Sqrt[1 + c^2*x^2] + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - 11*b*
(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^6*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [C] time = 0.425, size = 231, normalized size = 1.6

$$\frac{ax^4}{\pi c^2}\left(\pi c^2x^2+\pi\right)^{-\frac{3}{2}}+4\frac{ax^2}{c^4\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{8a}{3c^6\pi}\left(\pi c^2x^2+\pi\right)^{-\frac{3}{2}}+\frac{b\text{Arcsinh}(cx)}{\pi^{\frac{5}{2}}c^6}\sqrt{c^2x^2+1}-\frac{bx}{c^5\pi^{\frac{5}{2}}}+2\frac{b\text{Arcsin}}{\pi^{5/2}\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] a*x^4/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+4*a/c^4*x^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)+8/3
*a/c^6/Pi/(Pi*c^2*x^2+Pi)^(3/2)+b/Pi^(5/2)/c^6*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)-b*x/c^5/Pi^(5/2)+2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)*x^2+1/6
*b*x/c^5/Pi^(5/2)/(c^2*x^2+1)+5/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^6*arcsinh(
c*x)+11/6*I*b/c^6/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/6*I*b/c^6/Pi^(5/2
)*ln(c*x+(c^2*x^2+1)^(1/2)+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,b\left(\frac{\left(3\,\sqrt{\pi}c^4x^4+12\,\sqrt{\pi}c^2x^2+8\,\sqrt{\pi}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi^3c^8x^2+\pi^3c^6\right)\sqrt{c^2x^2+1}}+3\int\frac{3\,\sqrt{\pi}c^4x^4+12\,\sqrt{\pi}c^2x^2+8\,\sqrt{\pi}}{3\left(\pi^3c^{11}x^6+2\,\pi^3c^9x^4+\pi^3c^7x^2+\left(\pi^3c^{10}x^5+2\,\pi^3c^8x\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/3*b*((3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))*log(c*x + sq
rt(c^2*x^2 + 1))/((pi^3*c^8*x^2 + pi^3*c^6)*sqrt(c^2*x^2 + 1)) + 3*integrat
e(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sqrt(pi)*c^2*x^2 + 8*sqrt(pi))/(pi^3*c^11*x^
6 + 2*pi^3*c^9*x^4 + pi^3*c^7*x^2 + (pi^3*c^10*x^5 + 2*pi^3*c^8*x^3 + pi^3*
c^6*x)*sqrt(c^2*x^2 + 1)), x) - 3*integrate(1/3*(3*sqrt(pi)*c^4*x^4 + 12*sq
rt(pi)*c^2*x^2 + 8*sqrt(pi))/((pi^3*c^8*x^3 + pi^3*c^6*x)*sqrt(c^2*x^2 + 1)
), x)) + 1/3*a*(3*x^4/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 12*x^2/(pi*(pi + p
i*c^2*x^2)^(3/2)*c^4) + 8/(pi*(pi + pi*c^2*x^2)^(3/2)*c^6))
```

Fricas [A] time = 3.24203, size = 502, normalized size = 3.44

$$\frac{11\,\sqrt{\pi}\left(bc^4x^4+2\,bc^2x^2+b\right)\arctan\left(-\frac{2\,\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\,\sqrt{\pi+\pi c^2x^2}\left(3\,bc^4x^4+12\,bc^2x^2+8\,b\right)\log\left(cx+\sqrt{c^2x^2+\cdots}\right.}{12\left(\pi^3c^{10}x^4+2\,\pi^3c^8x^2+\pi^3c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/12*(11*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^4*x^4 + 12*b*c^2*x^2 + 8*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*s
qrt(pi + pi*c^2*x^2)*(6*a*c^4*x^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*
sqrt(c^2*x^2 + 1) + 16*a))/(pi^3*c^10*x^4 + 2*pi^3*c^8*x^2 + pi^3*c^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^5}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^5/(pi + pi*c^2*x^2)^(5/2), x)
```

3.102 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{5/2}bc^5}+\frac{b}{6\pi^{5/2}c^5\left(c^2x^2+1\right)}+\frac{2b\log\left(c^2x^2+1\right)}{3\pi^{5/2}c^5}$$

```
[Out] b/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)) - (x^3*(a + b*ArcSinh[c*x]))/(3*c^2*Pi*(Pi
 + c^2*Pi*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*Pi^2*Sqrt[Pi + c^2*Pi
*x^2]) + (a + b*ArcSinh[c*x])^2/(2*b*c^5*Pi^(5/2)) + (2*b*Log[1 + c^2*x^2])
/(3*c^5*Pi^(5/2))
```

Rubi [A] time = 0.282824, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5751, 5675, 260, 266, 43}

$$-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2\pi^{5/2}bc^5}+\frac{b}{6\pi^2c^5\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}+\frac{2b\sqrt{c^2x^2+1}\log\left(\right.}{3\pi^2c^5\sqrt{\pi c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (x^3*(a + b*ArcSin
h[c*x]))/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4
*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (a + b*ArcSinh[c*x])^2/(2*b*c^5*Pi^(5/2)) +
(2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

```
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{c^2\pi} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{\pi+c^2\pi x^2}}\,dx}{c^4\pi^2} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}}{c^3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5\pi^{5/2}} + \frac{b\sqrt{1+c^2x^2}\log\left(1+\right.}{2c^5\pi^2\sqrt{\pi+c^2\pi}}$$

$$= \frac{b}{6c^5\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\left(a+b\sinh\right.}{2bc^5\pi}$$

Mathematica [A] time = 0.350127, size = 166, normalized size = 1.19

$$\frac{2\sinh^{-1}(cx)\left(3a\left(c^2x^2+1\right)^2-bcx\sqrt{c^2x^2+1}\left(4c^2x^2+3\right)\right)-8ac^3x^3\sqrt{c^2x^2+1}-6acx\sqrt{c^2x^2+1}+bc^2x^2+4b\left(c^2x^2+1\right)^2\text{lo}}{6\pi^{5/2}c^5\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b + b*c^2*x^2 - 6*a*c*x*Sqrt[1 + c^2*x^2] - 8*a*c^3*x^3*Sqrt[1 + c^2*x^2]
+ 2*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^2*x^2))*ArcSinh
[c*x] + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]^2 + 4*b*(1 + c^2*x^2)^2*Log[1 + c^
2*x^2])/(6*c^5*Pi^(5/2)*(1 + c^2*x^2)^2)
```

Maple [B] time = 0.203, size = 897, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a*x^3/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-a/Pi^2/c^4*x/(Pi*c^2*x^2+Pi)^(1/2)+
a/Pi^2/c^4*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)
+1/2*b/c^5/Pi^(5/2)*arcsinh(c*x)^2-8/3*b/c^5/Pi^(5/2)*arcsinh(c*x)+32*b/Pi^
(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^8-32*b/Pi
^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^7+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2*c^3*x^8-8/3*b/Pi^(5/2
)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)*c*x^6+116*b/Pi^(5/2)/(24*c^4*x^4+3
9*c^2*x^2+16)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^6-76*b/Pi^(5/2)/(24*c^4*x^4+39
*c^2*x^2+16)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5+32/3*b/Pi^(5/2)/(24*c^4*x^4
+39*c^2*x^2+16)/(c^2*x^2+1)^2*c*x^6-4*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)
/(c^2*x^2+1)/c*x^4+472/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^
2/c*arcsinh(c*x)*x^4-181/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1
)^(3/2)/c^2*arcsinh(c*x)*x^3+16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*
x^2+1)^2/c*x^4-3/2*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)/c^3*x^
2+284/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*arcsinh(c*x
)*x^2-16*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^(3/2)/c^4*arcsin
h(c*x)*x+32/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^3*x^2+6
4/3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5*arcsinh(c*x)+8/
3*b/Pi^(5/2)/(24*c^4*x^4+39*c^2*x^2+16)/(c^2*x^2+1)^2/c^5+4/3*b/c^5/Pi^(5/2
)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\left(x\left(\frac{3\,x^2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}+\frac{2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^4}\right)+\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^4}-\frac{3\,\operatorname{arsinh}\left(\frac{c^2x}{\sqrt{c^2}}\right)}{\pi^2\sqrt{\pi c^2}c^4}\right)a+b\int\frac{x^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*(x*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2)^(
3/2)*c^4)) + x/(pi^2*sqrt(pi + pi*c^2*x^2)*c^4) - 3*arcsinh(c^2*x/sqrt(c^2)
)/(pi^2*sqrt(pi*c^2)*c^4))*a + b*integrate(x^4*log(c*x + sqrt(c^2*x^2 + 1))
/(pi + pi*c^2*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^4\operatorname{arsinh}(cx)+ax^4\right)}{\pi^3c^6x^6+3\,\pi^3c^4x^4+3\,\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^4*arcsinh(c*x) + a*x^4)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{ax^4}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int\frac{bx^4\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**4*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)x^4}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(pi + pi*c^2*x^2)^(5/2), x)
```

3.103 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{a+b\sinh^{-1}(cx)}{\pi^2c^4\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi c^4\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bx}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}+\frac{5b\tan^{-1}(cx)}{6\pi^{5/2}c^4}$$

```
[Out] -(b*x)/(6*c^3*Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*c^4*Pi*(Pi
+ c^2*Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*Pi^2*Sqrt[Pi + c^2*Pi*x^2]
) + (5*b*ArcTan[c*x])/(6*c^4*Pi^(5/2))
```

Rubi [A] time = 0.146583, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 43, 5732, 12, 385, 203}

$$-\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}c^4\sqrt{c^2x^2+1}}+\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}c^4\left(c^2x^2+1\right)^{3/2}}-\frac{bx}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}+\frac{5b\tan^{-1}(cx)}{6\pi^{5/2}c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -(b*x)/(6*c^3*Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*c^4*Pi^(5/2
)*(1 + c^2*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*Pi^(5/2)*Sqrt[1 + c^2*x^
2]) + (5*b*ArcTan[c*x])/(6*c^4*Pi^(5/2))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3 \left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-2-3c^2x^2}{3c^4\left(1+c^2x^2\right)^2}\,dx}{\pi^{5/2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{b\int \frac{-2-3c^2x^2}{\left(1+c^2x^2\right)^2}\,dx}{3c^3\pi^{5/2}}$$

$$= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)} + \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} + \frac{(5b)\int \frac{1}{1+c^2x^2}\,dx}{6c^3\pi^{5/2}}$$

$$= -\frac{bx}{6c^3\pi^{5/2}\left(1+c^2x^2\right)} + \frac{a+b\sinh^{-1}(cx)}{3c^4\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{c^4\pi^{5/2}\sqrt{1+c^2x^2}} + \frac{5b\tan^{-1}(cx)}{6c^4\pi^{5/2}}$$

Mathematica [A] time = 0.184065, size = 93, normalized size = 0.89

$$\frac{-6ac^2x^2-4a-bcx\sqrt{c^2x^2+1}+5b\left(c^2x^2+1\right)^{3/2}\tan^{-1}(cx)-2b\left(3c^2x^2+2\right)\sinh^{-1}(cx)}{6\pi^{5/2}c^4\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2] - 2*b*(2 + 3*c^2*x^2)*ArcSinh
[c*x] + 5*b*(1 + c^2*x^2)^(3/2)*ArcTan[c*x])/(6*c^4*Pi^(5/2)*(1 + c^2*x^2)^
(3/2))
```

Maple [C] time = 0.221, size = 175, normalized size = 1.7

$$-\frac{ax^2}{\pi c^2}\left(\pi c^2x^2+\pi\right)^{-\frac{3}{2}} - \frac{2a}{3\pi c^4}\left(\pi c^2x^2+\pi\right)^{-\frac{3}{2}} - \frac{b\mathrm{Arcsinh}\,(cx)\,x^2}{\pi^{\frac{5}{2}}c^2}\left(c^2x^2+1\right)^{-\frac{3}{2}} - \frac{bx}{6\,c^3\pi^{5/2}\left(c^2x^2+1\right)} - \frac{2\,b\mathrm{Arcsinh}\,(cx)}{3\,\pi^{5/2}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -a*x^2/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)-2/3*a/Pi/c^4/(Pi*c^2*x^2+Pi)^(3/2)-b/Pi
^(5/2)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)*x^2-1/6*b*x/c^3/Pi^(5/2)/(c^2*x^2
+1)-2/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/c^4*arcsinh(c*x)+5/6*I*b/c^4/Pi^(5/2)*
ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b/c^4/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I
)
```

Maxima [A] time = 1.8499, size = 186, normalized size = 1.77

$$-\frac{1}{6}\,bc\left(\frac{x}{\pi^{\frac{5}{2}}c^{6}x^{2}+\pi^{\frac{5}{2}}c^{4}}-\frac{5\,\arctan\left(cx\right)}{\pi^{\frac{5}{2}}c^{5}}\right)-\frac{1}{3}\,b\left(\frac{3\,x^{2}}{\pi\left(\pi+\pi c^{2}x^{2}\right)^{\frac{3}{2}}c^{2}}+\frac{2}{\pi\left(\pi+\pi c^{2}x^{2}\right)^{\frac{3}{2}}c^{4}}\right)\operatorname{arsinh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{3\,x^{2}}{\pi\left(\pi+\pi c^{2}x^{2}\right)^{\frac{3}{2}}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(x/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) - 5*arctan(c*x)/(pi^(5/2)*c^5
)) - 1/3*b*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) + 2/(pi*(pi + pi*c^2*x^2
)^(3/2)*c^4))*arcsinh(c*x) - 1/3*a*(3*x^2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
+ 2/(pi*(pi + pi*c^2*x^2)^(3/2)*c^4))
```

Fricas [B] time = 3.11247, size = 435, normalized size = 4.14

$$-\frac{5\,\sqrt{\pi}\left(bc^{4}x^{4}+2\,bc^{2}x^{2}+b\right)\arctan\left(-\frac{2\,\sqrt{\pi}\sqrt{\pi+\pi c^{2}x^{2}}\sqrt{c^{2}x^{2}+1}cx}{\pi-\pi c^{4}x^{4}}\right)+4\,\sqrt{\pi+\pi c^{2}x^{2}}\left(3\,bc^{2}x^{2}+2\,b\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+2\,\sqrt{\pi\cdots}}{12\left(\pi^{3}c^{8}x^{4}+2\,\pi^{3}c^{6}x^{2}+\pi^{3}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(5*sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi
+ pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2
*x^2)*(3*b*c^2*x^2 + 2*b)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(pi + pi*c^2
*x^2)*(6*a*c^2*x^2 + sqrt(c^2*x^2 + 1)*b*c*x + 4*a))/(pi^3*c^8*x^4 + 2*pi^3
*c^6*x^2 + pi^3*c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^3\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**3*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^3}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(pi + pi*c^2*x^2)^(5/2), x)
```

3.104 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{b}{6\pi^{5/2}c^3\left(c^2x^2+1\right)}-\frac{b\log\left(c^2x^2+1\right)}{6\pi^{5/2}c^3}$$

```
[Out] -b/(6*c^3*Pi^(5/2)*(1 + c^2*x^2)) + (x^3*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi +
c^2*Pi*x^2)^(3/2)) - (b*Log[1 + c^2*x^2])/(6*c^3*Pi^(5/2))
```

Rubi [A] time = 0.128864, antiderivative size = 119, normalized size of antiderivative = 1.49, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5723, 266, 43}

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{b}{6\pi^2c^3\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{6\pi^2c^3\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^3*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (x^3*(a + b*ArcSi
nh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2
*x^2])/(6*c^3*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx &= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{x}{\left(1+c^2x\right)^2}\,dx,x,x^2\right)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{c^2\left(1+c^2x\right)^2}+\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^2\right)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}\\
&= -\frac{b}{6c^3\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{6c^3\pi^2\sqrt{\pi+c^2\pi x^2}}
\end{aligned}$$

Mathematica [A] time = 0.17163, size = 88, normalized size = 1.1

$$-\frac{-2ac^3x^3+b\sqrt{c^2x^2+1}+b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)-2bc^3x^3\sinh^{-1}(cx)}{6\pi^{5/2}c^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] - 2*b*c^3*x^3*ArcSinh[c*x] + b*(1 + c^
2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c^3*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] time = 0.158, size = 707, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a/c^2/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+1/3*a/c^2/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2
)+2/3*b/c^3/Pi^(5/2)*arcsinh(c*x)-b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x
^2+1)^2*c^5*arcsinh(c*x)*x^8+b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)
^(3/2)*c^4*arcsinh(c*x)*x^7-1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2
+1)^2*c^5*x^8+1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)*c^3*x^6-3*
b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^3*arcsinh(c*x)*x^6+b/Pi^
(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*c^2*arcsinh(c*x)*x^5-2/3*b/
Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c^3*x^6-10/3*b/Pi^(5/2)/(3*c
^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2*c*arcsinh(c*x)*x^4+1/3*b/Pi^(5/2)/(3*c^4*
x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3-b/Pi^(5/2)/(3*c^4*x^4+3
*c^2*x^2+1)/(c^2*x^2+1)^2*c*x^4-5/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2
*x^2+1)^2/c*arcsinh(c*x)*x^2-2/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^
2+1)^2/c*x^2-1/3*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3*arcsi
nh(c*x)-1/6*b/Pi^(5/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^2/c^3-1/3*b/c^3/
Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [B] time = 1.26992, size = 185, normalized size = 2.31

$$-\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^6x^2+\pi^{\frac{5}{2}}c^4}+\frac{\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^4}\right)-\frac{1}{3}\,b\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}-\frac{x}{\pi^2\sqrt{\pi+\pi c^2x^2}c^2}\right)\operatorname{arsinh}(cx)-\frac{1}{3}\,a\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(1/(pi^(5/2)*c^6*x^2 + pi^(5/2)*c^4) + log(c^2*x^2 + 1)/(pi^(5/2)*
c^4)) - 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(pi^2*sqrt(pi + pi*c^
2*x^2)*c^2))*arcsinh(c*x) - 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2) - x/(
pi^2*sqrt(pi + pi*c^2*x^2)*c^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right)}{\pi^3c^6x^6+3\,\pi^3c^4x^4+3\,\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*x^2*arcsinh(c*x) + a*x^2)/(pi^3*c^6*x^6 +
 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{bx^2\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**
2 + 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x**2*asinh(c*x)/(c**4*x**4*s
qrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1))
, x))/pi**(5/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(pi + pi*c^2*x^2)^(5/2), x)
```

3.105 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a+b\sinh^{-1}(cx)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bx}{6\pi^{5/2}c\left(c^2x^2+1\right)}+\frac{b\tan^{-1}(cx)}{6\pi^{5/2}c^2}$$

```
[Out] (b*x)/(6*c*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(3*c^2*Pi*(Pi + c
^2*Pi*x^2)^(3/2)) + (b*ArcTan[c*x])/(6*c^2*Pi^(5/2))
```

Rubi [A] time = 0.0802926, antiderivative size = 114, normalized size of antiderivative = 1.52, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5717, 199, 203}

$$-\frac{a+b\sinh^{-1}(cx)}{3\pi c^2\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bx}{6\pi^2c\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}+\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{6\pi^2c^2\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (a + b*ArcSinh[c
*x])/(3*c^2*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])
/(6*c^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{bx}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6c\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$=\frac{bx}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2\pi\left(\pi+c^2\pi x^2\right)^{3/2}}+\frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6c^2\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.135417, size = 72, normalized size = 0.96

$$\frac{-2a+bcx\sqrt{c^2x^2+1}+b\left(c^2x^2+1\right)^{3/2}\tan^{-1}(cx)-2b\sinh^{-1}(cx)}{6\pi^{5/2}c^2\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (-2*a + b*c*x*Sqrt[1 + c^2*x^2] - 2*b*ArcSinh[c*x] + b*(1 + c^2*x^2)^(3/2)*
ArcTan[c*x])/(6*c^2*Pi^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [C] time = 0.088, size = 124, normalized size = 1.7

$$-\frac{a}{3\pi c^2}\left(\pi c^2x^2+\pi\right)^{-\frac{3}{2}}+\frac{bx}{6c\pi^{5/2}\left(c^2x^2+1\right)}-\frac{b\mathrm{Arcsinh}\,(cx)}{3\pi^{5/2}c^2}\left(c^2x^2+1\right)^{-\frac{3}{2}}+\frac{\frac{i}{6}b}{\pi^{\frac{5}{2}}c^2}\ln\left(cx+\sqrt{c^2x^2+1}+i\right)-\frac{\frac{i}{6}b}{\pi^{\frac{5}{2}}c^2}\ln\left(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/3*a/Pi/c^2/(Pi*c^2*x^2+Pi)^(3/2)+1/6*b*x/c/Pi^(5/2)/(c^2*x^2+1)-1/3*b/Pi
^(5/2)/(c^2*x^2+1)^(3/2)/c^2*arcsinh(c*x)+1/6*I*b/c^2/Pi^(5/2)*ln(c*x+(c^2*
x^2+1)^(1/2)+I)-1/6*I*b/c^2/Pi^(5/2)*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx-\frac{a}{3\,\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(pi + pi*c^2*x^2)^(5/2), x) - 1/
3*a/(pi*(pi + pi*c^2*x^2)^(3/2)*c^2)
```

Fricas [B] time = 3.11376, size = 389, normalized size = 5.19

$$-\frac{\sqrt{\pi}\left(bc^4x^4+2\,bc^2x^2+b\right)\arctan\left(-\frac{2\,\sqrt{\pi}\sqrt{\pi+\pi c^2x^2}\sqrt{c^2x^2+1}cx}{\pi-\pi c^4x^4}\right)+4\,\sqrt{\pi+\pi c^2x^2}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\,\sqrt{\pi+\pi c^2x^2}\left(\sqrt{c^2x\cdots}\right.}{12\left(\pi^3c^6x^4+2\,\pi^3c^4x^2+\pi^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(pi)*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*arctan(-2*sqrt(pi)*sqrt(pi +
pi*c^2*x^2)*sqrt(c^2*x^2 + 1)*c*x/(pi - pi*c^4*x^4)) + 4*sqrt(pi + pi*c^2*x
^2)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(pi + pi*c^2*x^2)*(sqrt(c^2*x^2
+ 1)*b*c*x - 2*a))/(pi^3*c^6*x^4 + 2*pi^3*c^4*x^2 + pi^3*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{bx\operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a*x/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 +
 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b*x*asinh(c*x)/(c**4*x**4*sqrt(c*
*2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/
pi**(5/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)x}{\left(\pi + \pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(pi + pi*c^2*x^2)^(5/2), x)
```

3.106 $\int \frac{a+b\sinh^{-1}(cx)}{(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b}{6\pi^{5/2}c\left(c^2x^2+1\right)}-\frac{b\log\left(c^2x^2+1\right)}{3\pi^{5/2}c}$$

```
[Out] b/(6*c*Pi^(5/2)*(1 + c^2*x^2)) + (x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*P
i*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) -
 (b*Log[1 + c^2*x^2])/(3*c*Pi^(5/2))
```

Rubi [A] time = 0.0873414, antiderivative size = 147, normalized size of antiderivative = 1.36, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5690, 5687, 260, 261}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{b}{6\pi^2c\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3\pi^2c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] b/(6*c*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (x*(a + b*ArcSinh[c*
x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sq
rt[Pi + c^2*Pi*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*Pi^2*Sqr
t[Pi + c^2*Pi*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2\int \frac{a+b\sinh^{-1}(cx)}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{b}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)}{3\pi^2\sqrt{\pi+}}$$

$$= \frac{b}{6c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{b\sqrt{1+c^2x^2}\log}{3c\pi^2\sqrt{\pi+}}$$

Mathematica [A] time = 0.146641, size = 100, normalized size = 0.93

$$\frac{4ac^3x^3+6acx+b\sqrt{c^2x^2+1}-2b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)+2bcx\left(2c^2x^2+3\right)\sinh^{-1}(cx)}{6\pi^{5/2}c\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*c*x*(3 + 2*c^2*x^2)*ArcS
inh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(6*c*Pi^(5/2)*(1 + c^2
```

```
*x^2)^(3/2))
```

Maple [B] time = 0.09, size = 618, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] 1/3*a/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+2/3*a/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)+4/3*b/c
/Pi^(5/2)*arcsinh(c*x)+2/3*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8-2
/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-2*b/Pi^(5/2)*c^5/(3*c^2*x^2
+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6+2*b/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+
1)^(3/2)*arcsinh(c*x)*x^5+8/3*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^
6-2*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4-20/3*b/Pi^(5/2)*c^3/(3*c^2
*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+17/3*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c
^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3+4*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)
^2*x^4-3/2*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x^2-22/3*b/Pi^(5/2)*c/(3*
c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4*b/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x
^2+1)^(3/2)*arcsinh(c*x)*x+8/3*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2
-8/3*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+2/3*b/Pi^(5/2)/c
/(3*c^2*x^2+4)/(c^2*x^2+1)^2-2/3*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)
```

Maxima [A] time = 1.12087, size = 170, normalized size = 1.57

$$\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2}-\frac{2\,\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^2}\right)+\frac{1}{3}\,b\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{2\,x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/2)
*c^2)) + 1/3*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi*c^2
*x^2)))*arcsinh(c*x) + 1/3*a*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sq
rt(pi + pi*c^2*x^2)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\,\text{arsinh}\,(cx)+a)}{\pi^3c^6x^6+3\,\pi^3c^4x^4+3\,\pi^3c^2x^2+\pi^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^6 + 3*pi^3*
c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int\frac{b\,\text{asinh}\,(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1
) + sqrt(c**2*x**2 + 1)), x) + Integral(b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x
**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**
(5/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(pi + pi*c^2*x^2)^(5/2), x)
```

3.107 $\int \frac{a+b\sinh^{-1}(cx)}{x(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}$$

```
[Out] -(b*c*x)/(6*Pi^(5/2)*(1 + c^2*x^2)) + (a + b*ArcSinh[c*x])/(3*Pi*(Pi + c^2*
Pi*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (7*b*A
rcTan[c*x])/(6*Pi^(5/2)) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])
/Pi^(5/2) - (b*PolyLog[2, -E^ArcSinh[c*x]])/Pi^(5/2) + (b*PolyLog[2, E^ArcS
inh[c*x]])/Pi^(5/2)
```

Rubi [A] time = 0.339061, antiderivative size = 187, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5755, 5760, 4182, 2279, 2391, 203, 199}

$$-\frac{b\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{b\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}+\frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -(b*c*x)/(6*Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (a + b*ArcSinh[
c*x])/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(Pi^2*Sqrt[Pi +
 c^2*Pi*x^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*Pi^2*Sqrt[Pi + c^2*P
i*x^2]) - (2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(5/2) - (b*Po
lyLog[2, -E^ArcSinh[c*x]])/Pi^(5/2) + (b*PolyLog[2, E^ArcSinh[c*x]])/Pi^(5/
2)
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
```

```
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{\pi} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{\pi+c^2\pi x^2}}\,dx}{\pi^2} - \frac{(bc}{}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= -\frac{bcx}{6\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{a+b\sinh^{-1}(cx)}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6\pi^2\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.822381, size = 209, normalized size = 1.41

$$6b\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 6b\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + \frac{6a}{\sqrt{c^2x^2+1}} + \frac{2a}{\left(c^2x^2+1\right)^{3/2}} - 6a\log\left(\pi\left(\sqrt{c^2x^2+1}+1\right)\right) + 6a\log(x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((2*a)/(1 + c^2*x^2)^(3/2) - (b*c*x)/(1 + c^2*x^2) + (6*a)/Sqrt[1 + c^2*x^2
] + (8*b*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (6*b*c^2*x^2*ArcSinh[c*x])/(1
+ c^2*x^2)^(3/2) - 14*b*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*b*ArcSinh[c*x]*Log
[1 - E^(-ArcSinh[c*x])] - 6*b*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 6*a
*Log[x] - 6*a*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] + 6*b*PolyLog[2, -E^(-ArcSinh
[c*x])] - 6*b*PolyLog[2, E^(-ArcSinh[c*x])])/(6*Pi^(5/2))
```

Maple [A] time = 0.171, size = 220, normalized size = 1.5

$$\frac{a}{3\pi}\left(\pi c^2x^2+\pi\right)^{-\frac{3}{2}}+\frac{a}{\pi^2}\frac{1}{\sqrt{\pi c^2x^2+\pi}}-\frac{a}{\pi^{\frac{5}{2}}}\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}}\right)+\frac{b\mathrm{Arcsinh}\left(cx\right)x^2c^2}{\pi^{\frac{5}{2}}}\left(c^2x^2+1\right)^{-\frac{3}{2}}-\frac{bc}{6\,\pi^{5/2}\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] 1/3*a/Pi/(Pi*c^2*x^2+Pi)^(3/2)+a/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)-a/Pi^(5/2)*arct
anh(Pi^(1/2)/(Pi*c^2*x^2+Pi)^(1/2))+b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*
x)*x^2*c^2-1/6*b*c*x/Pi^(5/2)/(c^2*x^2+1)+4/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*
arcsinh(c*x)-7/3*b/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)*dilog(
c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b/Pi^(5/2)
*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}a\left(\frac{3\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\pi^{\frac{5}{2}}}-\frac{1}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}-\frac{3}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(1/(sqrt(c^2)*abs(x)))/pi^(5/2) - 1/(pi*(pi + pi*c^2*x^2)^
(3/2)) - 3/(pi^2*sqrt(pi + pi*c^2*x^2))) + b*integrate(log(c*x + sqrt(c^2*x
^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}(b\,\mathrm{arsinh}\left(cx\right)+a)}{\pi^3c^6x^7+3\,\pi^3c^4x^5+3\,\pi^3c^2x^3+\pi^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^7 + 3*pi^3*
c^4*x^5 + 3*pi^3*c^2*x^3 + pi^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{(\pi + \pi c^2 x^2)^{\frac{5}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x), x)
```

3.108 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{bc}{6\pi^{5/2}\left(c^2x^2+1\right)}+\frac{5bc\log\left(c^2x^2+1\right)}{6\pi^{5/2}}+\frac{bc\,l}{\pi}$$

```
[Out] -(b*c)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(Pi*x*(Pi + c^2*Pi
*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)
) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*c*Lo
g[x])/Pi^(5/2) + (5*b*c*Log[1 + c^2*x^2])/(6*Pi^(5/2))
```

Rubi [A] time = 0.176281, antiderivative size = 153, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {271, 192, 191, 5732, 12, 1251, 893}

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{c^2x^2+1}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(c^2x^2+1\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(c^2x^2+1\right)^{3/2}}-\frac{bc}{6\pi^{5/2}\left(c^2x^2+1\right)}+\frac{5bc\log\left(c^2x^2+1\right)}{6\pi^{5/2}}+\frac{bc\,l}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -(b*c)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])/(Pi^(5/2)*x*(1 + c
^2*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^(5/2)*(1 + c^2*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (b*
c*Log[x])/Pi^(5/2) + (5*b*c*Log[1 + c^2*x^2])/(6*Pi^(5/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-3-12c^2x^2-8}{3x\left(1+c^2x^2\right.}}{\pi^{5/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\int \frac{-3-12c^2x^2-8}{x\left(1+c^2x^2\right)}}{3\pi^{5/2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\mathrm{Subst}\left(\int \frac{-3}{}\right.}{6}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}} - \frac{(bc)\,\mathrm{Subst}\left(\int\left(-\right.\right.}{}$$

$$= -\frac{bc}{6\pi^{5/2}\left(1+c^2x^2\right)} - \frac{a+b\sinh^{-1}(cx)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.207856, size = 123, normalized size = 0.82

$$\frac{-2a\left(8c^4x^4+12c^2x^2+3\right)-bcx\sqrt{c^2x^2+1}-2b\left(8c^4x^4+12c^2x^2+3\right)\sinh^{-1}(cx)}{6\pi^{5/2}x\left(c^2x^2+1\right)^{3/2}}+\frac{\frac{5}{2}bc\log\left(c^2x^2+1\right)+3bc\log(x)}{3\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) - 2*a*(3 + 12*c^2*x^2 + 8*c^4*x^4) - 2*b*(3 + 1
2*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x])/(6*Pi^(5/2)*x*(1 + c^2*x^2)^(3/2)) + (
3*b*c*Log[x] + (5*b*c*Log[1 + c^2*x^2])/2)/(3*Pi^(5/2))
```

Maple [B] time = 0.179, size = 778, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -a/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)-4/3*a*c^2/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)-8/3*a*c^
2/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-16/3*b*c/Pi^(5/2)*arcsinh(c*x)+32/3*b/Pi^(5/
2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^10*c^11-32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2
*x^2+1)*x^8*c^9+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^8*c^9-32*b/P
i^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^6*c^7+64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^
2*x^2+1)^2*x^6*arcsinh(c*x)*c^7-64/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^(
3/2)*x^5*arcsinh(c*x)*c^6+64*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^6*c^7
-32*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^4*c^5+200/3*b/Pi^(5/2)/(8*c^2*x^
2+9)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^5-56*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)*x^3*arcsinh(c*x)*c^4+128/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)^
2*x^4*c^5-12*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*x^2*c^3+208/3*b/Pi^(5/2)/
(8*c^2*x^2+9)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^3-44*b/Pi^(5/2)/(8*c^2*x^2+9
)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^2+32/3*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x
^2+1)^2*x^2*c^3-3/2*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^2+1)*c+24*b/Pi^(5/2)/(8
*c^2*x^2+9)/(c^2*x^2+1)^2*arcsinh(c*x)*c-9*b/Pi^(5/2)/(8*c^2*x^2+9)/(c^2*x^
2+1)^(3/2)/x*arcsinh(c*x)+b*c/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)+5/3*
b*c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\,a\left(\frac{4\,c^2x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{8\,c^2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{3}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*(4*c^2*x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 8*c^2*x/(pi^2*sqrt(pi + pi*c
^2*x^2)) + 3/(pi*(pi + pi*c^2*x^2)^(3/2)*x)) + b*integrate(log(c*x + sqrt(c
^2*x^2 + 1))/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\,(b\,\mathrm{arsinh}\,(cx)+a)}{\pi^3c^6x^8+3\,\pi^3c^4x^6+3\,\pi^3c^2x^4+\pi^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^8 + 3*pi^3*
c^4*x^6 + 3*pi^3*c^2*x^4 + pi^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^2), x)
```

3.109 $\int \frac{a+b\sinh^{-1}(cx)}{x^3(\pi+c^2\pi x^2)^{5/2}}\,dx$

Optimal. Leaf size=247

$$\frac{5bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b}{2\pi x^2(}$$

```
[Out] (-3*b*c)/(4*Pi^(5/2)*x) + (b*c)/(4*Pi^(5/2)*x*(1 + c^2*x^2)) + (5*b*c^3*x)/
(12*Pi^(5/2)*(1 + c^2*x^2)) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*Pi*(Pi + c^2*
Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*(Pi + c^2*Pi*x^2)^(3/2)) -
(5*c^2*(a + b*ArcSinh[c*x]))/(2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (13*b*c^2*Arc
Tan[c*x])/(6*Pi^(5/2)) + (5*c^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/Pi^(5/2) + (5*b*c^2*PolyLog[2, -E^ArcSinh[c*x]])/(2*Pi^(5/2)) - (5*b*c^2
*PolyLog[2, E^ArcSinh[c*x]])/(2*Pi^(5/2))
```

Rubi [A] time = 0.47513, antiderivative size = 325, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5747, 5755, 5760, 4182, 2279, 2391, 203, 199, 290, 325}

$$\frac{5bc^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5bc^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\pi^{5/2}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2\pi^2\sqrt{\pi c^2x^2+\pi}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b}{2\pi x^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (b*c)/(4*Pi^2*x*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (5*b*c^3*x)/(12*
Pi^2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(
4*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*Pi*(Pi +
c^2*Pi*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(2*Pi*x^2*(Pi + c^2*Pi*x^2)^(3/2)
) - (5*c^2*(a + b*ArcSinh[c*x]))/(2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (13*b*c^2
*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (5*c^2*(a
+ b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Pi^(5/2) + (5*b*c^2*PolyLog[2, -
E^ArcSinh[c*x]])/(2*Pi^(5/2)) - (5*b*c^2*PolyLog[2, E^ArcSinh[c*x]])/(2*Pi^
(5/2))
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)^2}\,dx}{2\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6\pi\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{2\pi x^2\left(\pi+c^2\pi x^2\right)^{3/2}} - \frac{\left(5c^2\right)\int \frac{a+b\sin}{x(\pi+c}}{2\pi}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\right.}{6\pi\left(\pi+\right.}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\right.}{6\pi\left(\pi+\right.}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\right.}{6\pi\left(\pi+\right.}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\right.}{6\pi\left(\pi+\right.}$$

$$= \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}} - \frac{5c^2\left(a+b\right.}{6\pi\left(\pi+\right.}$$

Mathematica [A] time = 6.15641, size = 331, normalized size = 1.34

$$-60bc^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+60bc^2\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\frac{48ac^2}{\sqrt{c^2x^2+1}}-\frac{8ac^2}{\left(c^2x^2+1\right)^{3/2}}-\frac{12a\sqrt{c^2x^2+1}}{x^2}+60ac^2\log\left(\pi\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] ((-8*a*c^2)/(1 + c^2*x^2)^(3/2) + (4*b*c^3*x)/(1 + c^2*x^2) - (48*a*c^2)/Sq
rt[1 + c^2*x^2] - (12*a*Sqrt[1 + c^2*x^2])/x^2 - (56*b*c^2*ArcSinh[c*x])/(1
 + c^2*x^2)^(3/2) - (48*b*c^4*x^2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + 104*b
*c^2*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*b*c^2*Coth[ArcSinh[c*x]/2] - 3*b*c^2*
ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*b*c^2*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] + 60*b*c^2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*a*c^2*L
```

```
og[x] + 60*a*c^2*Log[Pi*(1 + Sqrt[1 + c^2*x^2])] - 60*b*c^2*PolyLog[2, -E^(
-ArcSinh[c*x])] + 60*b*c^2*PolyLog[2, E^(-ArcSinh[c*x])] + 6*b*c^2*Tanh[Arc
Sinh[c*x]/2] - (6*b*c*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2])/x)/(24*Pi^(5/2))
```

Maple [A] time = 0.239, size = 314, normalized size = 1.3

$$-\frac{a}{2\,\pi\,x^2}\left(\pi\,c^2x^2+\pi\right)^{-\frac{3}{2}}-\frac{5\,ac^2}{6\,\pi}\left(\pi\,c^2x^2+\pi\right)^{-\frac{3}{2}}-\frac{5\,ac^2}{2\,\pi^2}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}+\frac{5\,ac^2}{2\,\pi^{5/2}}\mathrm{Artanh}\left(\sqrt{\pi}\frac{1}{\sqrt{\pi\,c^2x^2+\pi}}\right)-\frac{5\,bx^2\mathrm{Arcsinh}}{2\,\pi^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -1/2*a/Pi/x^2/(Pi*c^2*x^2+Pi)^(3/2)-5/6*a*c^2/Pi/(Pi*c^2*x^2+Pi)^(3/2)-5/2*
a*c^2/Pi^2/(Pi*c^2*x^2+Pi)^(1/2)+5/2*a*c^2/Pi^(5/2)*arctanh(Pi^(1/2)/(Pi*c^
2*x^2+Pi)^(1/2))-5/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*x^2*arcsinh(c*x)*c^4-1/3*
b*c^3*x/Pi^(5/2)/(c^2*x^2+1)-10/3*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)
*c^2-1/2*b*c/Pi^(5/2)/x/(c^2*x^2+1)-1/2*b/Pi^(5/2)/(c^2*x^2+1)^(3/2)/x^2*ar
csinh(c*x)+13/3*b*c^2/Pi^(5/2)*arctan(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(
5/2)*dilog(c*x+(c^2*x^2+1)^(1/2))+5/2*b*c^2/Pi^(5/2)*dilog(1+c*x+(c^2*x^2+1
)^(1/2))+5/2*b*c^2/Pi^(5/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\,a\left(\frac{15\,c^2\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{\pi^{\frac{5}{2}}}-\frac{5\,c^2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}-\frac{15\,c^2}{\pi^2\sqrt{\pi+\pi c^2x^2}}-\frac{3}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x^2}\right)+b\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(\pi+\pi c^2x^2\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/6*a*(15*c^2*arcsinh(1/(sqrt(c^2)*abs(x)))/pi^(5/2) - 5*c^2/(pi*(pi + pi*c
^2*x^2)^(3/2)) - 15*c^2/(pi^2*sqrt(pi + pi*c^2*x^2)) - 3/(pi*(pi + pi*c^2*x
^2)^(3/2)*x^2)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/((pi + pi*c^2*x^
2)^(5/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{\pi^3 c^6 x^9 + 3\,\pi^3 c^4 x^7 + 3\,\pi^3 c^2 x^5 + \pi^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^9 + 3*pi^3*
c^4*x^7 + 3*pi^3*c^2*x^5 + pi^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^3), x)
```

3.110 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi x\left(\pi c^2x^2+\pi\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3\pi x^3\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{bc^3}{6\pi^{5/2}\left(c^2x^2+1\right)}-$$

```
[Out] -(b*c)/(6*Pi^(5/2)*x^2) + (b*c^3)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSi
nh[c*x])/(3*Pi*x^3*(Pi + c^2*Pi*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x]))/
(Pi*x*(Pi + c^2*Pi*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi*(Pi +
 c^2*Pi*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^2*Sqrt[Pi + c^2
*Pi*x^2]) - (8*b*c^3*Log[x])/(3*Pi^(5/2)) - (4*b*c^3*Log[1 + c^2*x^2])/(3*P
i^(5/2))
```

Rubi [A] time = 0.239877, antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {271, 192, 191, 5732, 12, 1799, 1620}

$$\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\sqrt{c^2x^2+1}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(c^2x^2+1\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(c^2x^2+1\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(c^2x^2+1\right)^{3/2}}+\frac{bc^3}{6\pi^{5/2}\left(c^2x^2+1\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] -(b*c)/(6*Pi^(5/2)*x^2) + (b*c^3)/(6*Pi^(5/2)*(1 + c^2*x^2)) - (a + b*ArcSi
nh[c*x])/(3*Pi^(5/2)*x^3*(1 + c^2*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x])
)/(Pi^(5/2)*x*(1 + c^2*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^(
5/2)*(1 + c^2*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x]))/(3*Pi^(5/2)*Sqr
t[1 + c^2*x^2]) - (8*b*c^3*Log[x])/(3*Pi^(5/2)) - (4*b*c^3*Log[1 + c^2*x^2]
)/(3*Pi^(5/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh\right.}{3\pi^{5/2}\sqrt{1+c}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh\right.}{3\pi^{5/2}\sqrt{1+c}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh\right.}{3\pi^{5/2}\sqrt{1+c}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}\left(1+c^2x^2\right)^{3/2}}+\frac{16c^4x\left(a+b\sinh\right.}{3\pi^{5/2}\sqrt{1+c}}$$

$$= -\frac{bc}{6\pi^{5/2}x^2}+\frac{bc^3}{6\pi^{5/2}\left(1+c^2x^2\right)}-\frac{a+b\sinh^{-1}(cx)}{3\pi^{5/2}x^3\left(1+c^2x^2\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{\pi^{5/2}x\left(1+c^2x^2\right)^{3/2}}+\frac{8c^4x\left(a+b\,\mathrm{si}\right.}{3\pi^{5/2}\left(1+\right.}$$

Mathematica [A] time = 0.237217, size = 142, normalized size = 0.68

$$\frac{2a\left(16c^6x^6+24c^4x^4+6c^2x^2-1\right)-bcx\sqrt{c^2x^2+1}+2b\left(16c^6x^6+24c^4x^4+6c^2x^2-1\right)\sinh^{-1}(cx)}{6\pi^{5/2}x^3\left(c^2x^2+1\right)^{3/2}}-\frac{8\left(\frac{1}{2}bc^3\log\left(c^2x^2+1\right)\right.}{3\pi^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(Pi + c^2*Pi*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*x*Sqrt[1 + c^2*x^2]) + 2*a*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6
) + 2*b*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x])/(6*Pi^(5/2
)*x^3*(1 + c^2*x^2)^(3/2)) - (8*(b*c^3*Log[x] + (b*c^3*Log[1 + c^2*x^2])/2)
)/(3*Pi^(5/2))
```

Maple [B] time = 0.158, size = 1153, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*c^7+1/3*b/Pi^
(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)/x^3*arcsinh(c*x)+128/3*b/
Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^10*c^13+128*b/Pi^(5/2)/(12
*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^8*c^11+128*b/Pi^(5/2)/(12*c^4*x^4+12*c
^2*x^2-1)/(c^2*x^2+1)*x^6*c^9-2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x
^2+1)*x^2*c^5+128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*x^4*c^
7+1/6*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)/x^2*c-1/3*a/Pi/x^3/(
Pi*c^2*x^2+Pi)^(3/2)-8/3*b*c^3/Pi^(5/2)*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)+32/
3*b*c^3/Pi^(5/2)*arcsinh(c*x)+16/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^
2*x^2+1)^2*arcsinh(c*x)*c^3-128/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2
*x^2+1)^2*x^12*c^15-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^
2*x^10*c^13-256*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^8*c^11
-512/3*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^6*c^9-2*b/Pi^(5
/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)*c^3-6*b/Pi^(5/2)/(12*c^4*x^4+12*c
^2*x^2-1)/(c^2*x^2+1)^(3/2)/x*arcsinh(c*x)*c^2+64*b/Pi^(5/2)/(12*c^4*x^4+12
*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^7*arcsinh(c*x)*c^10+160*b/Pi^(5/2)/(12*c^4*
x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^5*arcsinh(c*x)*c^8+344/3*b/Pi^(5/2)/(
12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x^3*arcsinh(c*x)*c^6+12*b/Pi^(5/
2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^(3/2)*x*arcsinh(c*x)*c^4-64*b/Pi^(
5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^8*arcsinh(c*x)*c^11-560/3*b/
Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^4*arcsinh(c*x)*c^7-160/3
*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^2*arcsinh(c*x)*c^5-19
2*b/Pi^(5/2)/(12*c^4*x^4+12*c^2*x^2-1)/(c^2*x^2+1)^2*x^6*arcsinh(c*x)*c^9+8
/3*a*c^4/Pi*x/(Pi*c^2*x^2+Pi)^(3/2)+16/3*a*c^4/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)
+2*a*c^2/Pi/x/(Pi*c^2*x^2+Pi)^(3/2)
```

Maxima [A] time = 1.21905, size = 319, normalized size = 1.53

$$-\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{\pi^{\frac{5}{2}}}+\frac{1}{\pi^{\frac{5}{2}}c^2x^4+\pi^{\frac{5}{2}}x^2}\right)+\frac{1}{3}\left(\frac{8\,c^4x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{16\,c^4x}{\pi^2\sqrt{\pi+\pi c^2x^2}}+\frac{6\,c^2}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/6*b*c*(8*c^2*log(c^2*x^2 + 1)/pi^(5/2) + 16*c^2*log(x)/pi^(5/2) + 1/(pi^
(5/2)*c^2*x^4 + pi^(5/2)*x^2)) + 1/3*(8*c^4*x/(pi*(pi + pi*c^2*x^2)^(3/2))
```

```
+ 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(pi*(pi + pi*c^2*x^2)^(3/2)
*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*b*arcsinh(c*x) + 1/3*(8*c^4*x/(pi
*(pi + pi*c^2*x^2)^(3/2)) + 16*c^4*x/(pi^2*sqrt(pi + pi*c^2*x^2)) + 6*c^2/(
pi*(pi + pi*c^2*x^2)^(3/2)*x) - 1/(pi*(pi + pi*c^2*x^2)^(3/2)*x^3))*a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}(b \operatorname{arsinh}(cx) + a)}{\pi^3 c^6 x^{10} + 3\,\pi^3 c^4 x^8 + 3\,\pi^3 c^2 x^6 + \pi^3 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)/(pi^3*c^6*x^10 + 3*pi^3
*c^4*x^8 + 3*pi^3*c^2*x^6 + pi^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((pi + pi*c^2*x^2)^(5/2)*x^4), x)
```

3.111 $\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\sin}{15c^2\left(a^2c\right.}$$

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.12299, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5690, 5687, 260, 261}

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\sin}{15c^2\left(a^2c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

```
&& GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{15c^2} - \cdots$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}}{15c^2\left(c+a^2c\right.}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}}{15c^2\left(c+a^2c\right.}$$

Mathematica [A] time = 0.172358, size = 121, normalized size = 0.6

$$\frac{\sqrt{a^2cx^2+c}\left(4ax\sqrt{a^2x^2+1}\left(8a^4x^4+20a^2x^2+15\right)\sinh^{-1}(ax)-\left(a^2x^2+1\right)\left(-8a^2x^2+16\left(a^2x^2+1\right)^2\log\left(a^2x^2+1\right)-11\right.\right.}{60ac^4\left(a^2x^2+1\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

Maple [B] time = 0.161, size = 363, normalized size = 1.8

$$\frac{16\,\mathrm{Arcsinh}\,(ax)}{15\,ac^4}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{1}{\left(2400\,a^{10}x^{10}+12900\,x^8a^8+28140\,x^6a^6+31020\,x^4a^4+17220\,a^2x^2+3840\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*x^5*a^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*x^3*a^3-16*a^2*x
^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*x^8*a^8-64*(a^2*x^2+1
)^(1/2)*x^7*a^7-280*x^6*a^6-248*(a^2*x^2+1)^(1/2)*x^5*a^5+160*a^4*x^4*arcsi
nh(a*x)-456*x^4*a^4-340*a^3*x^3*(a^2*x^2+1)^(1/2)+380*a^2*x^2*arcsinh(a*x)-
328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [A] time = 1.14265, size = 201, normalized size = 1.

$$-\frac{1}{60}\,a\left(\frac{16\sqrt{\frac{1}{a^4c}}\log\left(x^2+\frac{1}{a^2}\right)}{c^3}-\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}-\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}c^3}+\frac{4\,x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*a*(16*sqrt(1/(a^4*c))*log(x^2 + 1/a^2)/c^3 - 3/((a^6*c^(5/2)*x^4 + 2*
a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) - 8/((a^4*c^(3/2)*x^2 + a^2*c^(3/2))*c^2)
```

```
) + 1/15*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) +
 3*x/((a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*
a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37617, size = 167, normalized size = 0.84

$$-\frac{1}{60}\sqrt{c}\left(\frac{16\,\log\left(a^2x^2+1\right)}{ac^4}-\frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right)+\frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

3.112 $\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=86

$$\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a^4}+\frac{3\sinh^{-1}(ax)^2}{16a^5}-\frac{x^4}{16a}$$

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rubi [A] time = 0.154876, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a^4}+\frac{3\sinh^{-1}(ax)^2}{16a^5}-\frac{x^4}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\,dx}{4a}$$
$$= -\frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2} + \frac{3\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4} + \frac{3\int x\,dx}{8a^3}$$
$$= \frac{3x^2}{16a^3} - \frac{x^4}{16a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2} + \frac{3\sinh^{-1}(ax)^2}{16a^5}$$

Mathematica [A] time = 0.046194, size = 63, normalized size = 0.73

$$\frac{-a^4x^4+3a^2x^2+2ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)+3\sinh^{-1}(ax)^2}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

Maple [A] time = 0.038, size = 74, normalized size = 0.9

$$\frac{1}{16\,a^5}\left(4\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}a^3x^3-x^4a^4-6\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}ax+3\,a^2x^2+3\,(\mathrm{Arcsinh}\,(ax))^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/16*(4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-x^4*a^4-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+4)/a^5
```

Maxima [A] time = 1.12058, size = 138, normalized size = 1.6

$$-\frac{1}{16}\left(\frac{x^4}{a^2}-\frac{3\,x^2}{a^4}+\frac{3\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)^2}{a^6}\right)a+\frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2}-\frac{3\,\sqrt{a^2x^2+1}x}{a^4}+\frac{3\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^4}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a^2*x/sqrt(a^2))^2/a^6)*a + 1/8*(2*s
qrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a^2*x/sqrt
(a^2))/(sqrt(a^2)*a^4))*arcsinh(a*x)
```

Fricas [A] time = 2.5808, size = 188, normalized size = 2.19

$$-\frac{a^4x^4-3\,a^2x^2-2\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*
x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5
```

Sympy [A] time = 2.97125, size = 82, normalized size = 0.95

$$\begin{cases} -\frac{x^4}{16a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2}+\frac{3x^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a^4}+\frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

3.113 $\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^4}+\frac{2x}{3a^3}-\frac{x^3}{9a}$$

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rubi [A] time = 0.111001, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^2}-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^4}+\frac{2x}{3a^3}-\frac{x^3}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a} \\
&= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3} \\
&= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^2}
\end{aligned}
$$

Mathematica [A] time = 0.0395997, size = 48, normalized size = 0.69

$$
\frac{-a^3x^3+3\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)+6ax}{9a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

Maple [A] time = 0.031, size = 82, normalized size = 1.2

$$
\frac{1}{9\,a^4}\left(3\,a^4x^4\mathrm{Arcsinh}\,(ax)-3\,a^2x^2\mathrm{Arcsinh}\,(ax)-a^3x^3\sqrt{a^2x^2+1}-6\,\mathrm{Arcsinh}\,(ax)+6\,ax\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/9/a^4/(a^2*x^2+1)^(1/2)*(3*a^4*x^4*arcsinh(a*x)-3*a^2*x^2*arcsinh(a*x)-a^
3*x^3*(a^2*x^2+1)^(1/2)-6*arcsinh(a*x)+6*a*x*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.1451, size = 80, normalized size = 1.14

$$-\frac{1}{9}a\left(\frac{x^3}{a^2}-\frac{6x}{a^4}\right)+\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

Fricas [A] time = 2.53258, size = 126, normalized size = 1.8

$$-\frac{a^3x^3-3\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-6\,ax}{9\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

Sympy [A] time = 1.52718, size = 65, normalized size = 0.93

$$\begin{cases} -\frac{x^3}{9a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^2}+\frac{2x}{3a^3}-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{3a^4} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

Giac [A] time = 1.33975, size = 85, normalized size = 1.21

$$-\frac{a^2x^3 - 6x}{9a^3} + \frac{\left(\left(a^2x^2 + 1\right)^{\frac{3}{2}} - 3\sqrt{a^2x^2 + 1}\right)\log\left(ax + \sqrt{a^2x^2 + 1}\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/9*(a^2*x^3 - 6*x)/a^3 + 1/3*((a^2*x^2 + 1)^(3/2) - 3*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))/a^4
```

3.114 $\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3} - \frac{x^2}{4a}$$

```
[Out] -x^2/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rubi [A] time = 0.103116, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3} - \frac{x^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -x^2/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\,dx}{2a}$$
$$= -\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3}$$

Mathematica [A] time = 0.038447, size = 42, normalized size = 0.86

$$-\frac{a^2x^2 - 2ax\sqrt{a^2x^2+1}\sinh^{-1}(ax) + \sinh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/(4*a^3)
```

Maple [A] time = 0.03, size = 40, normalized size = 0.8

$$-\frac{1}{4a^3}\left(-2\,\mathrm{Arcsinh}\,(ax)\sqrt{a^2x^2+1}ax + a^2x^2 + (\mathrm{Arcsinh}\,(ax))^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

Maxima [A] time = 1.15993, size = 100, normalized size = 2.04

$$-\frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\mathrm{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\mathrm{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(x^2/a^2 - arcsinh(a^2*x/sqrt(a^2))^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*
x/a^2 - arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))*arcsinh(a*x)
```

Fricas [A] time = 2.54241, size = 146, normalized size = 2.98

$$-\frac{a^2x^2 - 2\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A] time = 0.934297, size = 42, normalized size = 0.86

$$\begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a^2} - \frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

3.115 $\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rubi [A] time = 0.0459697, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5717, 8}

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2} - \frac{\int 1\,dx}{a}$$
$$= -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2}$$

Mathematica [A] time = 0.0271409, size = 28, normalized size = 1.

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2} - \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Maple [A] time = 0.026, size = 47, normalized size = 1.7

$$\frac{1}{a^2}\left(a^2x^2\mathrm{Arcsinh}\,(ax) + \mathrm{Arcsinh}\,(ax) - ax\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+arcsinh(a*x)-a*x*(a^2*x^2+1)^
(1/2))
```

Maxima [A] time = 1.24676, size = 35, normalized size = 1.25

$$-\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

Fricas [A] time = 2.50549, size = 82, normalized size = 2.93

$$-\frac{ax-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] time = 0.560833, size = 24, normalized size = 0.86

$$\begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a^2} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

Giac [A] time = 1.34296, size = 51, normalized size = 1.82

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

3.116 $\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rubi [A] time = 0.021915, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^2}{2a}$$

Mathematica [A] time = 0.0062127, size = 13, normalized size = 1.

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$\frac{(\text{Arcsinh}\,(ax))^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Maxima [A] time = 1.08765, size = 15, normalized size = 1.15

$$\frac{\text{arsinh}\,(ax)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Fricas [B] time = 2.47326, size = 51, normalized size = 3.92

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

Sympy [A] time = 0.409135, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Giac [B] time = 1.39752, size = 31, normalized size = 2.38

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

3.117 $\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=34

$$-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.0916102, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5760, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx &= \text{Subst}\left(\int x\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + \text{Subst}\left(\int \log\left(1+\right.\right.\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right) + \text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - \text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + \text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)\end{aligned}$$

Mathematica [A] time = 0.096044, size = 57, normalized size = 1.68

$$\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)+\sinh^{-1}(ax)\left(\log\left(1-e^{-\sinh^{-1}(ax)}\right)-\log\left(e^{-\sinh^{-1}(ax)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

Maple [A] time = 0.03, size = 42, normalized size = 1.2

$$2\,\text{dilog}\left(\left(ax+\sqrt{a^2x^2+1}\right)^{-1}\right)-\frac{1}{2}\text{dilog}\left(\left(ax+\sqrt{a^2x^2+1}\right)^{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 2*dilog(1/(a*x+(a^2*x^2+1)^(1/2)))-1/2*dilog(1/(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

3.118 $\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=27

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}$$

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rubi [A] time = 0.0636175, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5723, 29}

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x} + a\int \frac{1}{x}\,dx$$
$$= -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x} + a\log(x)$$

Mathematica [A] time = 0.0364905, size = 29, normalized size = 1.07

$$a\log(ax) - \frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

Maple [B] time = 0.056, size = 56, normalized size = 2.1

$$-2\,a\mathrm{Arcsinh}\,(ax) + \frac{\mathrm{Arcsinh}\,(ax)}{x}\left(ax - \sqrt{a^2x^2+1}\right) + a\ln\left(\left(ax + \sqrt{a^2x^2+1}\right)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

Maxima [A] time = 1.16535, size = 34, normalized size = 1.26

$$a\log(x) - \frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

Fricas [A] time = 2.65206, size = 88, normalized size = 3.26

$$\frac{ax\log(x)-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [B] time = 1.38914, size = 113, normalized size = 4.19

$$-a\left(\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}-\frac{|a|\log(|x|)}{a^2}\right)|a|+\frac{2|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*(log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - abs(a)*log(abs(x))/a^2)*abs
(a) + 2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))
^2 - 1)
```

3.119 $\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -a/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rubi [A] time = 0.148335, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5747, 5760, 4182, 2279, 2391, 30}

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```


```
[Out] -a/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
```

```
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2} - \frac{1}{2}a^2\,\mathrm{Subst}\left(\int x\,\mathrm{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2} + a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int \log\left(1-e^x\right)\,dx, :\right.$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2} + a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x\right.$$

$$= -\frac{a}{2x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2} + a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\mathrm{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) - \frac{1}{2}a^2\mathrm{Li}_2$$

Mathematica [A] time = 0.671114, size = 126, normalized size = 1.58

$$\frac{1}{8}a^2\left(-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-4\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)+4\sinh^{-1}(ax)\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Arc
Sinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])]
- ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A] time = 0.052, size = 150, normalized size = 1.9

$$-\frac{1}{2x^2}\left(a^2x^2\text{Arcsinh}(ax)+ax\sqrt{a^2x^2+1}+\text{Arcsinh}(ax)\right)\frac{1}{\sqrt{a^2x^2+1}}+\frac{a^2\text{Arcsinh}(ax)}{2}\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+\frac{a^2}{2}\text{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+a*x*(a^2*x^2+1)^(1/2)+arcsinh(
a*x))/x^2+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(
2,-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-
1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}\,(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

3.120 $\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=175

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bx\sqrt{c^2dx^2}}{15c^3\sqrt{c^2x^2}}$$

```
[Out] (2*b*x*Sqrt[d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d + c^
2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (b*c*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt
[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*d) + (
(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d^2)
```

Rubi [A] time = 0.160971, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43, 5734, 12}

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bx^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bx\sqrt{c^2dx^2}}{15c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (2*b*x*Sqrt[d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d + c^
2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (b*c*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt
[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^4*d) + (
(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d^2)
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5734

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[a + b*ArcS
inh[c*x], Int[x^m*(d + e*x^2)^p, x], x] - Dist[(b*c*d^(p - 1/2)*Sqrt[d + e*
x^2])/Sqrt[1 + c^2*x^2], Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x],
 x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && (
IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = -\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{-2+c^2x^2+3c^4x^4}{15c^4}\,dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(cx)\right)\int x^3\sqrt{d+c^2dx^2}\,dx$$

$$= -\frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+3c^4x^4\right)dx}{15c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sinh^{-1}(cx)\right)\text{Subst}\left(\int x\right.$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sinh^{-1}(cx)\right)\text{Su}$$

$$= \frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}}-\frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sin\right.}{3c^4d}$$

Mathematica [A] time = 0.117843, size = 120, normalized size = 0.69

$$\frac{\sqrt{c^2dx^2+d}\left(15a\left(3c^2x^2-2\right)\left(c^2x^2+1\right)^2+bcx\left(-9c^4x^4-5c^2x^2+30\right)\sqrt{c^2x^2+1}+15b\left(3c^2x^2-2\right)\left(c^2x^2+1\right)^2\sinh^{-1}(cx)\right)}{225c^4\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) + b*c*x*Sqrt[1
+ c^2*x^2]*(30 - 5*c^2*x^2 - 9*c^4*x^4) + 15*b*(1 + c^2*x^2)^2*(-2 + 3*c^2*
x^2)*ArcSinh[c*x]))/(225*c^4*(1 + c^2*x^2))
```

Maple [B] time = 0.219, size = 578, normalized size = 3.3

$$a\left(\frac{x^2}{5c^2d}\left(c^2dx^2+d\right)^{\frac{3}{2}}-\frac{2}{15dc^4}\left(c^2dx^2+d\right)^{\frac{3}{2}}\right)+b\left(\frac{-1+5\,\mathrm{Arcsinh}\,(cx)}{800\,c^4\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(16\,c^6x^6+16\,c^5x^5\sqrt{c^2x^2+1}+28\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b*(1/8
00*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^
4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*
arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*
x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*
x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/
2)+1)*(-1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2
-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^
2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2
+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)
*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)
^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65268, size = 346, normalized size = 1.98

$$\frac{15\left(3\,bc^6x^6+4\,bc^4x^4-bc^2x^2-2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(45\,ac^6x^6+60\,ac^4x^4-15\,ac^2x^2-\left(9\,bc^5x^5+5\,bc\right.\right.}{225\left(c^6x^2+c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 + 4*b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 + 60*a*c^4*x^4 - 15*a*c^2*x^2
 - (9*b*c^5*x^5 + 5*b*c^3*x^3 - 30*b*c*x)*sqrt(c^2*x^2 + 1) - 30*a)*sqrt(c^
2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.121 $\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=181

$$\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{c^2dx^2+}}{16\sqrt{c^2x^2+}}$$

```
[Out] -(b*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[d + c
^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 - (Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.194137, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5742, 5758, 5675, 30}

$$\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcx^4\sqrt{c^2dx^2+}}{16\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4*Sqrt[d + c
^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 - (Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{4}x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx}{4\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d}\right.}{4}$$

$$= -\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\right.$$

$$= -\frac{bx^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{}$$

Mathematica [A] time = 0.78889, size = 129, normalized size = 0.71

$$-\frac{-16acx\left(2c^2x^2+1\right)\sqrt{c^2dx^2+d}+16a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+\frac{b\sqrt{c^2dx^2+d}\left(8\sinh^{-1}(cx)^2-4\sinh\left(4\sinh^{-1}(cx)\right)\sinh^{-1}(cx)+\cos\right.}{\sqrt{c^2x^2+1}}}{128c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(-16*a*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 16*a*Sqrt[d]*Log[c*d*x +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + C
osh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^
2])/(128*c^3)
```

Maple [B] time = 0.148, size = 320, normalized size = 1.8

$$\frac{ax}{4\,c^2d}\left(c^2dx^2+d\right)^{\frac{3}{2}}-\frac{ax}{8\,c^2}\sqrt{c^2dx^2+d}-\frac{ad}{8\,c^2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}-\frac{b}{128\,c^3}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}-\frac{b(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a/c^2
*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/128*b*(d*(
c^2*x^2+1))^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^3*arcsinh(c*x)^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*
arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*x^4+3/8*b
*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/16*b*(d*(c^2*x^2+1))^
(1/2)/c/(c^2*x^2+1)^(1/2)*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2+1)*a
rcsinh(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^2, x)
```

3.122 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=105

$$\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{bcx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}$$

```
[Out] -(b*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[d + c^2*
d*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])
)/(3*c^2*d)
```

Rubi [A] time = 0.0674952, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5717}

$$\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{bcx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}-\frac{bx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*x^3*Sqrt[d + c^2*
d*x^2])/(9*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])
)/(3*c^2*d)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} - \frac{\left(b\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)dx}{3c\sqrt{1+c^2x^2}}$$
$$= -\frac{bx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}} + \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}$$

Mathematica [A] time = 0.0915695, size = 92, normalized size = 0.88

$$\frac{\sqrt{c^2dx^2+d}\left(3a\left(c^2x^2+1\right)^2 - bcx\left(c^2x^2+3\right)\sqrt{c^2x^2+1} + 3b\left(c^2x^2+1\right)^2\sinh^{-1}(cx)\right)}{9c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(3*a*(1 + c^2*x^2)^2 - b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^
2*x^2) + 3*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(9*c^2*(1 + c^2*x^2))
```

Maple [B] time = 0.122, size = 321, normalized size = 3.1

$$\frac{a}{3c^2d}\left(c^2dx^2+d\right)^{\frac{3}{2}} + b\left(\frac{-1+3\,\mathrm{Arcsinh}\,(cx)}{72\,c^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(4\,c^4x^4+4\,c^3x^3\sqrt{c^2x^2+1}+5\,c^2x^2+3\,cx\sqrt{c^2x^2+1}+1\right)+\frac{-1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a/c^2/d*(c^2*d*x^2+d)^(3/2)+b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsin
h(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^
(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/72*(d*(c^2*x
^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^
2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^2/(c^2*x^2+1))
```

Maxima [A] time = 1.13595, size = 99, normalized size = 0.94

$$\frac{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}b\,\mathrm{arsinh}\left(cx\right)}{3\,c^{2}d}-\frac{\left(c^{2}d^{\frac{3}{2}}x^{3}+3\,d^{\frac{3}{2}}x\right)b}{9\,cd}+\frac{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}a}{3\,c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*d*x^2 + d)^(3/2)*b*arcsinh(c*x)/(c^2*d) - 1/9*(c^2*d^(3/2)*x^3 + 3
*d^(3/2)*x)*b/(c*d) + 1/3*(c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Fricas [A] time = 2.58918, size = 273, normalized size = 2.6

$$\frac{3\left(bc^{4}x^{4}+2\,bc^{2}x^{2}+b\right)\sqrt{c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(3\,ac^{4}x^{4}+6\,ac^{2}x^{2}-\left(bc^{3}x^{3}+3\,bcx\right)\sqrt{c^{2}x^{2}+1}+3\,a\right)\sqrt{c^{2}dx^{2}}}{9\left(c^{4}x^{2}+c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 + 6*a*c^2*x^2 - (b*c^3*x^3 + 3*b*c*x)*sqrt(c^2*x^
2 + 1) + 3*a)*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.123 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=111

$$\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

```
[Out] -(b*c*x^2*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.0611233, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5682, 5675, 30}

$$\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -(b*c*x^2*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(
4*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\int\frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}dx}{2\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d+c^2dx\right.}{2\sqrt{1+c}}$$

$$= -\frac{bcx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{4bc\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.440367, size = 120, normalized size = 1.08

$$\frac{1}{8}\left(4ax\sqrt{c^2dx^2+d}+\frac{4a\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{c}+\frac{b\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)-\right.}{c\sqrt{c^2x^2+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d + c^2*d*x^2] + (4*a*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/c + (b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]
*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2]))/8
```

Maple [B] time = 0.091, size = 222, normalized size = 2.

$$\frac{ax}{2}\sqrt{c^2dx^2+d}+\frac{ad}{2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{4c}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+\frac{bc^2\mathrm{Arcsinh}\left(cx\right)}{2c^2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*x*(c^2*d*x^2+d)^(1/2)+1/2*a*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^
(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsin
```

```
h(c*x)^2+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b
*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)*arcsinh(c*x)*x-1/8*b*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

3.124 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=177

$$-\frac{b\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{c^2}}{}$$

```
[Out] -((b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^Ar
cSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSi
nh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.195302, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{b\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{b\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{2\sqrt{c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```


```
[Out] -((b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^2]) + Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^Ar
cSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSi
nh[c*x]])/Sqrt[1 + c^2*x^2] + (b*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c
*x]])/Sqrt[1 + c^2*x^2]
```


Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
```

& EqQ[e, c^2*d] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{\sqrt{d+c^2dx^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(bc\sqrt{d+c^2dx^2}\right.}{\sqrt{1+c^2x}}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{\sqrt{d+c^2dx^2}\,\mathrm{Subst}\left(\int(a+b\right.}{\sqrt{1+}}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c}}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c}}$$

$$= -\frac{bcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c}}$$

Mathematica [A] time = 0.451983, size = 168, normalized size = 0.95

$$\frac{b\sqrt{c^2dx^2+d}\left(\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\left(1-\right.\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d + c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 + c^2*d*x^2]] + (b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh
[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*
x])]))/Sqrt[1 + c^2*x^2]
```

Maple [A] time = 0.146, size = 331, normalized size = 1.9

$$-\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)\sqrt{d}a+a\sqrt{c^2dx^2+d}+\frac{b\mathrm{Arcsinh}\,(cx)\,x^2c^2}{c^2x^2+1}\sqrt{d\left(c^2x^2+1\right)}-bcx\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] -ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*d^(1/2)*a+a*(c^2*d*x^2+d)^(1/2)+
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*c*x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x
)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+
1)^(1/2))-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2
+1)^(1/2))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-
(c^2*x^2+1)^(1/2))+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\text{asinh}\,(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}(b \operatorname{arsinh}(cx)+a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x, x)
```

3.125 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=105

$$\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Lo
g[x])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.114713, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5737, 29, 5675}

$$\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x) + (c*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(2*b*Sqrt[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Lo
g[x])/Sqrt[1 + c^2*x^2]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{1}{x}\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(c^2\sqrt{d+c^2dx^2}\right)\int \frac{}{}}{\sqrt{1+c}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{1+c^2x^2}} + \frac{bc\sqrt{d+c}}{\sqrt{1}}$$

Mathematica [A] time = 0.320564, size = 129, normalized size = 1.23

$$-\frac{a\sqrt{d\left(c^2x^2+1\right)}}{x} + ac\sqrt{d}\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+cdx\right) + \frac{bc\sqrt{d\left(c^2x^2+1\right)}\left(-\frac{2\sqrt{c^2x^2+1}\sinh^{-1}(cx)}{cx}+2\log(cx)+\sinh^{-1}(cx)\right.}{2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d*(1 + c^2*x^2)])/x) + (b*c*Sqrt[d*(1 + c^2*x^2)]*((-2*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x])/(c*x) + ArcSinh[c*x]^2 + 2*Log[c*x]))/(2*Sqrt[1 + c^2
*x^2]) + a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]]
```

Maple [B] time = 0.128, size = 263, normalized size = 2.5

$$-\frac{a}{dx}\left(c^2dx^2+d\right)^{\frac{3}{2}} + ac^2x\sqrt{c^2dx^2+d} + ac^2d\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}} + \frac{b\left(\mathrm{Arcsinh}\left(cx\right)\right)^2c}{2}\sqrt{d\left(c^2x^2+1\right)}\frac{}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(3/2)+a*c^2*x*(c^2*d*x^2+d)^(1/2)+a*c^2*d*ln(x*c^2*d/(
c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)
```

```
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*c-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*x/(c^2*x^2+1)*c^2-b*
(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\text{asinh}\,(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.126 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=201

$$-\frac{bc^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2}{}$$

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.195166, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5737, 30, 5760, 4182, 2279, 2391}

$$-\frac{bc^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}-\frac{c^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(2*x^2) - (c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]) + (b*c^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2])
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
```

```
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1}{x^2}\,dx}{2\sqrt{1+c^2x^2}} + \frac{\left(c^2\sqrt{d+c^2dx^2}\right)\int\frac{a}{}}{2\sqrt{1+c^2}}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2} + \frac{\left(c^2\sqrt{d+c^2dx^2}\right)\mathrm{Subst}\left(\int(a+\right.}{2\sqrt{1-}}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x}}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x}}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2} - \frac{c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x}}$$

Mathematica [A] time = 2.90418, size = 223, normalized size = 1.11

$$\frac{1}{8}\left(\frac{bc^2\sqrt{c^2dx^2+d}\left(4\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\sinh\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 + 4*a*c^2*Sqrt[d]*Log[x] - 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*Sqrt[d + c^2*d*x^2]*(-2*Coth[
ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[
1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*Poly
Log[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]
*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

Maple [A] time = 0.178, size = 377, normalized size = 1.9

$$-\frac{a}{2\,dx^2}\left(c^2dx^2+d\right)^{\frac{3}{2}} - \frac{ac^2}{2}\sqrt{d}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right) + \frac{ac^2}{2}\sqrt{c^2dx^2+d} - \frac{b\mathrm{Arcsinh}\,(cx)\,c^2}{2\,c^2x^2+2}\sqrt{d\left(c^2x^2+1\right)} - \frac{bc}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(3/2)-1/2*a*d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2
+d)^(1/2))/x)*c^2+1/2*a*(c^2*d*x^2+d)^(1/2)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/x/(c^2*x^2+1)^(1/
2)*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/x^2/(c^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*c^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\text{asinh}\,(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.127 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(3*
Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.0945453, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 14}

$$-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2dx^2+d}}{6x^2\sqrt{c^2x^2+1}}+\frac{bc^3\log(x)\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(3*
Sqrt[1 + c^2*x^2])
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\frac{1+c^2x^2}{x^3}\,dx}{3\sqrt{1+c^2x^2}}$$
$$= -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{\left(bc\sqrt{d+c^2dx^2}\right)\int\left(\frac{1}{x^3}+\frac{c^2}{x}\right)dx}{3\sqrt{1+c^2x^2}}$$
$$= -\frac{bc\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{bc^3\sqrt{d+c^2dx^2}\log(x)}{3\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.186435, size = 131, normalized size = 1.24

$$\frac{bc^3\log(x)\sqrt{d\left(c^2x^2+1\right)}}{3\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(2a\left(c^2x^2+1\right)^2+bcx\left(3c^2x^2+1\right)\sqrt{c^2x^2+1}+2b\left(c^2x^2+1\right)^2\sinh^{-1}(cx)\right)}{6x^3\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(2*a*(1 + c^2*x^2)^2 + b*c*x*Sqrt[1 + c^2*x^2]*(1 + 3
*c^2*x^2) + 2*b*(1 + c^2*x^2)^2*ArcSinh[c*x]))/(6*x^3*(1 + c^2*x^2)) + (b*c
^3*Sqrt[d*(1 + c^2*x^2)]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.171, size = 946, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(3/2)-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)*c^3-b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c
^2*x^2+1)*arcsinh(c*x)*c^8+b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*
```

```
x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+
3*c^2*x^2+1)*x^3*c^6-3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/
(c^2*x^2+1)*arcsinh(c*x)*c^6+b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+3*c^2*x^2+1)*x*c^4-10/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1
)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c
^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/
(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-1/3*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+1/3*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.04533, size = 470, normalized size = 4.43

$$-\frac{2\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}\left(x^4-1\right)\cdots}{c^2x^4+x^2}\right.}{6\left(c^2x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 + 1)) - (b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
```

```
d*x^4 + sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x
^4 + x^2)) + (2*a*c^4*x^4 + 4*a*c^2*x^2 - (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
1) + 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*x^5 + x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.128 $\int x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=217

$$\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d}-\frac{bc^3dx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{8bcdx^5\sqrt{c^2dx^2+d}}{175\sqrt{c^2x^2+1}}-\frac{bdx^3}{105c}$$

```
[Out] (2*b*d*x*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^3*Sqrt[d
+ c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2]) - (8*b*c*d*x^5*Sqrt[d + c^2*d*x^2])
/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c
^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d) + ((d + c
^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d^2)
```

Rubi [A] time = 0.17704, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5734, 12, 373}

$$\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d^2}-\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^4d}-\frac{bc^3dx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{8bcdx^5\sqrt{c^2dx^2+d}}{175\sqrt{c^2x^2+1}}-\frac{bdx^3}{105c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (2*b*d*x*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^3*Sqrt[d
+ c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2]) - (8*b*c*d*x^5*Sqrt[d + c^2*d*x^2])
/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c
^2*x^2]) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^4*d) + ((d + c
^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d^2)
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5734

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[a + b*ArcS
inh[c*x], Int[x^m*(d + e*x^2)^p, x], x] - Dist[(b*c*d^(p - 1/2)*Sqrt[d + e*
x^2])/Sqrt[1 + c^2*x^2], Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x],
 x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && (
IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=-\frac{\left(bcd\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)}{35c^4}\,dx}{\sqrt{1+c^2x^2}}+\left(a+b\sinh^{-1}(cx)\right)\int x^3\left(d+c^2d\right.$$

$$=-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2\left(-2+5c^2x^2\right)dx}{35c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sinh^{-1}(cx)\right)\mathrm{Sub}$$

$$=-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(-2+c^2x^2+8c^4x^4+5c^6x^6\right)dx}{35c^3\sqrt{1+c^2x^2}}+\frac{1}{2}\left(a+b\sinh^{-1}(cx)\right)$$

$$=\frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}}-\frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}}-\frac{bc^3dx^7\sqrt{d+c^2d}}{49\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.164445, size = 130, normalized size = 0.6

$$\frac{d\sqrt{c^2dx^2+d}\left(105a\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^3-bcx\left(75c^6x^6+168c^4x^4+35c^2x^2-210\right)\sqrt{c^2x^2+1}+105b\left(5c^2x^2-2\right)\left(c^2x^2\right.\right.}{3675c^4\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - b*c*x*Sqrt
[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 105*b*(1 + c
^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]))/(3675*c^4*(1 + c^2*x^2))
```

Maple [B] time = 0.211, size = 872, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b*(1/6
272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*
x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+
25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c
^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4
+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arc
sinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1
/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2
-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*
(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c
^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh
(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^
7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-5
6*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcs
inh(c*x))*d/c^4/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51518, size = 460, normalized size = 2.12

$$\frac{105\left(5\,bc^8dx^8+13\,bc^6dx^6+9\,bc^4dx^4-bc^2dx^2-2\,bd\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(525\,ac^8dx^8+1365\,ac^6dx^6+94\right.}{3675\left(c^6x^2+c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*b*c^8*d*x^8 + 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 - b*c^2*d*x^2 -
 2*b*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (525*a*c^8*d*x^8
 + 1365*a*c^6*d*x^6 + 945*a*c^4*d*x^4 - 105*a*c^2*d*x^2 - 210*a*d - (75*b*c
^7*d*x^7 + 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 - 210*b*c*d*x)*sqrt(c^2*x^2 + 1
))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.129 $\int x^2 \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=254

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}-\frac{d\sqrt{c^2dx}}{}$$

```
[Out] -(b*d*x^2*Sqrt[d + c^2*d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt
[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*Sqrt[d + c^2*d*x^2])
/(36*Sqrt[1 + c^2*x^2]) + (d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(1
6*c^2) + (d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(d + c^2
*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.310098, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5744, 5742, 5758, 5675, 30, 14}

$$\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}-\frac{d\sqrt{c^2dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -(b*d*x^2*Sqrt[d + c^2*d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt
[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*Sqrt[d + c^2*d*x^2])
/(36*Sqrt[1 + c^2*x^2]) + (d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(1
6*c^2) + (d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (x^3*(d + c^2
*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
```

```
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{1}{6}x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.$$

$$= \frac{1}{8}dx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right.}{16c^2}$$

$$= -\frac{bdx^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}} - \frac{7bcdx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{bc^3dx^6\sqrt{d+c^2dx^2}}{36\sqrt{1+c^2x^2}} + \frac{dx\sqrt{d+c^2d}}{}$$

Mathematica [A] time = 0.712962, size = 251, normalized size = 0.99

$$\frac{-144ad^{3/2}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+48acdx\sqrt{c^2x^2+1}\left(8c^4x^4+14c^2x^2+3\right)\sqrt{c^2dx^2+d}-18bd\sqrt{c^2dx^2+d}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(3 + 14*c^2*x^2 + 8*c^4*x
^4) - 144*a*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^
2]] - 18*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] -
 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + b*d*Sqrt[d + c^2*d*x^2]*(72*ArcSinh
[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSi
nh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]
] + Sinh[6*ArcSinh[c*x]])))/(2304*c^3*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.214, size = 421, normalized size = 1.7

$$\frac{ax}{6\,c^2d}\left(c^2dx^2+d\right)^{\frac{5}{2}} - \frac{ax}{24\,c^2}\left(c^2dx^2+d\right)^{\frac{3}{2}} - \frac{adx}{16\,c^2}\sqrt{c^2dx^2+d} - \frac{ad^2}{16\,c^2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}} - \frac{b\left(\mathrm{Arcsinh}\,(cx\right.}{32\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*a*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a/c^2*x*(c^2*d*x^2+d)^(3/2)-1/16*a/c
^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a/c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x
^2+d)^(1/2))/(c^2*d)^(1/2)-1/32*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
^3*arcsinh(c*x)^2*d+7/2304*b*(d*(c^2*x^2+1))^(1/2)*d/c^3/(c^2*x^2+1)^(1/2)-
1/32*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)*x^2+1/16*b*(d*(c^2*x^2+1
))^(1/2)*d/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+1/6*b*(d*(c^2*x^2+1))^(1/2)*d*c^4
/(c^2*x^2+1)*arcsinh(c*x)*x^7-1/36*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1
)^(1/2)*x^6+11/24*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^
5-7/96*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^4+17/48*b*(d*(c^2*x^
2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^2dx^4 + adx^2 + \left(bc^2dx^4 + bdx^2\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^4 + a*d*x^2 + (b*c^2*d*x^4 + b*d*x^2)*arcsinh(c*x))*sqr
t(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

3.130 $\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=146

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{bc^3dx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{2bcdx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bdx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}$$

```
[Out] -(b*d*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*x^3*Sqrt[d
+ c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^5*Sqrt[d + c^2*d*x^2])/(2
5*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*
d)
```

Rubi [A] time = 0.0740795, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{bc^3dx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{2bcdx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bdx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*x^3*Sqrt[d
+ c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^5*Sqrt[d + c^2*d*x^2])/(2
5*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(5*c^2*
d)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2dx}{5c\sqrt{1+c^2x^2}}$$
$$=\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{\left(bd\sqrt{d+c^2dx^2}\right)\int\left(1+2c^2x^2+c^4x^4\right)dx}{5c\sqrt{1+c^2x^2}}$$
$$=-\frac{bdx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{2bcdx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+\frac{\left(d+c^2dx^2\right)^{5/2}}{5\ldots}$$

Mathematica [A] time = 0.12883, size = 102, normalized size = 0.7

$$\frac{d\sqrt{c^2dx^2+d}\left(15a\left(c^2x^2+1\right)^3-bcx\left(3c^4x^4+10c^2x^2+15\right)\sqrt{c^2x^2+1}+15b\left(c^2x^2+1\right)^3\sinh^{-1}(cx)\right)}{75c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15
+ 10*c^2*x^2 + 3*c^4*x^4) + 15*b*(1 + c^2*x^2)^3*ArcSinh[c*x]))/(75*c^2*(1
+ c^2*x^2))
```

Maple [B] time = 0.149, size = 559, normalized size = 3.8

$$\frac{a}{5\,c^2d}\left(c^2dx^2+d\right)^{\frac{5}{2}}+b\left(\frac{(-1+5\,\mathrm{Arcsinh}\,(cx))\,d}{800\,c^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(16\,c^6x^6+16\,c^5x^5\sqrt{c^2x^2+1}+28\,c^4x^4+20\,c^3x^3\sqrt{c^2x^2+1}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/5*a/c^2/d*(c^2*d*x^2+d)^(5/2)+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+
16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2
*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*
(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*
x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^2/(c^2
*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arc
sinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^
3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))
*d/c^2/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*
x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*
x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

Maxima [A] time = 1.205, size = 115, normalized size = 0.79

$$\frac{\left(c^{2}dx^{2}+d\right)^{\frac{5}{2}}b\operatorname{arsinh}\left(cx\right)}{5\,c^{2}d}+\frac{\left(c^{2}dx^{2}+d\right)^{\frac{5}{2}}a}{5\,c^{2}d}-\frac{\left(3\,c^{4}d^{\frac{5}{2}}x^{5}+10\,c^{2}d^{\frac{5}{2}}x^{3}+15\,d^{\frac{5}{2}}x\right)b}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b*arcsinh(c*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2
)*a/(c^2*d) - 1/75*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*
b/(c*d)
```

Fricas [A] time = 2.4402, size = 373, normalized size = 2.55

$$\frac{15\left(bc^{6}dx^{6}+3\,bc^{4}dx^{4}+3\,bc^{2}dx^{2}+bd\right)\sqrt{c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(15\,ac^{6}dx^{6}+45\,ac^{4}dx^{4}+45\,ac^{2}dx^{2}+15\,ad-\right.}{75\left(c^{4}x^{2}+c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*(b*c^6*d*x^6 + 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 + b*d)*sqrt(c^2*d*x^2
 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (15*a*c^6*d*x^6 + 45*a*c^4*d*x^4 + 45*
a*c^2*d*x^2 + 15*a*d - (3*b*c^5*d*x^5 + 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqrt(c
```

```
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.131 $\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=180

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{bc^3dx^4}{16\sqrt{}}$$

```
[Out] (-5*b*c*d*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^4*Sq
rt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/8 + (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.108896, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5682, 5675, 30, 14}

$$\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{bc^3dx^4}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-5*b*c*d*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^4*Sq
rt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x]))/8 + (x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/4 + (3
*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}(3d)\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$=\frac{3}{8}dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{(3}{}$$

$$=-\frac{5bcdx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^3dx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+\frac{3}{8}dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)$$

Mathematica [A] time = 0.792922, size = 200, normalized size = 1.11

$$\frac{3ad^{3/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{8c}+\frac{1}{8}adx\left(2c^2x^2+5\right)\sqrt{c^2dx^2+d}+\frac{bd\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\,\mathrm{si}\right.\right.\right.}{8c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*d*x*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/8 + (3*a*d^(3/2)*Log[c*d*x + Sq
rt[d]*Sqrt[d + c^2*d*x^2]])/(8*c) + (b*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSi
nh[c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt
[1 + c^2*x^2]) - (b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSin
h[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.13, size = 318, normalized size = 1.8

$$\frac{ax}{4}\left(c^2dx^2+d\right)^{\frac{3}{2}}+\frac{3\,adx}{8}\sqrt{c^2dx^2+d}+\frac{3\,ad^2}{8}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{bdc^4\mathrm{Arcsinh}\,(cx)\,x^5}{4\,c^2x^2+4}\sqrt{d\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a+3/8*a*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a*d^2*ln(x*c^
2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(c^2*x^2+1))^
(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*d*c^3
/(c^2*x^2+1)^(1/2)*x^4+7/8*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsin
h(c*x)*x^3-5/16*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^2+5/8*b*(d*
(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x+3/16*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d-17/128*b*(d*(c^2*x^2+1))^(1/2)*d/c/(
c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}} \left(a + b\operatorname{asinh}\left(cx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right) + a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a), x)
```

3.132 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=249

$$-\frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] (-4*b*c*d*x*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[
d + c^2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (
b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.303196, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8}

$$-\frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{1}{3}\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```


```
[Out] (-4*b*c*d*x*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[
d + c^2*d*x^2])/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 - (2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (
b*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
```

```
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx -$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{4bcdx\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+$$

Mathematica [A] time = 0.739786, size = 248, normalized size = 1.

$$\frac{bd\sqrt{c^2dx^2+d}\left(\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\left(1-\right.\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (a*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 + (b*d*Sqrt[d + c^2*d*x^2]*(-(c*x
*(3 + c^2*x^2)) + 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]
) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*
d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*
x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + P
olyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^
2*x^2]
```

Maple [A] time = 0.158, size = 428, normalized size = 1.7

$$\frac{a}{3}\left(c^2dx^2+d\right)^{\frac{3}{2}}-ad^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)+a\sqrt{c^2dx^2+d}d+b\mathrm{Arcsinh}\left(cx\right)d\sqrt{d\left(c^2x^2+1\right)}\ln\left(1-cx-\sqrt{c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/3*(c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/
x)+a*(c^2*d*x^2+d)^(1/2)*d+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2
+1)*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(
c^2*x^2+1)^(1/2))*d+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x
+(c^2*x^2+1)^(1/2))*d+1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x
)*x^4*c^4-1/9*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c^3*x^3+5/3*b*(d*
(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2-4/3*b*(d*(c^2*x^2+1))
^(1/2)*d/(c^2*x^2+1)^(1/2)*c*x-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\mathrm{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)/x, x)
```

3.133 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=177

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3cd\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{bc^3dx^2\sqrt{\cdots}}{4\sqrt{c^2\cdots}}$$

```
[Out] -(b*c^3*d*x^2*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (3*c^2*d*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/x + (3*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (b*c*d*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.170462, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5682, 5675, 30, 14}

$$\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3cd\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{bc^3dx^2\sqrt{\cdots}}{4\sqrt{c^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] -(b*c^3*d*x^2*Sqrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (3*c^2*d*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/x + (3*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt
[1 + c^2*x^2]) + (b*c*d*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\left(3c^2d\right)\int\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{(b}{}$$

$$= -\frac{bc^3dx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}+\frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{3/2}(}{}$$

Mathematica [A] time = 0.777871, size = 200, normalized size = 1.13

$$\frac{1}{8}\left(12acd^{3/2}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+\frac{4ad\left(c^2x^2-2\right)\sqrt{c^2dx^2+d}}{x}+\frac{4bd\sqrt{c^2dx^2+d}\left(-2\sqrt{c^2x^2+1}\sinh^{-1}(cx)+2cx\,\mathrm{l}\right.}{x\sqrt{c^2x^2+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] ((4*a*d*(-2 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/x + (4*b*d*Sqrt[d + c^2*d*x^2]*
(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/
(x*Sqrt[1 + c^2*x^2]) + 12*a*c*d^(3/2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] + (b*c*d*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(
ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2])/8
```

Maple [B] time = 0.146, size = 392, normalized size = 2.2

$$-\frac{a}{dx}\left(c^2dx^2+d\right)^{\frac{5}{2}}+ac^2x\left(c^2dx^2+d\right)^{\frac{3}{2}}+\frac{3\,ac^2dx}{2}\sqrt{c^2dx^2+d}+\frac{3\,ac^2d^2}{2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{3\,b\,(\mathrm{Arcsin}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(5/2)+a*c^2*x*(c^2*d*x^2+d)^(3/2)+3/2*a*c^2*d*x*(c^2*d
*x^2+d)^(1/2)+3/2*a*c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)+3/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c
*d+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*
(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x^2-1/2*b*(d*(c^2*x^2+1))^(1/2)*
c^2*d/(c^2*x^2+1)*arcsinh(c*x)*x-b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)-b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)*d/x/(c^2*x^2+1)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\,\text{asinh}\left(cx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.134 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=270

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}c^2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}\right.$$

```
[Out] -(b*c*d*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x*Sqrt[d +
c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (3*c^2*d
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 +
 c^2*x^2] - (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*
Sqrt[1 + c^2*x^2]) + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*
x]])/(2*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.31003, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5742, 5760, 4182, 2279, 2391, 8, 14}

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}c^2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```


```
[Out] -(b*c*d*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x*Sqrt[d +
c^2*d*x^2])/Sqrt[1 + c^2*x^2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/2 - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (3*c^2*d
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 +
 c^2*x^2] - (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*
Sqrt[1 + c^2*x^2]) + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*
x]])/(2*Sqrt[1 + c^2*x^2])
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
```

```
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}\left(3c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right.}{x}$$

$$= \frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{\left(bcd\right.}{}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{bc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}}+\frac{3}{2}c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)-$$

Mathematica [A] time = 4.07929, size = 352, normalized size = 1.3

$$\frac{bc^2d\sqrt{c^2dx^2+d}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\left(1\right.\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] a*(c^2*d - d/(2*x^2))*Sqrt[d + c^2*d*x^2] + (3*a*c^2*d^(3/2)*Log[x])/2 - (3
*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/2 + (b*c^2*d*Sqrt[d +
c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 -
```

```
E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -
E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b
*c^2*d*Sqrt[d + c^2*d*x^2]*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[Arc
Sinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]
*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[
2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSin
h[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.184, size = 472, normalized size = 1.8

$$-\frac{a}{2\,dx^2}\left(c^2dx^2+d\right)^{\frac{5}{2}}+\frac{ac^2}{2}\left(c^2dx^2+d\right)^{\frac{3}{2}}-\frac{3\,ac^2}{2}d^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)+\frac{3\,ac^2d}{2}\sqrt{c^2dx^2+d}+\frac{bc^4d\,\mathrm{Arcsinh}}{c^2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(5/2)+1/2*a*c^2*(c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d^
(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+3/2*a*c^2*(c^2*d*x^2+d)^(1/
2)*d+b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x
^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c
^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)*c-
1/2*b*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)*arcsinh(c*x)-3/2*b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*
d-3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*c^2*d+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(
1-c*x-(c^2*x^2+1)^(1/2))*c^2*d+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.135 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=184

$$\frac{c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{c^2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{c^2dx}}{6x^2\sqrt{c^2x}}$$

```
[Out] -(b*c*d*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (c^2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/x - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/(3*x^3) + (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*Sqrt
[1 + c^2*x^2]) + (4*b*c^3*d*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2
])
```

Rubi [A] time = 0.231355, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5739, 5737, 29, 5675, 14}

$$\frac{c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2b\sqrt{c^2x^2+1}}-\frac{c^2d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}-\frac{bcd\sqrt{c^2dx}}{6x^2\sqrt{c^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```


```
[Out] -(b*c*d*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (c^2*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/x - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x]))/(3*x^3) + (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*Sqrt
[1 + c^2*x^2]) + (4*b*c^3*d*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2
])
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
```

```
] && LtQ[m, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\left(c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$$

$$= -\frac{c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3}+\frac{\left(bcd\right.}{}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}}-\frac{c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.}{3x^3}$$

Mathematica [A] time = 0.721007, size = 214, normalized size = 1.16

$$\frac{1}{6}d\left(-\frac{2a\left(4c^2x^2+1\right)\sqrt{c^2dx^2+d}}{x^3}+6ac^3\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+\frac{3bc^2\sqrt{c^2dx^2+d}\left(-2\sqrt{c^2x^2+1}\sinh^{-1}(cx)+2c\right.}{x\sqrt{c^2x^2+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d*((-2*a*(1 + 4*c^2*x^2)*Sqrt[d + c^2*d*x^2])/x^3 + (3*b*c^2*Sqrt[d + c^2*
d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[
c*x]))/(x*Sqrt[1 + c^2*x^2]) - (b*Sqrt[d + c^2*d*x^2]*(c*x + 2*(1 + c^2*x^2
)^(3/2)*ArcSinh[c*x] - 2*c^3*x^3*Log[c*x]))/(x^3*Sqrt[1 + c^2*x^2]) + 6*a*c
^3*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]]))/6
```

Maple [B] time = 0.176, size = 1107, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(5/2)-2/3*a*c^2/d/x*(c^2*d*x^2+d)^(5/2)+2/3*a*c^
4*x*(c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(c^2*d*x^2+d)^(1/2)+a*c^4*d^2*ln(x*c^2*d/
(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d-8/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*c^3*d-32*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8+32*b*(d*(c^2*x^2+1))^(1/2)*d/(24
*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7-8/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8+8/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3*c^6-52*b*(d*(c^2*x^2+1))^(1/2)
*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6+12*b*(d*(c^2*x
^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
c^5-10/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)
*c^6-4*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(
1/2)*c^5+2/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x*c^4-73/3*
b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*
x)*c^4+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*c^3-2/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1
```

```
)*x/(c^2*x^2+1)*c^4-3/2*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/
(c^2*x^2+1)^(1/2)*c^3-14/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+
1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-1/3*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4
*x^4+9*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+4/3*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^2dx^2+ad+\left(bc^2dx^2+bd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.136 $\int x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=266

$$\frac{\left(c^2dx^2+d\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d}-\frac{bc^5d^2x^9\sqrt{c^2dx^2+d}}{81\sqrt{c^2x^2+1}}-\frac{19bc^3d^2x^7\sqrt{c^2dx^2+d}}{441\sqrt{c^2x^2+1}}-\frac{bc}{}$$

```
[Out] (2*b*d^2*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(189*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(21*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2])/(441*S
qrt[1 + c^2*x^2]) - (b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2])/(81*Sqrt[1 + c^2*x^
2]) - ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d) + ((d + c^2*d*
x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^4*d^2)
```

Rubi [A] time = 0.192028, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {266, 43, 5734, 12, 373}

$$\frac{\left(c^2dx^2+d\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{9c^4d^2}-\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^4d}-\frac{bc^5d^2x^9\sqrt{c^2dx^2+d}}{81\sqrt{c^2x^2+1}}-\frac{19bc^3d^2x^7\sqrt{c^2dx^2+d}}{441\sqrt{c^2x^2+1}}-\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (2*b*d^2*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*x^3*Sqr
t[d + c^2*d*x^2])/(189*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^5*Sqrt[d + c^2*d*x
^2])/(21*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2])/(441*S
qrt[1 + c^2*x^2]) - (b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2])/(81*Sqrt[1 + c^2*x^
2]) - ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^4*d) + ((d + c^2*d*
x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(9*c^4*d^2)
```


Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5734

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[a + b*ArcS
inh[c*x], Int[x^m*(d + e*x^2)^p, x], x] - Dist[(b*c*d^(p - 1/2)*Sqrt[d + e*
x^2])/Sqrt[1 + c^2*x^2], Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x],
 x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p + 1/2, 0] && (
IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = -\frac{\left(bcd^2\sqrt{d+c^2dx^2}\right)\int \frac{\left(1+c^2x^2\right)^3\left(-2+7c^2x^2\right)}{63c^4}\,dx}{\sqrt{1+c^2x^2}} + \left(a+b\sinh^{-1}(cx)\right)\int x^3 \left(d+\right.$$

$$= -\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^3\left(-2+7c^2x^2\right) dx}{63c^3\sqrt{1+c^2x^2}} + \frac{1}{2}\left(a+b\sinh^{-1}(cx)\right)\mathrm{S}$$

$$= -\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int \left(-2+c^2x^2+15c^4x^4+19c^6x^6+7c^8x^8\right) dx}{63c^3\sqrt{1+c^2x^2}} + \frac{1}{2}\left(a+b\right.$$

$$= \frac{2bd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{bd^2x^3\sqrt{d+c^2dx^2}}{189c\sqrt{1+c^2x^2}} - \frac{bcd^2x^5\sqrt{d+c^2dx^2}}{21\sqrt{1+c^2x^2}} - \frac{19bc^3d^2x^7\sqrt{}}{441\sqrt{1+}}$$

Mathematica [A] time = 0.197109, size = 140, normalized size = 0.53

$$\frac{d^2\sqrt{c^2dx^2+d}\left(63a\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^4-bcx\left(49c^8x^8+171c^6x^6+189c^4x^4+21c^2x^2-126\right)\sqrt{c^2x^2+1}+63b\left(7c^2x^2-\right.\right.}{3969c^4\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - b*c*x*Sqr
t[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 49*c^8*x^8)
 + 63*b*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSinh[c*x]))/(3969*c^4*(1 + c^2*
x^2))
```

Maple [B] time = 0.216, size = 996, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] a*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b*(1/4
1472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2)+704
*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^2+1)^
(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*x^2+1
)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^
(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2
*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^
2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2
+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+
1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(
c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*
(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d
^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^
2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3
*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c
*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c
^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*
```

```
x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)
+41*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+1)*(1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.43565, size = 585, normalized size = 2.2

$$\frac{63\left(7\,bc^{10}d^2x^{10}+26\,bc^8d^2x^8+34\,bc^6d^2x^6+16\,bc^4d^2x^4-bc^2d^2x^2-2\,bd^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(441\,ac^{10}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 + 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 + 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 - 2*b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 + 1)) + (441*a*c^10*d^2*x^10 + 1638*a*c^8*d^2*x^8 + 2142*a*c^6*d^2*x
^6 + 1008*a*c^4*d^2*x^4 - 63*a*c^2*d^2*x^2 - 126*a*d^2 - (49*b*c^9*d^2*x^9
+ 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 + 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.137 $\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=337

$$\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{128c^2}-\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{256bc^3\sqrt{c^2x^2+1}}+\frac{1}{8}x$$

```
[Out] (-5*b*d^2*x^2*Sqrt[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^2*x^6*Sqrt[d
 + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]
)/(64*Sqrt[1 + c^2*x^2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(128*c^2) + (5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5
*d*x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (x^3*(d + c^2*d*x^2
)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.466263, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5758, 5675, 30, 14, 266, 43}

$$\frac{5}{64}d^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{128c^2}-\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{256bc^3\sqrt{c^2x^2+1}}+\frac{1}{8}x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-5*b*d^2*x^2*Sqrt[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(768*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^2*x^6*Sqrt[d
 + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]
)/(64*Sqrt[1 + c^2*x^2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(128*c^2) + (5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5
*d*x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/48 + (x^3*(d + c^2*d*x^2
)^(5/2)*(a + b*ArcSinh[c*x]))/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}(5d)\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sin\right.$$

$$=\frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}\right.$$

$$=\frac{5}{64}d^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{48}dx^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

$$=-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^8\sqrt{d+c^2dx^2}}{64\sqrt{1+c^2x^2}}+\frac{5a}{}$$

$$=-\frac{5bd^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{59bcd^2x^4\sqrt{d+c^2dx^2}}{768\sqrt{1+c^2x^2}}-\frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}}{288\sqrt{1+c^2x^2}}-\frac{bc^5}{}$$

Mathematica [A] time = 0.940361, size = 388, normalized size = 1.15

$$\frac{d^2\left(9216ac^7x^7\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+26112ac^5x^5\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+22656ac^3x^3\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+2880acx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(2880*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 22656*a*c^3*x^3*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 26112*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqr
```

```
t[d + c^2*d*x^2] + 9216*a*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 1
440*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 + 576*b*Sqrt[d + c^2*d*x^2]*Cosh[2
*ArcSinh[c*x]] - 144*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 64*b*Sqrt
[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 9*b*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSi
nh[c*x]] - 2880*a*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^
2*d*x^2]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(-48*Sinh[2*ArcSinh[c*x]]
 + 24*Sinh[4*ArcSinh[c*x]] + 16*Sinh[6*ArcSinh[c*x]] + 3*Sinh[8*ArcSinh[c*x
]]))/(73728*c^3*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.259, size = 537, normalized size = 1.6

$$\frac{ax}{8\,c^2d}\left(c^2dx^2+d\right)^{\frac{7}{2}}-\frac{ax}{48\,c^2}\left(c^2dx^2+d\right)^{\frac{5}{2}}-\frac{5\,adx}{192\,c^2}\left(c^2dx^2+d\right)^{\frac{3}{2}}-\frac{5\,ad^2x}{128\,c^2}\sqrt{c^2dx^2+d}-\frac{5\,ad^3}{128\,c^2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/8*a*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a/c^2*x*(c^2*d*x^2+d)^(5/2)-5/192*a/
c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)-5/128*a/c
^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-5/256*b*
(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)*x^2+5/128*b*(d*(c^2*x^2+1))^(
1/2)*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+1/8*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6
/(c^2*x^2+1)*arcsinh(c*x)*x^9-1/64*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2
+1)^(1/2)*x^8+23/48*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x
)*x^7-17/288*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*x^6+127/192*
b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-59/768*b*(d*(c
^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^4+133/384*b*(d*(c^2*x^2+1))^(1/2
)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+359/73728*b*(d*(c^2*x^2+1))^(1/2)*d^2/c^
3/(c^2*x^2+1)^(1/2)-5/256*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arc
sinh(c*x)^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2x^6 + 2\,ac^2d^2x^4 + ad^2x^2 + \left(bc^4d^2x^6 + 2\,bc^2d^2x^4 + bd^2x^2\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^6 + 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 + 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^2, x)
```

3.138 $\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=193

$$\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d}-\frac{bc^5d^2x^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{3bc^3d^2x^5\sqrt{c^2dx^2+d}}{35\sqrt{c^2x^2+1}}-\frac{bcd^2x^3\sqrt{c^2dx^2+d}}{7\sqrt{c^2x^2+1}}-\frac{bd^2x\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}}$$

```
[Out] -(b*d^2*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^3*Sqrt[
d + c^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2
])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1
+ c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2*d)
```

Rubi [A] time = 0.0881364, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 194}

$$\frac{\left(c^2dx^2+d\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d}-\frac{bc^5d^2x^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{3bc^3d^2x^5\sqrt{c^2dx^2+d}}{35\sqrt{c^2x^2+1}}-\frac{bcd^2x^3\sqrt{c^2dx^2+d}}{7\sqrt{c^2x^2+1}}-\frac{bd^2x\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*d^2*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^3*Sqrt[
d + c^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2
])/(35*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1
+ c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(7*c^2*d)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d}-\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^3dx}{7c\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{7c^2d}-\frac{\left(bd^2\sqrt{d+c^2dx^2}\right)\int\left(1+3c^2x^2+3c^4x^4+\right.}{7c\sqrt{1+c^2x^2}}$$

$$=-\frac{bd^2x\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{bcd^2x^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3d^2x^5\sqrt{d+c^2dx^2}}{35\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^7\sqrt{}}{49\sqrt{1}}$$

Mathematica [A] time = 0.147508, size = 112, normalized size = 0.58

$$\frac{d^2\sqrt{c^2dx^2+d}\left(35a\left(c^2x^2+1\right)^4-bcx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)\sqrt{c^2x^2+1}+35b\left(c^2x^2+1\right)^4\sinh^{-1}(cx)\right)}{245c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(35*a*(1 + c^2*x^2)^4 - b*c*x*Sqrt[1 + c^2*x^2]*(3
5 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 35*b*(1 + c^2*x^2)^4*ArcSinh[c*x
]))/(245*c^2*(1 + c^2*x^2))
```

Maple [B] time = 0.18, size = 863, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/7*a/c^2/d*(c^2*d*x^2+d)^(7/2)+b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8
+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104
```

```
*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)
*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+
1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2
*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/1
28*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x)
)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^
(1/2)+1)*(1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(
4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*
(1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*
x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13
*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+
1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c
^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/
2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^
2+1))
```

Maxima [A] time = 1.14682, size = 130, normalized size = 0.67

$$\frac{\left(c^2dx^2+d\right)^{\frac{7}{2}}b\operatorname{arsinh}(cx)}{7\,c^2d}+\frac{\left(c^2dx^2+d\right)^{\frac{7}{2}}a}{7\,c^2d}-\frac{\left(5\,c^6d^{\frac{7}{2}}x^7+21\,c^4d^{\frac{7}{2}}x^5+35\,c^2d^{\frac{7}{2}}x^3+35\,d^{\frac{7}{2}}x\right)b}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b*arcsinh(c*x)/(c^2*d) + 1/7*(c^2*d*x^2 + d)^(7/2
)*a/(c^2*d) - 1/245*(5*c^6*d^(7/2)*x^7 + 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2
)*x^3 + 35*d^(7/2)*x)*b/(c*d)
```

Fricas [A] time = 2.61141, size = 483, normalized size = 2.5

$$\frac{35\left(bc^8d^2x^8+4\,bc^6d^2x^6+6\,bc^4d^2x^4+4\,bc^2d^2x^2+bd^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(35\,ac^8d^2x^8+140\,ac^6d^2x^6+2\right.}{245\left(c^4x^2+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 + 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 + 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (35*a*c^8*d
^2*x^8 + 140*a*c^6*d^2*x^6 + 210*a*c^4*d^2*x^4 + 140*a*c^2*d^2*x^2 + 35*a*d
^2 - (5*b*c^7*d^2*x^7 + 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 + 35*b*c*d^2*x)
*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.139 $\int \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=254

$$\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}dx$$

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2
*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*d^2*(1 + c^2*x^2)^(5/
2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/16 + (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(d
 + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.159522, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5684, 5682, 5675, 30, 14, 261}

$$\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}dx$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2
*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*d^2*(1 + c^2*x^2)^(5/
2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/16 + (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/24 + (x*(d
 + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx=\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}(5d)\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.$$

$$=-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}x\left(\right.$$

$$=-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{16}d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}dx\left(\right.$$

$$=-\frac{25bcd^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bc^3d^2x^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}-$$

Mathematica [A] time = 0.690686, size = 317, normalized size = 1.25

$$\frac{d^2\left(384ac^5x^5\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+1248ac^3x^3\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+1584acx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+720a\sqrt{d}\sqrt{c^2x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(1584*a*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 1248*a*c^3*x^3*Sqr
t[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 384*a*c^5*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 360*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2 - 270*b*Sqrt[d + c
^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 27*b*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*
x]] - 2*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 720*a*Sqrt[d]*Sqrt[1 +
 c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 12*b*Sqrt[d + c^2*d*x^
2]*ArcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*
ArcSinh[c*x]])))/(2304*c*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.161, size = 427, normalized size = 1.7

$$\frac{ax}{6}\left(c^2dx^2+d\right)^{\frac{5}{2}}+\frac{5\,adx}{24}\left(c^2dx^2+d\right)^{\frac{3}{2}}+\frac{5\,ad^2x}{16}\sqrt{c^2dx^2+d}+\frac{5\,ad^3}{16}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}-\frac{299\,bd^2}{2304\,c}\sqrt{d\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/6*x*(c^2*d*x^2+d)^(5/2)*a+5/24*a*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(c^
2*d*x^2+d)^(1/2)+5/16*a*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(
c^2*d)^(1/2)-299/2304*b*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)+5/32*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d^2+1/6*b*(d*(c^
2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7-1/36*b*(d*(c^2*x^2+1))
^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^6+17/24*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/
(c^2*x^2+1)*arcsinh(c*x)*x^5-13/96*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2
+1)^(1/2)*x^4+59/48*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x
)*x^3-11/32*b*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^2+11/16*b*(d*
(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a), x)
```

3.140 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=329

$$-\frac{bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] (-23*b*c*d^2*x*Sqrt[d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*d^2*
x^3*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c
^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) + d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]) + (d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((d + c^2*d*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*d^2*Sqrt[d + c^2*d*x^
2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*d^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.439084, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5744, 5742, 5760, 4182, 2279, 2391, 8, 194}

$$-\frac{bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```


```
[Out] (-23*b*c*d^2*x*Sqrt[d + c^2*d*x^2])/(15*Sqrt[1 + c^2*x^2]) - (11*b*c^3*d^2*
x^3*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c
^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) + d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]) + (d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/3 + ((d + c^2*d*x^2)
^(5/2)*(a + b*ArcSinh[c*x]))/5 - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b*d^2*Sqrt[d + c^2*d*x^
2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b*d^2*Sqrt[d + c^2*d*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx = \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+d\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,d$$

$$= \frac{1}{3}d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+d$$

$$= -\frac{8bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+d^2\sqrt{d+}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+d^2\sqrt{d}$$

$$= -\frac{23bcd^2x\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{11bc^3d^2x^3\sqrt{d+c^2dx^2}}{45\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}+d^2\sqrt{d}$$

Mathematica [A] time = 1.23533, size = 361, normalized size = 1.1

$$\frac{bd^2\sqrt{c^2dx^2+d}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\left(1\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x,x]
```

```
[Out] (-8*b*c*d^2*x*(3 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(45*Sqrt[1 + c^2*x^2]) - (
b*c^3*d^2*x^3*(5 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(75*Sqrt[1 + c^2*x^2]) +
 (a*d^2*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c^4*x^4))/15 + (2*b*d^2*(1
 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/3 + (b*d*(-2 + 3*c^2*x^2)*(d
+ c^2*d*x^2)^(3/2)*ArcSinh[c*x])/15 + a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1
+ c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh
[c*x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog
[2, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

Maple [A] time = 0.202, size = 540, normalized size = 1.6

$$\frac{a}{5}\left(c^2dx^2+d\right)^{\frac{5}{2}}+\frac{ad}{3}\left(c^2dx^2+d\right)^{\frac{3}{2}}-ad^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)+a\sqrt{c^2dx^2+d}d^2+\frac{23\,bd^2\mathrm{Arcsinh}\,(cx)}{15\,c^2x^2+15}\sqrt{d\left(c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x)
```

```
[Out] 1/5*(c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2*d
^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a*(c^2*d*x^2+d)^(1/2)*d^2+23/15*b*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*d^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2+1
/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6-1/25*b*(d*(
c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5+14/15*b*(d*(c^2*x^2+1))^(1/
2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/(
c^2*x^2+1)^(1/2)*c^3*x^3+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcs
inh(c*x)*x^2*c^2-23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4+2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4+2\,bc^2d^2x^2+bd^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)/x, x)
```

3.141 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=257

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{15cd^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16b\sqrt{c^2x^2+1}}+\frac{5}{4}c^2dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] (-9*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*d^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x + (15*c*d
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[1 + c^2*x^2]) + (
b*c*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.23772, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5739, 5684, 5682, 5675, 30, 14, 266, 43}

$$\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{15cd^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16b\sqrt{c^2x^2+1}}+\frac{5}{4}c^2dx\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```


```
[Out] (-9*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*
x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (15*c^2*d^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*
ArcSinh[c*x]))/4 - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x + (15*c*d
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*Sqrt[1 + c^2*x^2]) + (
b*c*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/Sqrt[1 + c^2*x^2]
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
```

```
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \left(5c^2d\right)\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + \frac{1}{4}$$

$$= \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.$$

$$= -\frac{9bc^3d^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}} + \frac{15}{8}c^2d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh\right.$$

Mathematica [A] time = 1.34322, size = 270, normalized size = 1.05

$$\frac{1}{128}d^2\left(\frac{16a\left(2c^4x^4+9c^2x^2-8\right)\sqrt{c^2dx^2+d}}{x} + 240ac\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) + \frac{64b\sqrt{c^2dx^2+d}\left(-2\sqrt{c^2x^2+1}\sinh\right.}{x\sqrt{c}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d + c^2*d*x^2]*(-8 + 9*c^2*x^2 + 2*c^4*x^4))/x + (64*b*Sqr
t[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 +
2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x^2]) + 240*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (32*b*c*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]]
 + 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/Sqrt[1 + c^2*x^2]
 - (b*c*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*Ar
```

```
cSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2]))/128
```

Maple [B] time = 0.185, size = 506, normalized size = 2.

$$-\frac{a}{dx}\left(c^2dx^2+d\right)^{\frac{7}{2}}+ac^2x\left(c^2dx^2+d\right)^{\frac{5}{2}}+\frac{5\,ac^2dx}{4}\left(c^2dx^2+d\right)^{\frac{3}{2}}+\frac{15\,ac^2d^2x}{8}\sqrt{c^2dx^2+d}+\frac{15\,ac^2d^3}{8}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(7/2)+a*c^2*x*(c^2*d*x^2+d)^(5/2)+5/4*a*c^2*d*x*(c^2*d
*x^2+d)^(3/2)+15/8*a*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+15/8*a*c^2*d^3*ln(x*c^2*
d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+15/16*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c*d^2+11/8*b*(d*(c^2*x^2+1))^(1/2)*c
^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-9/16*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c
^2*x^2+1)^(1/2)*x^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh
(c*x)*x-b*(d*(c^2*x^2+1))^(1/2)*c*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-b*(d*(
c^2*x^2+1))^(1/2)*arcsinh(c*x)*d^2/x/(c^2*x^2+1)+1/4*b*(d*(c^2*x^2+1))^(1/2
)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/2)*c^5*d^2
/(c^2*x^2+1)^(1/2)*x^4+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c
^2*x^2+1)^(1/2))^2-1)*c*d^2-33/128*b*(d*(c^2*x^2+1))^(1/2)*c*d^2/(c^2*x^2+1
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4+2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4+2\,bc^2d^2x^2+bd^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)/x^2, x)
```

3.142 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=355

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}\left(a+b\sinh\right.$$

```
[Out] -(b*c*d^2*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*d^2*x*Sqr
t[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2
])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/2 + (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((d + c^2
*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b*c^
2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]
) + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1
+ c^2*x^2])
```

Rubi [A] time = 0.447402, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5739, 5744, 5742, 5760, 4182, 2279, 2391, 8, 270}

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^2d^2\sqrt{c^2dx^2+d}\left(a+b\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```


```
[Out] -(b*c*d^2*Sqrt[d + c^2*d*x^2])/(2*x*Sqrt[1 + c^2*x^2]) - (7*b*c^3*d^2*x*Sqr
t[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2
])/(9*Sqrt[1 + c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/2 + (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/6 - ((d + c^2
*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b*c^
2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[1 + c^2*x^2]
) + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[1
+ c^2*x^2])
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{x}$$

$$=\frac{5}{6}c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}$$

$$=-\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}+\frac{bc^3d^2x\sqrt{d+c^2dx^2}}{6\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2d}$$

$$=-\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2}$$

$$=-\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2}$$

$$=-\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2}$$

$$=-\frac{bcd^2\sqrt{d+c^2dx^2}}{2x\sqrt{1+c^2x^2}}-\frac{7bc^3d^2x\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}-\frac{bc^5d^2x^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{5}{2}c^2d^2\sqrt{d+c^2}$$

Mathematica [A] time = 6.73012, size = 467, normalized size = 1.32

$$\frac{2bc^2d^2\sqrt{d\left(c^2x^2+1\right)}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^3,x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((7*a*c^2*d^2)/3 - (a*d^2)/(2*x^2) + (a*c^4*d^2*x^2)/
3) + b*c^2*d^2*(-(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(9*Sqrt[1 + c^2*
x^2]) + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3) + (5*a*c^2*d^
(5/2)*Log[x])/2 - (5*a*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/
2 + (2*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-A
rcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2] + (b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[ArcSinh
[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^(
```

```
-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2,
-E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[A
rcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [A] time = 0.217, size = 588, normalized size = 1.7

$$-\frac{a}{2\,dx^2}\left(c^2dx^2+d\right)^{\frac{7}{2}}+\frac{ac^2}{2}\left(c^2dx^2+d\right)^{\frac{5}{2}}+\frac{5\,ac^2d}{6}\left(c^2dx^2+d\right)^{\frac{3}{2}}-\frac{5\,ac^2}{2}d^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)+\frac{5\,ac^2d^2}{2}\sqrt{c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(7/2)+1/2*a*c^2*(c^2*d*x^2+d)^(5/2)+5/6*a*c^2*d*
(c^2*d*x^2+d)^(3/2)-5/2*a*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2)
)/x)+5/2*a*c^2*(c^2*d*x^2+d)^(1/2)*d^2-1/9*b*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/
(c^2*x^2+1)^(1/2)*x^3+8/3*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsi
nh(c*x)*x^2-7/3*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*x-1/2*b*(
d*(c^2*x^2+1))^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c-5/2*b*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2+1/3*b*(d*(c^2*x
^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4-5/2*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d^2+11/6*
b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+
1))^(1/2)*d^2/x^2/(c^2*x^2+1)*arcsinh(c*x)+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2+5/2*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ac^4d^2x^4+2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4+2\,bc^2d^2x^2+bd^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)/x^3, x)
```

3.143 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=266

$$\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5c^3d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{5c^2d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-$$

```
[Out] -(b*c*d^2*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^2*S
qrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x]))/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (5*c^3
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt[1 + c^2*x^2]) +
(7*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.299715, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5739, 5682, 5675, 30, 14, 266, 43}

$$\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5c^3d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4b\sqrt{c^2x^2+1}}-\frac{5c^2d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}-$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```


```
[Out] -(b*c*d^2*Sqrt[d + c^2*d*x^2])/(6*x^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^2*S
qrt[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]))/2 - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x]))/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(3*x^3) + (5*c^3
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*Sqrt[1 + c^2*x^2]) +
(7*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*Log[x])/(3*Sqrt[1 + c^2*x^2])
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
```

```
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3} + \frac{1}{3}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{x^2}$$

$$= -\frac{5c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x} - \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^3} +$$

$$= \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{5c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}}{6x^2\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} + \frac{5}{2}c^4d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.$$

Mathematica [A] time = 0.961127, size = 287, normalized size = 1.08

$$\frac{d^2\left(4a\sqrt{c^2x^2+1}\left(3c^4x^4-14c^2x^2-2\right)\sqrt{c^2dx^2+d}+60ac^3\sqrt{d}x^3\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+24bc^2x^2\sqrt{c^2dx^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x^4,x]
```

```
[Out] (d^2*(4*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-2 - 14*c^2*x^2 + 3*c^4*x^
4) + 24*b*c^2*x^2*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]) + 4*b*Sqrt[d + c^2*d*x^2]*(-(c*x) - 2*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] + 2*c^3*x^3*Log[c*x]) + 60*a*c^3*Sqrt[d]*x
^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3*b*c^3*x^3
*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] +
 Sinh[2*ArcSinh[c*x]]))))/(24*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.208, size = 1316, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x)
```

```
[Out] -1/8*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)-147*b*(d*(c^2*x^2+1)
)^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-203*
b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsi
nh(c*x)*c^6-190/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(
c^2*x^2+1)*arcsinh(c*x)*c^4-23/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15
*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2+147*b*(d*(c^2*x^2+1))^(1/2)*d^2/
(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7+35*b*(d*(c
^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*c^5-4/3*a*c^2/d/x*(c^2*d*x^2+d)^(7/2)+5/3*a*c^4*d*x*(c^2*d*x^2+d)^(3
/2)+5/2*a*c^4*d^2*x*(c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3*ln(x*c^2*d/(c^2*d)^(1
/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+49/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63
*c^4*x^4+15*c^2*x^2+1)*x^3*c^6+7/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+
15*c^2*x^2+1)*x*c^4-1/4*b*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x
^2-5/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1
/2)*c^3-14/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3*d^2
+7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*c^3-21/2*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+
1)*x^2/(c^2*x^2+1)^(1/2)*c^5-1/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15
*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^
2*x^2+1)*arcsinh(c*x)*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*a
rcsinh(c*x)*x-49/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^
5/(c^2*x^2+1)*c^8-28/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1
)*x^3/(c^2*x^2+1)*c^6-7/6*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^
2+1)*x/(c^2*x^2+1)*c^4-1/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x
^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)+7/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3*d^2+5/4*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d^2-1/3*a/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3
*a*c^4*x*(c^2*d*x^2+d)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right) + a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)/x^4, x)
```

3.144 $\int \sqrt{1+x^2} \sinh^{-1}(x) \, dx$

Optimal. Leaf size=32

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

```
[Out] -x^2/4 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rubi [A] time = 0.0290022, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5682, 5675, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{x^2+1}x\sinh^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] -x^2/4 + (x*Sqrt[1 + x^2]*ArcSinh[x])/2 + ArcSinh[x]^2/4
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+x^2}\sinh^{-1}(x)\,dx = \frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x) - \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{x^2}{4} + \frac{1}{2}x\sqrt{1+x^2}\sinh^{-1}(x) + \frac{1}{4}\sinh^{-1}(x)^2$$

Mathematica [A] time = 0.0128693, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-x^2 + 2\sqrt{x^2+1}x\sinh^{-1}(x) + \sinh^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*ArcSinh[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 + x^2]*ArcSinh[x] + ArcSinh[x]^2)/4
```

Maple [A] time = 0., size = 26, normalized size = 0.8

$$\frac{x\mathrm{Arcsinh}(x)}{2}\sqrt{x^2+1} + \frac{(\mathrm{Arcsinh}(x))^2}{4} - \frac{x^2}{4} - \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x)*(x^2+1)^(1/2),x)
```

```
[Out] 1/2*x*arcsinh(x)*(x^2+1)^(1/2)+1/4*arcsinh(x)^2-1/4*x^2-1/4
```

Maxima [A] time = 1.65747, size = 38, normalized size = 1.19

$$-\frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{x^2+1}x + \mathrm{arsinh}(x)\right)\mathrm{arsinh}(x) - \frac{1}{4}\,\mathrm{arsinh}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(x^2 + 1)*x + arcsinh(x))*arcsinh(x) - 1/4*arcsinh(x)^2
```

Fricas [A] time = 2.40515, size = 115, normalized size = 3.59

$$\frac{1}{2}\sqrt{x^2+1}x\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2+\frac{1}{4}\log\left(x+\sqrt{x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - 1/4*x^2 + 1/4*log(x + sqrt(x^2
 + 1))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^2+1}\operatorname{asinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x)*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*asinh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^2+1}\operatorname{arsinh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x)*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)*arcsinh(x), x)
```

3.145 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=215

$$\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15c^4d}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{bx^5\sqrt{c^2x^2+}}{25c\sqrt{c^2dx^2}}$$

```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (4*b*x^3*Sqrt[1 +
 c^2*x^2])/(45*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25*c*S
qrt[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*
d) - (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*d) + (x^4*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*d)
```

Rubi [A] time = 0.264369, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}-\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15c^4d}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}-\frac{bx^5\sqrt{c^2x^2+}}{25c\sqrt{c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```


```
[Out] (-8*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (4*b*x^3*Sqrt[1 +
 c^2*x^2])/(45*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^5*Sqrt[1 + c^2*x^2])/(25*c*S
qrt[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^6*
d) - (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*c^4*d) + (x^4*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2*d)
```


Rule *5758*

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^4\,dx}{5c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c^2d}$$

$$= \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d} - \frac{4x^2\sqrt{d+c^2dx^2}\left(a\right.}{15c^4}$$

$$= -\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{d+c^2dx^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15c^6d}$$

Mathematica [A] time = 0.171129, size = 119, normalized size = 0.55

$$\frac{15a\left(3c^6x^6-c^4x^4+4c^2x^2+8\right)+bcx\sqrt{c^2x^2+1}\left(-9c^4x^4+20c^2x^2-120\right)+15b\left(3c^6x^6-c^4x^4+4c^2x^2+8\right)\sinh^{-1}(cx)}{225c^6\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4*x^4) + 15*a*(8 + 4*c^2*
x^2 - c^4*x^4 + 3*c^6*x^6) + 15*b*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*Arc
Sinh[c*x])/(225*c^6*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.214, size = 625, normalized size = 2.9

$$a\left(\frac{x^4}{5c^2d}\sqrt{c^2dx^2+d}-\frac{4}{5c^2}\left(\frac{x^2}{3c^2d}\sqrt{c^2dx^2+d}-\frac{2}{3dc^4}\sqrt{c^2dx^2+d}\right)\right)+b\left(\frac{-1+5\,\mathrm{Arcsinh}\,(cx)}{800\,dc^6\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(16\,c^6x^6+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(
1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6
*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+1
3*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-
5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^6/
d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*
(1+arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4
*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsin
h(c*x))/c^6/d/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^
5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*
x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5436, size = 355, normalized size = 1.65

$$\frac{15\left(3\,bc^{6}x^{6}-bc^{4}x^{4}+4\,bc^{2}x^{2}+8\,b\right)\sqrt{c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(45\,ac^{6}x^{6}-15\,ac^{4}x^{4}+60\,ac^{2}x^{2}-\left(9\,bc^{5}x^{5}-20\,bc\right.\right.}{225\left(c^{8}dx^{2}+c^{6}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (45*a*c^6*x^6 - 15*a*c^4*x^4 + 60*a*c^2*x^2
 - (9*b*c^5*x^5 - 20*b*c^3*x^3 + 120*b*c*x)*sqrt(c^2*x^2 + 1) + 120*a)*sqrt
(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\sqrt{d\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^{5}}{\sqrt{c^{2}dx^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^5/sqrt(c^2*d*x^2 + d), x)
```

3.146 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=192

$$\frac{x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{4c^2d}-\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d}+\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^5\sqrt{c^2dx^2+d}}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+}}$$

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 +
c^2*x^2])/(16*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*c^
2*d) + (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[d + c^2*
d*x^2])
```

Rubi [A] time = 0.252467, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5677, 5675, 30}

$$\frac{x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{4c^2d}-\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d}+\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^5\sqrt{c^2dx^2+d}}-\frac{bx^4\sqrt{c^2x^2+1}}{16c\sqrt{c^2dx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (3*b*x^2*Sqrt[1 + c^2*x^2])/(16*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 +
c^2*x^2])/(16*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x]))/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*c^
2*d) + (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^5*Sqrt[d + c^2*
d*x^2])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{4c^2d} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3\,dx}{4c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{4c^2d} +$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}\left(a\right.}{4c^2}$$

$$= \frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}\left(a\right.}{4c^2}$$

Mathematica [A] time = 0.6033, size = 151, normalized size = 0.79

$$\frac{\frac{16acx\left(2c^2x^2-3\right)\sqrt{c^2dx^2+d}}{d} + \frac{48a\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{\sqrt{d}} + \frac{b\sqrt{c^2x^2+1}\left(4\sinh^{-1}(cx)\left(6\sinh^{-1}(cx)-8\sinh\left(2\sinh^{-1}(cx)\right)+\sinh\left(4\sinh^{-1}(cx)\right)\right)+16\cosh\left(2\sinh\right.}{\sqrt{c^2dx^2+d}}}{128c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] ((16*a*c*x*(-3 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2])/d + (48*a*Log[c*d*x + Sqrt
[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh
[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*A
rcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/(128*c^5)
```

Maple [B] time = 0.253, size = 347, normalized size = 1.8

$$\frac{ax^3}{4\,c^2d}\sqrt{c^2dx^2+d}-\frac{3\,ax}{8\,c^4d}\sqrt{c^2dx^2+d}+\frac{3\,a}{8\,c^4}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\mathrm{Arcsinh}\,(cx)\,x^5}{4\,d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-\frac{bx}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3/8*a
/c^4*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b*(d*(
c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/16*b*(d*(c^2*x^2+1))^(1/
2)/c/d/(c^2*x^2+1)^(1/2)*x^4-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2+1)*
arcsinh(c*x)*x^3+3/16*b*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+1)^(1/2)*x^2-3
/8*b*(d*(c^2*x^2+1))^(1/2)/c^4/d/(c^2*x^2+1)*arcsinh(c*x)*x+3/16*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^2+15/128*b*(d*(c^2*x^2+1
))^(1/2)/c^5/d/(c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^4\,\mathrm{arsinh}\,(cx)+ax^4}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{asinh}(cx))}{\sqrt{d (c^2x^2 + 1)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{\sqrt{c^2dx^2 + d}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/sqrt(c^2*d*x^2 + d), x)
```

3.147 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=142

$$\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c^2
*x^2])/(9*c*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/(3*c^4*d) + (x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2*d)
```

Rubi [A] time = 0.158251, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d}-\frac{bx^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c^2
*x^2])/(9*c*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/(3*c^4*d) + (x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2*d)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^2\,dx}{3c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^2d} + \Big(2$$

$$= \frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\right.}{3c^2d}$$

Mathematica [A] time = 0.13315, size = 93, normalized size = 0.65

$$\frac{3a\left(c^4x^4-c^2x^2-2\right)+bcx\sqrt{c^2x^2+1}\left(6-c^2x^2\right)+3b\left(c^4x^4-c^2x^2-2\right)\sinh^{-1}(cx)}{9c^4\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b*c*x*(6 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-2 - c^2*x^2 + c^4*x^4) + 3*b
*(-2 - c^2*x^2 + c^4*x^4)*ArcSinh[c*x])/(9*c^4*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.158, size = 358, normalized size = 2.5

$$a\left(\frac{x^2}{3c^2d}\sqrt{c^2dx^2+d}-\frac{2}{3dc^4}\sqrt{c^2dx^2+d}\right)+b\left(\frac{-1+3\operatorname{Arcsinh}(cx)}{72dc^4(c^2x^2+1)}\sqrt{d(c^2x^2+1)}\left(4c^4x^4+4c^3x^3\sqrt{c^2x^2+1}+5c^2x^2+3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b*(1/72
*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c
*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcs
inh(c*x))/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3
*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/
c^4/d/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.514, size = 278, normalized size = 1.96

$$\frac{3\left(bc^4x^4-bc^2x^2-2b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(3ac^4x^4-3ac^2x^2-\left(bc^3x^3-6bcx\right)\sqrt{c^2x^2+1}-6a\right)\sqrt{c^2dx^2}}{9\left(c^6dx^2+c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - b*c^2*x^2 - 2*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + (3*a*c^4*x^4 - 3*a*c^2*x^2 - (b*c^3*x^3 - 6*b*c*x)*sqrt(c^2*x^
2 + 1) - 6*a)*sqrt(c^2*d*x^2 + d))/(c^6*d*x^2 + c^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)x^3}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/sqrt(c^2*d*x^2 + d), x)
```

3.148 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=119

$$\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.145791, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5758, 5677, 5675, 30}

$$\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x]))/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
```

```
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^2}-\frac{\left(b\sqrt{1+c^2x^2}\right)\int x\,dx}{2c\sqrt{d+c^2dx^2}}\\ &=-\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{2c^2\sqrt{d+c^2dx^2}}\\ &=-\frac{bx^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^2d}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.723047, size = 121, normalized size = 1.02

$$-\frac{-\frac{4acx\sqrt{c^2dx^2+d}}{d}+\frac{4a\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{\sqrt{d}}+\frac{b\sqrt{c^2x^2+1}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-\sinh\left(2\sinh^{-1}(cx)\right)\right)+\cosh\left(2\sinh^{-1}(cx)\right)\right)}{\sqrt{c^2dx^2+d}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((-4*a*c*x*Sqrt[d + c^2*d*x^2])/d + (4*a*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c
*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])))/Sqrt[d + c^2*d*x^2])/(8*c^3)
```

Maple [B] time = 0.171, size = 247, normalized size = 2.1

$$\frac{ax}{2c^2d}\sqrt{c^2dx^2+d}-\frac{a}{2c^2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}-\frac{b\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{4c^3d}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+\frac{b\mathrm{Arcsinh}\left(\right.}{2d\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d
*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
c^3/d*arcsinh(c*x)^2+1/2*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)
*x^3-1/4*b*(d*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*x^2+1/2*b*(d*(c^2*x^
2+1))^(1/2)/c^2/d/(c^2*x^2+1)*arcsinh(c*x)*x-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^
3/d/(c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arsinh}\left(cx\right)+ax^2}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsinh(c*x) + a*x^2)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{asinh}(cx))}{\sqrt{d(c^2x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^2}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/sqrt(c^2*d*x^2 + d), x)
```

3.149 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(c^2*d)
```

Rubi [A] time = 0.0609169, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 8}

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```


```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(c^2*d)
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2d} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int 1\,dx}{c\sqrt{d+c^2dx^2}}$$
$$= -\frac{bx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^2d}$$

Mathematica [A] time = 0.102264, size = 74, normalized size = 1.16

$$\frac{\sqrt{c^2dx^2+d}\left(a\sqrt{c^2x^2+1}+b\sqrt{c^2x^2+1}\sinh^{-1}(cx)-bcx\right)}{c^2d\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) + a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]))/(c^2*d*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.086, size = 148, normalized size = 2.3

$$\frac{a}{c^2d}\sqrt{c^2dx^2+d}+b\left(\frac{-1+\mathrm{Arcsinh}\,(cx)}{2\,c^2d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2+cx\sqrt{c^2x^2+1}+1\right)+\frac{1+\mathrm{Arcsinh}\,(cx)}{2\,c^2d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a/c^2/d*(c^2*d*x^2+d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/
2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/d/(c^2*x^2+1))
```

Maxima [A] time = 1.14788, size = 74, normalized size = 1.16

$$-\frac{bx}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}b\,\mathrm{arsinh}\,(cx)}{c^2d}+\frac{\sqrt{c^2dx^2+d}a}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*
x^2 + d)*a/(c^2*d)
```

Fricas [A] time = 2.39403, size = 205, normalized size = 3.2

$$\frac{\left(bc^2x^2+b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2x^2-\sqrt{c^2x^2+1}bcx+a\right)\sqrt{c^2dx^2+d}}{c^4dx^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*c^2*x^2 + b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*
x^2 - sqrt(c^2*x^2 + 1)*b*c*x + a)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}\left(cx\right)+a)x}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/sqrt(c^2*d*x^2 + d), x)
```

3.150 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.0569614, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.0485878, size = 48, normalized size = 1.02

$$\frac{\sqrt{c^2x^2+1}\sinh^{-1}(cx)\left(2a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(2*a + b*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d
*x^2])
```

Maple [A] time = 0.036, size = 77, normalized size = 1.6

$$a\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\left(\mathrm{Arcsinh}\,(cx)\right)^2}{2\,dc}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{asinh}\,(cx)}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\mathrm{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

3.151 $\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{\sqrt{c^2dx^2+d}}$$

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d
+ c^2*d*x^2] - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[d + c
^2*d*x^2] + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d
*x^2]
```

Rubi [A] time = 0.188167, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5764, 5760, 4182, 2279, 2391}

$$-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```


```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d
+ c^2*d*x^2] - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[d + c
^2*d*x^2] + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d
*x^2]
```


Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
```

```
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx,\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{s}\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\mathbf{Li}_2\left(-e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c}}{}$$

Mathematica [A] time = 0.361731, size = 129, normalized size = 1.06

$$\frac{b\sqrt{c^2x^2+1}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\left(\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\log\left(e^{-\sinh^{-1}(cx)}\right.\right.\right.}{\sqrt{d\left(c^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/Sqrt[d] + (
b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(
-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]))/Sqrt[d*(1 + c^2*x^2)]
```

Maple [A] time = 0.124, size = 234, normalized size = 1.9

$$-a\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)\frac{1}{\sqrt{d}}-\frac{b\text{Arcsinh}\,(cx)}{d}\sqrt{d\left(c^2x^2+1\right)}\ln\left(1+cx+\sqrt{c^2x^2+1}\right)\frac{1}{\sqrt{c^2x^2+1}}-\frac{b}{d}\sqrt{d\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))
+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^2dx^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^3 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{asinh}\,(cx)}{x\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{c^2dx^2+d}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x), x)
```

3.152 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=63

$$\frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^2
]*Log[x])/Sqrt[d + c^2*d*x^2]
```

Rubi [A] time = 0.0891756, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5723, 29}

$$\frac{bc\sqrt{c^2x^2+1}\log(x)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[1 + c^2*x^2
]*Log[x])/Sqrt[d + c^2*d*x^2]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} + \frac{bc\sqrt{1+c^2x^2}\log(x)}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.158615, size = 67, normalized size = 1.06

$$\frac{bc\log(x)\sqrt{d\left(c^2x^2+1\right)}}{d\sqrt{c^2x^2+1}} - \frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) + (b*c*Sqrt[d*(1 + c^2*
x^2)]*Log[x])/(d*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.118, size = 183, normalized size = 2.9

$$-\frac{a}{dx}\sqrt{c^2dx^2+d} - \frac{b\mathrm{Arcsinh}\,(cx)\,c}{d}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}} - \frac{b\mathrm{Arcsinh}\,(cx)\,xc^2}{d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)} - \frac{b\mathrm{Arcsinh}\,(cx)}{\left(c^2x^2+1\right)dx}\sqrt{d\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d/x*(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcs
inh(c*x)*c-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/d*x*c^2-b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/d/x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.12669, size = 301, normalized size = 4.78

$$\frac{bc\sqrt{d}x\log\left(\frac{c^2dx^6+c^2dx^2+dx^4+\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}\left(x^4-1\right)\sqrt{d}+d}{c^2x^4+x^2}\right)-2\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2+d}a}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 + d*x^4 + sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*x^4 + x^2)) - 2*sqrt(c^2*d*x
^2 + d)*b*log(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*a)/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{asinh}(cx)}{x^2\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

3.153 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{c^2\ldots}}{\ldots}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*x*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2]) - (b*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.291936, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5764, 5760, 4182, 2279, 2391, 30}

$$\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{c^2\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*x*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]
)*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[2, -E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2]) - (b*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} - \frac{1}{2}c^2\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx\right.}{2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sin}\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sin}\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sin}\right.}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 3.02521, size = 229, normalized size = 1.13

$$\frac{bc^2d^2\left(c^2x^2+1\right)^{3/2}\left(-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(e^{-\sinh^{-1}(cx)}+1\right)+2\tanh\left(\frac{1}{2}\sin\right.\right.}{\left(c^2dx^2+d\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a*Sqrt[d + c^2*d*x^2])/x^2 - 4*a*c^2*Sqrt[d]*Log[x] + 4*a*c^2*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-2*C
oth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]*
Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 4*
PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[
c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2
))/(8*d)
```

Maple [A] time = 0.187, size = 380, normalized size = 1.9

$$-\frac{a}{2\,dx^2}\sqrt{c^2dx^2+d}+\frac{ac^2}{2}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)\frac{1}{\sqrt{d}}-\frac{b\text{Arcsinh}\,(cx)\,c^2}{2\,d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-\frac{bc}{2\,dx}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)-1/2*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*c^
2-1/2*b*(d*(c^2*x^2+1))^(1/2)/x/d/(c^2*x^2+1)^(1/2)*c-1/2*b*(d*(c^2*x^2+1))
^(1/2)/x^2/d/(c^2*x^2+1)*arcsinh(c*x)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2+1/2*b*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(
1/2))*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^
2*x^2+1)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{asinh}(cx)}{x^3\sqrt{d(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{c^2dx^2+d}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

3.154 $\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=141

$$\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}-\frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{c^2dx^2+d}}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*x^2*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[d + c^2*d*x^2
])
```

Rubi [A] time = 0.18419, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5747, 5723, 29, 30}

$$\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2+1}}{6x^2\sqrt{c^2dx^2+d}}-\frac{2bc^3\sqrt{c^2x^2+1}\log(x)}{3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*x^2*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x]))/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/(3*d*x) - (2*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*Sqrt[d + c^2*d*x^2
])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}-\frac{1}{3}\left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{d+c^2dx^2}}\,dx+\frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3}\,dx}{3\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{2c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{\left(2bc^3\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6x^2\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^3}+\frac{2c^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx}-\frac{2bc^3}{3}$$

Mathematica [A] time = 0.209244, size = 135, normalized size = 0.96

$$\frac{2a\left(2c^4x^4+c^2x^2-1\right)+bcx\sqrt{c^2x^2+1}\left(6c^2x^2-1\right)+2b\left(2c^4x^4+c^2x^2-1\right)\sinh^{-1}(cx)}{6x^3\sqrt{c^2dx^2+d}}-\frac{2bc^3\log(x)\sqrt{d\left(c^2x^2+1\right)}}{3d\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b*c*x*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2) + 2*a*(-1 + c^2*x^2 + 2*c^4*x^4)
+ 2*b*(-1 + c^2*x^2 + 2*c^4*x^4)*ArcSinh[c*x])/(6*x^3*Sqrt[d + c^2*d*x^2])
- (2*b*c^3*Sqrt[d*(1 + c^2*x^2)]*Log[x])/(3*d*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.17, size = 791, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a/d/x^3*(c^2*d*x^2+d)^(1/2)+2/3*a*c^2/d/x*(c^2*d*x^2+d)^(1/2)+4/3*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*c^3+2/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8-2/3*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*c^6+2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*c^6-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5+1/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^4*x^4+2*c^2*x^2-1)/d*x*(c^2*x^2+1)*c^4+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c^4+2/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-1/3*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c^4-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)^(1/2)-4/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x
^4+2*c^2*x^2-1)/d/x*arcsinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+2*c^2*x^2-1)/d/x^2*c*(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+2*c^2*x^2-1)/d/x^3*arcsinh(c*x)-2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.18487, size = 477, normalized size = 3.38

$$\frac{2\left(2\,bc^4x^4+bc^2x^2-b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(bc^5x^5+bc^3x^3\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2+dx^4-\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}\left(x^4-1\right)}{c^2x^4+x^2}\right.}{6\left(c^2dx^5+dx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*b*c^4*x^4 + b*c^2*x^2 - b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*(b*c^5*x^5 + b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 +
 d*x^4 - sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) + d)/(c^2*
x^4 + x^2)) + (4*a*c^4*x^4 + 2*a*c^2*x^2 + (b*c*x^3 - b*c*x)*sqrt(c^2*x^2 +
 1) - 2*a)*sqrt(c^2*d*x^2 + d))/(c^2*d*x^5 + d*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asinh}(cx)}{x^4 \sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\sqrt{c^2dx^2 + d}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

3.155 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c^6d^3}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2}-\frac{a+b\sinh^{-1}(cx)}{c^6d\sqrt{c^2dx^2+d}}-\frac{bx^3\sqrt{c^2dx^2+d}}{9c^3d^2\sqrt{c^2x^2+1}}+\frac{5bx\sqrt{c^2dx}}{3c^5d^2\sqrt{c^2x}}$$

```
[Out] (5*b*x*Sqrt[d + c^2*d*x^2])/(3*c^5*d^2*Sqrt[1 + c^2*x^2]) - (b*x^3*Sqrt[d +
 c^2*d*x^2])/(9*c^3*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(c^6*d*Sq
rt[d + c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^6*d^2)
 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c^6*d^3) + (b*Sqrt[d + c
^2*d*x^2]*ArcTan[c*x])/(c^6*d^2*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.293835, antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5751, 5758, 5717, 8, 30, 302, 203}

$$\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2}-\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^6d^2}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx^3\sqrt{c^2x^2+1}}{9c^3d\sqrt{c^2dx^2+d}}+\frac{5bx}{3c^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (5*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^3*Sqrt[1 + c
^2*x^2])/(9*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x]))/(c^2*d*
Sqrt[d + c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^6*
d^2) + (4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4*d^2) + (b*Sq
rt[1 + c^2*x^2]*ArcTan[c*x])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

```
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5 \left(a + b \sinh^{-1}(cx)\right)}{\left(d + c^2dx^2\right)^{3/2}} dx = -\frac{x^4 \left(a + b \sinh^{-1}(cx)\right)}{c^2d\sqrt{d + c^2dx^2}} + \frac{4 \int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}} dx}{c^2d} + \frac{\left(b\sqrt{1 + c^2x^2}\right) \int \frac{x^4}{1+c^2x^2} dx}{cd\sqrt{d + c^2dx^2}}$$

$$= -\frac{x^4 \left(a + b \sinh^{-1}(cx)\right)}{c^2d\sqrt{d + c^2dx^2}} + \frac{4x^2\sqrt{d + c^2dx^2} \left(a + b \sinh^{-1}(cx)\right)}{3c^4d^2} - \frac{8 \int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}} dx}{3c^4d} - \frac{(4}{}$$

$$= -\frac{bx\sqrt{1 + c^2x^2}}{c^5d\sqrt{d + c^2dx^2}} - \frac{bx^3\sqrt{1 + c^2x^2}}{9c^3d\sqrt{d + c^2dx^2}} - \frac{x^4 \left(a + b \sinh^{-1}(cx)\right)}{c^2d\sqrt{d + c^2dx^2}} - \frac{8\sqrt{d + c^2dx^2} \left(a + b \sinh^{-1}\right.}{3c^6d^2}$$

$$= \frac{5bx\sqrt{1 + c^2x^2}}{3c^5d\sqrt{d + c^2dx^2}} - \frac{bx^3\sqrt{1 + c^2x^2}}{9c^3d\sqrt{d + c^2dx^2}} - \frac{x^4 \left(a + b \sinh^{-1}(cx)\right)}{c^2d\sqrt{d + c^2dx^2}} - \frac{8\sqrt{d + c^2dx^2} \left(a + b \sinh^{-1}\right.}{3c^6d^2}$$

Mathematica [A] time = 0.264626, size = 148, normalized size = 0.7

$$\frac{\sqrt{c^2dx^2 + d}\left(3a\left(c^4x^4 - 4c^2x^2 - 8\right) + bcx\sqrt{c^2x^2 + 1}\left(15 - c^2x^2\right) + 3b\left(c^4x^4 - 4c^2x^2 - 8\right)\sinh^{-1}(cx)\right)}{9c^6d^2\left(c^2x^2 + 1\right)} + \frac{b\sqrt{d\left(c^2x^2 + 1\right)}\,\text{ta}}{c^6d^2\sqrt{c^2x^2 +}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x*(15 - c^2*x^2)*Sqrt[1 + c^2*x^2] + 3*a*(-8 - 4*
c^2*x^2 + c^4*x^4) + 3*b*(-8 - 4*c^2*x^2 + c^4*x^4)*ArcSinh[c*x]))/(9*c^6*d
^2*(1 + c^2*x^2)) + (b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^6*d^2*Sqrt[1 +
 c^2*x^2])
```

Maple [C] time = 0.206, size = 362, normalized size = 1.7

$$\frac{ax^4}{3c^2d}\frac{1}{\sqrt{c^2dx^2 + d}} - \frac{4\,ax^2}{3\,dc^4}\frac{1}{\sqrt{c^2dx^2 + d}} - \frac{8\,a}{3\,dc^6}\frac{1}{\sqrt{c^2dx^2 + d}} + \frac{b\text{Arcsinh}\,(cx)\,x^4}{3\,c^2d^2\left(c^2x^2 + 1\right)}\sqrt{d\left(c^2x^2 + 1\right)} - \frac{bx^3}{9\,c^3d^2}\sqrt{d\left(c^2x^2 + 1\right)}\frac{}{\sqrt{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/3*a*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3*a/c^4*x^2/d/(c^2*d*x^2+d)^(1/2)-8/3
*a/c^6/d/(c^2*d*x^2+d)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1
```

```
)*arcsinh(c*x)*x^4-1/9*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^
3-4/3*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+5/3*b*(d
*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*x+I*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)+I)-I*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)-8/3*b*(d*(c^2*x^2
+1))^(1/2)/c^6/d^2/(c^2*x^2+1)*arcsinh(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.61838, size = 436, normalized size = 2.06

$$-\frac{9\left(bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)-6\left(bc^4x^4-4\,bc^2x^2-8\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)-2\left(3\,ac^4x^4\right.}{18\left(c^8d^2x^2+c^6d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18*(9*(b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2
+ 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 6*(b*c^4*x^4 - 4*b*c^2*x^2 - 8*b)*sqrt(
c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(3*a*c^4*x^4 - 12*a*c^2*x^2
 - (b*c^3*x^3 - 15*b*c*x)*sqrt(c^2*x^2 + 1) - 24*a)*sqrt(c^2*d*x^2 + d))/(c
^8*d^2*x^2 + c^6*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^5}{(c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^5/(c^2*d*x^2 + d)^(3/2), x)
```

3.156 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=206

$$\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^5d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c^3d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2}\cdots}{2c^5\cdots}$$

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcS
inh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x]))/(2*c^4*d^2) - (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^
5*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.282246, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5751, 5758, 5677, 5675, 30, 266, 43}

$$\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^5d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{4c^3d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2}\cdots}{2c^5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -(b*x^2*Sqrt[1 + c^2*x^2])/(4*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcS
inh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x]))/(2*c^4*d^2) - (3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*c^
5*d*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
```

n, 0] && LtQ[p, -1] && GtQ[m, 1]

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4 \left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2} - \frac{3\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{2c^4d} - \frac{\left(3b\sqrt{1\right.}{2c^3d}$$

$$= -\frac{3bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2} - \frac{\left(3\sqrt{1+c^2\right.}{2c}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}}{4c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2} - \frac{3\sqrt{1+c^2x}}{4bc}$$

Mathematica [A] time = 0.471689, size = 161, normalized size = 0.78

$$\frac{4ac\sqrt{d}x\left(c^2x^2+3\right)-12a\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+b\sqrt{d}\left(8cx\sinh^{-1}(cx)-\sqrt{c^2x^2+1}\left(4\log\left(c^2x^2+1\right)+6\right.\right.}{8c^5d^{3/2}\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[
d]*Sqrt[d + c^2*d*x^2]] + b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]
*(6*ArcSinh[c*x]^2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[
c*x]*Sinh[2*ArcSinh[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.246, size = 366, normalized size = 1.8

$$\frac{ax^3}{2c^2d}\frac{1}{\sqrt{c^2dx^2+d}} + \frac{3ax}{2dc^4}\frac{1}{\sqrt{c^2dx^2+d}} - \frac{3a}{2dc^4}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}} - \frac{3b\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{4c^5d^2}\sqrt{d\left(c^2x^2+1\right)}\frac{}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/2*a*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3/2*a
/c^4/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-3/4*b*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^2+1/2*b*(d*(c^2*
x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/4*b*(d*(c^2*x^2+1))^(1
/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*
x^2+1)*arcsinh(c*x)*x+b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)-1/8*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^4 \operatorname{arsinh}(cx) + ax^4\right)\sqrt{c^2dx^2 + d}}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a + b \operatorname{asinh}(cx)\right)}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^4}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^(3/2), x)
```

3.157 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}+\frac{a+b\sinh^{-1}(cx)}{c^4d\sqrt{c^2dx^2+d}}-\frac{bx\sqrt{c^2dx^2+d}}{c^3d^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\tan^{-1}(cx)}{c^4d^2\sqrt{c^2x^2+1}}$$

```
[Out] -((b*x*Sqrt[d + c^2*d*x^2])/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c
*x])/(c^4*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(c^4*d^2) - (b*Sqrt[d + c^2*d*x^2]*ArcTan[c*x])/(c^4*d^2*Sqrt[1 + c^2*x^
2])
```

Rubi [A] time = 0.181724, antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5751, 5717, 8, 321, 203}

$$\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx\sqrt{c^2x^2+1}}{c^3d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{c^4d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2])) - (x^2*(a + b*ArcSin
h[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x]))/(c^4*d^2) - (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(c^4*d*Sqrt[d + c^2*
d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^2}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{bx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\ldots}{c^3d\sqrt{d+\ldots}}$$

$$= -\frac{bx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2} - \frac{b\sqrt{1+c^2x^2}\,\mathrm{t}\ldots}{c^4d\sqrt{d+\ldots}}$$

Mathematica [A] time = 0.242831, size = 143, normalized size = 1.05

$$\frac{\sqrt{c^2dx^2+d}\left(a\sqrt{c^2x^2+1}\left(c^2x^2+2\right)-b\left(c^3x^3+cx\right)+b\sqrt{c^2x^2+1}\left(c^2x^2+2\right)\sinh^{-1}(cx)\right)}{c^4d^2\left(c^2x^2+1\right)^{3/2}}-\frac{b\sqrt{d\left(c^2x^2+1\right)}\tan^{-1}(cx)}{c^4d^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2) - b*(c*x + c^3*x^3)
 + b*Sqrt[1 + c^2*x^2]*(2 + c^2*x^2)*ArcSinh[c*x]))/(c^4*d^2*(1 + c^2*x^2)^
(3/2)) - (b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^4*d^2*Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.159, size = 260, normalized size = 1.9

$$\frac{ax^2}{c^2d}\frac{1}{\sqrt{c^2dx^2+d}}+2\frac{a}{dc^4\sqrt{c^2dx^2+d}}+\frac{b\mathrm{Arcsinh}\,(cx)\,x^2}{c^2d^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-\frac{bx}{c^3d^2}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+2\frac{b\sqrt{d\left(c^2x^2+\right.}}{d^2c^4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2*a/d/c^4/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2
+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^3
/d^2/(c^2*x^2+1)^(1/2)*x+2*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcs
inh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*ln(c*x+(c^2*x^
2+1)^(1/2)-I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*ln(c*x+(c
^2*x^2+1)^(1/2)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.3151, size = 365, normalized size = 2.68

$$\frac{\left(bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+2\left(bc^2x^2+2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(ac^2x^2-\sqrt{c^2x^2+1}bc\right.}{2\left(c^6d^2x^2+c^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)
*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*(b*c^2*x^2 + 2*b)*sqrt(c^2*d*x^2 + d)*log
(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*c^2*x^2 - sqrt(c^2*x^2 + 1)*b*c*x + 2*a)*s
qrt(c^2*d*x^2 + d))/(c^6*d^2*x^2 + c^4*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}\left(cx\right)+a)x^3}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(c^2*d*x^2 + d)^(3/2), x)
```

3.158 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^3d\sqrt{c^2dx^2+d}}$$

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.168188, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5751, 5677, 5675, 260}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*ArcSinh[c*x]))/(c^2*d*Sqrt[d + c^2*d*x^2])) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^2)/(2*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2])/(2*c^3*d*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx &= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\\ &= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{c^2d\sqrt{d+c^2dx^2}}\\ &= -\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^3d\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.195132, size = 146, normalized size = 1.12

$$-\frac{ax\sqrt{d\left(c^2x^2+1\right)}}{c^2d^2\left(c^2x^2+1\right)}+\frac{a\log\left(\sqrt{d}\sqrt{d\left(c^2x^2+1\right)}+cdx\right)}{c^3d^{3/2}}+\frac{b\left(\sqrt{c^2x^2+1}\left(2\log\left(\sqrt{c^2x^2+1}\right)+\sinh^{-1}(cx)^2\right)-2cx\sinh^{-1}(cx\right.}{2c^3d\sqrt{d\left(c^2x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a*x*Sqrt[d*(1 + c^2*x^2)])/(c^2*d^2*(1 + c^2*x^2))) + (b*(-2*c*x*ArcSinh
[c*x] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]^2 + 2*Log[Sqrt[1 + c^2*x^2]])))/(2*
c^3*d*Sqrt[d*(1 + c^2*x^2)]) + (a*Log[c*d*x + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]
```

```
])/(c^3*d^(3/2))
```

Maple [A] time = 0.151, size = 232, normalized size = 1.8

$$-\frac{ax}{c^2d}\frac{1}{\sqrt{c^2dx^2+d}}+\frac{a}{c^2d}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b\left(\mathrm{Arcsinh}\,(cx)\right)^2}{2\,c^3d^2}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}-\frac{b\mathrm{Arcsinh}\,(cx}{c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a/c^2/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d
^2*arcsinh(c*x)^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh
(c*x)-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2/(c^2*x^2+1)*x+b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(bx^2\,\mathrm{arsinh}\,(cx)+ax^2\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^4*d^2*x^4 + 2*
c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{asinh}(cx))}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^2}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

3.159 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])) + (b*Sqrt[1 + c^2*x^2]*
ArcTan[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.0703006, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5717, 203}

$$\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])) + (b*Sqrt[1 + c^2*x^2]*
ArcTan[c*x])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$
$$= -\frac{a+b\sinh^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{c^2d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.162319, size = 82, normalized size = 1.17

$$\frac{b\sqrt{d\left(c^2x^2+1\right)}\tan^{-1}(cx)}{c^2d^2\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^2*d^2*(1 + c^2*x^2))) + (b*
Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(c^2*d^2*Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.083, size = 164, normalized size = 2.3

$$-\frac{a}{c^2d}\frac{1}{\sqrt{c^2dx^2+d}}-\frac{b\mathrm{Arcsinh}\,(cx)}{c^2d^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}+\frac{ib}{c^2d^2}\sqrt{d\left(c^2x^2+1\right)}\ln\left(cx+\sqrt{c^2x^2+1}+i\right)\frac{1}{\sqrt{c^2x^2+1}}-\frac{ib}{c^2d^2}\sqrt{d\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a/c^2/d/(c^2*d*x^2+d)^(1/2)-b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*ar
csinh(c*x)+I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+(c^2*
x^2+1)^(1/2)+I)-I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+
(c^2*x^2+1)^(1/2)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\left(\frac{-\operatorname{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)}{c^2 d^{\frac{3}{2}}} - \frac{\log\left(cx + \sqrt{c^2x^2+1}\right)}{\sqrt{c^2x^2+1}c^2 d^{\frac{3}{2}}} - \int \frac{1}{c^5 d^{\frac{3}{2}} x^4 + c^3 d^{\frac{3}{2}} x^2 + \left(c^4 d^{\frac{3}{2}} x^3 + c^2 d^{\frac{3}{2}} x\right)\sqrt{c^2x^2+1}}\,dx\right) - \frac{a}{\sqrt{c^2dx^2+d}c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(integrate(1/(sqrt(c^2*x^2 + 1)*x), x)/(c^2*d^(3/2)) - log(c*x + sqrt(c^2
*x^2 + 1))/(sqrt(c^2*x^2 + 1)*c^2*d^(3/2)) - integrate(1/(c^5*d^(3/2)*x^4 +
 c^3*d^(3/2)*x^2 + (c^4*d^(3/2)*x^3 + c^2*d^(3/2)*x)*sqrt(c^2*x^2 + 1)), x)
) - a/(sqrt(c^2*d*x^2 + d)*c^2*d)
```

Fricas [B] time = 3.18023, size = 288, normalized size = 4.11

$$-\frac{\left(bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right) + 2\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right) + 2\sqrt{c^2dx^2+d}a}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1
)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2
*x^2 + 1)) + 2*sqrt(c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + b\operatorname{asinh}(cx)\right)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x}{(c^2dx^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(3/2), x)
```

3.160 $\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log
[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.0382054, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5687, 260}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2cd\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log
[1 + c^2*x^2])/(2*c*d*Sqrt[d + c^2*d*x^2])
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$
$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2cd\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.159034, size = 100, normalized size = 1.32

$$\frac{\sqrt{c^2dx^2+d}\left(2acx\sqrt{c^2x^2+1}-\left(bc^2x^2+b\right)\log\left(c^2x^2+1\right)+2bcx\sqrt{c^2x^2+1}\sinh^{-1}(cx)\right)}{2cd^2\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(2*a*c*x*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - (b + b*c^2*x^2)*Log[1 + c^2*x^2]))/(2*c*d^2*(1 + c^2*x^2)^(
3/2))
```

Maple [B] time = 0.085, size = 143, normalized size = 1.9

$$\frac{ax}{d}\frac{1}{\sqrt{c^2dx^2+d}}+\frac{b\mathrm{Arcsinh}\,(cx)}{cd^2}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+\frac{b\mathrm{Arcsinh}\,(cx)\,x}{d^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-\frac{b}{cd^2}\sqrt{d\left(c^2x^2+1\right)}\ln\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*x/d/(c^2*d*x^2+d)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*a
rcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^2*x^2+1)*x-b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [A] time = 1.24511, size = 88, normalized size = 1.16

$$-\frac{bc\sqrt{\frac{1}{c^4d}}\log\left(x^2+\frac{1}{c^2}\right)}{2\,d}+\frac{bx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dx^2+d}d}+\frac{ax}{\sqrt{c^2dx^2+d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*sqrt(1/(c^4*d))*log(x^2 + 1/c^2)/d + b*x*arcsinh(c*x)/(sqrt(c^2*d*
x^2 + d)*d) + a*x/(sqrt(c^2*d*x^2 + d)*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^4 + 2*c^2*d^2*
x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{asinh}\,(cx)}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(3/2), x)
```

3.161 $\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\text{s}}\right.}{d\sqrt{c}}$$

```
[Out] (a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan[
c*x])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*A
rcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Poly
Log[2, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.30334, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5755, 5764, 5760, 4182, 2279, 2391, 203}

$$-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{d\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(e^{\text{s}}\right.}{d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```


```
[Out] (a + b*ArcSinh[c*x])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*ArcTan[
c*x])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*A
rcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Poly
Log[2, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*Pol
yLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```


Rule *5755*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)\mathbf{csch}(x)\,dx, x, \sinh^{-1}(c\right.}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} -$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} -$$

$$= \frac{a+b\sinh^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} -$$

Mathematica [A] time = 0.883428, size = 231, normalized size = 1.19

$$\frac{\frac{bd\left(\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\sqrt{c^2x^2+1}\sinh^{-1}(cx)\log\left(e^{-\sinh^{-1}(cx)}+1\right)\right.}{\sqrt{c^2dx^2+d}}}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((a*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2) + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d
 + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d*(ArcSinh[c*x] - 2*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-A
rcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] +
 Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*PolyL
og[2, E^(-ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2])/d^2
```

Maple [A] time = 0.138, size = 274, normalized size = 1.4

$$\frac{a}{d}\frac{1}{\sqrt{c^2dx^2+d}} - a\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)d^{-\frac{3}{2}} + \frac{b\text{Arcsinh}(cx)}{d^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)} - 2\frac{b\sqrt{d\left(c^2x^2+1\right)}\arctan\left(cx+\sqrt{\right.}}{\sqrt{c^2x^2+1}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a/d/(c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)
+b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^2+1)*arcsinh(c*x)-2*b*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^5+2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^5 + 2*c^2*d^2*
x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{asinh}\,(cx)}{x\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x), x)
```

3.162 $\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{c^2dx^2+d}}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^2\sqrt{c^2x^2+1}}+\frac{bc\sqrt{c^2dx^2+d}\log\left(c^2x^2+1\right)}{2d^2\sqrt{c^2x^2+1}}$$

```
[Out] -((a + b*ArcSinh[c*x])/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSinh
[c*x]))/(d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])/(d^2*Sqr
t[1 + c^2*x^2]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(2*d^2*Sqrt[1
+ c^2*x^2])
```

Rubi [A] time = 0.153653, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5687, 260, 266, 36, 29, 31}

$$-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\log(x)}{d\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{2d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSinh
[c*x]))/(d*Sqrt[d + c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[x])/(d*Sqrt[d
+ c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*d*Sqrt[d + c^2*
d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx,x,x^2\right)}{2d\sqrt{d+c^2dx^2}} + \cdots$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{d\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{S}\cdots}{2d\sqrt{\cdots}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log(x)}{d\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log\left(1+c^2\cdots\right.}{2d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.278813, size = 163, normalized size = 1.14

$$-\frac{\sqrt{c^2dx^2+d}\left(4ac^2x^2\sqrt{c^2x^2+1}+2a\sqrt{c^2x^2+1}-2bc^3x^3\log\left(c^2x^2+1\right)+bcx\left(c^2x^2+1\right)\log\left(\frac{1}{c^2x^2}+1\right)-2bcx\log\left(c^2x^2+\cdots\right.\right.}{2d^2x\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 + c^2*x^2
] + 2*b*Sqrt[1 + c^2*x^2]*(1 + 2*c^2*x^2)*ArcSinh[c*x] + b*c*x*(1 + c^2*x^2
)*Log[1 + 1/(c^2*x^2)] - 2*b*c*x*Log[1 + c^2*x^2] - 2*b*c^3*x^3*Log[1 + c^2
*x^2]))/(2*d^2*x*(1 + c^2*x^2)^(3/2))
```

Maple [A] time = 0.118, size = 205, normalized size = 1.4

$$-\frac{a}{dx}\frac{1}{\sqrt{c^2dx^2+d}}-2\frac{ac^2x}{d\sqrt{c^2dx^2+d}}-2\frac{b\sqrt{d\left(c^2x^2+1\right)}\text{Arcsinh}\left(cx\right)c}{\sqrt{c^2x^2+1}d^2}-2\frac{b\sqrt{d\left(c^2x^2+1\right)}\text{Arcsinh}\left(cx\right)xc^2}{\left(c^2x^2+1\right)d^2}-\frac{b\text{Arcsinh}\left(\cdots\right.}{\left(c^2x^2+1\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a/d/x/(c^2*d*x^2+d)^(1/2)-2*a*c^2/d*x/(c^2*d*x^2+d)^(1/2)-2*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c-2*b*(d*(c^2*x^2+1))^(1/2)*ar
csinh(c*x)/(c^2*x^2+1)/d^2*x*c^2-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*
x^2+1)/d^2/x+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2
+1)^(1/2))^4-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^6 + 2*c^2*d^2*
x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{asinh}\,(cx)}{x^2\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2 d x^2 + d\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

3.163 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=287

$$\frac{3bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}}{2dx^2\sqrt{c^2dx^2}}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*d*x*Sqrt[d + c^2*d*x^2]) - (3*c^2*(a + b*ArcSin
h[c*x]))/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(2*d*x^2*Sqrt[d +
 c^2*d*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(d*Sqrt[d + c^2*d*x^2]
) + (3*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/
(d*Sqrt[d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[
c*x]])/(2*d*Sqrt[d + c^2*d*x^2]) - (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^
ArcSinh[c*x]])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.430531, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5747, 5755, 5764, 5760, 4182, 2279, 2391, 203, 325}

$$\frac{3bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2d\sqrt{c^2dx^2+d}}-\frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}}{2dx^2\sqrt{c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(2*d*x*Sqrt[d + c^2*d*x^2]) - (3*c^2*(a + b*ArcSin
h[c*x]))/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(2*d*x^2*Sqrt[d +
 c^2*d*x^2]) + (b*c^2*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(d*Sqrt[d + c^2*d*x^2]
) + (3*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/
(d*Sqrt[d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[
c*x]])/(2*d*Sqrt[d + c^2*d*x^2]) - (3*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^
ArcSinh[c*x]])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
```

```
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx}{2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} - \frac{\left(3c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{2d} - \frac{(bc}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{(3}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} - \frac{(3}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{3c}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{3c}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{2dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)}{2d\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\sqrt{d+c^2dx^2}} + \frac{bc^2\sqrt{1+c^2x^2}\tan^{-1}(cx)}{d\sqrt{d+c^2dx^2}} + \frac{3c}{}$$

Mathematica [A] time = 6.36783, size = 381, normalized size = 1.33

$$\frac{bc^2\left(-12\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+12\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-12\sqrt{c^2x^2+1}\sinh^{-1}(cx)\log\left(1-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-a/(2*d^2*x^2) - (a*c^2)/(d^2*(1 + c^2*x^2))) - (3*a
*c^2*Log[x])/(2*d^(3/2)) + (3*a*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])
/(2*d^(3/2)) + (b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[A
rcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x
^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(
-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] + 12*
Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh
```

```
[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(
8*d*Sqrt[d*(1 + c^2*x^2)])
```

Maple [A] time = 0.172, size = 389, normalized size = 1.4

$$-\frac{a}{2\,dx^2}\frac{1}{\sqrt{c^2dx^2+d}}-\frac{3\,ac^2}{2\,d}\frac{1}{\sqrt{c^2dx^2+d}}+\frac{3\,ac^2}{2}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)d^{-\frac{3}{2}}-\frac{3\,b\mathrm{Arcsinh}\,(cx)\,c^2}{2\,d^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(1/2)-3/2*a*c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a*c^2/
d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-3/2*b*(d*(c^2*x^2+1))^(1/
2)/d^2/(c^2*x^2+1)*arcsinh(c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^
2+1)^(1/2)/x*c-1/2*b*(d*(c^2*x^2+1))^(1/2)/d^2/(c^2*x^2+1)/x^2*arcsinh(c*x)
+2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arctan(c*x+(c^2*x^2+1)^(1/
2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(c*x+(c^2*x^
2+1)^(1/2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*dilog(1+c
*x+(c^2*x^2+1)^(1/2))*c^2+3/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2
*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^7 + 2*c^2*d^2*
x^5 + d^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2 d x^2 + d\right)^{\frac{3}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

3.164 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=228

$$\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^2x^2\sqrt{c^2x^2+1}}-\frac{5bc^3\log(x)\sqrt{c^2dx^2+d}}{3d^2\sqrt{c^2x^2+1}}-\frac{bc^3\sqrt{}}{}$$

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(6*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c
*x])/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*d*x*Sq
rt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*Sqrt[d + c^2*d*x^2
]) - (5*b*c^3*Sqrt[d + c^2*d*x^2]*Log[x])/(3*d^2*Sqrt[1 + c^2*x^2]) - (b*c^
3*Sqrt[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(2*d^2*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.288848, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5747, 5687, 260, 266, 36, 29, 31, 44}

$$\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{c^2dx^2+d}}+\frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}}{6dx^2\sqrt{c^2dx^2+d}}-\frac{5bc^3\sqrt{c^2x^2+1}\log(x)}{3d\sqrt{c^2dx^2+d}}-\frac{bc^3\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(6*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x
])/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x]))/(3*d*x*Sqrt
[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*Sqrt[d + c^2*d*x^2])
 - (5*b*c^3*Sqrt[1 + c^2*x^2]*Log[x])/(3*d*Sqrt[d + c^2*d*x^2]) - (b*c^3*Sq
rt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(2*d*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} - \frac{1}{3}\left(4c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3(1+c^2x^2)}\,dx}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{1}{3}\left(8c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)}{6}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}}{6d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{d+c^2dx^2}} - \frac{bc^3}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6dx^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)}{3dx\sqrt{d+c^2dx^2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\sqrt{d+c^2dx^2}} - \frac{5b}{}$$

Mathematica [A] time = 0.299031, size = 216, normalized size = 0.95

$$\frac{\sqrt{c^2dx^2+d}\left(16ac^4x^4\sqrt{c^2x^2+1}+8ac^2x^2\sqrt{c^2x^2+1}-2a\sqrt{c^2x^2+1}-bc^3x^3-8bc^5x^5\log\left(c^2x^2+1\right)+5bc^3x^3\left(c^2x^2+1\right)\log\right.}{6d^2x^3\left(c^2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 8*a*c^
2*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2
*x^2]*(-1 + 4*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 5*b*c^3*x^3*(1 + c^2*x^2)
*Log[1 + 1/(c^2*x^2)] - 8*b*c^3*x^3*Log[1 + c^2*x^2] - 8*b*c^5*x^5*Log[1 +
c^2*x^2]))/(6*d^2*x^3*(1 + c^2*x^2)^(3/2))
```

Maple [B] time = 0.171, size = 965, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a/d/x^3/(c^2*d*x^2+d)^(1/2)+4/3*a*c^2/d/x/(c^2*d*x^2+d)^(1/2)+8/3*a*c^
4/d*x/(c^2*d*x^2+d)^(1/2)+16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^
2*arcsinh(c*x)*c^3+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
*x^7*c^10-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2
*x^2+1)*c^8+16*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8-
16/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^
6+64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)
*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)*c^5+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)
/d^2*x^3*c^6+4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*(c^2
*x^2+1)*c^4+8*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh
(c*x)*c^4+8/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-
1)/d^2*x*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(c
^2*x^2+1)^(1/2)-4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arc
sinh(c*x)*c^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*c
*(c^2*x^2+1)^(1/2)+1/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/
x^3*arcsinh(c*x)-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)*c^3-5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*l
n((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^4d^2x^8+2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^4*d^2*x^8 + 2*c^2*d^2*
x^6 + d^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

3.165 $\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^6d^3}-\frac{5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^7d^2\sqrt{c^2dx^2+d}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

```
[Out] -b/(6*c^7*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*
x^2])/(4*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3*c^2*d
*(d + c^2*d*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqrt[d +
c^2*d*x^2]) + (5*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*c^6*d^3) -
(5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*d^2*Sqrt[d + c^2*d*x^
2]) - (7*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*d^2*Sqrt[d + c^2*d*x^
2])
```

Rubi [A] time = 0.434799, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5751, 5758, 5677, 5675, 30, 266, 43}

$$-\frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2c^6d^3}-\frac{5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^7d^2\sqrt{c^2dx^2+d}}-\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```


```
[Out] -b/(6*c^7*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*
x^2])/(4*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^5*(a + b*ArcSinh[c*x]))/(3*c^2*d
*(d + c^2*d*x^2)^(3/2)) - (5*x^3*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqrt[d +
c^2*d*x^2]) + (5*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*c^6*d^3) -
(5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^7*d^2*Sqrt[d + c^2*d*x^
2]) - (7*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(6*c^7*d^2*Sqrt[d + c^2*d*x^
2])
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
```

```
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{5\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^5}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{c^4d^2} + \frac{\left(5b\sqrt{1+c^2x\cdots\right.}{3c^3d^2\sqrt{\cdots}}$$

$$= -\frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{5x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{5x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^6d^3} - \frac{5\cdots}{\cdots}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{13bx^2\sqrt{1+c^2x^2}}{12c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{5x^3\left(a+b\,\mathrm{s}\cdots\right.}{3c^4d^2\sqrt{d\cdots}}$$

$$= -\frac{b}{6c^7d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}}{4c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{5x^3\left(a+b\sin\cdots\right.}{3c^4d^2\sqrt{d+\cdots}}$$

Mathematica [A] time = 1.00256, size = 222, normalized size = 0.79

$$\frac{4acdx\left(3c^4x^4+20c^2x^2+15\right)-60a\sqrt{d}\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+bd\left(-\sqrt{c^2x^2+1}\left(6c^4x^4+9c^2\cdots\right.\right.}{24c^7d^3\left(c^2x^2+1\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a*c*d*x*(15 + 20*c^2*x^2 + 3*c^4*x^4) + b*d*(4*c*x*(15 + 20*c^2*x^2 + 3*
c^4*x^4)*ArcSinh[c*x] - 30*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - Sqrt[1 + c^
2*x^2]*(7 + 9*c^2*x^2 + 6*c^4*x^4 + 28*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - 6
0*a*Sqrt[d]*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]])/(24*c^7*d^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.341, size = 1607, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -5/2*a/c^6/d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-
1/8*b*(d*(c^2*x^2+1))^(1/2)/c^7/d^3/(c^2*x^2+1)^(1/2)+1/2*a*x^5/c^2/d/(c^2*
d*x^2+d)^(3/2)+5/6*a/c^4*x^3/d/(c^2*d*x^2+d)^(3/2)+5/2*a/c^6/d^2*x/(c^2*d*x
^2+d)^(1/2)+49/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^
4+209*c^2*x^2+49)/d^3*x^7+1009/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^
6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/d^3*arcsinh(c*x)*x^3-37/2*b*(d*(c^2*x
^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^5/d^3*x^
2*(c^2*x^2+1)^(1/2)-7*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c
^4*x^4+209*c^2*x^2+49)/c^6/d^3*(c^2*x^2+1)*x+98*b*(d*(c^2*x^2+1))^(1/2)/(63
*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^6/d^3*arcsinh(c*x)*x-343
/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+
49)/c^7/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+1/2*b*(d*(c^2*x^2+1))^(1/2)/c^4/
d^3/(c^2*x^2+1)*arcsinh(c*x)*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x
^2+1)*arcsinh(c*x)*x-49/6*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+3
34*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*(c^2*x^2+1)*x^5+385*b*(d*(c^2*x^2+1))^(1
/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*arcsinh(c*x
)*x^5-21/2*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*
c^2*x^2+49)/c^3/d^3*(c^2*x^2+1)^(1/2)*x^4-91/6*b*(d*(c^2*x^2+1))^(1/2)/(63*
c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/d^3*(c^2*x^2+1)*x^3-147
*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49
)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6-406*b*(d*(c^2*x^2+1))^(1/2)/(63*
c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^3/d^3*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x^4-1120/3*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*
c^4*x^4+209*c^2*x^2+49)/c^5/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-1/4*b*(d
*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x^2+7*b*(d*(c^2*x^2+1))^(1/2)
/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^6/d^3*x-49/6*b*(d*(c
^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^7/d^
3*(c^2*x^2+1)^(1/2)-5/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^7/d^3*a
rcsinh(c*x)^2+147*b*(d*(c^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x
^4+209*c^2*x^2+49)/d^3*arcsinh(c*x)*x^7+14/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^7/d^3*arcsinh(c*x)-7/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^7/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+70/3*b*(d*(c^2*x^2+1))^(1/2)/(6
3*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^2/d^3*x^5+133/6*b*(d*(c
^2*x^2+1))^(1/2)/(63*c^8*x^8+237*c^6*x^6+334*c^4*x^4+209*c^2*x^2+49)/c^4/d^
```

```
3*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^6 \operatorname{arsinh}(cx) + ax^6\right)\sqrt{c^2dx^2 + d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arcsinh(c*x) + a*x^6)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^6}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^6/(c^2*d*x^2 + d)^(5/2), x)
```

3.166 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^6d^3}+\frac{2\left(a+b\sinh^{-1}(cx)\right)}{c^6d^2\sqrt{c^2dx^2+d}}-\frac{a+b\sinh^{-1}(cx)}{3c^6d\left(c^2dx^2+d\right)^{3/2}}-\frac{bx\sqrt{c^2dx^2+d}}{c^5d^3\sqrt{c^2x^2+1}}+\frac{bx\sqrt{c^2dx^2+d}}{6c^5d^3\left(c^2x^2+1\right)^{3/2}}-1$$

```
[Out] (b*x*Sqrt[d + c^2*d*x^2])/(6*c^5*d^3*(1 + c^2*x^2)^(3/2)) - (b*x*Sqrt[d + c
^2*d*x^2])/(c^5*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*c^6*d*(d +
 c^2*d*x^2)^(3/2)) + (2*(a + b*ArcSinh[c*x]))/(c^6*d^2*Sqrt[d + c^2*d*x^2])
 + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(c^6*d^3) - (11*b*Sqrt[d + c^
2*d*x^2]*ArcTan[c*x])/(6*c^6*d^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.308664, antiderivative size = 225, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5717, 8, 321, 203, 288}

$$-\frac{4x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^6d^3}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{bx^3}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{5b}{6c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b*x^3)/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*b*x*Sqrt[1
+ c^2*x^2])/(6*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x]))/(3
*c^2*d*(d + c^2*d*x^2)^(3/2)) - (4*x^2*(a + b*ArcSinh[c*x]))/(3*c^4*d^2*Sqr
t[d + c^2*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^6*d^3
) - (11*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
```

```
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^4}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{4x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{8\int \frac{x(a+}{}}{}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{11bx\sqrt{1+c^2x^2}}{6c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{4x^2\left(a+b\,\mathrm{si}\right.}{3c^4d^2\sqrt{d+}}$$

$$= -\frac{bx^3}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5bx\sqrt{1+c^2x^2}}{6c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{4x^2\left(a+b\,\mathrm{si}\right.}{3c^4d^2\sqrt{d+}}$$

Mathematica [A] time = 0.271615, size = 154, normalized size = 0.73

$$\frac{\sqrt{c^2dx^2+d}\left(2a\left(3c^4x^4+12c^2x^2+8\right)-bcx\sqrt{c^2x^2+1}\left(6c^2x^2+5\right)+2b\left(3c^4x^4+12c^2x^2+8\right)\sinh^{-1}(cx)\right)}{6c^6d^3\left(c^2x^2+1\right)^2} - \frac{11b\sqrt{d\left(c^2x^2\right.}}{6c^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 6*c^2*x^2)) + 2*a*(8 +
 12*c^2*x^2 + 3*c^4*x^4) + 2*b*(8 + 12*c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]))/
(6*c^6*d^3*(1 + c^2*x^2)^2) - (11*b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(6*c
^6*d^3*Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.194, size = 394, normalized size = 1.9

$$\frac{ax^4}{c^2d}\left(c^2dx^2+d\right)^{-\frac{3}{2}} + 4\frac{ax^2}{dc^4\left(c^2dx^2+d\right)^{3/2}} + \frac{8\,a}{3\,dc^6}\left(c^2dx^2+d\right)^{-\frac{3}{2}} + \frac{b\mathrm{Arcsinh}\,(cx)\,x^2}{c^4d^3\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)} - \frac{bx}{c^5d^3}\sqrt{d\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a*x^4/c^2/d/(c^2*d*x^2+d)^(3/2)+4*a/c^4*x^2/d/(c^2*d*x^2+d)^(3/2)+8/3*a/c^6
/d/(c^2*d*x^2+d)^(3/2)+b*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(
c*x)*x^2-b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^(1/2)*x+b*(d*(c^2*x^2+
1))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)+2*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c
^2*x^2+1)^2/c^4*arcsinh(c*x)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1
)^(3/2)/c^5*x+5/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c*x
)+11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+
1)^(1/2)-I)-11/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x
+(c^2*x^2+1)^(1/2)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.3151, size = 483, normalized size = 2.3

$$\frac{11\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\,\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\left(3\,bc^4x^4+12\,bc^2x^2+8\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)}{12\left(c^{10}d^3x^4+2\,c^8d^3x^2+c^6d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(11*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^4*x^4 + 12*b*c^2
*x^2 + 8*b)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^4*x
^4 + 24*a*c^2*x^2 - (6*b*c^3*x^3 + 5*b*c*x)*sqrt(c^2*x^2 + 1) + 16*a)*sqrt(
c^2*d*x^2 + d))/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^5}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^5/(c^2*d*x^2 + d)^(5/2), x)
```

3.167 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{c^2dx^2+d}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6c^5d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2\cdots}}{3c^5\cdots}$$

```
[Out] b/(6*c^5*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c
*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*d^2*S
qrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.29639, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5751, 5677, 5675, 260, 266, 43}

$$-\frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{c^2dx^2+d}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6c^5d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2\cdots}}{3c^5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c^5*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c
*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x]))/(c^4*d^2*S
qrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
```

```
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}dx}{c^2d} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^3}{\left(1+c^2x^2\right)^2}dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}dx}{c^4d^2} + \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x}{1+c^2x^2}}{c^3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{2c^5d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int}{c^4d^2\sqrt{d}}$$

$$= \frac{b}{6c^5d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a\right.}{2bc^5d^2}$$

Mathematica [A] time = 0.532382, size = 191, normalized size = 0.94

$$\frac{-2ac\sqrt{d}x\left(4c^2x^2+3\right)+6a\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+b\sqrt{d}\left(\sqrt{c^2x^2+1}-8cx\left(c^2x^2+1\right)\sinh^{-1}(c\right.}{6c^5d^{5/2}\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-2*a*c*Sqrt[d]*x*(3 + 4*c^2*x^2) + b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*x*Ar
cSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*ArcS
inh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 6*a*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(6*c^5*d^(5/2)*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])
```

Maple [B] time = 0.238, size = 1430, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+a/c^4/
d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2-8/3*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)-32*b*(d*(c^2*x^2+1))^(1/2
)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)*x^
7+32*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+
16)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6-8/3*b*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7+8/3*b*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*(c^2*x^2
+1)*x^5-76*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^
2*x^2+16)/d^3*arcsinh(c*x)*x^5+84*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^
6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-2
2/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+1
6)/d^3*x^5+4*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*
c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4+14/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^
8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)*x^3-181/3*b
*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^
2/d^3*arcsinh(c*x)*x^3+220/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6
+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^2-20/3
*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/
c^2/d^3*x^3+13/2*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4
+71*c^2*x^2+16)/c^3/d^3*x^2*(c^2*x^2+1)^(1/2)+2*b*(d*(c^2*x^2+1))^(1/2)/(24
*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+1)*x-16*b*(
d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/
d^3*arcsinh(c*x)*x+64/3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*
c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2*b*(d*(c^2*x
^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*x+8/
3*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)
/c^5/d^3*(c^2*x^2+1)^(1/2)+4/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^
5/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^4\,\text{arsinh}\,(cx)+ax^4\right)\sqrt{c^2dx^2+d}}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arcsinh(c*x) + a*x^4)*sqrt(c^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)x^4}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

3.168 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{a+b\sinh^{-1}(cx)}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{3c^4d\left(c^2dx^2+d\right)^{3/2}}-\frac{bx\sqrt{c^2dx^2+d}}{6c^3d^3\left(c^2x^2+1\right)^{3/2}}+\frac{5b\sqrt{c^2dx^2+d}\tan^{-1}(cx)}{6c^4d^3\sqrt{c^2x^2+1}}$$

```
[Out] -(b*x*Sqrt[d + c^2*d*x^2])/(6*c^3*d^3*(1 + c^2*x^2)^(3/2)) + (a + b*ArcSinh
[c*x])/(3*c^4*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])/(c^4*d^2*Sqrt
[d + c^2*d*x^2]) + (5*b*Sqrt[d + c^2*d*x^2]*ArcTan[c*x])/(6*c^4*d^3*Sqrt[1
+ c^2*x^2])
```

Rubi [A] time = 0.182464, antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5751, 5717, 203, 288}

$$-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}-\frac{bx}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{5b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{6c^4d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b*x)/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcS
inh[c*x]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x]))/(3*c^
4*d^2*Sqrt[d + c^2*d*x^2]) + (5*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^4*d^2
*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{x^2}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1+c^2x\cdots\right.}{6c^3d^2\sqrt{\cdots}}$$

$$= -\frac{bx}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{3c^4d^2\sqrt{d+c^2dx^2}}+\frac{5b\sqrt{1+c^2x\cdots}}{6c^4d^2\sqrt{d\cdots}}$$

Mathematica [A] time = 0.239703, size = 151, normalized size = 1.05

$$\frac{5b\sqrt{d\left(c^2x^2+1\right)}\tan^{-1}(cx)}{6c^4d^3\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx^2+d}\left(2a\sqrt{c^2x^2+1}\left(3c^2x^2+2\right)+b\left(c^3x^3+cx\right)+2b\sqrt{c^2x^2+1}\left(3c^2x^2+2\right)\sinh^{-1}(cx\cdots\right.}{6c^4d^3\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(2*a*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2) + b*(c*x + c^3
*x^3) + 2*b*Sqrt[1 + c^2*x^2]*(2 + 3*c^2*x^2)*ArcSinh[c*x]))/(6*c^4*d^3*(1
+ c^2*x^2)^(5/2)) + (5*b*Sqrt[d*(1 + c^2*x^2)]*ArcTan[c*x])/(6*c^4*d^3*Sqrt
[1 + c^2*x^2])
```

Maple [C] time = 0.151, size = 262, normalized size = 1.8

$$-\frac{ax^2}{c^2d}\left(c^2dx^2+d\right)^{-\frac{3}{2}}-\frac{2a}{3dc^4}\left(c^2dx^2+d\right)^{-\frac{3}{2}}-\frac{b\mathrm{Arcsinh}\,(cx)\,x^2}{d^3\left(c^2x^2+1\right)^2c^2}\sqrt{d\left(c^2x^2+1\right)}-\frac{bx}{6\,d^3c^3}\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2+1\right)^{-\frac{3}{2}}-\frac{2\,b}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a*x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3*a/d/c^4/(c^2*d*x^2+d)^(3/2)-b*(d*(c^2*
x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)*x^2-1/6*b*(d*(c^2*x^2+1))^
(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*x-2/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+
1)^2/c^4*arcsinh(c*x)+5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d
^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-5/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [A] time = 1.78168, size = 186, normalized size = 1.29

$$-\frac{1}{6}\,bc\left(\frac{x}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}-\frac{5\,\arctan\,(cx)}{c^5d^{\frac{5}{2}}}\right)-\frac{1}{3}\,b\left(\frac{3\,x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)\mathrm{arsinh}\,(cx)-\frac{1}{3}\,a\left(\frac{3\,x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2)))
- 1/3*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)*c^4
*d))*arcsinh(c*x) - 1/3*a*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*
x^2 + d)^(3/2)*c^4*d))
```

Fricas [A] time = 3.23417, size = 416, normalized size = 2.89

$$-\frac{5\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\left(3\,bc^2x^2+2\,b\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(6\,ac^2\right.}{12\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(5*(b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)
*sqrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x^2 + 2*b)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(6*a*c^2*x^2 + sqrt(c^2*x
^2 + 1)*b*c*x + 4*a)*sqrt(c^2*d*x^2 + d))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^
4*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)x^3}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^3/(c^2*d*x^2 + d)^(5/2), x)
```

3.169 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{b}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{6c^3d^2\sqrt{c^2dx^2+d}}$$

```
[Out] -b/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[
c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])
/(6*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.124617, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5723, 266, 43}

$$\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{b}{6c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{6c^3d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -b/(6*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[
c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])
/(6*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx &= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}\\ &= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\frac{x}{\left(1+c^2x\right)^2}\,dx,x,x^2\right)}{6d^2\sqrt{d+c^2dx^2}}\\ &= \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{c^2\left(1+c^2x\right)^2}+\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^2\right)}{6d^2\sqrt{d+c^2dx^2}}\\ &= -\frac{b}{6c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}+\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}-\frac{b\sqrt{1+c^2x^2}\log\left(1+c^2x^2\right)}{6c^3d^2\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.196037, size = 118, normalized size = 0.99

$$-\frac{\sqrt{c^2dx^2+d}\left(-2ac^3x^3\sqrt{c^2x^2+1}+bc^2x^2+b\left(c^2x^2+1\right)^2\log\left(c^2x^2+1\right)-2bc^3x^3\sqrt{c^2x^2+1}\sinh^{-1}(cx)+b\right)}{6c^3d^3\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 - 2*a*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*b*
c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]
```

```
))/(6*c^3*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] time = 0.161, size = 1153, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a/c^2*x/d/(c^2*d*x^2+d)^(3/2)+1/3*a/c^2/d^2*x/(c^2*d*x^2+d)^(1/2)+2/3*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^3*arcsinh(c*x)+b*(d*(c^2*x^
2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*
x)*x^7-b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
*c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*
c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7-1/6*b*(d*(c^2*x^2+1))
^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5
+b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d
^3*arcsinh(c*x)*x^5-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x
^4+5*c^2*x^2+1)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4+1/3*b*(d*(c^2*x^2+
1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/2*b*(d
*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2
*x^2+1)^(1/2)*x^4-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x
^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x^3+1/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+
9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)*x^3-4/3*b*(d*(c^2*x^2+1)
)^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)*x^2+1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*
x^4+5*c^2*x^2+1)/d^3*x^3-1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+1
0*c^4*x^4+5*c^2*x^2+1)/c/d^3*x^2*(c^2*x^2+1)^(1/2)-1/3*b*(d*(c^2*x^2+1))^(1
/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)-1/6*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*
c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)/c^3/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [A] time = 1.25048, size = 185, normalized size = 1.55

$$-\frac{1}{6}\,bc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{\sqrt{c^2dx^2+d}c^2d^2}-\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{x}{\sqrt{c^2dx^2+dc^2d^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/2)
)) + 1/3*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*c^2*
d))*arcsinh(c*x) + 1/3*a*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 +
 d)^(3/2)*c^2*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(bx^2\operatorname{arsinh}(cx)+ax^2\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*x^2*arcsinh(c*x) + a*x^2)/(c^6*d^3*x^6 + 3*
c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)x^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

3.170 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{bx}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{6c^2d^2\sqrt{c^2dx^2+d}}$$

```
[Out] (b*x)/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x]
)/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^
2*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.0782282, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5717, 199, 203}

$$-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(c^2dx^2+d\right)^{3/2}}+\frac{bx}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\tan^{-1}(cx)}{6c^2d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x)/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x]
)/(3*c^2*d*(d + c^2*d*x^2)^(3/2)) + (b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*c^
2*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}\\
&= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\int\frac{1}{1+c^2x^2}\,dx}{6cd^2\sqrt{d+c^2dx^2}}\\
&= \frac{bx}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{a+b\sinh^{-1}(cx)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6c^2d^2\sqrt{d+c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.183021, size = 130, normalized size = 1.14

$$\frac{\sqrt{c^2dx^2+d}\left(-2a\sqrt{c^2x^2+1}+bc^3x^3-2b\sqrt{c^2x^2+1}\sinh^{-1}(cx)+bcx\right)}{6c^2d^3\left(c^2x^2+1\right)^{5/2}}+\frac{b\sqrt{d\left(c^2x^2+1\right)}\tan^{-1}(cx)}{6c^2d^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] - 2*b*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]))/(6*c^2*d^3*(1 + c^2*x^2)^(5/2)) + (b*Sqrt[d*(1
+ c^2*x^2)]*ArcTan[c*x])/(6*c^2*d^3*Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.114, size = 198, normalized size = 1.7

$$-\frac{a}{3c^2d}\left(c^2dx^2+d\right)^{-\frac{3}{2}}+\frac{bx}{6d^3c}\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2+1\right)^{-\frac{3}{2}}-\frac{b\mathrm{Arcsinh}\,(cx)}{3d^3\left(c^2x^2+1\right)^2c^2}\sqrt{d\left(c^2x^2+1\right)}+\frac{\frac{i}{6}b}{c^2d^3}\sqrt{d\left(c^2x^2+1\right)}\ln\left(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a/c^2/d/(c^2*d*x^2+d)^(3/2)+1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1
)^(3/2)/c*x-1/3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)+
1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^
(1/2)+I)-1/6*I*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^
2*x^2+1)^(1/2)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int\frac{x\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx-\frac{a}{3\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d)^(5/2), x) - 1/3*
a/((c^2*d*x^2 + d)^(3/2)*c^2*d)
```

Fricas [A] time = 3.12803, size = 370, normalized size = 3.25

$$-\frac{\left(bc^4x^4+2\,bc^2x^2+b\right)\sqrt{d}\arctan\left(\frac{2\sqrt{c^2dx^2+d}\sqrt{c^2x^2+1}c\sqrt{d}x}{c^4dx^4-d}\right)+4\sqrt{c^2dx^2+d}b\log\left(cx+\sqrt{c^2x^2+1}\right)-2\sqrt{c^2dx^2+d}\left(\sqrt{c^2x^2+\right.}{12\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*((b*c^4*x^4 + 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(c^2*d*x^2 + d)*s
qrt(c^2*x^2 + 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*sqrt(c^2*d*x^2 + d)*b*log
(c*x + sqrt(c^2*x^2 + 1)) - 2*sqrt(c^2*d*x^2 + d)*(sqrt(c^2*x^2 + 1)*b*c*x
- 2*a))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x}{\left(c^{2}dx^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x/(c^2*d*x^2 + d)^(5/2), x)
```

3.171 $\int \frac{a+b\sinh^{-1}(cx)}{(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

```
[Out] b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])
)/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*d^2*Sqrt[d + c^2*
d*x^2])
```

Rubi [A] time = 0.0794904, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5690, 5687, 260, 261}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{b}{6cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{3cd^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] b/(6*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])
)/(3*d*(d + c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d +
c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2])/(3*c*d^2*Sqrt[d + c^2*
d*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\ldots}{3d^2\sqrt{d+c\ldots}}$$

$$= \frac{b}{6cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}} - \frac{b\sqrt{1+c^2x^2}\log\left(\ldots\right.}{3cd^2\sqrt{d+c\ldots}}$$

Mathematica [A] time = 0.193406, size = 143, normalized size = 0.97

$$\frac{\sqrt{c^2dx^2+d}\left(4ac^3x^3\sqrt{c^2x^2+1}+6acx\sqrt{c^2x^2+1}+bc^2x^2-2b\left(c^2x^2+1\right)^2\log\left(c^2x^2+1\right)+2bcx\sqrt{c^2x^2+1}\left(2c^2x^2+3\right)\sin\ldots\right.}{6cd^3\left(c^2x^2+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(b + b*c^2*x^2 + 6*a*c*x*Sqrt[1 + c^2*x^2] + 4*a*c^3*x
^3*Sqrt[1 + c^2*x^2] + 2*b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2)*ArcSinh[c*
x] - 2*b*(1 + c^2*x^2)^2*Log[1 + c^2*x^2]))/(6*c*d^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] time = 0.109, size = 1005, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*x/d/(c^2*d*x^2+d)^(3/2)+2/3*a/d^2*x/(c^2*d*x^2+d)^(1/2)+4/3*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3*arcsinh(c*x)-2/3*b*(d*(c^2*x^2+1))^(
1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7+2/3*b*(d*(c^2*x^2+1))^
(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5+2*b*(d*(c
^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x
^5-2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*ar
csinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-7/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*
c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5+5/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10
*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3+17/3*b*(d*(c^2*x^2+1))^(1/2)
/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3-14/3*b*(d*(c^
2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*x^2-8/3*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*
x^2+4)*c^2/d^3*x^3+1/2*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2
*x^2+4)*c/d^3*x^2*(c^2*x^2+1)^(1/2)+b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c
^4*x^4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x+4*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x-8/3*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-b*(d
*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x+2/3*b*(d*(c^2
*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)-
2/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d^3*ln(1+(c*x+(c^2*x^2+1)^(
1/2))^2)
```

Maxima [A] time = 1.28324, size = 170, normalized size = 1.16

$$\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arsinh}\left(cx\right)+\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{}{\left(c^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5/2
))) + 1/3*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d))*a
rcsinh(c*x) + 1/3*a*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/
2)*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^6 + 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(c^2*d*x^2 + d)^(5/2), x)
```

3.172 $\int \frac{a+b\sinh^{-1}(cx)}{x(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(\right.}{d^2}$$

```
[Out] -(b*c*x)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x
])/(3*d*(d + c^2*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(d^2*Sqrt[d + c^2*d*x
^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) - (2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[
d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sq
rt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.415274, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5755, 5764, 5760, 4182, 2279, 2391, 203, 199}

$$-\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{c^2dx^2+d}}-\frac{2\sqrt{c^2x^2+1}\tanh^{-1}\left(\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```


```
[Out] -(b*c*x)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x
])/(3*d*(d + c^2*d*x^2)^(3/2)) + (a + b*ArcSinh[c*x])/(d^2*Sqrt[d + c^2*d*x
^2]) - (7*b*Sqrt[1 + c^2*x^2]*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) - (2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[
d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sq
rt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2])
```


Rule *5755*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{3/2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{d+c^2dx^2}}\,dx}{d^2} - \frac{(bc}{}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{a+b\sinh^{-1}(cx)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{a+b\sinh^{-1}(cx)}{d^2\sqrt{d+c^2dx^2}} - \frac{7b\sqrt{1+c^2x^2}\tan^{-1}(cx)}{6d^2\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 1.2341, size = 247, normalized size = 0.94

$$\frac{\frac{bd^2\left(c^2x^2+1\right)^{3/2}\left(6\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-6\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\frac{cx}{c^2x^2+1}+\frac{6\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+\frac{2\sinh^{-1}(cx)}{\left(c^2x^2+1\right)^{3/2}}+6\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-6\sinh^{-1}(cx)\log\right.}{\left(c^2dx^2+d\right)^{3/2}}}{6d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((2*a*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 6*a*Sqrt[d]*Lo
g[x] - 6*a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b*d^2*(1 + c^2*x
^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) +
(6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]] + 6*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[
c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSinh[c*x])]
))/(d + c^2*d*x^2)^(3/2))/(6*d^3)
```

Maple [A] time = 0.146, size = 364, normalized size = 1.4

$$\frac{a}{3\,d}\left(c^2dx^2+d\right)^{-\frac{3}{2}}+\frac{a}{d^2}\frac{1}{\sqrt{c^2dx^2+d}}-a\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)d^{-\frac{5}{2}}+\frac{b\text{Arcsinh}\,(cx)\,x^2c^2}{d^3\left(c^2x^2+1\right)^2}\sqrt{d\left(c^2x^2+1\right)}-\frac{bcx}{6\,d^3}\sqrt{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/d/(c^2*d*x^2+d)^(3/2)+a/d^2/(c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2*d
^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2*ar
csinh(c*x)*x^2*c^2-1/6*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)*c*x+4/
3*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2*arcsinh(c*x)-7/3*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(1+c*x+(c^2*x^2+1)^(1/2))-b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^7 + 3*c^4*d^3*
x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x), x)
```

3.173 $\int \frac{a+b\sinh^{-1}(cx)}{x^2(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{bc\sqrt{c^2dx^2+d}}{6d^3\left(c^2x^2+1\right)^{3/2}}+\frac{bc\log(x)\sqrt{c^2dx^2+d}}{d^3\sqrt{c^2x^2+1}}+\frac{5\ldots}{\ldots}$$

```
[Out] -(b*c*Sqrt[d + c^2*d*x^2])/(6*d^3*(1 + c^2*x^2)^(3/2)) - (a + b*ArcSinh[c*x
])/(d*x*(d + c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c
^2*d*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2
]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[x])/(d^3*Sqrt[1 + c^2*x^2]) + (5*b*c*Sqrt
[d + c^2*d*x^2]*Log[1 + c^2*x^2])/(6*d^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.213078, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5690, 5687, 260, 261, 266, 44}

$$-\frac{8c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{bc}{6d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{bc\sqrt{c^2x^2+1}\log(\ldots}{d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b*c)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])
/(d*x*(d + c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2
*d*x^2)^(3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2])
 + (b*c*Sqrt[1 + c^2*x^2]*Log[x])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c*Sqrt[1
 + c^2*x^2]*Log[1 + c^2*x^2])/(6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
```

```
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \left(4c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x\left(1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(8c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Sub}}{2d^2\sqrt{}}$$

$$= -\frac{2bc}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh\right.}{3d^2\sqrt{d+c^2d}}$$

$$= -\frac{bc}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{8c^2x\left(a+b\sinh\right.}{3d^2\sqrt{d+c^2d}}$$

Mathematica [A] time = 0.310772, size = 227, normalized size = 1.06

$$-\frac{\sqrt{c^2dx^2+d}\left(16ac^4x^4\sqrt{c^2x^2+1}+24ac^2x^2\sqrt{c^2x^2+1}+6a\sqrt{c^2x^2+1}+bc^3x^3-8bc^5x^5\log\left(c^2x^2+1\right)-16bc^3x^3\log\left(c^2x\right.\right.}{6d^3x\left(c^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(b*c*x + b*c^3*x^3 + 6*a*Sqrt[1 + c^2*x^2] + 24*a*c^2
*x^2*Sqrt[1 + c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*
x^2]*(3 + 12*c^2*x^2 + 8*c^4*x^4)*ArcSinh[c*x] + 3*b*c*x*(1 + c^2*x^2)^2*Lo
g[1 + 1/(c^2*x^2)] - 8*b*c*x*Log[1 + c^2*x^2] - 16*b*c^3*x^3*Log[1 + c^2*x^
2] - 8*b*c^5*x^5*Log[1 + c^2*x^2]))/(6*d^3*x*(1 + c^2*x^2)^(5/2))
```

Maple [B] time = 0.151, size = 1257, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a/d/x/(c^2*d*x^2+d)^(3/2)-4/3*a*c^2*x/d/(c^2*d*x^2+d)^(3/2)-8/3*a*c^2/d^2*
x/(c^2*d*x^2+d)^(1/2)-16/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ar
csinh(c*x)*c-32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+
9)/d^3*x^9*c^10+32/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x
^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-112/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c
^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8+80/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-64/3*b*(d*(c^2*x^2+1))^(1/2)
/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6+64/3*b*(d*(c^
2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*c^5-140/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^5*c^6+20*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*
c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-56*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4+136/3*b*(d*(c^2*x^2+1))^(1/
2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*c^3-24*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x
^3*c^4-4/3*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*
x^2*c^3*(c^2*x^2+1)^(1/2)+4*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+2
6*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-44*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25
*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+24*b*(d*(c^2*x^2+1))^(1/2)/(8
*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-4*b*
(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-3/2*b*(
d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^
(1/2)-9*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*a
rcsinh(c*x)+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+(c^2*x^2+
1)^(1/2))^2-1)*c+5/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^8 + 3*c^4*d^3*
x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

3.174 $\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=400

$$\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{5c^2\sqrt{c^2x^2+}}{}$$

```
[Out] (b*c)/(4*d^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (5*b*c^3*x)/(12*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(4*d^2*
x*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*d*(d + c^2*d*x^2)^
(3/2)) - (a + b*ArcSinh[c*x])/(2*d*x^2*(d + c^2*d*x^2)^(3/2)) - (5*c^2*(a +
 b*ArcSinh[c*x]))/(2*d^2*Sqrt[d + c^2*d*x^2]) + (13*b*c^2*Sqrt[1 + c^2*x^2]
*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*d^2*Sqrt[d + c^2*d*x^2]
) - (5*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*d^2*Sqrt[d +
c^2*d*x^2])
```

Rubi [A] time = 0.559931, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5747, 5755, 5764, 5760, 4182, 2279, 2391, 203, 199, 290, 325}

$$\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)}{2d^2\sqrt{c^2dx^2+d}}-\frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{2d^2\sqrt{c^2dx^2+d}}+\frac{5c^2\sqrt{c^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```



```
[Out] (b*c)/(4*d^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (5*b*c^3*x)/(12*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (3*b*c*Sqrt[1 + c^2*x^2])/(4*d^2*
x*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x]))/(6*d*(d + c^2*d*x^2)^
(3/2)) - (a + b*ArcSinh[c*x])/(2*d*x^2*(d + c^2*d*x^2)^(3/2)) - (5*c^2*(a +
 b*ArcSinh[c*x]))/(2*d^2*Sqrt[d + c^2*d*x^2]) + (13*b*c^2*Sqrt[1 + c^2*x^2]
*ArcTan[c*x])/(6*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^
2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^ArcSinh[c*x]])/(2*d^2*Sqrt[d + c^2*d*x^2]
) - (5*b*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, E^ArcSinh[c*x]])/(2*d^2*Sqrt[d +
```

```
c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^2\left(1+c^2x^2\right)^2}\,dx}{2d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{a+b\sinh^{-1}(cx)}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(5c^2\right)\int \frac{a+b\sinh^{-1}}{x\left(d+c^2dx^2\right.}}{2d}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

$$= \frac{bc}{4d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{5bc^3x}{12d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4d^2x\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx\right.}$$

Mathematica [A] time = 6.57009, size = 437, normalized size = 1.09

$$bc^2\left(-60\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+60\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\frac{4cx}{\sqrt{c^2x^2+1}}-\frac{8\sinh^{-1}(cx)}{c^2x^2+1}-60\sqrt{c^2x^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-a/(2*d^3*x^2) - (a*c^2)/(3*d^3*(1 + c^2*x^2)^2) - (
2*a*c^2)/(d^3*(1 + c^2*x^2))) - (5*a*c^2*Log[x])/(2*d^(5/2)) + (5*a*c^2*Log
[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (b*c^2*((4*c*x)/Sqrt[1 +
 c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x^2) + 104*Sqrt[1 +
 c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c
*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 60*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1 + c^2*x^2]*PolyLog[2,
-E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] -
3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 6*Sqrt[1 + c^2*x^
2]*Tanh[ArcSinh[c*x]/2]))/(24*d^2*Sqrt[d*(1 + c^2*x^2)])
```

Maple [A] time = 0.204, size = 546, normalized size = 1.4

$$-\frac{a}{2\,dx^2}\left(c^2dx^2+d\right)^{-\frac{3}{2}}-\frac{5\,ac^2}{6\,d}\left(c^2dx^2+d\right)^{-\frac{3}{2}}-\frac{5\,ac^2}{2\,d^2}\frac{1}{\sqrt{c^2dx^2+d}}+\frac{5\,ac^2}{2}\ln\left(\frac{1}{x}\left(2\,d+2\,\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)d^{-\frac{5}{2}}-\frac{5\,bx^2\mathrm{Arc}}{\left(2\,c^4x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/2*a/d/x^2/(c^2*d*x^2+d)^(3/2)-5/6*a*c^2/d/(c^2*d*x^2+d)^(3/2)-5/2*a*c^2/
d^2/(c^2*d*x^2+d)^(1/2)+5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(
1/2))/x)-5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*x^2*arcsinh(
c*x)*c^4-1/3*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*x*c^3*(c^2*x
^2+1)^(1/2)-10/3*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3*arcsinh(
c*x)*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3/x*c*(c^2*x^2
+1)^(1/2)-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^4*x^4+2*c^2*x^2+1)/d^3/x^2*arcsinh
(c*x)+13/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arctan(c*x+(c^2*x^
2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*dilog(c*x
+(c^2*x^2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*d
ilog(1+c*x+(c^2*x^2+1)^(1/2))*c^2+5/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^9 + 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{arsinh}\,(cx)+a}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

3.175 $\int \frac{a+b\sinh^{-1}(cx)}{x^4(d+c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{bc^3\sqrt{c^2dx^2+d}}{6d^3\left(c^2x^2+1\right)^{3/2}}-$$

```
[Out] (b*c^3*Sqrt[d + c^2*d*x^2])/(6*d^3*(1 + c^2*x^2)^(3/2)) - (b*c*Sqrt[d + c^2
*d*x^2])/(6*d^3*x^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])/(3*d*x^3*(d +
 c^2*d*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x]))/(d*x*(d + c^2*d*x^2)^(3/2
)) + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x
*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b*c^3*Sqrt[d + c^2*
d*x^2]*Log[x])/(3*d^3*Sqrt[1 + c^2*x^2]) - (4*b*c^3*Sqrt[d + c^2*d*x^2]*Log
[1 + c^2*x^2])/(3*d^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.372379, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5747, 5690, 5687, 260, 261, 266, 44}

$$\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(c^2dx^2+d\right)^{3/2}}+\frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(c^2dx^2+d\right)^{3/2}}-\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(c^2dx^2+d\right)^{3/2}}+\frac{bc^3}{6d^2\sqrt{c^2x^2+1}\sqrt{c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c^3)/(6*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x
^2])/(6*d^2*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])/(3*d*x^3*(d + c
^2*d*x^2)^(3/2)) + (2*c^2*(a + b*ArcSinh[c*x]))/(d*x*(d + c^2*d*x^2)^(3/2))
 + (8*c^4*x*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(
a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b*c^3*Sqrt[1 + c^2*x^
2]*Log[x])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4*b*c^3*Sqrt[1 + c^2*x^2]*Log[1 +
 c^2*x^2])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
```

```
1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} - \left(2c^2\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{1}{x^3\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} + \left(8c^4\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)}{}$$

$$= -\frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} + \frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(16c^4\right)\int \frac{a+b\sinh^{-1}(c}{\left(d+c^2dx^2\right)^{3/}}}{3d}$$

$$= \frac{7bc^3}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}}{6d^2x^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} +$$

$$= \frac{bc^3}{6d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}}{6d^2x^2\sqrt{d+c^2dx^2}} - \frac{a+b\sinh^{-1}(cx)}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)}{dx\left(d+c^2dx^2\right)^{3/2}} +$$

Mathematica [A] time = 0.356522, size = 267, normalized size = 0.9

$$\sqrt{c^2dx^2+d}\left(32ac^6x^6\sqrt{c^2x^2+1} + 48ac^4x^4\sqrt{c^2x^2+1} + 12ac^2x^2\sqrt{c^2x^2+1} - 2a\sqrt{c^2x^2+1} - bc^3x^3 - 16bc^7x^7\log\left(c^2x^2+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-(b*c*x) - b*c^3*x^3 - 2*a*Sqrt[1 + c^2*x^2] + 12*a*c
^2*x^2*Sqrt[1 + c^2*x^2] + 48*a*c^4*x^4*Sqrt[1 + c^2*x^2] + 32*a*c^6*x^6*Sq
rt[1 + c^2*x^2] + 2*b*Sqrt[1 + c^2*x^2]*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c
^6*x^6)*ArcSinh[c*x] + 8*b*c^3*x^3*(1 + c^2*x^2)^2*Log[1 + 1/(c^2*x^2)] - 1
6*b*c^3*x^3*Log[1 + c^2*x^2] - 32*b*c^5*x^5*Log[1 + c^2*x^2] - 16*b*c^7*x^7
*Log[1 + c^2*x^2]))/(6*d^3*x^3*(1 + c^2*x^2)^(5/2))
```

Maple [B] time = 0.175, size = 1790, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 32/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*c^3-64*b*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6
*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^9-128*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8
+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
*c^7-176/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2
*x^2-1)/d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5-1/3*a/d/x^3/(c^2*d*x^2+d
)^(3/2)+2*a*c^2/d/x/(c^2*d*x^2+d)^(3/2)-6*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x
^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*arcsinh(c*x)*c^2+1/6*b*(d*(c^2
*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^2*c*(c
^2*x^2+1)^(1/2)+16/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-128/3*b*(d*(c^2*x^
2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^
2+1)*c^12-320/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+1
0*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*c^10+64*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10-80*b*(d*(c^
2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*(c^
2*x^2+1)*c^8+160*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+
10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*c^8-40/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8
*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6+344/3*b*(d
*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3
*arcsinh(c*x)*c^6-2*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x
^4+10*c^2*x^2-1)/d^3*x^2*c^5*(c^2*x^2+1)^(1/2)+8/3*b*(d*(c^2*x^2+1))^(1/2)/
(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*(c^2*x^2+1)*c^4+12*b*
(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x
*arcsinh(c*x)*c^4-8/3*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x
+(c^2*x^2+1)^(1/2))^4-1)*c^3-2*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x
^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*c^3*(c^2*x^2+1)^(1/2)+128/3*b*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+448/
3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d
^3*x^9*c^12+560/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4
+10*c^2*x^2-1)/d^3*x^7*c^10+280/3*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^
6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8+32/3*b*(d*(c^2*x^2+1))^(1/2)/(12
*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-8/3*b*(d*(c^2*x^2+
1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4+1/3*b*(
d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^
```

```
3*arcsinh(c*x)+8/3*a*c^4*x/d/(c^2*d*x^2+d)^(3/2)+16/3*a*c^4/d^2*x/(c^2*d*x^
2+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^6*d^3*x^10 + 3*c^4*d^3
*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/x**4/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(c^2dx^2 + d\right)^{\frac{5}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

3.176 $\int \frac{\sinh^{-1}(ax)}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\sin}{15c^2\left(a^2c\right.}$$

```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.112084, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5690, 5687, 260, 261}

$$\frac{2}{15ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}+\frac{1}{20ac^3\left(a^2x^2+1\right)^{3/2}\sqrt{a^2cx^2+c}}-\frac{4\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{15ac^3\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)}{15c^3\sqrt{a^2cx^2+c}}+\frac{4x\sin}{15c^2\left(a^2c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```



```
[Out] 1/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + 2/(15*a*c^3*Sqrt[1 +
 a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x])/(5*c*(c + a^2*c*x^2)^(5/2
)) + (4*x*ArcSinh[a*x])/(15*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x])
/(15*c^3*Sqrt[c + a^2*c*x^2]) - (4*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(15*
a*c^3*Sqrt[c + a^2*c*x^2])
```


Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

```
&& GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{15c^2} - \cdots$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}}{15c^2\left(c+a^2c\right.}\cdots$$

$$= \frac{1}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}}{15c^2\left(c+a^2c\right.}\cdots$$

Mathematica [A] time = 0.140835, size = 121, normalized size = 0.6

$$\frac{\sqrt{a^2cx^2+c}\left(4ax\sqrt{a^2x^2+1}\left(8a^4x^4+20a^2x^2+15\right)\sinh^{-1}(ax)-\left(a^2x^2+1\right)\left(-8a^2x^2+16\left(a^2x^2+1\right)^2\log\left(a^2x^2+1\right)-11\cdots\right.\right.}{60ac^4\left(a^2x^2+1\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*(4*a*x*Sqrt[1 + a^2*x^2]*(15 + 20*a^2*x^2 + 8*a^4*x^4)
*ArcSinh[a*x] - (1 + a^2*x^2)*(-11 - 8*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[1 +
 a^2*x^2])))/(60*a*c^4*(1 + a^2*x^2)^(7/2))
```

Maple [B] time = 0., size = 363, normalized size = 1.8

$$\frac{16\,\mathrm{Arcsinh}\,(ax)}{15\,ac^4}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{1}{\left(2400\,a^{10}x^{10}+12900\,x^8a^8+28140\,x^6a^6+31020\,x^4a^4+17220\,a^2x^2+3840\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 16/15*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*arcsinh(a*x)+1/60*(c*(a
^2*x^2+1))^(1/2)*(8*x^5*a^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*x^3*a^3-16*a^2*x
^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*x^8*a^8-64*(a^2*x^2+1
)^(1/2)*x^7*a^7-280*x^6*a^6-248*(a^2*x^2+1)^(1/2)*x^5*a^5+160*a^4*x^4*arcsi
nh(a*x)-456*x^4*a^4-340*a^3*x^3*(a^2*x^2+1)^(1/2)+380*a^2*x^2*arcsinh(a*x)-
328*a^2*x^2-165*a*x*(a^2*x^2+1)^(1/2)+256*arcsinh(a*x)-88)/(40*a^10*x^10+21
5*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-8/15*(c*(a^2*x^2+1)
)^(1/2)/(a^2*x^2+1)^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [A] time = 1.26929, size = 201, normalized size = 1.

$$-\frac{1}{60}\,a\left(\frac{16\sqrt{\frac{1}{a^4c}}\log\left(x^2+\frac{1}{a^2}\right)}{c^3}-\frac{3}{\left(a^6c^{\frac{5}{2}}x^4+2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}-\frac{8}{\left(a^4c^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}\right)c^2}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{a^2cx^2+c}c^3}+\frac{4\,x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*a*(16*sqrt(1/(a^4*c))*log(x^2 + 1/a^2)/c^3 - 3/((a^6*c^(5/2)*x^4 + 2*
a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) - 8/((a^4*c^(3/2)*x^2 + a^2*c^(3/2))*c^2)
```

```
) + 1/15*(8*x/(sqrt(a^2*c*x^2 + c)*c^3) + 4*x/((a^2*c*x^2 + c)^(3/2)*c^2) +
 3*x/((a^2*c*x^2 + c)^(5/2)*c))*arcsinh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 + 6*
a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4841, size = 167, normalized size = 0.84

$$-\frac{1}{60}\sqrt{c}\left(\frac{16\log\left(a^2x^2+1\right)}{ac^4}-\frac{24\,a^4x^4+56\,a^2x^2+35}{\left(a^2x^2+1\right)^2ac^4}\right)+\frac{\left(4\left(\frac{2\,a^4x^2}{c}+\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2+1}\right)}{15\left(a^2cx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(c)*(16*log(a^2*x^2 + 1)/(a*c^4) - (24*a^4*x^4 + 56*a^2*x^2 + 35)
/((a^2*x^2 + 1)^2*a*c^4)) + 1/15*(4*(2*a^4*x^2/c + 5*a^2/c)*x^2 + 15/c)*x*l
og(a*x + sqrt(a^2*x^2 + 1))/(a^2*c*x^2 + c)^(5/2)
```

3.177 $\int \frac{x^4 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a^4}+\frac{3\sinh^{-1}(ax)^2}{16a^5}-\frac{x^4}{16a}$$

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rubi [A] time = 0.145946, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{3x^2}{16a^3}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{8a^4}+\frac{3\sinh^{-1}(ax)^2}{16a^5}-\frac{x^4}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*x^2)/(16*a^3) - x^4/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a^4
) + (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) + (3*ArcSinh[a*x]^2)/(16*a
^5)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2}-\frac{3\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{4a^2}-\frac{\int x^3\,dx}{4a}\\ &= -\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2}+\frac{3\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^4}+\frac{3\int x\,dx}{8a^3}\\ &= \frac{3x^2}{16a^3}-\frac{x^4}{16a}-\frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{8a^4}+\frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2}+\frac{3\sinh^{-1}(ax)^2}{16a^5}\end{aligned}$$

Mathematica [A] time = 0.0448544, size = 63, normalized size = 0.73

$$\frac{-a^4x^4+3a^2x^2+2ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)+3\sinh^{-1}(ax)^2}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a^2*x^2 - a^4*x^4 + 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x
] + 3*ArcSinh[a*x]^2)/(16*a^5)
```

Maple [A] time = 0., size = 74, normalized size = 0.9

$$\frac{1}{16\,a^5}\left(4\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}a^3x^3-x^4a^4-6\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}ax+3\,a^2x^2+3\,\left(\mathrm{Arcsinh}\,(ax)\right)^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

[Out] 1/16*(4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-x^4*a^4-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+4)/a^5

Maxima [A] time = 1.15374, size = 138, normalized size = 1.6

$$-\frac{1}{16}\left(\frac{x^4}{a^2}-\frac{3\,x^2}{a^4}+\frac{3\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)^2}{a^6}\right)a+\frac{1}{8}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2}-\frac{3\,\sqrt{a^2x^2+1}x}{a^4}+\frac{3\,\operatorname{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^4}\right)\operatorname{arsinh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -1/16*(x^4/a^2 - 3*x^2/a^4 + 3*arcsinh(a^2*x/sqrt(a^2))^2/a^6)*a + 1/8*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4))*arcsinh(a*x)

Fricas [A] time = 2.71582, size = 188, normalized size = 2.19

$$-\frac{a^4x^4-3\,a^2x^2-2\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)-3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] -1/16*(a^4*x^4 - 3*a^2*x^2 - 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)) - 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^5

Sympy [A] time = 3.31726, size = 82, normalized size = 0.95

$$\begin{cases}-\frac{x^4}{16a}+\frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2}+\frac{3x^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a^4}+\frac{3\operatorname{asinh}^2(ax)}{16a^5} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4/(16*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) + 3*
x**2/(16*a**3) - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(8*a**4) + 3*asinh(a*x)
**2/(16*a**5), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

3.178 $\int \frac{x^3 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=70

$$\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^2} - \frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^4} + \frac{2x}{3a^3} - \frac{x^3}{9a}$$

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rubi [A] time = 0.102941, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5758, 5717, 8, 30}

$$\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^2} - \frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{3a^4} + \frac{2x}{3a^3} - \frac{x^3}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*x)/(3*a^3) - x^3/(9*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^4) + (x
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*a^2)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\,dx}{3a}\\ &= -\frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^2} + \frac{2\int 1\,dx}{3a^3}\\ &= \frac{2x}{3a^3} - \frac{x^3}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.0408277, size = 48, normalized size = 0.69

$$\frac{-a^3x^3+3\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)+6ax}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (6*a*x - a^3*x^3 + 3*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^4)
```

Maple [A] time = 0., size = 82, normalized size = 1.2

$$\frac{1}{9\,a^4}\left(3\,a^4x^4\mathrm{Arcsinh}\,(ax)-3\,a^2x^2\mathrm{Arcsinh}\,(ax)-a^3x^3\sqrt{a^2x^2+1}-6\,\mathrm{Arcsinh}\,(ax)+6\,ax\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/9/a^4/(a^2*x^2+1)^(1/2)*(3*a^4*x^4*arcsinh(a*x)-3*a^2*x^2*arcsinh(a*x)-a^
3*x^3*(a^2*x^2+1)^(1/2)-6*arcsinh(a*x)+6*a*x*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.24391, size = 80, normalized size = 1.14

$$-\frac{1}{9}\,a\left(\frac{x^3}{a^2}-\frac{6\,x}{a^4}\right)+\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(x^3/a^2 - 6*x/a^4) + 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^
2 + 1)/a^4)*arcsinh(a*x)
```

Fricas [A] time = 2.57303, size = 126, normalized size = 1.8

$$-\frac{a^3x^3-3\,\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-6\,ax}{9\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^3*x^3 - 3*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 +
1)) - 6*a*x)/a^4
```

Sympy [A] time = 1.66434, size = 65, normalized size = 0.93

$$\begin{cases} -\frac{x^3}{9a}+\frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}\left(ax\right)}{3a^2}+\frac{2x}{3a^3}-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}\left(ax\right)}{3a^4} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**2) + 2*x
/(3*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(3*a**4), Ne(a, 0)), (0, True)
)
```

Giac [A] time = 1.35864, size = 85, normalized size = 1.21

$$-\frac{a^2x^3 - 6\,x}{9\,a^3} + \frac{\left(\left(a^2x^2 + 1\right)^{\frac{3}{2}} - 3\,\sqrt{a^2x^2 + 1}\right)\log\left(ax + \sqrt{a^2x^2 + 1}\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/9*(a^2*x^3 - 6*x)/a^3 + 1/3*((a^2*x^2 + 1)^(3/2) - 3*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))/a^4
```

3.179 $\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}-\frac{x^2}{4a}$$

```
[Out] -x^2/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rubi [A] time = 0.0807102, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5758, 5675, 30}

$$\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2a^2}-\frac{\sinh^{-1}(ax)^2}{4a^3}-\frac{x^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -x^2/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a^2) - ArcSinh[a*x]^2/(4
*a^3)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{\int x\,dx}{2a}$$
$$= -\frac{x^2}{4a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2a^2} - \frac{\sinh^{-1}(ax)^2}{4a^3}$$

Mathematica [A] time = 0.0394678, size = 42, normalized size = 0.86

$$-\frac{a^2x^2 - 2ax\sqrt{a^2x^2+1}\sinh^{-1}(ax) + \sinh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2)/(4*a^3)
```

Maple [A] time = 0., size = 40, normalized size = 0.8

$$-\frac{1}{4a^3}\left(-2\,\mathrm{Arcsinh}\,(ax)\sqrt{a^2x^2+1}ax + a^2x^2 + (\mathrm{Arcsinh}\,(ax))^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsinh(a*x)^2+1)/a^3
```

Maxima [A] time = 1.29806, size = 100, normalized size = 2.04

$$-\frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\mathrm{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)^2}{a^4}\right) + \frac{1}{2}\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\mathrm{arsinh}\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)\mathrm{arsinh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(x^2/a^2 - arcsinh(a^2*x/sqrt(a^2))^2/a^4) + 1/2*(sqrt(a^2*x^2 + 1)*
x/a^2 - arcsinh(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))*arcsinh(a*x)
```

Fricas [A] time = 2.55848, size = 146, normalized size = 2.98

$$-\frac{a^2x^2 - 2\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + log(
a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A] time = 0.956813, size = 42, normalized size = 0.86

$$\begin{cases} -\frac{x^2}{4a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a^2} - \frac{\operatorname{asinh}^2(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2/(4*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a**2) - asinh(
a*x)**2/(4*a**3), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

3.180 $\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rubi [A] time = 0.0419967, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5717, 8}

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2} - \frac{\int 1\,dx}{a}$$
$$= -\frac{x}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2}$$

Mathematica [A] time = 0.0262278, size = 28, normalized size = 1.

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2} - \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(x/a) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2
```

Maple [A] time = 0., size = 47, normalized size = 1.7

$$\frac{1}{a^2}\left(a^2x^2\text{Arcsinh}\,(ax) + \text{Arcsinh}\,(ax) - ax\sqrt{a^2x^2+1}\right)\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+arcsinh(a*x)-a*x*(a^2*x^2+1)^
(1/2))
```

Maxima [A] time = 1.24198, size = 35, normalized size = 1.25

$$-\frac{x}{a} + \frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a^2
```

Fricas [A] time = 3.10367, size = 82, normalized size = 2.93

$$-\frac{ax-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(a*x - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] time = 0.538699, size = 24, normalized size = 0.86

$$\begin{cases} -\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True)
)
```

Giac [A] time = 1.41754, size = 51, normalized size = 1.82

$$-\frac{x}{a}+\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -x/a + sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2
```

3.181 $\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rubi [A] time = 0.0198992, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}} \, dx = \frac{\sinh^{-1}(ax)^2}{2a}$$

Mathematica [A] time = 0.0059901, size = 13, normalized size = 1.

$$\frac{\sinh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^2/(2*a)
```

Maple [A] time = 0., size = 12, normalized size = 0.9

$$\frac{(\text{Arcsinh}\,(ax))^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Maxima [A] time = 1.15359, size = 15, normalized size = 1.15

$$\frac{\text{arsinh}\,(ax)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(a*x)^2/a
```

Fricas [B] time = 3.00367, size = 51, normalized size = 3.92

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

Sympy [A] time = 0.414252, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^{2}{\left(a x \right)}}{2 a} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Giac [B] time = 1.37943, size = 31, normalized size = 2.38

$$\frac{\log\left(ax + \sqrt{a^{2} x^{2} + 1}\right)^{2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(a*x + sqrt(a^2*x^2 + 1))^2/a
```

3.182 $\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=34

$$-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.0854683, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5760, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - PolyLog[2, -E^ArcSinh[a*x]] + Pol
yLog[2, E^ArcSinh[a*x]]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx &= \text{Subst}\left(\int x\,\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right)+\text{Subst}\left(\int \log(1+\right.\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\sinh^{-1}(ax)}\right)+\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\ &= -2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)-\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)+\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right)\end{aligned}$$

Mathematica [A] time = 0.041103, size = 57, normalized size = 1.68

$$\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)+\sinh^{-1}(ax)\left(\log\left(1-e^{-\sinh^{-1}(ax)}\right)-\log\left(e^{-\sinh^{-1}(ax)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]*(Log[1 - E^(-ArcSinh[a*x])] - Log[1 + E^(-ArcSinh[a*x])]) + Po
lyLog[2, -E^(-ArcSinh[a*x])] - PolyLog[2, E^(-ArcSinh[a*x])]
```

Maple [A] time = 0., size = 42, normalized size = 1.2

$$2\,\text{dilog}\left(\left(ax+\sqrt{a^2x^2+1}\right)^{-1}\right)-\frac{1}{2}\text{dilog}\left(\left(ax+\sqrt{a^2x^2+1}\right)^{-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 2*dilog(1/(a*x+(a^2*x^2+1)^(1/2)))-1/2*dilog(1/(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x), x)
```

3.183 $\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=27

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}$$

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rubi [A] time = 0.058963, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5723, 29}

$$a\log(x)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[x]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x} + a\int \frac{1}{x}\,dx$$
$$= -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{x} + a\log(x)$$

Mathematica [A] time = 0.036604, size = 29, normalized size = 1.07

$$a\log(ax) - \frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) + a*Log[a*x]
```

Maple [B] time = 0., size = 56, normalized size = 2.1

$$-2\,a\mathrm{Arcsinh}\,(ax) + \frac{\mathrm{Arcsinh}\,(ax)}{x}\left(ax - \sqrt{a^2x^2+1}\right) + a\ln\left(\left(ax + \sqrt{a^2x^2+1}\right)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -2*a*arcsinh(a*x)+(a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)+a*ln((a*x+(a^2*x^2
+1)^(1/2))^2-1)
```

Maxima [A] time = 1.15529, size = 34, normalized size = 1.26

$$a\log(x) - \frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x) - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/x
```

Fricas [A] time = 2.64203, size = 88, normalized size = 3.26

$$\frac{ax\log(x) - \sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [B] time = 1.45234, size = 113, normalized size = 4.19

$$-a\left(\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} - \frac{|a|\log(|x|)}{a^2}\right)|a| + \frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*(log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - abs(a)*log(abs(x))/a^2)*abs
(a) + 2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))
^2 - 1)
```

3.184 $\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -a/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rubi [A] time = 0.14391, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5747, 5760, 4182, 2279, 2391, 30}

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -a/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*x^2) + a^2*ArcSinh[a*x]*ArcT
anh[E^ArcSinh[a*x]] + (a^2*PolyLog[2, -E^ArcSinh[a*x]])/2 - (a^2*PolyLog[2,
 E^ArcSinh[a*x]])/2
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
```

```
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2}\,dx-\frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2}-\frac{1}{2}a^2\,\text{Subst}\left(\int x\,\mathbf{csch}(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx,\right.\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x\right.\\
&= -\frac{a}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{2x^2}+a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}a^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right)-\frac{1}{2}a^2\mathbf{Li}_2
\end{aligned}$$

Mathematica [A] time = 0.0258279, size = 126, normalized size = 1.58

$$\frac{1}{8}a^2\left(-4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right)-4\sinh^{-1}(ax)\log\left(1-e^{-\sinh^{-1}(ax)}\right)+4\sinh^{-1}(ax)\log$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]*Csch[ArcSinh[a*x]/2]^2 - 4*Arc
Sinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a
*x])] - 4*PolyLog[2, -E^(-ArcSinh[a*x])] + 4*PolyLog[2, E^(-ArcSinh[a*x])]
- ArcSinh[a*x]*Sech[ArcSinh[a*x]/2]^2 + 2*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A] time = 0., size = 150, normalized size = 1.9

$$-\frac{1}{2x^2}\left(a^2x^2\text{Arcsinh}(ax)+ax\sqrt{a^2x^2+1}+\text{Arcsinh}(ax)\right)\frac{1}{\sqrt{a^2x^2+1}}+\frac{a^2\text{Arcsinh}(ax)}{2}\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+\frac{a^2}{2}\text{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)*(a^2*x^2*arcsinh(a*x)+a*x*(a^2*x^2+1)^(1/2)+arcsinh(
a*x))/x^2+1/2*a^2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/2*a^2*polylog(
2,-a*x-(a^2*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-
1/2*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arsinh}(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}{a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)/(a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asinh}\,(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)/(sqrt(a^2*x^2 + 1)*x^3), x)
```

3.185 $\int x^m \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=313

$$-\frac{3bcd^3\left(35m^3+455m^2+1813m+2161\right)x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)^2(m+7)^2}+\frac{3c^2d^3x^{m+3}\left(a+b\sinh^{-1}(c\right.}{m+3}$$

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*x^(2 + m)*Sqrt[1 + c^2*
x^2])/((3 + m)^2*(5 + m)^2*(7 + m)^2)) - (b*c^3*d^3*(9 + m)*(13 + 2*m)*x^(4
 + m)*Sqrt[1 + c^2*x^2])/((5 + m)^2*(7 + m)^2) - (b*c^5*d^3*x^(6 + m)*Sqrt[
1 + c^2*x^2])/(7 + m)^2 + (d^3*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (3
*c^2*d^3*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a
+ b*ArcSinh[c*x]))/(5 + m) + (c^6*d^3*x^(7 + m)*(a + b*ArcSinh[c*x]))/(7 +
m) - (3*b*c*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 +
 m)^2*(7 + m)^2)
```

Rubi [A] time = 2.17226, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {270, 5730, 12, 1809, 1267, 459, 364}

$$\frac{3c^2d^3x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{3c^4d^3x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{c^6d^3x^{m+7}\left(a+b\sinh^{-1}(cx)\right)}{m+7}+\frac{d^3x^{m+1}\left(a+b\sinh^{-1}(c\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*x^(2 + m)*Sqrt[1 + c^2*
x^2])/((3 + m)^2*(5 + m)^2*(7 + m)^2)) - (b*c^3*d^3*(9 + m)*(13 + 2*m)*x^(4
 + m)*Sqrt[1 + c^2*x^2])/((5 + m)^2*(7 + m)^2) - (b*c^5*d^3*x^(6 + m)*Sqrt[
1 + c^2*x^2])/(7 + m)^2 + (d^3*x^(1 + m)*(a + b*ArcSinh[c*x]))/(1 + m) + (3
*c^2*d^3*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a
+ b*ArcSinh[c*x]))/(5 + m) + (c^6*d^3*x^(7 + m)*(a + b*ArcSinh[c*x]))/(7 +
m) - (3*b*c*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric
2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 +
 m)^2*(7 + m)^2)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{3c^2d^3x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{3c^4d^3x^{5+m}\left(a\right.}{5}$$

$$= \frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{3c^2d^3x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{3c^4d^3x^{5+m}\left(a\right.}{5}$$

$$= -\frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} + \frac{d^3x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{3c^2d^3x^{3+m}\left(a+b\sinh\right.}{3+m}$$

$$= -\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2(7+m)^2} - \frac{bc^5d^3x^{6+m}\sqrt{1+c^2x^2}}{(7+m)^2} + \frac{d^3x^{1+m}\left(\right.}{}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} - \frac{bc^3d^3(9+}{}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} - \frac{bc^3d^3(9+}{}$$

Mathematica [A] time = 0.51656, size = 257, normalized size = 0.82

$$x^{m+1}\left(\frac{6d\left(\frac{4d^2\left(-bc(m+1)x\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,-c^2x^2\right)-2bcx\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,-c^2x^2\right)+(m+2)\left(c^2mx^2+c^2x^2+m+3\right)\left(a+b\sinh^{-1}(cx)\right)\right)}{(m+1)(m+2)(m+3)}-\frac{bcd^2x\text{Hyper}}{}\right.}{m+5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x]) - (b*c*d^3*x*Hypergeomet
ric2F1[-5/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (6*d*((d + c^2*d*x^2)
```

```
^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2 + m
/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a
 + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2
, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)
]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)))/(7 + m)
```

Maple [F] time = 2.484, size = 0, normalized size = 0.

$$\int x^m \left(c^2 d x^2 + d\right)^3 (a + b\text{Arcsinh}\,(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ac^6d^3x^6 + 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 + ad^3 + \left(bc^6d^3x^6 + 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 + bd^3\right)\text{arsinh}\,(cx)\right)x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^6*d^3*x^6 + 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 + a*d^3 + (b*c^
6*d^3*x^6 + 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 + b*d^3)*arcsinh(c*x))*x^m, x
```

)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^3(b\operatorname{arsinh}(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)*x^m, x)
```

3.186 $\int x^m \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=217

$$-\frac{bcd^2\left(15m^2+100m+149\right)x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)^2}+\frac{2c^2d^2x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{c^4d^2x^{m+}\cdots}{\cdots}$$

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m
)^2)) - (b*c^3*d^2*x^(4 + m)*Sqrt[1 + c^2*x^2])/(5 + m)^2 + (d^2*x^(1 + m)*
(a + b*ArcSinh[c*x]))/(1 + m) + (2*c^2*d^2*x^(3 + m)*(a + b*ArcSinh[c*x]))/
(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 + m) - (b*c*d^2*(149
+ 100*m + 15*m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(
c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)^2)
```

Rubi [A] time = 0.296669, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {270, 5730, 12, 1267, 459, 364}

$$\frac{2c^2d^2x^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{c^4d^2x^{m+5}\left(a+b\sinh^{-1}(cx)\right)}{m+5}+\frac{d^2x^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{bcd^2\left(15m^2+100m+149\right)x\cdots}{(m+1)(m+2)(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*x^(2 + m)*Sqrt[1 + c^2*x^2])/((3 + m)^2*(5 + m
)^2)) - (b*c^3*d^2*x^(4 + m)*Sqrt[1 + c^2*x^2])/(5 + m)^2 + (d^2*x^(1 + m)*
(a + b*ArcSinh[c*x]))/(1 + m) + (2*c^2*d^2*x^(3 + m)*(a + b*ArcSinh[c*x]))/
(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcSinh[c*x]))/(5 + m) - (b*c*d^2*(149
+ 100*m + 15*m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(
c^2*x^2)])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)^2)
```


Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{c^4d^2x^{5+m}\left(a+b\ldots\right.}{5+\ldots}$$

$$= \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m} + \frac{c^4d^2x^{5+m}\left(a+b\ldots\right.}{5+\ldots}$$

$$= -\frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m} + \frac{2c^2d^2x^{3+m}\left(a+b\sinh\ldots\right.}{3+m}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}\left(a\ldots\right.}{1\ldots}$$

$$= -\frac{bcd^2\left(38+13m+m^2\right)x^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2(5+m)^2} - \frac{bc^3d^2x^{4+m}\sqrt{1+c^2x^2}}{(5+m)^2} + \frac{d^2x^{1+m}\left(a\ldots\right.}{1\ldots}$$

Mathematica [A] time = 0.0786517, size = 188, normalized size = 0.87

$$\frac{x^{m+1}\left(\frac{4d^2\left(-bc(m+1)x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,-c^2x^2\right)-2bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,-c^2x^2\right)+(m+2)\left(c^2mx^2+c^2x^2+m+3\right)\left(a+b\sinh^{-1}(\ldots\right.\right.}{(m+1)(m+2)(m+3)}\ldots\right.}{m+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x]) - (b*c*d^2*x*Hypergeomet
ric2F1[-3/2, 1 + m/2, 2 + m/2, -(c^2*x^2)])/(2 + m) + (4*d^2*((2 + m)*(3 +
m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Hypergeometri
c2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c*x*Hypergeometric2F1[1/2, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*(3 + m))))/(5 + m)
```

Maple [F] time = 2.119, size = 0, normalized size = 0.

$$\int x^m \left(c^2dx^2+d\right)^2 (a+b\text{Arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ac^4d^2x^4 + 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 + 2\,bc^2d^2x^2 + bd^2\right)\mathrm{arsinh}\,(cx)\right)x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^2 (b\,\mathrm{arsinh}\,(cx)+a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)*x^m, x)
```

3.187 $\int x^m \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=128

$$-\frac{bcd(3m+7)x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{(m+1)(m+2)(m+3)^2}+\frac{c^2dx^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{dx^{m+1}\left(a+b\sinh^{-1}(cx\right.}{m+1}$$

```
[Out] -((b*c*d*x^(2 + m)*Sqrt[1 + c^2*x^2])/(3 + m)^2) + (d*x^(1 + m)*(a + b*ArcS
inh[c*x]))/(1 + m) + (c^2*d*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) - (b*c*
d*(7 + 3*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^
2)])/((1 + m)*(2 + m)*(3 + m)^2)
```

Rubi [A] time = 0.129572, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {14, 5730, 12, 459, 364}

$$\frac{c^2dx^{m+3}\left(a+b\sinh^{-1}(cx)\right)}{m+3}+\frac{dx^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{m+1}-\frac{bcd(3m+7)x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{(m+1)(m+2)(m+3)^2}-\frac{bcd\sqrt{c^2x^2+1}x^{m}}{(m+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -((b*c*d*x^(2 + m)*Sqrt[1 + c^2*x^2])/(3 + m)^2) + (d*x^(1 + m)*(a + b*ArcS
inh[c*x]))/(1 + m) + (c^2*d*x^(3 + m)*(a + b*ArcSinh[c*x]))/(3 + m) - (b*c*
d*(7 + 3*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c^2*x^
2)])/((1 + m)*(2 + m)*(3 + m)^2)
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int x^m\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)dx=\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-(bc)\int\frac{dx^{1+m}\left(\frac{1}{1+m}\right.}{\sqrt{1+c^2}}$$

$$=\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}-(bcd)\int\frac{x^{1+m}\left(\frac{1}{1+m}\right.}{\sqrt{1+c^2}}$$

$$=-\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}$$

$$=-\frac{bcdx^{2+m}\sqrt{1+c^2x^2}}{(3+m)^2}+\frac{dx^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{1+m}+\frac{c^2dx^{3+m}\left(a+b\sinh^{-1}(cx)\right)}{3+m}$$

Mathematica [A] time = 0.0686725, size = 118, normalized size = 0.92

$$\frac{dx^{m+1}\left(-bc(m+1)x\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,-c^2x^2\right)-2bcx\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2}+1,\frac{m}{2}+2,-c^2x\right.}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*((2 + m)*(3 + m + c^2*x^2 + c^2*m*x^2)*(a + b*ArcSinh[c*x]) -
b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)] - 2*b*c
*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)*
(3 + m))
```

Maple [F] time = 1.754, size = 0, normalized size = 0.

$$\int x^{m}\left(c^{2}d x^{2}+d\right)\left(a+b\,\mathrm{Arcsinh}\left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a c^{2} d x^{2}+a d+\left(b c^{2} d x^{2}+b d\right)\mathrm{arsinh}\left(c x\right)\right)x^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int ax^m\,dx + \int bx^m \operatorname{asinh}(cx)\,dx + \int ac^2x^2x^m\,dx + \int bc^2x^2x^m \operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] d*(Integral(a*x**m, x) + Integral(b*x**m*asinh(c*x), x) + Integral(a*c**2*x
**2*x**m, x) + Integral(b*c**2*x**2*x**m*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)(b\operatorname{arsinh}(cx) + a)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

3.188 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{c^2dx^2+d},x\right)$$

```
[Out] Unintegrable[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Rubi [A] time = 0.0707081, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Mathematica [A] time = 3.67025, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x]
```

Maple [A] time = 0.38, size = 0, normalized size = 0.

$$\int \frac{x^m (a + b\text{Arcsinh}(cx))}{c^2dx^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}(cx) + a)x^m}{c^2dx^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\,\text{arsinh}(cx) + a)x^m}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^m}{c^2x^2+1}\,dx + \int \frac{bx^m \operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a*x**m/(c**2*x**2 + 1), x) + Integral(b*x**m*asinh(c*x)/(c**2*x**
2 + 1), x))/d
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d), x)
```

3.189 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=115

$$\frac{(1-m)\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{c^2dx^2+d},x\right)}{2d}-\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{2d^2(m+2)}+\frac{x^{m+1}\left(a+b\sinh^{-1}(c\right.}{2d^2\left(c^2x^2+1\right)}$$

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (b*c*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2*d^2*(2 + m)) + ((
1 - m)*Unintegrable[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x])/(2*d)
```

Rubi [A] time = 0.158176, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(2*d^2*(1 + c^2*x^2)) - (b*c*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2*d^2*(2 + m)) + ((
1 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{(bc)\int \frac{x^{1+m}}{\left(1+c^2x^2\right)^{3/2}}\,dx}{2d^2}+\frac{(1-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{2d}\\ &= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{2d^2\left(1+c^2x^2\right)}-\frac{bcx^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{2d^2(2+m)}+\frac{(1-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{d+c^2dx^2}\,dx}{2d}\end{aligned}$$

Mathematica [A] time = 5.4083, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^2, x]
```

Maple [A] time = 0.435, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x^m}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x^m}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^m}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{bx^m\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**m/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b*x**m*asinh(
c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^2, x)
```

3.190 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=206

$$\frac{(1-m)(3-m)\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{c^2dx^2+d},x\right)}{8d^2}-\frac{bc(3-m)x^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{8d^3(m+2)}-\frac{bcx^m}{}$$

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + ((3 - m)*x^(1 +
m)*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) - (b*c*(3 - m)*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(8*d^3*(2 + m)) - (b
*c*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(4*d
^3*(2 + m)) + ((1 - m)*(3 - m)*Unintegrable[(x^m*(a + b*ArcSinh[c*x]))/(d +
 c^2*d*x^2), x])/(8*d^2)
```

Rubi [A] time = 0.253305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(4*d^3*(1 + c^2*x^2)^2) + ((3 - m)*x^(1 +
m)*(a + b*ArcSinh[c*x]))/(8*d^3*(1 + c^2*x^2)) - (b*c*(3 - m)*x^(2 + m)*Hyp
ergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(8*d^3*(2 + m)) - (b
*c*x^(2 + m)*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(4*d
^3*(2 + m)) + ((1 - m)*(3 - m)*Defer[Int][(x^m*(a + b*ArcSinh[c*x]))/(d + c
^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\int \frac{x^m \left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x^{1+m}}{\left(1+c^2x^2\right)^{5/2}}\,dx}{4d^3} + \frac{(3-m)\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{(3-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)} - \frac{bcx^{2+m}\,{}_2F_1\left(\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};-c^2\right.}{4d^3(2+m)}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{4d^3\left(1+c^2x^2\right)^2} + \frac{(3-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{8d^3\left(1+c^2x^2\right)} - \frac{bc(3-m)x^{2+m}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\right.}{8d^3(2+m)}$$

Mathematica [A] time = 5.66301, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3,x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^3, x]
```

Maple [A] time = 0.47, size = 0, normalized size = 0.

$$\int \frac{x^m \left(a+b\mathrm{Arcsinh}\,(cx)\right)}{\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{(c^2dx^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{arsinh}(cx) + a)x^m}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*
x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{(c^2dx^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^3, x)
```

3.191 $\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=618

$$-\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}\,\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)}{(m+1)(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}}+\frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\,\text{Hy}}{(m}$$

```
[Out] (-15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*(6 + m)*Sqrt
[1 + c^2*x^2]) - (5*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((6 + m)*(8 + 6*
m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((12
+ 8*m + m^2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2
])/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d^2*x^(4 + m)*Sqrt[d +
c^2*d*x^2])/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^(6 + m)*Sqrt
[d + c^2*d*x^2])/((6 + m)^2*Sqrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m
)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)) + (x^(1 + m
)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(6 + m) + (15*d^2*x^(1 + m)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)])/((6 + m)*(8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2])
 - (15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.556182, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5744, 5742, 5762, 30, 14, 270}

$$-\frac{15bcd^2x^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)(m+6)\sqrt{c^2x^2+1}}+\frac{15d^2x^{m+1}\sqrt{c^2dx^2+d}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a\right.}{(m+6)\left(m^3+7m^2+14m+8\right)\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*(6 + m)*Sqrt
[1 + c^2*x^2]) - (5*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((6 + m)*(8 + 6*
m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((12
+ 8*m + m^2)*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2
])/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d^2*x^(4 + m)*Sqrt[d +
```

```
c^2*d*x^2])/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^(6 + m)*Sqrt
[d + c^2*d*x^2])/((6 + m)^2*Sqrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m
)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)) + (x^(1 + m
)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(6 + m) + (15*d^2*x^(1 + m)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (
3 + m)/2, -(c^2*x^2)])/((6 + m)*(8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2])
 - (15*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{x^{1+m}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{6+m} + \frac{(5d)\int x^m\left(d+c^2dx^2\right)^{3/2}\left(a+b\sin\right.}{6+m}$$

$$= \frac{5dx^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(4+m)(6+m)} + \frac{x^{1+m}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh\right.}{6+m}$$

$$= -\frac{bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{\left(12+8m+m^2\right)\sqrt{1+c^2x^2}} - \frac{2bc^3d^2x^{4+m}\sqrt{d+c^2dx^2}}{(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{bc^5d^2x^{6+m}\sqrt{d}}{(6+m)^2\sqrt{1}}$$

$$= -\frac{15bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1+c^2x^2}} - \frac{5bcd^2x^{2+m}\sqrt{d+c^2dx^2}}{(2+m)(4+m)(6+m)\sqrt{1+c^2x^2}} -$$

Mathematica [A] time = 1.31886, size = 332, normalized size = 0.54

$$\frac{d^2x^{m+1}\sqrt{c^2dx^2+d}\left(-\frac{5\left(3(m+4)\left(bcx\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)-(m+2)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)\left(a+b\right.\right.\right.}{(m}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*((4 + m)*(6 + m) + 2*c^2*(2 +
m)*(6 + m)*x^2 + c^4*(2 + m)*(4 + m)*x^4))/((2 + m)*(4 + m)*(6 + m)*Sqrt[1
```

```
+ c^2*x^2])) + (1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]) - (5*(b*c*(1 + m)*(2 +
m)*x*(4 + m + c^2*(2 + m)*x^2) - (1 + m)*(2 + m)^2*(4 + m)*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]) + 3*(4 + m)*(b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2 + m)*(a + b*ArcSinh[c*x])*Hypergeome
tric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] + b*c*x*HypergeometricPFQ[{1
, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])))/((1 + m)*(2 + m)^
2*(4 + m)^2*Sqrt[1 + c^2*x^2])))/(6 + m)
```

Maple [F] time = 1.312, size = 0, normalized size = 0.

$$\int x^{m} \left(c^{2} d x^{2}+d\right)^{\frac{5}{2}} \left(a+b \operatorname{Arcsinh}\left(c x\right)\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^{2} d x^{2}+d\right)^{\frac{5}{2}} (b \operatorname{arsinh}\left(c x\right)+a) x^{m} \, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a c^{4} d^{2} x^{4}+2 \, a c^{2} d^{2} x^{2}+a d^{2}+\left(b c^{4} d^{2} x^{4}+2 \, b c^{2} d^{2} x^{2}+b d^{2}\right) \operatorname{arsinh}\left(c x\right)\right) \sqrt{c^{2} d x^{2}+d} x^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.192 $\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=390

$$-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)}{(m+1)(m+2)^2(m+4)\sqrt{c^2x^2+1}}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\text{Hyperge}}{\left(m^3\right.}$$

```
[Out] (-3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x^
2]) - (b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((8 + 6*m + m^2)*Sqrt[1 + c^2*x
^2]) - (b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*Sqrt[1 + c^2*x^2]
) + (3*d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8 + 6*m + m^2
) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(4 + m) + (3*d*x
^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2
]) - (3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*Sqr
t[1 + c^2*x^2])
```

Rubi [A] time = 0.330163, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5744, 5742, 5762, 30, 14}

$$-\frac{3bcdx^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2(m+4)\sqrt{c^2x^2+1}}+\frac{3dx^{m+1}\sqrt{c^2dx^2+d}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\,\text{si}\right.}{\left(m^3+7m^2+14m+8\right)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x^
2]) - (b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((8 + 6*m + m^2)*Sqrt[1 + c^2*x
^2]) - (b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*Sqrt[1 + c^2*x^2]
) + (3*d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8 + 6*m + m^2
) + (x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(4 + m) + (3*d*x
^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((8 + 14*m + 7*m^2 + m^3)*Sqrt[1 + c^2*x^2
]) - (3*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*(4 + m)*Sqr
t[1 + c^2*x^2])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) dx = \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{4+m} + \frac{(3d)\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{4+m}$$

$$= \frac{3dx^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8+6m+m^2} + \frac{x^{1+m}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{4+m}$$

$$= -\frac{3bcdx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2(4+m)\sqrt{1+c^2x^2}} - \frac{bcdx^{2+m}\sqrt{d+c^2dx^2}}{\left(8+6m+m^2\right)\sqrt{1+c^2x^2}} - \frac{bc^3dx^{4+m}\sqrt{d+c}}{(4+m)^2\sqrt{1+c}}$$

Mathematica [A] time = 0.528491, size = 233, normalized size = 0.6

$$\frac{dx^{m+1}\sqrt{c^2dx^2+d}\left(-\frac{3\left(bcx\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)-(m+2)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)\right.}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}\right.}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(-((b*c*x*(4 + m + c^2*(2 + m)*x^2))/((2 +
 m)*(4 + m)*Sqrt[1 + c^2*x^2])) + (1 + c^2*x^2)*(a + b*ArcSinh[c*x]) - (3*(
b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) - (2
 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c
^2*x^2)] + b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])))/(4 + m)
```

Maple [F] time = 1.136, size = 0, normalized size = 0.

$$\int x^m \left(c^2dx^2+d\right)^{\frac{3}{2}} (a+b\text{Arcsinh}\,(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(ac^2dx^2 + ad + \left(bc^2dx^2 + bd\right) \operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2 + d}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.193 $\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=240

$$-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}\,\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\,\text{Hypergeon}}{(n}$$

```
[Out] -((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 + c^2*x^2])) + (x^(
1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m) + (x^(1 + m)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*x^(2 + m)*Sqr
t[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.202771, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5742, 5762, 30}

$$-\frac{bcx^{m+2}\sqrt{c^2dx^2+d}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{(m+1)(m+2)^2\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2dx^2+d}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh\right.}{\left(m^2+3m+2\right)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2])/((2 + m)^2*Sqrt[1 + c^2*x^2])) + (x^(
1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2 + m) + (x^(1 + m)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[1 + c^2*x^2]) - (b*c*x^(2 + m)*Sqr
t[d + c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, -(c^2*x^2)])/((1 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
```

```
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+\frac{\sqrt{d+c^2dx^2}\int\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{(2+m)\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{\right.}{(}$$

$$=-\frac{bcx^{2+m}\sqrt{d+c^2dx^2}}{(2+m)^2\sqrt{1+c^2x^2}}+\frac{x^{1+m}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2+m}+\frac{x^{1+m}\sqrt{d+c^2dx^2}}{}$$

Mathematica [A] time = 0.0709386, size = 179, normalized size = 0.75

$$\frac{x^{m+1}\sqrt{c^2dx^2+d}\left(-bcx\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)+(m+2)\text{Hypergeometric2I}\right.}{(m+1)(m+2)^{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[d + c^2*d*x^2]*((1 + m)*(-(b*c*x) + a*(2 + m)*Sqrt[1 + c^2*
x^2] + b*(2 + m)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]) + (2 + m)*(a + b*ArcSinh[c
*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1
 + m)*(2 + m)^2*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 1.042, size = 0, normalized size = 0.

$$\int x^m \sqrt{c^2 d x^2 + d}\,(a + b \operatorname{Arcsinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2 d x^2 + d}(b \operatorname{arsinh}(cx) + a)x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{c^2 d x^2 + d}(b \operatorname{arsinh}(cx) + a)x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{d\left(c^2 x^2 + 1\right)}\,(a + b \operatorname{asinh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.194 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{c^2x^2+1}\,x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\text{HypergeometricPF}}{\left(m^2+3m\right.}$$

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2
+ m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.192269, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5764, 5762}

$$\frac{\sqrt{c^2x^2+1}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(m+1)\sqrt{c^2dx^2+d}}-\frac{bc\sqrt{c^2x^2+1}\,x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x\right.}{\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2
+ m)*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
 2 + m/2}, -(c^2*x^2)])/((2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$
$$= \frac{x^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{(1+m)\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_3F_2\left(1,1+\right.}{\left(2+3m+\right.}$$

Mathematica [A] time = 0.0563458, size = 129, normalized size = 0.8

$$\frac{\sqrt{c^2x^2+1}x^{m+1}\left((m+2)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-bcx\text{HypergeometricPFQ}\left(\left\{1,\right.\right.\right.}{(m+1)(m+2)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + c^2*x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 +
m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/((1 + m)*(2 + m)*Sqrt[d
+ c^2*d*x^2])
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int x^m\left(a+b\text{Arcsinh}\left(cx\right)\right)\frac{1}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)x^m}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arsinh}\,(cx) + a)x^m}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,(a + b\,\mathrm{asinh}\,(cx))}{\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/sqrt(c^2*d*x^2 + d), x)
```

3.195 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=268

$$\frac{bcm\sqrt{c^2x^2+1}x^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)}{d\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{m\sqrt{c^2x^2+1}x^{m+1}\text{Hypergeom}}{}$$

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (m*x^(1 + m)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/(d*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1
+ c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d*(2 +
m)*Sqrt[d + c^2*d*x^2]) + (b*c*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(d*(2 + 3*m +
 m^2)*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.330701, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5755, 5764, 5762, 364}

$$\frac{bcm\sqrt{c^2x^2+1}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{d\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{m\sqrt{c^2x^2+1}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)\left(a+b\sinh^{-}\right.}{d(m+1)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```


```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + c^2*d*x^2]) - (m*x^(1 + m)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/(d*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1
+ c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d*(2 +
m)*Sqrt[d + c^2*d*x^2]) + (b*c*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric
PFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(d*(2 + 3*m +
 m^2)*Sqrt[d + c^2*d*x^2])
```


Rule *5755*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m \left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{m\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{x^{1+m}}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d(2+m)\sqrt{d+c^2dx^2}} - \frac{\left(m\sqrt{1+c^2x^2}\right)\int}{d\sqrt{d}}$$

$$= \frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+c^2dx^2}} - \frac{mx^{1+m}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{d(1+m)\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.224382, size = 206, normalized size = 0.77

$$\frac{x^{m+1}\left(bcmx\sqrt{c^2x^2+1}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)-m(m+2)\sqrt{c^2x^2+1}\text{Hyperg}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(-(m*(2 + m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)]) + (1 + m)*((2 + m)*(a + b*Arc
Sinh[c*x]) - b*c*x*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2,
 -(c^2*x^2)]) + b*c*m*x*Sqrt[1 + c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)]))/(d*(1 + m)*(2 + m)*Sqrt[d + c^2
*d*x^2])
```

Maple [F] time = 0.424, size = 0, normalized size = 0.

$$\int x^m\left(a+b\text{Arcsinh}\,(cx)\right)\left(c^2dx^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{(c^2dx^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)x^m}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (a + b \operatorname{asinh}(cx))}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)x^m}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

3.196 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=402

$$\frac{bc(2-m)m\sqrt{c^2x^2+1}x^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2\right\},-c^2x^2\right)}{3d^2\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{(2-m)m\sqrt{c^2x^2+1}x^{m+1}}{}$$

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + ((2 - m)*x^(
1 + m)*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - ((2 - m)*m*x^(1
+ m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(3*d^2*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*(2 -
m)*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -
(c^2*x^2)])/(3*d^2*(2 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c
^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(3*d^2*(2 +
 m)*Sqrt[d + c^2*d*x^2]) + (b*c*(2 - m)*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hyper
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(3*d
^2*(2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.470745, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5755, 5764, 5762, 364}

$$\frac{bc(2-m)m\sqrt{c^2x^2+1}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-c^2x^2\right)}{3d^2\left(m^2+3m+2\right)\sqrt{c^2dx^2+d}}-\frac{(2-m)m\sqrt{c^2x^2+1}x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)}{3d^2(m+1)\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*(a + b*ArcSinh[c*x]))/(3*d*(d + c^2*d*x^2)^(3/2)) + ((2 - m)*x^(
1 + m)*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[d + c^2*d*x^2]) - ((2 - m)*m*x^(1
+ m)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(3*d^2*(1 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*(2 -
m)*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -
(c^2*x^2)])/(3*d^2*(2 + m)*Sqrt[d + c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 + c
^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(3*d^2*(2 +
 m)*Sqrt[d + c^2*d*x^2]) + (b*c*(2 - m)*m*x^(2 + m)*Sqrt[1 + c^2*x^2]*Hyper
```

```
geometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, -(c^2*x^2)])/(3*d
^2*(2 + 3*m + m^2)*Sqrt[d + c^2*d*x^2])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)\int\frac{x^m\left(a+b\sinh^{-1}(cx)\right)}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d}-\frac{\left(bc\sqrt{1+c^2x^2}\right)\int\frac{x^{1+m}}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$=\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}-\frac{bcx^{2+m}\sqrt{1+c^2x^2}\,{}_2F_1\left(2,\frac{2+m}{2}\right.}{3d^2(2+m)\sqrt{d+c^2}}$$

$$=\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}-\frac{bc(2-m)x^{2+m}\sqrt{1+c^2x^2}\,{}_2F_1}{3d^2(2+m)\sqrt{d}}$$

$$=\frac{x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+c^2dx^2\right)^{3/2}}+\frac{(2-m)x^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+c^2dx^2}}-\frac{(2-m)mx^{1+m}\sqrt{1+c^2x^2}\left(a+\right.}{3d^2(1}$$

Mathematica [A] time = 0.421123, size = 286, normalized size = 0.71

$$\frac{x^{m+1}\left((2-m)\left(c^2x^2+1\right)\left(-m\sqrt{c^2x^2+1}\left((m+2)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)-bcx\text{Hyp}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x]))/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (x^(1 + m)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*(1 + c^2*x
^2)^(3/2)*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, -(c^2*x^2)] + (2 - m)*(1 +
 c^2*x^2)*((1 + m)*(2 + m)*(a + b*ArcSinh[c*x]) - b*c*(1 + m)*x*Sqrt[1 + c^
2*x^2]*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(c^2*x^2)] - m*Sqrt[1 + c^2*
x^2]*((2 + m)*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m
)/2, -(c^2*x^2)] - b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/
2, 2 + m/2}, -(c^2*x^2)]))))/(3*d^2*(1 + m)*(2 + m)*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2])
```

Maple [F] time = 0.433, size = 0, normalized size = 0.

$$\int x^m\left(a+b\text{Arcsinh}\,(cx)\right)\left(c^2dx^2+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)x^m}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)*x^m/(c^6*d^3*x^6 + 3*c^4*
d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)x^m}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

3.197 $\int \frac{x^m \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=102

$$\frac{x^{m+1}\sinh^{-1}(ax)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{ax^{m+2}\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}+\frac{3}{2}\right.\right.}{m^2+3m+2}$$

```
[Out] (x^(1 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*
x^2)])/(1 + m) - (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rubi [A] time = 0.0715032, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5762}

$$\frac{x^{m+1}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-a^2x^2\right)}{m+1}-\frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;-a^2x^2\right)}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*
x^2)])/(1 + m) - (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 5762

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*ArcSinh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(Sqrt[d]*f*(m + 1)), x] -
Simp[(b*c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2
, 2 + m/2}, -(c^2*x^2)])/(Sqrt[d]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx = \frac{x^{1+m}\sinh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-a^2x^2\right)}{1+m} - \frac{ax^{2+m}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};-a^2x^2\right)}{2+3m+m^2}$$

Mathematica [A] time = 0.0312955, size = 97, normalized size = 0.95

$$\frac{x^{m+1}\left((m+2)\sinh^{-1}(ax)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right) - ax\text{HypergeometricPFQ}\left(\left\{1,\frac{m}{2}+1,\frac{m}{2}+1\right\},\left\{\frac{m}{2}\right.\right.\right.}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*ArcSinh[a*x])/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, -(a^2*x^2)] - a*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2
 + m/2}, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int x^m \text{Arcsinh}\,(ax)\,\frac{1}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\,\text{arsinh}\,(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{asinh}(ax)}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)/sqrt(a**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{arsinh}(ax)}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)/sqrt(a^2*x^2 + 1), x)
```

3.198 $\int x^4 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=283

$$\frac{1}{7} d x^5 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{4 b d x^4 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{175 c}+\frac{16 b d x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{525 c^3}-\frac{2 b d \left(c\right.}{}$$

```
[Out] (304*b^2*d*x)/(3675*c^4) - (152*b^2*d*x^3)/(11025*c^2) + (38*b^2*d*x^5)/612
5 + (2*b^2*c^2*d*x^7)/343 - (32*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(525*c^5) + (16*b*d*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(525*c^3)
- (4*b*d*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(21*c^5) + (4*b*d*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x]))/(35*c^5) - (2*b*d*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[
c*x]))/(49*c^5) + (2*d*x^5*(a + b*ArcSinh[c*x])^2)/35 + (d*x^5*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/7
```

Rubi [A] time = 0.476747, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12}

$$\frac{1}{7} d x^5 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{4 b d x^4 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{175 c}+\frac{16 b d x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{525 c^3}-\frac{2 b d \left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (304*b^2*d*x)/(3675*c^4) - (152*b^2*d*x^3)/(11025*c^2) + (38*b^2*d*x^5)/612
5 + (2*b^2*c^2*d*x^7)/343 - (32*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(525*c^5) + (16*b*d*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(525*c^3)
- (4*b*d*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d*(1 +
c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(21*c^5) + (4*b*d*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x]))/(35*c^5) - (2*b*d*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[
c*x]))/(49*c^5) + (2*d*x^5*(a + b*ArcSinh[c*x])^2)/35 + (d*x^5*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/7
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
```

```
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^4\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{7}dx^5\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(2d)\int x^4\left(a+b\sinh^{-1}(cx)\right)^2\,dx-\frac{1}{7}($$

$$=-\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{21c^5}+\frac{4bd\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{35c^5}-$$

$$=-\frac{4bdx^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{175c}-\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{21c^5}+$$

$$=\frac{16b^2dx}{735c^4}-\frac{8b^2dx^3}{2205c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7+\frac{16bdx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.}{525c^3}$$

$$=\frac{16b^2dx}{735c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7-\frac{32bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.}{525c^5}$$

$$=\frac{304b^2dx}{3675c^4}-\frac{152b^2dx^3}{11025c^2}+\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7-\frac{32bd\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{525c^5}$$

Mathematica [A] time = 0.270758, size = 201, normalized size = 0.71

$$\frac{d\left(11025a^2c^5x^5\left(5c^2x^2+7\right)-210ab\sqrt{c^2x^2+1}\left(75c^6x^6+57c^4x^4-76c^2x^2+152\right)-210b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(75c^6x^6\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(11025*a^2*c^5*x^5*(7 + 5*c^2*x^2) - 210*a*b*Sqrt[1 + c^2*x^2]*(152 - 76
*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x - 5320*c^3*x^3 + 2394*
c^5*x^5 + 2250*c^7*x^7) - 210*b*(-105*a*c^5*x^5*(7 + 5*c^2*x^2) + b*Sqrt[1
+ c^2*x^2]*(152 - 76*c^2*x^2 + 57*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 110
25*b^2*c^5*x^5*(7 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^5)
```

Maple [A] time = 0.043, size = 342, normalized size = 1.2

$$\frac{1}{c^5}\left(da^2\left(\frac{c^7x^7}{7}+\frac{c^5x^5}{5}\right)+db^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2\,c^3x^3\left(c^2x^2+1\right)^2}{7}-\frac{3\,(\mathrm{Arcsinh}\,(cx))^2\,cx\left(c^2x^2+1\right)^2}{35}+\frac{2\,(\mathrm{Arcsinh}\,(cx))^2}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^5*(d*a^2*(1/7*c^7*x^7+1/5*c^5*x^5)+d*b^2*(1/7*arcsinh(c*x)^2*c^3*x^3*(c
^2*x^2+1)^2-3/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+2/35*arcsinh(c*x)^2*c*x+1
/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(5
/2)+62/1225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(3/2)+116/3675*arcsinh(c*x)*c^
2*x^2*(c^2*x^2+1)^(1/2)-304/3675*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/343*c*x*(
c^2*x^2+1)^3+37384/385875*c*x-484/42875*c*x*(c^2*x^2+1)^2-3358/385875*c*x*(
c^2*x^2+1))+2*d*a*b*(1/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/49
*c^6*x^6*(c^2*x^2+1)^(1/2)-19/1225*c^4*x^4*(c^2*x^2+1)^(1/2)+76/3675*c^2*x^
2*(c^2*x^2+1)^(1/2)-152/3675*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.186, size = 595, normalized size = 2.1

$$\frac{1}{7}b^2c^2dx^7\,\mathrm{arsinh}\,(cx)^2+\frac{1}{7}a^2c^2dx^7+\frac{1}{5}b^2dx^5\,\mathrm{arsinh}\,(cx)^2+\frac{1}{5}a^2dx^5+\frac{2}{245}\left(35\,x^7\,\mathrm{arsinh}\,(cx)-\left(\frac{5\sqrt{c^2x^2+1}x^6}{c^2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^2*d*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcsin
h(c*x)^2 + 1/5*a^2*d*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1
)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*
sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6
/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(
c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3
- 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)
*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*d
```

Fricas [A] time = 2.79591, size = 613, normalized size = 2.17

$$\frac{1125\left(49\,a^2+2\,b^2\right)c^7dx^7+63\left(1225\,a^2+38\,b^2\right)c^5dx^5-5320\,b^2c^3dx^3+31920\,b^2cdx+11025\left(5\,b^2c^7dx^7+7\,b^2c^5dx^5\right)\log(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d*x^7 + 63*(1225*a^2 + 38*b^2)*c^5*d*x^
5 - 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 + 7*b^2*c
^5*d*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d*x^7 + 735*a*b
*c^5*d*x^5 - (75*b^2*c^6*d*x^6 + 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 + 152*
b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d*
x^6 + 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 + 152*a*b*d)*sqrt(c^2*x^2 + 1))/c
^5
```

Sympy [A] time = 15.7201, size = 388, normalized size = 1.37

$$\begin{cases} \frac{a^2c^2dx^7}{7}+\frac{a^2dx^5}{5}+\frac{2abc^2dx^7\operatorname{asinh}(cx)}{7}-\frac{2abcdx^6\sqrt{c^2x^2+1}}{49}+\frac{2abdx^5\operatorname{asinh}(cx)}{5}-\frac{38abdx^4\sqrt{c^2x^2+1}}{1225c}+\frac{152abdx^2\sqrt{c^2x^2+1}}{3675c^3}-\frac{304abd\sqrt{c^2x^2+1}}{3675c^5}+\frac{b^2c^2\cdots}{\cdots} \\ \frac{a^2dx^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**7/7 + a**2*d*x**5/5 + 2*a*b*c**2*d*x**7*asinh(c*x
)/7 - 2*a*b*c*d*x**6*sqrt(c**2*x**2 + 1)/49 + 2*a*b*d*x**5*asinh(c*x)/5 - 3
8*a*b*d*x**4*sqrt(c**2*x**2 + 1)/(1225*c) + 152*a*b*d*x**2*sqrt(c**2*x**2 +
 1)/(3675*c**3) - 304*a*b*d*sqrt(c**2*x**2 + 1)/(3675*c**5) + b**2*c**2*d*x
**7*asinh(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 - 2*b**2*c*d*x**6*sqrt(c**2*x*
*2 + 1)*asinh(c*x)/49 + b**2*d*x**5*asinh(c*x)**2/5 + 38*b**2*d*x**5/6125 -
 38*b**2*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1225*c) - 152*b**2*d*x**3/(
11025*c**2) + 152*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*c**3) +
304*b**2*d*x/(3675*c**4) - 304*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3675*
c**5), Ne(c, 0)), (a**2*d*x**5/5, True))
```

Giac [A] time = 2.43869, size = 599, normalized size = 2.12

$$\frac{1}{7}a^2c^2dx^7 + \frac{1}{5}a^2dx^5 + \frac{2}{245}\left(35\,x^7\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{5\left(c^2x^2+1\right)^{\frac{7}{2}} - 21\left(c^2x^2+1\right)^{\frac{5}{2}} + 35\left(c^2x^2+1\right)^{\frac{3}{2}} - 35\sqrt{c^2x^2+}}{c^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/7*a^2*c^2*d*x^7 + 1/5*a^2*d*x^5 + 2/245*(35*x^7*log(c*x + sqrt(c^2*x^2 +
1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1)^(3
/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*a*b*c^2*d + 1/25725*(3675*x^7*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 2*c*((75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)
/c^7 - 105*(5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 +
1)^(3/2) - 35*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^8))*b^2*c^2
*d + 2/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5/2) - 1
0*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b*d + 1/1125*(225*x^5*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^5
- 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*d
```

3.199 $\int x^3 \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=198

$$-\frac{1}{18} b c d x^5 \sqrt{c^2x^2+1} \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{6} d x^4 \left(c^2x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{b d x^3 \sqrt{c^2x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{18 c}+\frac{b d x^{}}{}$$

```
[Out] -(b^2*d*x^2)/(24*c^2) + (b^2*d*x^4)/72 + (b^2*c^2*d*x^6)/108 + (b*d*x*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(12*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(18*c) - (b*c*d*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/18 - (d*(a + b*ArcSinh[c*x])^2)/(24*c^4) + (d*x^4*(a + b*ArcSinh[c*x])
^2)/12 + (d*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6
```

Rubi [A] time = 0.568175, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5744, 5661, 5758, 5675, 30, 5742}

$$-\frac{1}{18} b c d x^5 \sqrt{c^2x^2+1} \left(a+b \sinh^{-1}(cx)\right)+\frac{1}{6} d x^4 \left(c^2x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{b d x^3 \sqrt{c^2x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{18 c}+\frac{b d x^{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(b^2*d*x^2)/(24*c^2) + (b^2*d*x^4)/72 + (b^2*c^2*d*x^6)/108 + (b*d*x*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(12*c^3) - (b*d*x^3*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(18*c) - (b*c*d*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*
x]))/18 - (d*(a + b*ArcSinh[c*x])^2)/(24*c^4) + (d*x^4*(a + b*ArcSinh[c*x])
^2)/12 + (d*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/6
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{6}dx^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}d\int x^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx-\frac{1}{3}(bc$$

$$=-\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{12}dx^4\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{6}dx^4\,(1$$

$$=\frac{1}{108}b^2c^2dx^6-\frac{bdx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{1}{18}bcdx^5\sqrt{1+c^2x^2}\,(a+b\,\mathrm{s}$$

$$=\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{12c^3}-\frac{bdx^3\sqrt{1+c^2x^2}\,(}{1}$$

$$=-\frac{b^2dx^2}{24c^2}+\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6+\frac{bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{12c^3}-\frac{bdx^3}{}$$

Mathematica [A] time = 0.259205, size = 186, normalized size = 0.94

$$\frac{d\left(cx\left(18a^2c^3x^3\left(2c^2x^2+3\right)-6ab\sqrt{c^2x^2+1}\left(2c^4x^4+2c^2x^2-3\right)+b^2cx\left(2c^4x^4+3c^2x^2-9\right)\right)+6b\sinh^{-1}(cx)\left(3a\left(4c^6x^6+\right.\right.\right.}{216c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(18*a^2*c^3*x^3*(3 + 2*c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-3 + 2*c
^2*x^2 + 2*c^4*x^4) + b^2*c*x*(-9 + 3*c^2*x^2 + 2*c^4*x^4)) + 6*b*(b*c*x*Sq
rt[1 + c^2*x^2]*(3 - 2*c^2*x^2 - 2*c^4*x^4) + 3*a*(-1 + 6*c^4*x^4 + 4*c^6*x
^6))*ArcSinh[c*x] + 9*b^2*(-1 + 6*c^4*x^4 + 4*c^6*x^6)*ArcSinh[c*x]^2))/(21
6*c^4)
```

Maple [A] time = 0.039, size = 298, normalized size = 1.5

$$\frac{1}{c^4}\left(da^2\left(\frac{c^6x^6}{6}+\frac{c^4x^4}{4}\right)+db^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2\,c^2x^2\left(c^2x^2+1\right)^2}{6}-\frac{(\mathrm{Arcsinh}\,(cx))^2\,c^2x^2\left(c^2x^2+1\right)}{12}-\frac{(\mathrm{Arcsinh}\,(cx))^2\,(c^2x^2}{12}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^4*(d*a^2*(1/6*c^6*x^6+1/4*c^4*x^4)+d*b^2*(1/6*arcsinh(c*x)^2*c^2*x^2*(c
^2*x^2+1)^2-1/12*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)-1/12*arcsinh(c*x)^2*(c^
2*x^2+1)-1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+1/18*arcsinh(c*x)*c*x*(c^2
*x^2+1)^(3/2)+1/108*c^2*x^2*(c^2*x^2+1)^2-1/216*c^2*x^2*(c^2*x^2+1)-5/108*c
^2*x^2-5/108+1/12*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/24*arcsinh(c*x)^2)+2
*d*a*b*(1/6*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^4-1/36*c^5*x^5*(c^2
*x^2+1)^(1/2)-1/36*c^3*x^3*(c^2*x^2+1)^(1/2)+1/24*c*x*(c^2*x^2+1)^(1/2)-1/2
4*arcsinh(c*x)))
```

Maxima [B] time = 1.25314, size = 680, normalized size = 3.43

$$\frac{1}{6}\,b^2c^2dx^6\,\mathrm{arsinh}\,(cx)^2+\frac{1}{6}\,a^2c^2dx^6+\frac{1}{4}\,b^2dx^4\,\mathrm{arsinh}\,(cx)^2+\frac{1}{4}\,a^2dx^4+\frac{1}{144}\left(48\,x^6\,\mathrm{arsinh}\,(cx)-\left(\frac{8\,\sqrt{c^2x^2+1}x^5}{c^2}-\frac{10\ \cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^2*d*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^2*d*x^6 + 1/4*b^2*d*x^4*arcsin
h(c*x)^2 + 1/4*a^2*d*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1
)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*
arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*a*b*c^2*d + 1/864*((8*x^6/c^2
- 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c
^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 1
5*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c*
arcsinh(c*x))*b^2*c^2*d + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x
^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*
c^4))*c)*a*b*d + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c^2*x/sqrt(c^2) + sqrt(
c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2
+ 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c*arcsinh(c*x))*b^
2*d
```

Fricas [A] time = 2.69882, size = 525, normalized size = 2.65

$$\frac{2\left(18\,a^2+b^2\right)c^6dx^6+3\left(18\,a^2+b^2\right)c^4dx^4-9\,b^2c^2dx^2+9\left(4\,b^2c^6dx^6+6\,b^2c^4dx^4-b^2d\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+6\left(12\,a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*a^2 + b^2)*c^6*d*x^6 + 3*(18*a^2 + b^2)*c^4*d*x^4 - 9*b^2*c^2*
d*x^2 + 9*(4*b^2*c^6*d*x^6 + 6*b^2*c^4*d*x^4 - b^2*d)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 6*(12*a*b*c^6*d*x^6 + 18*a*b*c^4*d*x^4 - 3*a*b*d - (2*b^2*c^5*d
*x^5 + 2*b^2*c^3*d*x^3 - 3*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2
*x^2 + 1)) - 6*(2*a*b*c^5*d*x^5 + 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x)*sqrt(c^2*x
^2 + 1))/c^4
```

Sympy [A] time = 10.7605, size = 332, normalized size = 1.68

$$\begin{cases} \frac{a^2c^2dx^6}{6} + \frac{a^2dx^4}{4} + \frac{abc^2dx^6 \operatorname{asinh}(cx)}{3} - \frac{abcdx^5\sqrt{c^2x^2+1}}{18} + \frac{abdx^4 \operatorname{asinh}(cx)}{2} - \frac{abdx^3\sqrt{c^2x^2+1}}{18c} + \frac{abdx\sqrt{c^2x^2+1}}{12c^3} - \frac{abd \operatorname{asinh}(cx)}{12c^4} + \frac{b^2c^2dx^6 \operatorname{asinh}^2 (}{6} \\ \frac{a^2dx^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**6/6 + a**2*d*x**4/4 + a*b*c**2*d*x**6*asinh(c*x)/
3 - a*b*c*d*x**5*sqrt(c**2*x**2 + 1)/18 + a*b*d*x**4*asinh(c*x)/2 - a*b*d*x
**3*sqrt(c**2*x**2 + 1)/(18*c) + a*b*d*x*sqrt(c**2*x**2 + 1)/(12*c**3) - a*
b*d*asinh(c*x)/(12*c**4) + b**2*c**2*d*x**6*asinh(c*x)**2/6 + b**2*c**2*d*x
**6/108 - b**2*c*d*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/18 + b**2*d*x**4*asi
nh(c*x)**2/4 + b**2*d*x**4/72 - b**2*d*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/
(18*c) - b**2*d*x**2/(24*c**2) + b**2*d*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(1
2*c**3) - b**2*d*asinh(c*x)**2/(24*c**4), Ne(c, 0)), (a**2*d*x**4/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)\left(b \operatorname{arsinh}(cx) + a\right)^2 x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^3, x)
```

3.200 $\int x^2 \left(d+c^2 d x^2\right) \left(a+b \sinh ^{-1}(c x)\right)^2 \, dx$

Optimal. Leaf size=206

$$\frac{1}{5} d x^3 \left(c^2 x^2+1\right) \left(a+b \sinh ^{-1}(c x)\right)^2-\frac{4 b d x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh ^{-1}(c x)\right)}{45 c}-\frac{2 b d \left(c^2 x^2+1\right)^{5/2} \left(a+b \sinh ^{-1}(c x)\right)}{25 c^3}+\frac{2 b d (}{}$$

```
[Out] (-52*b^2*d*x)/(225*c^2) + (26*b^2*d*x^3)/675 + (2*b^2*c^2*d*x^5)/125 + (8*b
*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3) - (4*b*d*x^2*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c) + (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(15*c^3) - (2*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2
5*c^3) + (2*d*x^3*(a + b*ArcSinh[c*x])^2)/15 + (d*x^3*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/5
```

Rubi [A] time = 0.341996, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12}

$$\frac{1}{5} d x^3 \left(c^2 x^2+1\right) \left(a+b \sinh ^{-1}(c x)\right)^2-\frac{4 b d x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh ^{-1}(c x)\right)}{45 c}-\frac{2 b d \left(c^2 x^2+1\right)^{5/2} \left(a+b \sinh ^{-1}(c x)\right)}{25 c^3}+\frac{2 b d (}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (-52*b^2*d*x)/(225*c^2) + (26*b^2*d*x^3)/675 + (2*b^2*c^2*d*x^5)/125 + (8*b
*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3) - (4*b*d*x^2*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c) + (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*Ar
cSinh[c*x]))/(15*c^3) - (2*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(2
5*c^3) + (2*d*x^3*(a + b*ArcSinh[c*x])^2)/15 + (d*x^3*(1 + c^2*x^2)*(a + b*
ArcSinh[c*x])^2)/5
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{5}dx^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(2d)\int x^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx-\frac{1}{5}\ldots$$

$$=\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15c^3}-\frac{2bd\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c^3}+\ldots$$

$$=-\frac{4bdx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c}+\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15c^3}\ldots$$

$$=-\frac{4b^2dx}{75c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3}-\frac{4b\ldots}{\ldots}$$

$$=-\frac{52b^2dx}{225c^2}+\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5+\frac{8bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3}-\frac{4\ldots}{\ldots}$$

Mathematica [A] time = 0.235485, size = 177, normalized size = 0.86

$$\frac{d\left(225a^2c^3x^3\left(3c^2x^2+5\right)-30ab\sqrt{c^2x^2+1}\left(9c^4x^4+13c^2x^2-26\right)-30b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(9c^4x^4+13c^2x^2-26\right)-\ldots\right.\right.}{3375c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(225*a^2*c^3*x^3*(5 + 3*c^2*x^2) - 30*a*b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^
2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(-390 + 65*c^2*x^2 + 27*c^4*x^4) - 30*b*(-15
*a*c^3*x^3*(5 + 3*c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(-26 + 13*c^2*x^2 + 9*c^4*
x^4))*ArcSinh[c*x] + 225*b^2*c^3*x^3*(5 + 3*c^2*x^2)*ArcSinh[c*x]^2))/(3375
*c^3)
```

Maple [A] time = 0.037, size = 260, normalized size = 1.3

$$\frac{1}{c^3}\left(da^2\left(\frac{c^5x^5}{5}+\frac{c^3x^3}{3}\right)+db^2\left(\frac{(\text{Arcsinh}(cx))^2\,cx\left(c^2x^2+1\right)^2}{5}-\frac{2\,(\text{Arcsinh}(cx))^2\,cx}{15}-\frac{(\text{Arcsinh}(cx))^2\,cx\left(c^2x^2+1\right)}{15}-\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^3*(d*a^2*(1/5*c^5*x^5+1/3*c^3*x^3)+d*b^2*(1/5*arcsinh(c*x)^2*c*x*(c^2*x
^2+1)^2-2/15*arcsinh(c*x)^2*c*x-1/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/25*ar
csinh(c*x)*c^2*x^2*(c^2*x^2+1)^(3/2)-8/225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)
^(1/2)+52/225*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/125*c*x*(c^2*x^2+1)^2-856/33
75*c*x+22/3375*c*x*(c^2*x^2+1))+2*d*a*b*(1/5*arcsinh(c*x)*c^5*x^5+1/3*arcsi
nh(c*x)*c^3*x^3-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)-13/225*c^2*x^2*(c^2*x^2+1)^(
1/2)+26/225*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.18843, size = 467, normalized size = 2.27

$$\frac{1}{5}\,b^2c^2dx^5\,\text{arsinh}(cx)^2+\frac{1}{5}\,a^2c^2dx^5+\frac{1}{3}\,b^2dx^3\,\text{arsinh}(cx)^2+\frac{2}{75}\left(15\,x^5\,\text{arsinh}(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+1}x^2}{c^4}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^2*d*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcsin
h(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d - 2/1125*(15
*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
```

```
2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*
d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d
```

Fricas [A] time = 2.70919, size = 509, normalized size = 2.47

$$\frac{27\left(25\,a^2+2\,b^2\right)c^5dx^5+5\left(225\,a^2+26\,b^2\right)c^3dx^3-780\,b^2cdx+225\left(3\,b^2c^5dx^5+5\,b^2c^3dx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d*x^5 + 5*(225*a^2 + 26*b^2)*c^3*d*x^3 - 78
0*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 + 5*b^2*c^3*d*x^3)*log(c*x + sqrt(c^2*x^
2 + 1))^2 + 30*(45*a*b*c^5*d*x^5 + 75*a*b*c^3*d*x^3 - (9*b^2*c^4*d*x^4 + 13
*b^2*c^2*d*x^2 - 26*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(9*a*b*c^4*d*x^4 + 13*a*b*c^2*d*x^2 - 26*a*b*d)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 5.56227, size = 313, normalized size = 1.52

$$\begin{cases}\frac{a^2c^2dx^5}{5}+\frac{a^2dx^3}{3}+\frac{2abc^2dx^5\operatorname{asinh}(cx)}{5}-\frac{2abcdx^4\sqrt{c^2x^2+1}}{25}+\frac{2abdx^3\operatorname{asinh}(cx)}{3}-\frac{26abdx^2\sqrt{c^2x^2+1}}{225c}+\frac{52abd\sqrt{c^2x^2+1}}{225c^3}+\frac{b^2c^2dx^5\operatorname{asinh}^2(cx)}{5}+\frac{2b}{} \\ \frac{a^2dx^3}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**5/5 + a**2*d*x**3/3 + 2*a*b*c**2*d*x**5*asinh(c*x
)/5 - 2*a*b*c*d*x**4*sqrt(c**2*x**2 + 1)/25 + 2*a*b*d*x**3*asinh(c*x)/3 - 2
6*a*b*d*x**2*sqrt(c**2*x**2 + 1)/(225*c) + 52*a*b*d*sqrt(c**2*x**2 + 1)/(22
5*c**3) + b**2*c**2*d*x**5*asinh(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 - 2*b**
2*c*d*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + b**2*d*x**3*asinh(c*x)**2/3
+ 26*b**2*d*x**3/675 - 26*b**2*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c
) - 52*b**2*d*x/(225*c**2) + 52*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*
c**3), Ne(c, 0)), (a**2*d*x**3/3, True))
```

Giac [B] time = 2.20009, size = 501, normalized size = 2.43

$$\frac{1}{5}a^2c^2dx^5 + \frac{2}{75}\left(15\,x^5\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{3\left(c^2x^2+1\right)^{\frac{5}{2}} - 10\left(c^2x^2+1\right)^{\frac{3}{2}} + 15\,\sqrt{c^2x^2+1}}{c^5}\right)abc^2d + \frac{1}{1125}\left(225\,x^5\log\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/5*a^2*c^2*d*x^5 + 2/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2
 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b*c^2*d
 + 1/1125*(225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 - 20*c^
2*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 1
5*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*c^2*d + 1/3*a^2
*d*x^3 + 2/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3
*sqrt(c^2*x^2 + 1))/c^3)*a*b*d + 1/27*(9*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2
 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*d
```

3.201 $\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=135

$$-\frac{bdx\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bdx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c}+\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{3d\left(a+\right.}{}$$

```
[Out] (5*b^2*d*x^2)/32 + (b^2*c^2*d*x^4)/32 - (3*b*d*x*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/(16*c) - (b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(8*
c) - (3*d*(a + b*ArcSinh[c*x])^2)/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcS
inh[c*x])^2)/(4*c^2)
```

Rubi [A] time = 0.134246, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5717, 5684, 5682, 5675, 30, 14}

$$-\frac{bdx\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3bdx\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c}+\frac{d\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{3d\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (5*b^2*d*x^2)/32 + (b^2*c^2*d*x^4)/32 - (3*b*d*x*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x]))/(16*c) - (b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(8*
c) - (3*d*(a + b*ArcSinh[c*x])^2)/(32*c^2) + (d*(1 + c^2*x^2)^2*(a + b*ArcS
inh[c*x])^2)/(4*c^2)
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
```

```
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{(bd)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{2c}$$

$$=-\frac{bdx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{d\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}+\frac{1}{8}$$

$$=-\frac{3bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8c}+$$

$$=\frac{5}{32}b^2dx^2+\frac{1}{32}b^2c^2dx^4-\frac{3bdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{16c}-\frac{bdx\left(1+c^2x^2\right)^{3/2}}{}$$

Mathematica [A] time = 0.258247, size = 155, normalized size = 1.15

$$\frac{d\left(cx\left(8a^2cx\left(c^2x^2+2\right)-2ab\sqrt{c^2x^2+1}\left(2c^2x^2+5\right)+b^2cx\left(c^2x^2+5\right)\right)+2b\sinh^{-1}(cx)\left(a\left(8c^4x^4+16c^2x^2+5\right)-bcx\sqrt{c}\right.\right.}{32c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(c*x*(8*a^2*c*x*(2 + c^2*x^2) + b^2*c*x*(5 + c^2*x^2) - 2*a*b*Sqrt[1 + c
^2*x^2]*(5 + 2*c^2*x^2)) + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2))
+ a*(5 + 16*c^2*x^2 + 8*c^4*x^4))*ArcSinh[c*x] + b^2*(5 + 16*c^2*x^2 + 8*c^
4*x^4)*ArcSinh[c*x]^2))/(32*c^2)
```

Maple [A] time = 0.036, size = 216, normalized size = 1.6

$$\frac{1}{c^2}\left(da^2\left(\frac{c^4x^4}{4}+\frac{c^2x^2}{2}\right)+db^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2c^2x^2\left(c^2x^2+1\right)}{4}+\frac{(\mathrm{Arcsinh}\,(cx))^2\left(c^2x^2+1\right)}{4}-\frac{\mathrm{Arcsinh}\,(cx)\,cx}{8}\left(c^2x^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^2*(d*a^2*(1/4*c^4*x^4+1/2*c^2*x^2)+d*b^2*(1/4*arcsinh(c*x)^2*c^2*x^2*(c
^2*x^2+1)+1/4*arcsinh(c*x)^2*(c^2*x^2+1)-1/8*arcsinh(c*x)*c*x*(c^2*x^2+1)^(
3/2)-3/16*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-3/32*arcsinh(c*x)^2+1/32*c^2*x
```

```
^2*(c^2*x^2+1)+1/8*c^2*x^2+1/8)+2*d*a*b*(1/4*arcsinh(c*x)*c^4*x^4+1/2*arcsi
nh(c*x)*c^2*x^2-1/16*c^3*x^3*(c^2*x^2+1)^(1/2)-5/32*c*x*(c^2*x^2+1)^(1/2)+5
/32*arcsinh(c*x)))
```

Maxima [B] time = 1.22708, size = 552, normalized size = 4.09

$$\frac{1}{4}\,b^2c^2dx^4\,\operatorname{arsinh}(cx)^2+\frac{1}{4}\,a^2c^2dx^4+\frac{1}{2}\,b^2dx^2\,\operatorname{arsinh}(cx)^2+\frac{1}{16}\left(8\,x^4\,\operatorname{arsinh}(cx)-\left(\frac{2\,\sqrt{c^2x^2+1}x^3}{c^2}-\frac{3\,\sqrt{c^2x^2+1}x}{c^4}+\frac{3}{\ }\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c^2*d*x^4*arcsinh(c*x)^2 + 1/4*a^2*c^2*d*x^4 + 1/2*b^2*d*x^2*arcsin
h(c*x)^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt
(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*a*b*c^
2*d + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1
))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4
+ 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c*arcsinh(c*x))*b^2*c^2*d + 1
/2*a^2*d*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsi
nh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b*d + 1/4*(c^2*(x^2/c^2 - log(c^2*x
/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arc
sinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2))*arcsinh(c*x))*b^2*d
```

Fricas [A] time = 2.69083, size = 447, normalized size = 3.31

$$\frac{\left(8\,a^2+b^2\right)c^4dx^4+\left(16\,a^2+5\,b^2\right)c^2dx^2+\left(8\,b^2c^4dx^4+16\,b^2c^2dx^2+5\,b^2d\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(8\,abc^4dx^4+16\,abc^2\right.}{32\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*c^4*d*x^4 + (16*a^2 + 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d*x
^4 + 16*b^2*c^2*d*x^2 + 5*b^2*d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(8*a*b*
c^4*d*x^4 + 16*a*b*c^2*d*x^2 + 5*a*b*d - (2*b^2*c^3*d*x^3 + 5*b^2*c*d*x)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 2*(2*a*b*c^3*d*x^3 + 5*a*b*
```

```
c*d*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [A] time = 3.39426, size = 269, normalized size = 1.99

$$\begin{cases} \frac{a^2c^2dx^4}{4} + \frac{a^2dx^2}{2} + \frac{abc^2dx^4\operatorname{asinh}(cx)}{2} - \frac{abcdx^3\sqrt{c^2x^2+1}}{8} + abdx^2\operatorname{asinh}(cx) - \frac{5abdx\sqrt{c^2x^2+1}}{16c} + \frac{5abd\operatorname{asinh}(cx)}{16c^2} + \frac{b^2c^2dx^4\operatorname{asinh}^2(cx)}{4} + \frac{b^2 \\ \frac{a^2dx^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**4/4 + a**2*d*x**2/2 + a*b*c**2*d*x**4*asinh(c*x)/
2 - a*b*c*d*x**3*sqrt(c**2*x**2 + 1)/8 + a*b*d*x**2*asinh(c*x) - 5*a*b*d*x*
sqrt(c**2*x**2 + 1)/(16*c) + 5*a*b*d*asinh(c*x)/(16*c**2) + b**2*c**2*d*x**
4*asinh(c*x)**2/4 + b**2*c**2*d*x**4/32 - b**2*c*d*x**3*sqrt(c**2*x**2 + 1)
*asinh(c*x)/8 + b**2*d*x**2*asinh(c*x)**2/2 + 5*b**2*d*x**2/32 - 5*b**2*d*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(16*c) + 5*b**2*d*asinh(c*x)**2/(32*c**2),
Ne(c, 0)), (a**2*d*x**2/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)(b\operatorname{arsinh}(cx) + a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x, x)
```

3.202 $\int \left(d+c^2 d x^2\right) \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=125

$$\frac{1}{3}dx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{4bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{2}{3}dx\left(a+\right.$$

```
[Out] (14*b^2*d*x)/9 + (2*b^2*c^2*d*x^3)/27 - (4*b*d*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(3*c) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(9*c)
+ (2*d*x*(a + b*ArcSinh[c*x])^2)/3 + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/3
```

Rubi [A] time = 0.144216, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5684, 5653, 5717, 8}

$$\frac{1}{3}dx\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}-\frac{4bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{2}{3}dx\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (14*b^2*d*x)/9 + (2*b^2*c^2*d*x^3)/27 - (4*b*d*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(3*c) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(9*c)
+ (2*d*x*(a + b*ArcSinh[c*x])^2)/3 + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/3
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{3}dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}(2d)\int\left(a+b\sinh^{-1}(cx)\right)^2\,dx-\frac{1}{3}(2bcd$$

$$= -\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{2}{3}dx\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{3}dx\left(1+c^2\right.$$

$$= \frac{2}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a\right.}{9c}$$

$$= \frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3-\frac{4bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{2bd\left(1+c^2x^2\right)^{3/2}\left(a\right.}{9c}$$

Mathematica [A] time = 0.201368, size = 135, normalized size = 1.08

$$\frac{d\left(9a^2cx\left(c^2x^2+3\right)-6ab\sqrt{c^2x^2+1}\left(c^2x^2+7\right)-6b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(c^2x^2+7\right)-3acx\left(c^2x^2+3\right)\right)+2b^2cx\left(c^2x^2\right.}{27c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*(9*a^2*c*x*(3 + c^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(7 + c^2*x^2) + 2*b^2
*c*x*(21 + c^2*x^2) - 6*b*(-3*a*c*x*(3 + c^2*x^2) + b*Sqrt[1 + c^2*x^2]*(7
```

```
+ c^2*x^2))*ArcSinh[c*x] + 9*b^2*c*x*(3 + c^2*x^2)*ArcSinh[c*x]^2))/(27*c)
```

Maple [A] time = 0.031, size = 172, normalized size = 1.4

$$\frac{1}{c}\left(da^{2}\left(\frac{c^{3}x^{3}}{3}+cx\right)+db^{2}\left(\frac{2\,(\mathrm{Arcsinh}\,(cx))^{2}\,cx}{3}+\frac{(\mathrm{Arcsinh}\,(cx))^{2}\,cx\left(c^{2}x^{2}+1\right)}{3}-\frac{14\,\mathrm{Arcsinh}\,(cx)}{9}\sqrt{c^{2}x^{2}+1}+\frac{40\,cx}{27}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(d*a^2*(1/3*c^3*x^3+c*x)+d*b^2*(2/3*arcsinh(c*x)^2*c*x+1/3*arcsinh(c*x)
^2*c*x*(c^2*x^2+1)-14/9*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+40/27*c*x-2/9*arcsin
h(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+2/27*c*x*(c^2*x^2+1))+2*d*a*b*(1/3*arcsinh
(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/9*c^2*x^2*(c^2*x^2+1)^(1/2)-7/9*(c^2*x^2+1
)^(1/2)))
```

Maxima [B] time = 1.10435, size = 311, normalized size = 2.49

$$\frac{1}{3}\,b^{2}c^{2}dx^{3}\,\mathrm{arsinh}\,(cx)^{2}+\frac{1}{3}\,a^{2}c^{2}dx^{3}+\frac{2}{9}\left(3\,x^{3}\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^{2}x^{2}+1}x^{2}}{c^{2}}-\frac{2\,\sqrt{c^{2}x^{2}+1}}{c^{4}}\right)\right)abc^{2}d-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{c^{2}x^{2}+1}x^{2}}{c^{2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*c^2*d*x^3*arcsinh(c*x)^2 + 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*arcsinh(c
*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d -
2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x
) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsinh(c*x)^2 + 2*b^2*d*(x -
sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^
2*x^2 + 1))*a*b*d/c
```

Fricas [A] time = 2.71419, size = 387, normalized size = 3.1

$$\frac{\left(9\,a^{2}+2\,b^{2}\right)c^{3}dx^{3}+3\left(9\,a^{2}+14\,b^{2}\right)cdx+9\left(b^{2}c^{3}dx^{3}+3\,b^{2}cdx\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+6\left(3\,abc^{3}dx^{3}+9\,abcdx-\left(b^{2}c^{2}d\right.\right.}{27\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*d*x^3 + 3*(9*a^2 + 14*b^2)*c*d*x + 9*(b^2*c^3*d*x
^3 + 3*b^2*c*d*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^3*d*x^3 + 9*a
*b*c*d*x - (b^2*c^2*d*x^2 + 7*b^2*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*
x^2 + 1)) - 6*(a*b*c^2*d*x^2 + 7*a*b*d)*sqrt(c^2*x^2 + 1))/c
```

Sympy [A] time = 1.56101, size = 224, normalized size = 1.79

$$\begin{cases} \frac{a^2c^2dx^3}{3} + a^2dx + \frac{2abc^2dx^3\operatorname{asinh}(cx)}{3} - \frac{2abcdx^2\sqrt{c^2x^2+1}}{9} + 2abdx\operatorname{asinh}(cx) - \frac{14abd\sqrt{c^2x^2+1}}{9c} + \frac{b^2c^2dx^3\operatorname{asinh}^2(cx)}{3} + \frac{2b^2c^2dx^3}{27} - \frac{2b^2cdx \\ a^2dx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**2*d*x**3/3 + a**2*d*x + 2*a*b*c**2*d*x**3*asinh(c*x)/3 -
 2*a*b*c*d*x**2*sqrt(c**2*x**2 + 1)/9 + 2*a*b*d*x*asinh(c*x) - 14*a*b*d*sqr
t(c**2*x**2 + 1)/(9*c) + b**2*c**2*d*x**3*asinh(c*x)**2/3 + 2*b**2*c**2*d*x
**3/27 - 2*b**2*c*d*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/9 + b**2*d*x*asinh(
c*x)**2 + 14*b**2*d*x/9 - 14*b**2*d*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c), N
e(c, 0)), (a**2*d*x, True))
```

Giac [B] time = 2.06326, size = 383, normalized size = 3.06

$$\frac{1}{3}a^2c^2dx^3 + \frac{2}{9}\left(3x^3\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\left(c^2x^2+1\right)^{\frac{3}{2}} - 3\sqrt{c^2x^2+1}}{c^3}\right)abc^2d + \frac{1}{27}\left(9x^3\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2c\left(\frac{c^2}{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c^2*d*x^3 + 2/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1
)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*a*b*c^2*d + 1/27*(9*x^3*log(c*x + sqrt(
```

```
c^2*x^2 + 1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2) - 3*sqr
t(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*c^2*d + 2*(x*log(c*x
 + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b*d + (x*log(c*x + sqrt(c^2*
x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))/c^2
))*b^2*d + a^2*d*x
```

3.203 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=166

$$-bd\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d\text{PolyLog}\left(3,e^{-2\sinh^{-1}(cx)}\right)+\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-$$

```
[Out] (b^2*c^2*d*x^2)/4 - (b*c*d*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (d
*(a + b*ArcSinh[c*x])^2)/4 + (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (
d*(a + b*ArcSinh[c*x])^3)/(3*b) + d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Ar
cSinh[c*x])] - b*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])] - (
b^2*d*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.246114, antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30}

$$bd\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{2}b$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (b^2*c^2*d*x^2)/4 - (b*c*d*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/2 - (d
*(a + b*ArcSinh[c*x])^2)/4 + (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 - (
d*(a + b*ArcSinh[c*x])^3)/(3*b) + d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*Arc
Sinh[c*x])] + b*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])] - (b^
2*d*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx - (bcd)\int \sqrt{1+$$

$$= -\frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2 + d\,\mathrm{S}$$

$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{4}d\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{2}d$$

$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{4}d\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{2}d$$

$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{4}d\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{2}d$$

$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{4}d\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{2}d$$

$$= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{4}d\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{2}d$$

Mathematica [A] time = 0.373892, size = 209, normalized size = 1.26

$$\frac{1}{8}d\left(-8ab\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+8b^2\left(\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{3}\sinh^{-1}(c\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d*(4*a^2*c^2*x^2 - 4*a*b*(c*x*Sqrt[1 + c^2*x^2] - ArcSinh[c*x]) + 8*a*b*c^
2*x^2*ArcSinh[c*x] + b^2*(1 + 2*ArcSinh[c*x]^2)*Cosh[2*ArcSinh[c*x]] + 8*a*
b*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a^2*Log[
x] - 8*a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 8*b^2*(-ArcSinh[c*x]^3/3 + Arc
Sinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcS
inh[c*x])] - PolyLog[3, E^(2*ArcSinh[c*x])]/2) - 2*b^2*ArcSinh[c*x]*Sinh[2*
ArcSinh[c*x]]))/8
```

Maple [B] time = 0.125, size = 425, normalized size = 2.6

$$\frac{da^2c^2x^2}{2}+da^2\ln(cx)-\frac{db^2(\text{Arcsinh}(cx))^3}{3}+\frac{db^2(\text{Arcsinh}(cx))^2c^2x^2}{2}-\frac{db^2\text{Arcsinh}(cx)cx}{2}\sqrt{c^2x^2+1}+\frac{db^2(\text{Arcsinh}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 1/2*d*a^2*c^2*x^2+d*a^2*ln(c*x)-1/3*d*b^2*arcsinh(c*x)^3+1/2*d*b^2*arcsinh(
c*x)^2*c^2*x^2-1/2*d*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+1/4*d*b^2*arcsi
nh(c*x)^2+1/4*b^2*c^2*d*x^2+1/8*d*b^2+d*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^
2+1)^(1/2))+2*d*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*d*b^2*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))+d*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)
^(1/2))+2*d*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*d*b^2*polyl
og(3,c*x+(c^2*x^2+1)^(1/2))-d*a*b*arcsinh(c*x)^2+d*a*b*arcsinh(c*x)*c^2*x^2
-1/2*d*a*b*c*x*(c^2*x^2+1)^(1/2)+1/2*d*a*b*arcsinh(c*x)+2*d*a*b*arcsinh(c*x
)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*d
*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d*a*b*polylog(2,c*x+(c^2*x^
2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2c^2dx^2 + a^2d\log(x) + \int b^2c^2dx\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2\,abc^2dx\log\left(cx + \sqrt{c^2x^2+1}\right) + \frac{b^2d\log\left(cx + \sqrt{c^2x^2+1}\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^2*d*x^2 + a^2*d*log(x) + integrate(b^2*c^2*d*x*log(c*x + sqrt(c^2
*x^2 + 1))^2 + 2*a*b*c^2*d*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d*log(c*x +
 sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int\frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int\frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int b^2c^2x\operatorname{asinh}^2(cx)\,dx + \int 2abc^2x\operatorname{asinh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(b**2*asinh(c*x
)**2/x, x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(b**2*c**2*x*asinh(c
*x)**2, x) + Integral(2*a*b*c**2*x*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2dx^2+d)(b\,\mathrm{arsinh}\,(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x, x)
```

3.204 $\int \frac{(d+c^2dx^2)\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=131

$$-2b^2cd\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-2bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(c^2x^2+1\right)\left(a+\right.}{x}$$

```
[Out] 2*b^2*c^2*d*x - 2*b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + 2*c^2*d*x*
(a + b*ArcSinh[c*x])^2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x - 4*b*c
*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d*PolyLog[2, -E^A
rcSinh[c*x]] + 2*b^2*c*d*PolyLog[2, E^ArcSinh[c*x]]
```

Rubi [A] time = 0.318425, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5739, 5653, 5717, 8, 5742, 5760, 4182, 2279, 2391}

$$-2b^2cd\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-2bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{d\left(c^2x^2+1\right)\left(a+\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] 2*b^2*c^2*d*x - 2*b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + 2*c^2*d*x*
(a + b*ArcSinh[c*x])^2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/x - 4*b*c
*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d*PolyLog[2, -E^A
rcSinh[c*x]] + 2*b^2*c*d*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x} + (2bcd)\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}\,dx$$

$$= 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d\left(1+c^2x^2\right)\left(a\right.}{}$$

$$= -2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(}{}$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(1}{}$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(1}{}$$

$$= 2b^2c^2dx - 2bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) + 2c^2dx\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{d(1}{}$$

Mathematica [A] time = 0.427355, size = 192, normalized size = 1.47

$$\frac{d\left(-b^2\left(-2cx\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+2cx\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2cx\left(\log\left(e^{-\sinh^{-1}(cx)}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d*(-a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x])
+ b^2*c*x*(2*c*x - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2) -
 2*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]) - b^2*(ArcSinh[c*x]*
(ArcSinh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*
```

```
x])])) - 2*c*x*PolyLog[2, -E^(-ArcSinh[c*x])] + 2*c*x*PolyLog[2, E^(-ArcSin
h[c*x])]))))/x
```

Maple [A] time = 0.105, size = 252, normalized size = 1.9

$$da^2c^2x - \frac{da^2}{x} + db^2 (\mathrm{Arcsinh}(cx))^2 c^2x - 2\,cdb^2\mathrm{Arcsinh}(cx)\sqrt{c^2x^2+1} + 2\,b^2c^2dx - \frac{db^2(\mathrm{Arcsinh}(cx))^2}{x} - 2\,cdb^2\mathrm{Arcsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] d*a^2*c^2*x-d*a^2/x+d*b^2*arcsinh(c*x)^2*c^2*x-2*c*d*b^2*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)+2*b^2*c^2*d*x-d*b^2*arcsinh(c*x)^2/x-2*c*d*b^2*arcsinh(c*x)*ln
(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*c*d
*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*c*d*polylog(2,c*x+(c^2*
x^2+1)^(1/2))+2*d*a*b*arcsinh(c*x)*c^2*x-2*d*a*b*arcsinh(c*x)/x-2*c*d*a*b*(
c^2*x^2+1)^(1/2)-2*c*d*a*b*arctanh(1/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2c^2dx\,\mathrm{arsinh}(cx)^2 + 2\,b^2c^2d\left(x - \frac{\sqrt{c^2x^2+1}\,\mathrm{arsinh}(cx)}{c}\right) + a^2c^2dx + 2\left(cx\,\mathrm{arsinh}(cx) - \sqrt{c^2x^2+1}\right)abcd - 2\left(c\,\mathrm{arsinh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] b^2*c^2*d*x*arcsinh(c*x)^2 + 2*b^2*c^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*c^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d - 2*(
c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcsinh(c*x)/x)*a*b*d - b^2*d*(log(c*x +
sqrt(c^2*x^2 + 1))^2/x - integrate(2*(c^3*x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c
)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^
2 + 1)), x)) - a^2*d/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\text{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^2 + abd\right)\text{arsinh}\left(cx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int b^2c^2\,\text{asinh}^2\left(cx\right)dx + \int \frac{b^2\,\text{asinh}^2\left(cx\right)}{x^2}\,dx + \int 2abc^2\,\text{asinh}\left(cx\right)dx + \int \frac{2ab\,\text{asinh}\left(cx\right)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(b**2*c**2*asi
nh(c*x)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2
*asinh(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)\left(b\,\text{arsinh}\left(cx\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x^2, x)
```

3.205 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=180

$$-bc^2d\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2c^2d\text{PolyLog}\left(3,e^{-2\sinh^{-1}(cx)}\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}$$

```
[Out] -((b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/x) + (c^2*d*(a + b*ArcSinh
[c*x])^2)/2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (c^2*d*(a
+ b*ArcSinh[c*x])^3)/(3*b) + c^2*d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Arc
Sinh[c*x])] + b^2*c^2*d*Log[x] - b*c^2*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^
(-2*ArcSinh[c*x])] - (b^2*c^2*d*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.310279, antiderivative size = 179, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5739, 5659, 3716, 2190, 2531, 2282, 6589, 5737, 29, 5675}

$$bc^2d\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2c^2d\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{d\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/x) + (c^2*d*(a + b*ArcSinh
[c*x])^2)/2 - (d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*d*(a
+ b*ArcSinh[c*x])^3)/(3*b) + c^2*d*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcS
inh[c*x])] + b^2*c^2*d*Log[x] + b*c^2*d*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])] - (b^2*c^2*d*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
```

```
] && LtQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+(bcd)\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^2}\,dx+($$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}-\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+\left(c^2d\right)\mathrm{Su}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d\left(1+c^2x^2\right)\left(a\right.}{2x}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d\left(1+c^2x^2\right)\left(a\right.}{2x}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d\left(1+c^2x^2\right)\left(a\right.}{2x}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d\left(1+c^2x^2\right)\left(a\right.}{2x}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2d\left(a+b\sinh^{-1}(cx)\right)^2-\frac{d\left(1+c^2x^2\right)\left(a\right.}{2x}$$

Mathematica [A] time = 0.366043, size = 212, normalized size = 1.18

$$\frac{1}{2}d\left(2abc^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)-\frac{1}{3}b^2c^2\left(-6\sinh^{-1}(cx)\text{Poly}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d*(-(a^2/x^2) - (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/x^2 + 2*a^2
*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 -
 2*c^2*x^2*Log[c*x]))/x^2 + 2*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1
 - E^(-2*ArcSinh[c*x])]) - PolyLog[2, E^(-2*ArcSinh[c*x])]) - (b^2*c^2*(2*A
rcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*ArcSinh[c*x])]) - 6*ArcSinh[c*
x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[3, E^(2*ArcSinh[c*x])]))/3))/
2
```

Maple [B] time = 0.246, size = 515, normalized size = 2.9

$$c^2da^2\ln(cx)-\frac{da^2}{2x^2}-\frac{c^2db^2(\text{Arcsinh}(cx))^3}{3}-\frac{cdb^2\text{Arcsinh}(cx)}{x}\sqrt{c^2x^2+1}+c^2db^2\text{Arcsinh}(cx)-\frac{db^2(\text{Arcsinh}(cx))^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] c^2*d*a^2*ln(c*x)-1/2*d*a^2/x^2-1/3*c^2*d*b^2*arcsinh(c*x)^3-c*d*b^2*arcsin
h(c*x)/x*(c^2*x^2+1)^(1/2)+c^2*d*b^2*arcsinh(c*x)-1/2*d*b^2*arcsinh(c*x)^2/
x^2+c^2*d*b^2*ln(c*x+(c^2*x^2+1)^(1/2)-1)-2*c^2*d*b^2*ln(c*x+(c^2*x^2+1)^(1
/2))+c^2*d*b^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+c^2*d*b^2*arcsinh(c*x)^2*ln(1+c*
x+(c^2*x^2+1)^(1/2))+2*c^2*d*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1
/2))-2*c^2*d*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+c^2*d*b^2*arcsinh(c*x)^2
*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*c^2*d*b^2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^
2+1)^(1/2))-2*c^2*d*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-c^2*d*a*b*arcsinh(
c*x)^2-c*d*a*b/x*(c^2*x^2+1)^(1/2)+c^2*d*a*b-d*a*b*arcsinh(c*x)/x^2+2*c^2*d
*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*c^2*d*a*b*polylog(2,-c*x-(c
^2*x^2+1)^(1/2))+2*c^2*d*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*c^2
*d*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2c^2d\log(x) - abd\left(\frac{\sqrt{c^2x^2+1}c}{x} + \frac{\operatorname{arsinh}(cx)}{x^2}\right) - \frac{a^2d}{2x^2} + \int \frac{b^2c^2d\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{x} + \frac{2abc^2d\log\left(cx+\sqrt{c^2x^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d*log(x) - a*b*d*(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a
^2*d/x^2 + integrate(b^2*c^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*c^2
*d*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d*log(c*x + sqrt(c^2*x^2 + 1))^2/x^
3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{a^2}{x^3}\,dx + \int \frac{a^2c^2}{x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{b^2c^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2abc^2\operatorname{asinh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(b**2*asinh(
c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(b**2*c**2*
```

```
asinh(c*x)**2/x, x) + Integral(2*a*b*c**2*asinh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2dx^2 + d)(b \operatorname{arsinh}(cx) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x^3, x)
```

3.206 $\int \frac{(d+c^2dx^2)(a+b\sinh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=158

$$-\frac{5}{3}b^2c^3d\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{5}{3}b^2c^3d\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{d\left(c^2x^2+1\right)(a}{3x}$$

```
[Out] -(b^2*c^2*d)/(3*x) - (b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2)
 - (2*c^2*d*(a + b*ArcSinh[c*x])^2)/(3*x) - (d*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(3*x^3) - (10*b*c^3*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/3 - (5*b^2*c^3*d*PolyLog[2, -E^ArcSinh[c*x]])/3 + (5*b^2*c^3*d*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rubi [A] time = 0.393379, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5739, 5661, 5760, 4182, 2279, 2391, 5737, 30}

$$-\frac{5}{3}b^2c^3d\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{5}{3}b^2c^3d\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{bcd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{d\left(c^2x^2+1\right)(a}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```


```
[Out] -(b^2*c^2*d)/(3*x) - (b*c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2)
 - (2*c^2*d*(a + b*ArcSinh[c*x])^2)/(3*x) - (d*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x])^2)/(3*x^3) - (10*b*c^3*d*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]
])/3 - (5*b^2*c^3*d*PolyLog[2, -E^ArcSinh[c*x]])/3 + (5*b^2*c^3*d*PolyLog[2
, E^ArcSinh[c*x]])/3
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
```

```
] && LtQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bcd)\int \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,dx$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{d\left(1+c^2x^2\right)\left(a+\cdots\right.}{3x\cdots}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{d\left(1+c\cdots\right.}{\cdots}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{d\left(1+c\cdots\right.}{\cdots}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{d\left(1+c\cdots\right.}{\cdots}$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d\left(a+b\sinh^{-1}(cx)\right)^2}{3x}-\frac{d\left(1+c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.793836, size = 245, normalized size = 1.55

$$-\frac{d\left(-5b^2c^3x^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+5b^2c^3x^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+3a^2c^2x^2+a^2+abcx\sqrt{c^2x^2+1}+6abc^2x^2\,\text{si}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(d*(a^2 + 3*a^2*c^2*x^2 + b^2*c^2*x^2 + a*b*c*x*Sqrt[1 + c^2*x^2] + 2*a*b*
ArcSinh[c*x] + 6*a*b*c^2*x^2*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x] + b^2*ArcSinh[c*x]^2 + 3*b^2*c^2*x^2*ArcSinh[c*x]^2 + 5*a*b*c^3*x^3
*ArcTanh[Sqrt[1 + c^2*x^2]] - 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-ArcSin
h[c*x])] + 5*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 5*b^2*c^
3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] + 5*b^2*c^3*x^3*PolyLog[2, E^(-ArcSinh
[c*x])]))/(3*x^3)
```

Maple [A] time = 0.238, size = 278, normalized size = 1.8

$$-\frac{c^2da^2}{x}-\frac{da^2}{3\,x^3}-\frac{c^2db^2\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{x}-\frac{cdb^2\mathrm{Arcsinh}\left(cx\right)}{3\,x^2}\sqrt{c^2x^2+1}-\frac{db^2\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{3\,x^3}-\frac{c^2db^2}{3\,x}-\frac{5\,c^3db^2\mathrm{Arcsinh}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -c^2*d*a^2/x-1/3*d*a^2/x^3-c^2*d*b^2*arcsinh(c*x)^2/x-1/3*c*d*b^2/x^2*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)-1/3*d*b^2/x^3*arcsinh(c*x)^2-1/3*b^2*c^2*d/x-5/3*
c^3*d*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-5/3*b^2*c^3*d*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))+5/3*c^3*d*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/
2))+5/3*b^2*c^3*d*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*c^2*d*a*b*arcsinh(c*x)
/x-2/3*d*a*b*arcsinh(c*x)/x^3-5/3*c^3*d*a*b*arctanh(1/(c^2*x^2+1)^(1/2))-1/
3*c*d*a*b/x^2*(c^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\left(c\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\mathrm{arsinh}\left(cx\right)}{x}\right)abc^2d+\frac{1}{3}\left(\left(c^2\,\mathrm{arsinh}\left(\frac{1}{\sqrt{c^2}|x|}\right)-\frac{\sqrt{c^2x^2+1}}{x^2}\right)c-\frac{2\,\mathrm{arsinh}\left(cx\right)}{x^3}\right)abd-\frac{a^2c^2d}{x}-\frac{a^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -2*(c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d + 1/3*((c^2
*arcsinh(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/
x^3)*a*b*d - a^2*c^2*d/x - 1/3*a^2*d/x^3 - 1/3*(3*b^2*c^2*d*x^2 + b^2*d)*lo
g(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(3*b^2*c^5*d*x^4 + 4*b^2*c
^3*d*x^2 + b^2*c*d + (3*b^2*c^4*d*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log
(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2c^2dx^2+a^2d+\left(b^2c^2dx^2+b^2d\right)\mathrm{arsinh}\left(cx\right)^2+2\left(abc^2dx^2+abd\right)\mathrm{arsinh}\left(cx\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{a^2}{x^4}\,dx + \int \frac{a^2c^2}{x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^4}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^4}\,dx + \int \frac{b^2c^2\operatorname{asinh}^2(cx)}{x^2}\,dx + \int \frac{2abc^2\operatorname{asinh}(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(b**2*asi
nh(c*x)**2/x**4, x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(b**2*c*
*2*asinh(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)(b\operatorname{arsinh}(cx) + a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x^4, x)
```

3.207 $\int x^4 \left(d + c^2 d x^2\right)^2 \left(a + b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=386

$$\frac{1}{9} d^2 x^5 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{4}{63} d^2 x^5 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{16 b d^2 x^4 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{1575 c}$$

```
[Out] (4208*b^2*d^2*x)/(99225*c^4) - (2104*b^2*d^2*x^3)/(297675*c^2) + (526*b^2*d
^2*x^5)/165375 + (212*b^2*c^2*d^2*x^7)/27783 + (2*b^2*c^4*d^2*x^9)/729 - (1
28*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^5) + (64*b*d^2*x^2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^3) - (16*b*d^2*x^4*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(1575*c) - (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x]))/(189*c^5) + (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh
[c*x]))/(315*c^5) + (20*b*d^2*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(44
1*c^5) - (2*b*d^2*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^5) + (8*d
^2*x^5*(a + b*ArcSinh[c*x])^2)/315 + (4*d^2*x^5*(1 + c^2*x^2)*(a + b*ArcSin
h[c*x])^2)/63 + (d^2*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/9
```

Rubi [A] time = 0.738931, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 1153}

$$\frac{1}{9} d^2 x^5 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{4}{63} d^2 x^5 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{16 b d^2 x^4 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{1575 c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4208*b^2*d^2*x)/(99225*c^4) - (2104*b^2*d^2*x^3)/(297675*c^2) + (526*b^2*d
^2*x^5)/165375 + (212*b^2*c^2*d^2*x^7)/27783 + (2*b^2*c^4*d^2*x^9)/729 - (1
28*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^5) + (64*b*d^2*x^2
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(4725*c^3) - (16*b*d^2*x^4*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(1575*c) - (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a
 + b*ArcSinh[c*x]))/(189*c^5) + (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh
[c*x]))/(315*c^5) + (20*b*d^2*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(44
1*c^5) - (2*b*d^2*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^5) + (8*d
^2*x^5*(a + b*ArcSinh[c*x])^2)/315 + (4*d^2*x^5*(1 + c^2*x^2)*(a + b*ArcSin
h[c*x])^2)/63 + (d^2*x^5*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/9
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \,dx = \frac{1}{9}d^2x^5\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}(4d)\int x^4\left(d+c^2dx^2\right)\left(a+b\sinh\right.$$

$$= -\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{45c^5}+\frac{4bd^2\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right.}{63c^5}$$

$$= -\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{189c^5}+\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right.}{315c^5}$$

$$= -\frac{16bd^2x^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{1575c}-\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.}{189c^5}$$

$$= \frac{304b^2d^2x}{19845c^4}-\frac{152b^2d^2x^3}{59535c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d^2x^9+\frac{64bd}{}$$

$$= \frac{304b^2d^2x}{19845c^4}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d^2x^9-\frac{128b}{}$$

$$= \frac{4208b^2d^2x}{99225c^4}-\frac{2104b^2d^2x^3}{297675c^2}+\frac{526b^2d^2x^5}{165375}+\frac{212b^2c^2d^2x^7}{27783}+\frac{2}{729}b^2c^4d^2x^9-\frac{128}{}$$

Mathematica [A] time = 0.3879, size = 251, normalized size = 0.65

$$\frac{d^2\left(99225a^2c^5x^5\left(35c^4x^4+90c^2x^2+63\right)-630ab\sqrt{c^2x^2+1}\left(1225c^8x^8+2650c^6x^6+789c^4x^4-1052c^2x^2+2104\right)-630b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(99225*a^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) - 630*a*b*Sqrt[1 + c
^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x^8) +
 2*b^2*c*x*(662760 - 110460*c^2*x^2 + 49707*c^4*x^4 + 119250*c^6*x^6 + 4287
5*c^8*x^8) - 630*b*(-315*a*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4) + b*Sqrt[
1 + c^2*x^2]*(2104 - 1052*c^2*x^2 + 789*c^4*x^4 + 2650*c^6*x^6 + 1225*c^8*x
^8))*ArcSinh[c*x] + 99225*b^2*c^5*x^5*(63 + 90*c^2*x^2 + 35*c^4*x^4)*ArcSin
h[c*x]^2))/(31255875*c^5)
```

Maple [A] time = 0.046, size = 446, normalized size = 1.2

$$\frac{1}{c^5}\left(d^2a^2\left(\frac{c^9x^9}{9}+\frac{2c^7x^7}{7}+\frac{c^5x^5}{5}\right)+d^2b^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2c^3x^3\left(c^2x^2+1\right)^3}{9}-\frac{(\mathrm{Arcsinh}\,(cx))^2cx\left(c^2x^2+1\right)^3}{21}+\frac{8\,(\mathrm{Arcsin}}{31}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^5*(d^2*a^2*(1/9*c^9*x^9+2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b^2*(1/9*arcsinh(c
*x)^2*c^3*x^3*(c^2*x^2+1)^3-1/21*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+8/315*arc
sinh(c*x)^2*c*x+1/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/315*arcsinh(c*x)^2
*c*x*(c^2*x^2+1)-2/81*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(7/2)+82/3969*arcsin
h(c*x)*c^2*x^2*(c^2*x^2+1)^(5/2)+1672/99225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1
)^(3/2)+832/99225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)-4208/99225*arcsinh
(c*x)*(c^2*x^2+1)^(1/2)+2/729*c*x*(c^2*x^2+1)^4+1493104/31255875*c*x-836/25
0047*c*x*(c^2*x^2+1)^3-33862/10418625*c*x*(c^2*x^2+1)^2-47248/31255875*c*x*
(c^2*x^2+1))+2*d^2*a*b*(1/9*arcsinh(c*x)*c^9*x^9+2/7*arcsinh(c*x)*c^7*x^7+1
/5*arcsinh(c*x)*c^5*x^5-1/81*c^8*x^8*(c^2*x^2+1)^(1/2)-106/3969*c^6*x^6*(c^
2*x^2+1)^(1/2)-263/33075*c^4*x^4*(c^2*x^2+1)^(1/2)+1052/99225*c^2*x^2*(c^2*
x^2+1)^(1/2)-2104/99225*(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.35261, size = 1026, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^4*d^2*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 + 2/7*b^2*c^2*d^2*
x^7*arcsinh(c*x)^2 + 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arcsinh(c*x)^2 +
 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2
*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2
/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 - 2/893025*(315*(35*sqrt(
c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*
x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*arcs
inh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 + 403
20*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 + 4/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 - 4/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^2 + 2/75*(15*x^5*arcsinh(c*x) -
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 + 1)/c^6)*c)*a*b*d^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^
2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 -
```

```
20*c^2*x^3 + 120*x)/c^4)*b^2*d^2
```

Fricas [A] time = 3.1998, size = 871, normalized size = 2.26

$$\frac{42875\left(81\,a^2+2\,b^2\right)c^9d^2x^9+2250\left(3969\,a^2+106\,b^2\right)c^7d^2x^7+189\left(33075\,a^2+526\,b^2\right)c^5d^2x^5-220920\,b^2c^3d^2x^3+13255}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^2*x^9 + 2250*(3969*a^2 + 106*b^2)*
c^7*d^2*x^7 + 189*(33075*a^2 + 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*x^
3 + 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 + 90*b^2*c^7*d^2*x^7 +
63*b^2*c^5*d^2*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^2
*x^9 + 28350*a*b*c^7*d^2*x^7 + 19845*a*b*c^5*d^2*x^5 - (1225*b^2*c^8*d^2*x^
8 + 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2*x^4 - 1052*b^2*c^2*d^2*x^2 + 210
4*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*
c^8*d^2*x^8 + 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 - 1052*a*b*c^2*d^2
*x^2 + 2104*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 44.9501, size = 563, normalized size = 1.46

$$\begin{cases} \frac{a^2c^4d^2x^9}{9}+\frac{2a^2c^2d^2x^7}{7}+\frac{a^2d^2x^5}{5}+\frac{2abc^4d^2x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^3d^2x^8\sqrt{c^2x^2+1}}{81}+\frac{4abc^2d^2x^7\operatorname{asinh}(cx)}{7}-\frac{212abcd^2x^6\sqrt{c^2x^2+1}}{3969}+\frac{2abd^2x^5\operatorname{asinh}(cx)}{5} \\ \frac{a^2d^2x^5}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**9/9 + 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**5
/5 + 2*a*b*c**4*d**2*x**9*asinh(c*x)/9 - 2*a*b*c**3*d**2*x**8*sqrt(c**2*x**
2 + 1)/81 + 4*a*b*c**2*d**2*x**7*asinh(c*x)/7 - 212*a*b*c*d**2*x**6*sqrt(c*
*2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*asinh(c*x)/5 - 526*a*b*d**2*x**4*sqrt(c
**2*x**2 + 1)/(33075*c) + 2104*a*b*d**2*x**2*sqrt(c**2*x**2 + 1)/(99225*c**
3) - 4208*a*b*d**2*sqrt(c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**2*x**9*a
sinh(c*x)**2/9 + 2*b**2*c**4*d**2*x**9/729 - 2*b**2*c**3*d**2*x**8*sqrt(c**
2*x**2 + 1)*asinh(c*x)/81 + 2*b**2*c**2*d**2*x**7*asinh(c*x)**2/7 + 212*b**
```

```
2*c**2*d**2*x**7/27783 - 212*b**2*c*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x
)/3969 + b**2*d**2*x**5*asinh(c*x)**2/5 + 526*b**2*d**2*x**5/165375 - 526*b
**2*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(33075*c) - 2104*b**2*d**2*x**
3/(297675*c**2) + 2104*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225
*c**3) + 4208*b**2*d**2*x/(99225*c**4) - 4208*b**2*d**2*sqrt(c**2*x**2 + 1)
*asinh(c*x)/(99225*c**5), Ne(c, 0)), (a**2*d**2*x**5/5, True))
```

Giac [B] time = 2.73084, size = 998, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/9*a^2*c^4*d^2*x^9 + 2/7*a^2*c^2*d^2*x^7 + 2/2835*(315*x^9*log(c*x + sqrt(
c^2*x^2 + 1)) - (35*(c^2*x^2 + 1)^(9/2) - 180*(c^2*x^2 + 1)^(7/2) + 378*(c^
2*x^2 + 1)^(5/2) - 420*(c^2*x^2 + 1)^(3/2) + 315*sqrt(c^2*x^2 + 1))/c^9)*a*
b*c^4*d^2 + 1/893025*(99225*x^9*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((1225
*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 + 40320*x)/c^9 - 315*
(35*(c^2*x^2 + 1)^(9/2) - 180*(c^2*x^2 + 1)^(7/2) + 378*(c^2*x^2 + 1)^(5/2)
 - 420*(c^2*x^2 + 1)^(3/2) + 315*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1))/c^10))*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 + 4/245*(35*x^7*log(c*x + sqrt(c
^2*x^2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^
2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*a*b*c^2*d^2 + 2/25725*(3675*x^7*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^
3 - 1680*x)/c^7 - 105*(5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*
(c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^
8))*b^2*c^2*d^2 + 2/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 +
 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b*d^2 + 1
/1125*(225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 - 20*c^2*x^
3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*d^2
```

3.208 $\int x^3 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=296

$$-\frac{1}{32}bcd^2x^5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{25}{576}bcd^2x^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(c\right.$$

```
[Out] (-73*b^2*d^2*x^2)/(3072*c^2) + (73*b^2*d^2*x^4)/9216 + (43*b^2*c^2*d^2*x^6)
/3456 + (b^2*c^4*d^2*x^8)/256 + (73*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(1536*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(2304*c) - (25*b*c*d^2*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/576 - (
b*c*d^2*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/32 - (73*d^2*(a + b*A
rcSinh[c*x])^2)/(3072*c^4) + (d^2*x^4*(a + b*ArcSinh[c*x])^2)/24 + (d^2*x^4
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/12 + (d^2*x^4*(1 + c^2*x^2)^2*(a + b
*ArcSinh[c*x])^2)/8
```

Rubi [A] time = 1.03698, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5744, 5661, 5758, 5675, 30, 5742, 14}

$$-\frac{1}{32}bcd^2x^5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{25}{576}bcd^2x^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}d^2x^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```



```
[Out] (-73*b^2*d^2*x^2)/(3072*c^2) + (73*b^2*d^2*x^4)/9216 + (43*b^2*c^2*d^2*x^6)
/3456 + (b^2*c^4*d^2*x^8)/256 + (73*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(1536*c^3) - (73*b*d^2*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(2304*c) - (25*b*c*d^2*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/576 - (
b*c*d^2*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/32 - (73*d^2*(a + b*A
rcSinh[c*x])^2)/(3072*c^4) + (d^2*x^4*(a + b*ArcSinh[c*x])^2)/24 + (d^2*x^4
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/12 + (d^2*x^4*(1 + c^2*x^2)^2*(a + b
*ArcSinh[c*x])^2)/8
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
```

```
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{8}d^2x^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}d\int x^3\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{12}d^2x^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}\right.$$

$$= -\frac{25}{576}bcd^2x^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^2x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.$$

$$= \frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8-\frac{73bd^2x^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2304c}-\frac{25}{576}bc$$

$$= \frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{1536c^3}$$

$$= -\frac{73b^2d^2x^2}{3072c^2}+\frac{73b^2d^2x^4}{9216}+\frac{43b^2c^2d^2x^6}{3456}+\frac{1}{256}b^2c^4d^2x^8+\frac{73bd^2x\sqrt{1+c^2x^2}\left(a\right.}{1536c}$$

Mathematica [A] time = 0.355593, size = 237, normalized size = 0.8

$$d^2\left(cx\left(1152a^2c^3x^3\left(3c^4x^4+8c^2x^2+6\right)-6ab\sqrt{c^2x^2+1}\left(144c^6x^6+344c^4x^4+146c^2x^2-219\right)+b^2cx\left(108c^6x^6+344c^4x^4\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(1152*a^2*c^3*x^3*(6 + 8*c^2*x^2 + 3*c^4*x^4) + b^2*c*x*(-657 + 2
19*c^2*x^2 + 344*c^4*x^4 + 108*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(-219 + 1
46*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(
-219 + 146*c^2*x^2 + 344*c^4*x^4 + 144*c^6*x^6)) + 3*a*(-73 + 768*c^4*x^4 +
 1024*c^6*x^6 + 384*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(-73 + 768*c^4*x^4 + 102
4*c^6*x^6 + 384*c^8*x^8)*ArcSinh[c*x]^2))/(27648*c^4)
```

Maple [A] time = 0.045, size = 406, normalized size = 1.4

$$\frac{1}{c^4}\left(d^2a^2\left(\frac{c^8x^8}{8}+\frac{c^6x^6}{3}+\frac{c^4x^4}{4}\right)+d^2b^2\left(\frac{(\operatorname{Arcsinh}(cx))^2c^2x^2\left(c^2x^2+1\right)^3}{8}-\frac{(\operatorname{Arcsinh}(cx))^2c^2x^2\left(c^2x^2+1\right)^2}{24}-\frac{(\operatorname{Arcsinl}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^4*(d^2*a^2*(1/8*c^8*x^8+1/3*c^6*x^6+1/4*c^4*x^4)+d^2*b^2*(1/8*arcsinh(c
*x)^2*c^2*x^2*(c^2*x^2+1)^3-1/24*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^2-1/24*
arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)-1/24*arcsinh(c*x)^2*(c^2*x^2+1)-1/32*arc
sinh(c*x)*c*x*(c^2*x^2+1)^(7/2)+11/576*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)+5
5/2304*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+55/1536*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*c*x+55/3072*arcsinh(c*x)^2+1/256*c^2*x^2*(c^2*x^2+1)^3+5/6912*c^2*x^2
*(c^2*x^2+1)^2-145/27648*c^2*x^2*(c^2*x^2+1)-5/216*c^2*x^2-5/216)+2*d^2*a*b
*(1/8*arcsinh(c*x)*c^8*x^8+1/3*arcsinh(c*x)*c^6*x^6+1/4*arcsinh(c*x)*c^4*x^
4-1/64*c^7*x^7*(c^2*x^2+1)^(1/2)-43/1152*c^5*x^5*(c^2*x^2+1)^(1/2)-73/4608*
c^3*x^3*(c^2*x^2+1)^(1/2)+73/3072*c*x*(c^2*x^2+1)^(1/2)-73/3072*arcsinh(c*x
)))
```

Maxima [B] time = 1.33794, size = 1154, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^4*d^2*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^4*d^2*x^8 + 1/3*b^2*c^2*d^2*
x^6*arcsinh(c*x)^2 + 1/3*a^2*c^2*d^2*x^6 + 1/4*b^2*d^2*x^4*arcsinh(c*x)^2 +
 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2
*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/
c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^8))*c)*a*b*c^4*d^2 + 1/9216
*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c^2*x/sqrt
(c^2) + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^2 -
56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*
x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^8))*c*arcsinh(c*
x))*b^2*c^4*d^2 + 1/4*a^2*d^2*x^4 + 1/72*(48*x^6*arcsinh(c*x) - (8*sqrt(c^2
*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c
^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*a*b*c^2*d^2 + 1/432*((
```

```
8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2
 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x
^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2
)*c^6))*c*arcsinh(c*x))*b^2*c^2*d^2 + 1/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^
2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))
/(sqrt(c^2)*c^4))*c)*a*b*d^2 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c^2*x/sqr
t(c^2) + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3
*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c*ar
csinh(c*x))*b^2*d^2
```

Fricas [A] time = 3.00251, size = 784, normalized size = 2.65

$$\frac{108\left(32\,a^2+b^2\right)c^8d^2x^8+8\left(1152\,a^2+43\,b^2\right)c^6d^2x^6+3\left(2304\,a^2+73\,b^2\right)c^4d^2x^4-657\,b^2c^2d^2x^2+9\left(384\,b^2c^8d^2x^8+1024\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27648*(108*(32*a^2 + b^2)*c^8*d^2*x^8 + 8*(1152*a^2 + 43*b^2)*c^6*d^2*x^6
 + 3*(2304*a^2 + 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c^8
*d^2*x^8 + 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 6*(1152*a*b*c^8*d^2*x^8 + 3072*a*b*c^6*d^2*x^6 +
2304*a*b*c^4*d^2*x^4 - 219*a*b*d^2 - (144*b^2*c^7*d^2*x^7 + 344*b^2*c^5*d^2
*x^5 + 146*b^2*c^3*d^2*x^3 - 219*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) - 6*(144*a*b*c^7*d^2*x^7 + 344*a*b*c^5*d^2*x^5 + 146*a*b
*c^3*d^2*x^3 - 219*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 31.4014, size = 515, normalized size = 1.74

$$\begin{cases} \frac{a^2c^4d^2x^8}{8}+\frac{a^2c^2d^2x^6}{3}+\frac{a^2d^2x^4}{4}+\frac{abc^4d^2x^8\operatorname{asinh}(cx)}{4}-\frac{abc^3d^2x^7\sqrt{c^2x^2+1}}{32}+\frac{2abc^2d^2x^6\operatorname{asinh}(cx)}{3}-\frac{43abcd^2x^5\sqrt{c^2x^2+1}}{576}+\frac{abd^2x^4\operatorname{asinh}(cx)}{2}-\frac{73a}{} \\ \frac{a^2d^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**8/8 + a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/4
 + a*b*c**4*d**2*x**8*asinh(c*x)/4 - a*b*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)
```

```
/32 + 2*a*b*c**2*d**2*x**6*asinh(c*x)/3 - 43*a*b*c*d**2*x**5*sqrt(c**2*x**2
 + 1)/576 + a*b*d**2*x**4*asinh(c*x)/2 - 73*a*b*d**2*x**3*sqrt(c**2*x**2 +
1)/(2304*c) + 73*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(1536*c**3) - 73*a*b*d**2*a
sinh(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*asinh(c*x)**2/8 + b**2*c**4*d**
2*x**8/256 - b**2*c**3*d**2*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c
**2*d**2*x**6*asinh(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 - 43*b**2*c*d**
2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/576 + b**2*d**2*x**4*asinh(c*x)**2/4
+ 73*b**2*d**2*x**4/9216 - 73*b**2*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)
/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) + 73*b**2*d**2*x*sqrt(c**2*x**2 +
 1)*asinh(c*x)/(1536*c**3) - 73*b**2*d**2*asinh(c*x)**2/(3072*c**4), Ne(c,
0)), (a**2*d**2*x**4/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^2 (b\operatorname{arsinh}(cx)+a)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2*x^3, x)
```

3.209 $\int x^2 \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=303

$$\frac{1}{7} d^2 x^3 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{4}{35} d^2 x^3 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{16 b d^2 x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{315 c}-$$

```
[Out] (-1636*b^2*d^2*x)/(11025*c^2) + (818*b^2*d^2*x^3)/33075 + (136*b^2*c^2*d^2*
x^5)/6125 + (2*b^2*c^4*d^2*x^7)/343 + (32*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(315*c^3) - (16*b*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(315*c) + (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105*c^3)
+ (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c^3) - (2*b*d^2*(
1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c^3) + (8*d^2*x^3*(a + b*ArcSi
nh[c*x])^2)/105 + (4*d^2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (d^
2*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/7
```

Rubi [A] time = 0.594148, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 373}

$$\frac{1}{7} d^2 x^3 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{4}{35} d^2 x^3 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx)\right)^2-\frac{16 b d^2 x^2 \sqrt{c^2 x^2+1} \left(a+b \sinh^{-1}(cx)\right)}{315 c}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (-1636*b^2*d^2*x)/(11025*c^2) + (818*b^2*d^2*x^3)/33075 + (136*b^2*c^2*d^2*
x^5)/6125 + (2*b^2*c^4*d^2*x^7)/343 + (32*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(315*c^3) - (16*b*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(315*c) + (8*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105*c^3)
+ (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c^3) - (2*b*d^2*(
1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c^3) + (8*d^2*x^3*(a + b*ArcSi
nh[c*x])^2)/105 + (4*d^2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (d^
2*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/7
```


Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
```

```
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{7}d^2x^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{7}(4d)\int x^2\left(d+c^2dx^2\right)\left(a+b\sinh\right.$$
$$=\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{35c^3}-\frac{2bd^2\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c^3}$$
$$=\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3}+\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{175c^3}$$
$$=-\frac{16bd^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{315c}+\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.}{105c^3}$$
$$=-\frac{172b^2d^2x}{3675c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x^2}}{3}$$
$$=-\frac{1636b^2d^2x}{11025c^2}+\frac{818b^2d^2x^3}{33075}+\frac{136b^2c^2d^2x^5}{6125}+\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1+c^2x}}{}$$

Mathematica [A] time = 0.396393, size = 227, normalized size = 0.75

$$\frac{d^2\left(11025a^2c^3x^3\left(15c^4x^4+42c^2x^2+35\right)-210ab\sqrt{c^2x^2+1}\left(225c^6x^6+612c^4x^4+409c^2x^2-818\right)-210b\sinh^{-1}(cx)\left(b\sqrt{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(11025*a^2*c^3*x^3*(35 + 42*c^2*x^2 + 15*c^4*x^4) - 210*a*b*Sqrt[1 + c
^2*x^2]*(-818 + 409*c^2*x^2 + 612*c^4*x^4 + 225*c^6*x^6) + 2*b^2*c*x*(-8589
0 + 14315*c^2*x^2 + 12852*c^4*x^4 + 3375*c^6*x^6) - 210*b*(-105*a*c^3*x^3*(
35 + 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(-818 + 409*c^2*x^2 + 6
12*c^4*x^4 + 225*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c^3*x^3*(35 + 42*c^2*x^
2 + 15*c^4*x^4)*ArcSinh[c*x]^2))/(1157625*c^3)
```

Maple [A] time = 0.046, size = 364, normalized size = 1.2

$$\frac{1}{c^3}\left(d^2a^2\left(\frac{c^7x^7}{7}+\frac{2\,c^5x^5}{5}+\frac{c^3x^3}{3}\right)+d^2b^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2\,cx\left(c^2x^2+1\right)^3}{7}-\frac{8\,(\mathrm{Arcsinh}\,(cx))^2\,cx}{105}-\frac{(\mathrm{Arcsinh}\,(cx))^2\,cx\,(c}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^3*(d^2*a^2*(1/7*c^7*x^7+2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b^2*(1/7*arcsinh(c
*x)^2*c*x*(c^2*x^2+1)^3-8/105*arcsinh(c*x)^2*c*x-1/35*arcsinh(c*x)^2*c*x*(c
^2*x^2+1)^2-4/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2/49*arcsinh(c*x)*c^2*x^2*
(c^2*x^2+1)^(5/2)-36/1225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(3/2)-44/11025*a
rcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+1636/11025*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)+2/343*c*x*(c^2*x^2+1)^3-181456/1157625*c*x+202/42875*c*x*(c^2*x^2+1)^2
-2528/1157625*c*x*(c^2*x^2+1))+2*d^2*a*b*(1/7*arcsinh(c*x)*c^7*x^7+2/5*arcs
inh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/49*c^6*x^6*(c^2*x^2+1)^(1/2)-68
/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-409/11025*c^2*x^2*(c^2*x^2+1)^(1/2)+818/110
25*(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.22513, size = 836, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^4*d^2*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 + 2/5*b^2*c^2*d^2*
x^5*arcsinh(c*x)^2 + 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*arcsinh(c*x)
^2 + 4/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 - 4/1125*(15*(3*
sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 +
1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*d^2
+ 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2
- 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^2
```

Fricas [A] time = 2.70777, size = 755, normalized size = 2.49

$$\frac{3375\left(49\,a^2+2\,b^2\right)c^7d^2x^7+378\left(1225\,a^2+68\,b^2\right)c^5d^2x^5+35\left(11025\,a^2+818\,b^2\right)c^3d^2x^3-171780\,b^2cd^2x+11025\left(15\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1157625*(3375*(49*a^2 + 2*b^2)*c^7*d^2*x^7 + 378*(1225*a^2 + 68*b^2)*c^5*
d^2*x^5 + 35*(11025*a^2 + 818*b^2)*c^3*d^2*x^3 - 171780*b^2*c*d^2*x + 11025
*(15*b^2*c^7*d^2*x^7 + 42*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 210*(1575*a*b*c^7*d^2*x^7 + 4410*a*b*c^5*d^2*x^5 + 36
75*a*b*c^3*d^2*x^3 - (225*b^2*c^6*d^2*x^6 + 612*b^2*c^4*d^2*x^4 + 409*b^2*c
^2*d^2*x^2 - 818*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 210*(225*a*b*c^6*d^2*x^6 + 612*a*b*c^4*d^2*x^4 + 409*a*b*c^2*d^2*x^2 - 818
*a*b*d^2)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 17.3361, size = 483, normalized size = 1.59

$$\begin{cases} \frac{a^2c^4d^2x^7}{7} + \frac{2a^2c^2d^2x^5}{5} + \frac{a^2d^2x^3}{3} + \frac{2abc^4d^2x^7\operatorname{asinh}(cx)}{7} - \frac{2abc^3d^2x^6\sqrt{c^2x^2+1}}{49} + \frac{4abc^2d^2x^5\operatorname{asinh}(cx)}{5} - \frac{136abcd^2x^4\sqrt{c^2x^2+1}}{1225} + \frac{2abd^2x^3\operatorname{asinh}(cx)}{3} \\ \frac{a^2d^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**7/7 + 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**3
/3 + 2*a*b*c**4*d**2*x**7*asinh(c*x)/7 - 2*a*b*c**3*d**2*x**6*sqrt(c**2*x**
2 + 1)/49 + 4*a*b*c**2*d**2*x**5*asinh(c*x)/5 - 136*a*b*c*d**2*x**4*sqrt(c*
*2*x**2 + 1)/1225 + 2*a*b*d**2*x**3*asinh(c*x)/3 - 818*a*b*d**2*x**2*sqrt(c
**2*x**2 + 1)/(11025*c) + 1636*a*b*d**2*sqrt(c**2*x**2 + 1)/(11025*c**3) +
b**2*c**4*d**2*x**7*asinh(c*x)**2/7 + 2*b**2*c**4*d**2*x**7/343 - 2*b**2*c*
*3*d**2*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/49 + 2*b**2*c**2*d**2*x**5*asin
h(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 - 136*b**2*c*d**2*x**4*sqrt(c**2
*x**2 + 1)*asinh(c*x)/1225 + b**2*d**2*x**3*asinh(c*x)**2/3 + 818*b**2*d**2
*x**3/33075 - 818*b**2*d**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(11025*c) -
 1636*b**2*d**2*x/(11025*c**2) + 1636*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c
*x)/(11025*c**3), Ne(c, 0)), (a**2*d**2*x**3/3, True))
```

Giac [B] time = 2.76466, size = 853, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/7*a^2*c^4*d^2*x^7 + 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*log(c*x + sqrt(c^
2*x^2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2
 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*a*b*c^4*d^2 + 1/25725*(3675*x^7*lo
g(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3
 - 1680*x)/c^7 - 105*(5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(
c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^8
))*b^2*c^4*d^2 + 4/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 +
1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b*c^2*d^2
+ 2/1125*(225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 - 20*c^2
*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*c^2*d^2 + 1/3*a^
2*d^2*x^3 + 2/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2)
- 3*sqrt(c^2*x^2 + 1))/c^3)*a*b*d^2 + 1/27*(9*x^3*log(c*x + sqrt(c^2*x^2 +
1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2
+ 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*d^2
```

3.210 $\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=204

$$-\frac{bd^2x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5bd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{5bd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{48c}+$$

```
[Out] (25*b^2*d^2*x^2)/288 + (5*b^2*c^2*d^2*x^4)/288 + (b^2*d^2*(1 + c^2*x^2)^3)/
(108*c^2) - (5*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(48*c) - (5*
b*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(72*c) - (b*d^2*x*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(18*c) - (5*d^2*(a + b*ArcSinh[c*x])^2)/
(96*c^2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(6*c^2)
```

Rubi [A] time = 0.205952, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5717, 5684, 5682, 5675, 30, 14, 261}

$$-\frac{bd^2x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5bd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c}-\frac{5bd^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{48c}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (25*b^2*d^2*x^2)/288 + (5*b^2*c^2*d^2*x^4)/288 + (b^2*d^2*(1 + c^2*x^2)^3)/
(108*c^2) - (5*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(48*c) - (5*
b*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(72*c) - (b*d^2*x*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(18*c) - (5*d^2*(a + b*ArcSinh[c*x])^2)/
(96*c^2) + (d^2*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(6*c^2)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{6c^2} - \frac{\left(bd^2\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3c}$$

$$= -\frac{bd^2x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{18c} + \frac{d^2\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{6c^2}$$

$$= \frac{b^2d^2\left(1+c^2x^2\right)^3}{108c^2} - \frac{5bd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{72c} - \frac{bd^2x\left(1+c^2x^2\right)^{5}}{}$$

$$= \frac{b^2d^2\left(1+c^2x^2\right)^3}{108c^2} - \frac{5bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{48c} - \frac{5bd^2x\left(1+c^2x^2\right)^{3/2}}{72}$$

$$= \frac{25}{288}b^2d^2x^2 + \frac{5}{288}b^2c^2d^2x^4 + \frac{b^2d^2\left(1+c^2x^2\right)^3}{108c^2} - \frac{5bd^2x\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{48c}$$

Mathematica [A] time = 0.487445, size = 208, normalized size = 1.02

$$\frac{d^2\left(cx\left(144a^2cx\left(c^4x^4+3c^2x^2+3\right)-6ab\sqrt{c^2x^2+1}\left(8c^4x^4+26c^2x^2+33\right)+b^2cx\left(8c^4x^4+39c^2x^2+99\right)\right)+6b\sinh^{-1}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(c*x*(144*a^2*c*x*(3 + 3*c^2*x^2 + c^4*x^4) - 6*a*b*Sqrt[1 + c^2*x^2]*
(33 + 26*c^2*x^2 + 8*c^4*x^4) + b^2*c*x*(99 + 39*c^2*x^2 + 8*c^4*x^4)) + 6*
b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(33 + 26*c^2*x^2 + 8*c^4*x^4)) + 3*a*(11 + 48*
c^2*x^2 + 48*c^4*x^4 + 16*c^6*x^6))*ArcSinh[c*x] + 9*b^2*(11 + 48*c^2*x^2 +
 48*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x]^2))/(864*c^2)
```

Maple [A] time = 0.038, size = 324, normalized size = 1.6

$$\frac{1}{c^2}\left(d^2a^2\left(\frac{c^6x^6}{6}+\frac{c^4x^4}{2}+\frac{c^2x^2}{2}\right)+d^2b^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2\,c^2x^2\left(c^2x^2+1\right)^2}{6}+\frac{(\mathrm{Arcsinh}\,(cx))^2\,c^2x^2\left(c^2x^2+1\right)}{6}+\frac{(\mathrm{Arcsinh}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^2*(d^2*a^2*(1/6*c^6*x^6+1/2*c^4*x^4+1/2*c^2*x^2)+d^2*b^2*(1/6*arcsinh(c
*x)^2*c^2*x^2*(c^2*x^2+1)^2+1/6*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)+1/6*arcs
inh(c*x)^2*(c^2*x^2+1)-1/18*arcsinh(c*x)*c*x*(c^2*x^2+1)^(5/2)-5/72*arcsinh
(c*x)*c*x*(c^2*x^2+1)^(3/2)-5/48*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-5/96*ar
csinh(c*x)^2+1/108*c^2*x^2*(c^2*x^2+1)^2+23/864*c^2*x^2*(c^2*x^2+1)+17/216*
c^2*x^2+17/216)+2*d^2*a*b*(1/6*arcsinh(c*x)*c^6*x^6+1/2*arcsinh(c*x)*c^4*x^
4+1/2*arcsinh(c*x)*c^2*x^2-1/36*c^5*x^5*(c^2*x^2+1)^(1/2)-13/144*c^3*x^3*(c
^2*x^2+1)^(1/2)-11/96*c*x*(c^2*x^2+1)^(1/2)+11/96*arcsinh(c*x)))
```

Maxima [B] time = 1.23769, size = 964, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*c^4*d^2*x^6*arcsinh(c*x)^2 + 1/6*a^2*c^4*d^2*x^6 + 1/2*b^2*c^2*d^2*
x^4*arcsinh(c*x)^2 + 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arcsinh(c*x) - (8*
sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2
+ 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*a*b*c^4*d^2 +
1/864*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c^2*x/sqrt(c^2) + sqrt
(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^
2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(
sqrt(c^2)*c^6))*c*arcsinh(c*x))*b^2*c^4*d^2 + 1/2*b^2*d^2*x^2*arcsinh(c*x)^
2 + 1/8*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2
 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*a*b*c^2*d^2 +
1/16*((x^4/c^2 - 3*x^2/c^4 + 3*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c
^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*ar
csinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c*arcsinh(c*x))*b^2*c^2*d^2 + 1/2*a
^2*d^2*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh
(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b*d^2 + 1/4*(c^2*(x^2/c^2 - log(c^2*x
/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arc
sinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2))*arcsinh(c*x))*b^2*d^2
```

Fricas [A] time = 2.73979, size = 663, normalized size = 3.25

$$\frac{8\left(18\,a^2+b^2\right)c^6d^2x^6+3\left(144\,a^2+13\,b^2\right)c^4d^2x^4+9\left(48\,a^2+11\,b^2\right)c^2d^2x^2+9\left(16\,b^2c^6d^2x^6+48\,b^2c^4d^2x^4+48\,b^2c^2d^2x^2+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/864*(8*(18*a^2 + b^2)*c^6*d^2*x^6 + 3*(144*a^2 + 13*b^2)*c^4*d^2*x^4 + 9*
(48*a^2 + 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 + 48*b^2*c^4*d^2*x^4
+ 48*b^2*c^2*d^2*x^2 + 11*b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(48*a
*b*c^6*d^2*x^6 + 144*a*b*c^4*d^2*x^4 + 144*a*b*c^2*d^2*x^2 + 33*a*b*d^2 - (
8*b^2*c^5*d^2*x^5 + 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(8*a*b*c^5*d^2*x^5 + 26*a*b*c^3*d^2*x^3 +
 33*a*b*c*d^2*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [A] time = 11.8526, size = 430, normalized size = 2.11

$$\begin{cases} \frac{a^2c^4d^2x^6}{6} + \frac{a^2c^2d^2x^4}{2} + \frac{a^2d^2x^2}{2} + \frac{abc^4d^2x^6\operatorname{asinh}(cx)}{3} - \frac{abc^3d^2x^5\sqrt{c^2x^2+1}}{18} + abc^2d^2x^4\operatorname{asinh}(cx) - \frac{13abcd^2x^3\sqrt{c^2x^2+1}}{72} + abd^2x^2\operatorname{asinh}(\\ \frac{a^2d^2x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**6/6 + a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/2
 + a*b*c**4*d**2*x**6*asinh(c*x)/3 - a*b*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)
/18 + a*b*c**2*d**2*x**4*asinh(c*x) - 13*a*b*c*d**2*x**3*sqrt(c**2*x**2 + 1
)/72 + a*b*d**2*x**2*asinh(c*x) - 11*a*b*d**2*x*sqrt(c**2*x**2 + 1)/(48*c)
+ 11*a*b*d**2*asinh(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*asinh(c*x)**2/6 +
b**2*c**4*d**2*x**6/108 - b**2*c**3*d**2*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x
)/18 + b**2*c**2*d**2*x**4*asinh(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/288 - 1
3*b**2*c*d**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/72 + b**2*d**2*x**2*asinh
(c*x)**2/2 + 11*b**2*d**2*x**2/96 - 11*b**2*d**2*x*sqrt(c**2*x**2 + 1)*asin
h(c*x)/(48*c) + 11*b**2*d**2*asinh(c*x)**2/(96*c**2), Ne(c, 0)), (a**2*d**2
*x**2/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^2 (b\operatorname{arsinh}(cx) + a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2*x, x)
```

3.211 $\int \left(d+c^2 d x^2\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=214

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{15}d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}-\frac{8}{}$$

```
[Out] (298*b^2*d^2*x)/225 + (76*b^2*c^2*d^2*x^3)/675 + (2*b^2*c^4*d^2*x^5)/125 -
(16*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(15*c) - (8*b*d^2*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(45*c) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x]))/(25*c) + (8*d^2*x*(a + b*ArcSinh[c*x])^2)/15 + (4*d^2*x
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/15 + (d^2*x*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/5
```

Rubi [A] time = 0.255873, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5653, 5717, 8, 194}

$$\frac{1}{5}d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4}{15}d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bd^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}-\frac{8}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```



```
[Out] (298*b^2*d^2*x)/225 + (76*b^2*c^2*d^2*x^3)/675 + (2*b^2*c^4*d^2*x^5)/125 -
(16*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(15*c) - (8*b*d^2*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(45*c) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x]))/(25*c) + (8*d^2*x*(a + b*ArcSinh[c*x])^2)/15 + (4*d^2*x
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/15 + (d^2*x*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/5
```


Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
```

```
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{5}d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(4d)\int \left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= -\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}+\frac{4}{15}d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2+$$

$$= -\frac{8bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{45c}-\frac{2bd^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{25c}+$$

$$= \frac{58}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{15c}$$

$$= \frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3+\frac{2}{125}b^2c^4d^2x^5-\frac{16bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{15c}$$

Mathematica [A] time = 0.403551, size = 191, normalized size = 0.89

$$d^2\left(225a^2cx\left(3c^4x^4+10c^2x^2+15\right)-30ab\sqrt{c^2x^2+1}\left(9c^4x^4+38c^2x^2+149\right)-30b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(9c^4x^4+38c\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(225*a^2*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]*
(149 + 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 + 190*c^2*x^2 + 27*c^4*x^4
) - 30*b*(-15*a*c*x*(15 + 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 + c^2*x^2]*(14
9 + 38*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*b^2*c*x*(15 + 10*c^2*x^2 +
3*c^4*x^4)*ArcSinh[c*x]^2))/(3375*c)
```

Maple [A] time = 0.036, size = 276, normalized size = 1.3

$$\frac{1}{c}\left(d^2a^2\left(\frac{c^5x^5}{5}+\frac{2\,c^3x^3}{3}+cx\right)+d^2b^2\left(\frac{8\,(\mathrm{Arcsinh}\,(cx))^2\,cx}{15}+\frac{(\mathrm{Arcsinh}\,(cx))^2\,cx\left(c^2x^2+1\right)^2}{5}+\frac{4\,(\mathrm{Arcsinh}\,(cx))^2\,cx\,(c^2}{15}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(d^2*a^2*(1/5*c^5*x^5+2/3*c^3*x^3+c*x)+d^2*b^2*(8/15*arcsinh(c*x)^2*c*x
+1/5*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+4/15*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-2
98/225*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4144/3375*c*x-2/25*arcsinh(c*x)*c^2*x
^2*(c^2*x^2+1)^(3/2)-58/225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+2/125*c*
x*(c^2*x^2+1)^2+272/3375*c*x*(c^2*x^2+1))+2*d^2*a*b*(1/5*arcsinh(c*x)*c^5*x
^5+2/3*arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/25*c^4*x^4*(c^2*x^2+1)^(1/2)
-38/225*c^2*x^2*(c^2*x^2+1)^(1/2)-149/225*(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.15617, size = 617, normalized size = 2.88

$$\frac{1}{5}\,b^2c^4d^2x^5\,\mathrm{arsinh}\,(cx)^2+\frac{1}{5}\,a^2c^4d^2x^5+\frac{2}{3}\,b^2c^2d^2x^3\,\mathrm{arsinh}\,(cx)^2+\frac{2}{75}\left(15\,x^5\,\mathrm{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}-\frac{4\,\sqrt{c^2x^2+}}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*c^4*d^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 + 2/3*b^2*c^2*d^2*
x^3*arcsinh(c*x)^2 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c
^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^2
- 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c
^4)*b^2*c^4*d^2 + 2/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c
^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 - 4/27*(3*c*(sq
rt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3
- 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arcsinh(c*x)^2 + 2*b^2*d^2*(x - sqrt(c^
2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2
 + 1))*a*b*d^2/c
```

Fricas [A] time = 2.71586, size = 616, normalized size = 2.88

$$\frac{27\left(25\,a^2+2\,b^2\right)c^5d^2x^5+10\left(225\,a^2+38\,b^2\right)c^3d^2x^3+15\left(225\,a^2+298\,b^2\right)cd^2x+225\left(3\,b^2c^5d^2x^5+10\,b^2c^3d^2x^3+15\,b^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*d^2*x^5 + 10*(225*a^2 + 38*b^2)*c^3*d^2*x^3
 + 15*(225*a^2 + 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 + 10*b^2*c^3*d^2
*x^3 + 15*b^2*c*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^5*d^2*
x^5 + 150*a*b*c^3*d^2*x^3 + 225*a*b*c*d^2*x - (9*b^2*c^4*d^2*x^4 + 38*b^2*c
^2*d^2*x^2 + 149*b^2*d^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) -
 30*(9*a*b*c^4*d^2*x^4 + 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2)*sqrt(c^2*x^2 + 1
))/c
```

Sympy [A] time = 5.98837, size = 389, normalized size = 1.82

$$\begin{cases} \frac{a^2c^4d^2x^5}{5}+\frac{2a^2c^2d^2x^3}{3}+a^2d^2x+\frac{2abc^4d^2x^5\operatorname{asinh}(cx)}{5}-\frac{2abc^3d^2x^4\sqrt{c^2x^2+1}}{25}+\frac{4abc^2d^2x^3\operatorname{asinh}(cx)}{3}-\frac{76abcd^2x^2\sqrt{c^2x^2+1}}{225}+2abd^2x\operatorname{asinh}(cx \\ a^2d^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**4*d**2*x**5/5 + 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
2*a*b*c**4*d**2*x**5*asinh(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(c**2*x**2 + 1
)/25 + 4*a*b*c**2*d**2*x**3*asinh(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(c**2*x**
2 + 1)/225 + 2*a*b*d**2*x*asinh(c*x) - 298*a*b*d**2*sqrt(c**2*x**2 + 1)/(22
5*c) + b**2*c**4*d**2*x**5*asinh(c*x)**2/5 + 2*b**2*c**4*d**2*x**5/125 - 2*
b**2*c**3*d**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/25 + 2*b**2*c**2*d**2*x*
*3*asinh(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**2*sqrt(
c**2*x**2 + 1)*asinh(c*x)/225 + b**2*d**2*x*asinh(c*x)**2 + 298*b**2*d**2*x
/225 - 298*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(225*c), Ne(c, 0)), (a*
*2*d**2*x, True))
```

Giac [B] time = 2.44213, size = 690, normalized size = 3.22

$$\frac{1}{5}a^2c^4d^2x^5 + \frac{2}{75}\left(15\,x^5\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{3\left(c^2x^2+1\right)^{\frac{5}{2}} - 10\left(c^2x^2+1\right)^{\frac{3}{2}} + 15\sqrt{c^2x^2+1}}{c^5}\right)abc^4d^2 + \frac{1}{1125}\left(225\,x^5\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/5*a^2*c^4*d^2*x^5 + 2/75*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x
^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b*c^4
*d^2 + 1/1125*(225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 - 2
0*c^2*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2)
 + 15*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*c^4*d^2 + 2
/3*a^2*c^2*d^2*x^3 + 4/9*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 +
1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*a*b*c^2*d^2 + 2/27*(9*x^3*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2) - 3*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*c^2*d^2 + 2*(x*lo
g(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b*d^2 + (x*log(c*x + sq
rt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 +
1))/c^2))*b^2*d^2 + a^2*d^2*x
```

3.212 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x}\,dx$

Optimal. Leaf size=257

$$-bd^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}(cx)}\right)-\frac{1}{8}bcd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

```
[Out] (13*b^2*c^2*d^2*x^2)/32 + (b^2*c^4*d^2*x^4)/32 - (11*b*c*d^2*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/16 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/8 - (11*d^2*(a + b*ArcSinh[c*x])^2)/32 + (d^2*(1 + c^2*x^2)*(a +
 b*ArcSinh[c*x])^2)/2 + (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 + (d
^2*(a + b*ArcSinh[c*x])^3)/(3*b) + d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2
*ArcSinh[c*x])] - b*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])
] - (b^2*d^2*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.440717, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 5684, 14}

$$bd^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{8}bcd^2x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (13*b^2*c^2*d^2*x^2)/32 + (b^2*c^4*d^2*x^4)/32 - (11*b*c*d^2*x*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/16 - (b*c*d^2*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/8 - (11*d^2*(a + b*ArcSinh[c*x])^2)/32 + (d^2*(1 + c^2*x^2)*(a +
 b*ArcSinh[c*x])^2)/2 + (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
^2*(a + b*ArcSinh[c*x])^3)/(3*b) + d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*
ArcSinh[c*x])] + b*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])]
 - (b^2*d^2*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
```

```
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{4}d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{1}{2}d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= -\frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(\right.$$

$$= \frac{13}{32}b^2c^2d^2x^2 + \frac{1}{32}b^2c^4d^2x^4 - \frac{11}{16}bcd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{8}bcd^2x\left(\right.$$

Mathematica [A] time = 0.379819, size = 323, normalized size = 1.26

$$\frac{1}{768}d^2\left(-768ab\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right) + 768b^2\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right) - 384b^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^2*(768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 - 624*a*b*c*x*Sqrt[1 + c^2*x^2] - 9
6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 624*a*b*ArcSinh[c*x] + 1536*a*b*c^2*x^2*A
rcSinh[c*x] + 384*a*b*c^4*x^4*ArcSinh[c*x] + 768*a*b*ArcSinh[c*x]^2 - 256*b
^2*ArcSinh[c*x]^3 + 144*b^2*Cosh[2*ArcSinh[c*x]] + 288*b^2*ArcSinh[c*x]^2*C
osh[2*ArcSinh[c*x]] + 3*b^2*Cosh[4*ArcSinh[c*x]] + 24*b^2*ArcSinh[c*x]^2*Co
sh[4*ArcSinh[c*x]] + 1536*a*b*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] + 7
68*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 768*a^2*Log[c*x] - 768*
a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 768*b^2*ArcSinh[c*x]*PolyLog[2, E^(2*
ArcSinh[c*x])] - 384*b^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 288*b^2*ArcSinh[c
*x]*Sinh[2*ArcSinh[c*x]] - 12*b^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/768
```

Maple [B] time = 0.162, size = 586, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 49/256*d^2*b^2-13/16*d^2*a*b*c*x*(c^2*x^2+1)^(1/2)+1/2*d^2*a*b*arcsinh(c*x)
*c^4*x^4+2*d^2*a*b*arcsinh(c*x)*c^2*x^2-13/16*d^2*b^2*arcsinh(c*x)*(c^2*x^2
+1)^(1/2)*c*x-1/8*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-1/8*d^2*a*
b*c^3*x^3*(c^2*x^2+1)^(1/2)+1/4*d^2*a^2*c^4*x^4+d^2*a^2*c^2*x^2+d^2*b^2*arc
sinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,c*
x+(c^2*x^2+1)^(1/2))+2*d^2*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))+d^2*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-d^2*a*b*arcsinh(c*x)^
2+13/16*d^2*a*b*arcsinh(c*x)+2*d^2*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*d
^2*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+13/32*b^2*c^2*d^2*x^2+1/32*b^2*c^4
*d^2*x^4+d^2*a^2*ln(c*x)+13/32*d^2*b^2*arcsinh(c*x)^2-1/3*d^2*b^2*arcsinh(c
*x)^3-2*d^2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(3,-c*x-(
c^2*x^2+1)^(1/2))+1/4*d^2*b^2*arcsinh(c*x)^2*c^4*x^4+d^2*b^2*arcsinh(c*x)^2
*c^2*x^2+2*d^2*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^2*a*b*arcsi
nh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a^2c^4d^2x^4 + a^2c^2d^2x^2 + a^2d^2\log(x) + \int b^2c^4d^2x^3\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2\,abc^4d^2x^3\log\left(cx + \sqrt{c^2x^2+1}\right) + 2\,b^2c^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^4*d^2*x^4 + a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(b^2*c^4*
d^2*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x^3*log(c*x + sqrt(c
^2*x^2 + 1)) + 2*b^2*c^2*d^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 4*a*b*c^2*d
^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
x + 2*a*b*d^2*log(c*x + sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a^2}{x}\,dx + \int 2a^2c^2x\,dx + \int a^2c^4x^3\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x}\,dx + \int 2b^2c^2x\operatorname{asinh}^2(cx)\,dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(a**2/x, x) + Integral(2*a**2*c**2*x, x) + Integral(a**2*c**4
*x**3, x) + Integral(b**2*asinh(c*x)**2/x, x) + Integral(2*a*b*asinh(c*x)/x
, x) + Integral(2*b**2*c**2*x*asinh(c*x)**2, x) + Integral(b**2*c**4*x**3*a
sinh(c*x)**2, x) + Integral(4*a*b*c**2*x*asinh(c*x), x) + Integral(2*a*b*c*
*4*x**3*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^2(b\,\text{arsinh}\,(cx) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2/x, x)
```

3.213 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=229

$$-2b^2cd^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{4}{3}c^2d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{9}bcd^2\left(c^2x^2\right.$$

```
[Out] (32*b^2*c^2*d^2*x)/9 + (2*b^2*c^4*d^2*x^3)/27 - (10*b*c*d^2*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/3 - (2*b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/9 + (8*c^2*d^2*x*(a + b*ArcSinh[c*x])^2)/3 + (4*c^2*d^2*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/3 - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/x - 4*b*c*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^2*
PolyLog[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^2*PolyLog[2, E^ArcSinh[c*x]]
```

Rubi [A] time = 0.522505, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5739, 5684, 5653, 5717, 8, 5744, 5742, 5760, 4182, 2279, 2391}

$$-2b^2cd^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{4}{3}c^2d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{9}bcd^2\left(c^2x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (32*b^2*c^2*d^2*x)/9 + (2*b^2*c^4*d^2*x^3)/27 - (10*b*c*d^2*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/3 - (2*b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x]))/9 + (8*c^2*d^2*x*(a + b*ArcSinh[c*x])^2)/3 + (4*c^2*d^2*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/3 - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/x - 4*b*c*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^2*
PolyLog[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^2*PolyLog[2, E^ArcSinh[c*x]]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
```

```
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(4c^2d\right)\int\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right.$$

$$= \frac{2}{3}bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{4}{3}c^2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= -\frac{2}{3}b^2c^2d^2x - \frac{2}{9}b^2c^4d^2x^3 + 2bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}bcd^2\left(1+c^2\right.$$

$$= -\frac{16}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}bcd^2\left(1\right.$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}bcd^2\left(1+\right.$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}bcd^2\left(1+\right.$$

$$= \frac{32}{9}b^2c^2d^2x + \frac{2}{27}b^2c^4d^2x^3 - \frac{10}{3}bcd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{9}bcd^2\left(1+\right.$$

Mathematica [A] time = 1.15082, size = 306, normalized size = 1.34

$$\frac{1}{54}d^2\left(108b^2c\,\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 108b^2c\,\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + 18a^2c^4x^3 + 108a^2c^2x - \frac{54a^2}{x} - 12abc\left(c^2x^2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*((-54*a^2)/x + 108*a^2*c^2*x + 18*a^2*c^4*x^3 - 12*a*b*c*(-2 + c^2*x^2
)*Sqrt[1 + c^2*x^2] + 36*a*b*c^4*x^3*ArcSinh[c*x] - 189*b^2*c*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 216*a*b*c*(-Sqrt[1 + c^2*x^2] + c*x*ArcSinh[c*x]) + 108
*b^2*c^2*x*(2 + ArcSinh[c*x]^2) + 2*b^2*c^2*x*(-12 + 2*c^2*x^2 + 9*c^2*x^2*
ArcSinh[c*x]^2) - (108*a*b*(ArcSinh[c*x] + c*x*ArcTanh[Sqrt[1 + c^2*x^2]]))
/x - 3*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] - (54*b^2*ArcSinh[c*x]*(ArcS
inh[c*x] + 2*c*x*(-Log[1 - E^(-ArcSinh[c*x])] + Log[1 + E^(-ArcSinh[c*x])])
))/x + 108*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 108*b^2*c*PolyLog[2, E^(-
ArcSinh[c*x])]))/54
```

Maple [A] time = 0.134, size = 400, normalized size = 1.8

$$\frac{d^2a^2c^4x^3}{3}+2\,d^2a^2c^2x-\frac{d^2a^2}{x}+\frac{32\,b^2c^2d^2x}{9}+\frac{2\,b^2c^4d^2x^3}{27}+2\,b^2cd^2\mathrm{polylog}\left(2,cx+\sqrt{c^2x^2+1}\right)-2\,b^2cd^2\mathrm{polylog}\left(2,-cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] 1/3*d^2*a^2*c^4*x^3+2*d^2*a^2*c^2*x-d^2*a^2/x+32/9*b^2*c^2*d^2*x+2/27*b^2*c
^4*d^2*x^3+2*b^2*c*d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*c*d^2*polylog
(2,-c*x-(c^2*x^2+1)^(1/2))+1/3*d^2*b^2*arcsinh(c*x)^2*c^4*x^3+2*d^2*b^2*arc
sinh(c*x)^2*c^2*x-32/9*c*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*d^2*b^2
*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-d^2*b^2*arcsinh(c*x)^2/x-2*c*d^2*
b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2/9*d^2*b^2*arcsinh(c*x)*(c^2*
x^2+1)^(1/2)*c^3*x^2+2/3*d^2*a*b*arcsinh(c*x)*c^4*x^3+4*d^2*a*b*arcsinh(c*x
)*c^2*x-2*d^2*a*b*arcsinh(c*x)/x-2/9*d^2*a*b*c^3*x^2*(c^2*x^2+1)^(1/2)-32/9
*c*d^2*a*b*(c^2*x^2+1)^(1/2)-2*c*d^2*a*b*arctanh(1/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a^2c^4d^2x^3+\frac{2}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^4d^2+2\,b^2c^2d^2x\,\mathrm{arsinh}\,(cx)^2+4\,b^2c^2d^2\left(x-\frac{\sqrt{c^2x^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 + 2*b^2*c^2*d^2*x*arcsinh(c*x)^2
+ 4*b^2*c^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 2*a^2*c^2*d^2*x +
4*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^2 - 2*(c*arcsinh(1/(sqrt(c
^2)*abs(x))) + arcsinh(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*(b^2*c^4*d^2*x^4 -
 3*b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate(2/3*(b^2*c^7*d^2*x
^6 + b^2*c^5*d^2*x^4 - 3*b^2*c^3*d^2*x^2 - 3*b^2*c*d^2 + (b^2*c^6*d^2*x^5 -
 3*b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4
+ c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arsinh}\left(cx\right)^2 + 2\,\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int 2a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int a^2c^4x^2\,dx + \int 2b^2c^2\operatorname{asinh}^2\left(cx\right)dx + \int \frac{b^2\operatorname{asinh}^2\left(cx\right)}{x^2}\,dx + \int 4abc^2\operatorname{asinh}\left(cx\right)dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c**
4*x**2, x) + Integral(2*b**2*c**2*asinh(c*x)**2, x) + Integral(b**2*asinh(c
*x)**2/x**2, x) + Integral(4*a*b*c**2*asinh(c*x), x) + Integral(2*a*b*asinh
(c*x)/x**2, x) + Integral(b**2*c**4*x**2*asinh(c*x)**2, x) + Integral(2*a*b
*c**4*x**2*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^2\left(b\operatorname{arsinh}\left(cx\right) + a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2/x^2, x)
```

3.214 $\int \frac{(d+c^2dx^2)^2(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=272

$$-2bc^2d^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-b^2c^2d^2\text{PolyLog}\left(3,e^{-2\sinh^{-1}(cx)}\right)+\frac{1}{2}bc^3d^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}\right.$$

```
[Out] (b^2*c^4*d^2*x^2)/4 + (b*c^3*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
2 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (c^2*d^2*(a + b*
ArcSinh[c*x])^2)/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2 - (d^2*(1
 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (2*c^2*d^2*(a + b*ArcSinh[c
*x])^3)/(3*b) + 2*c^2*d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*ArcSinh[c*x]
)] + b^2*c^2*d^2*Log[x] - 2*b*c^2*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2
*ArcSinh[c*x])] - b^2*c^2*d^2*PolyLog[3, E^(-2*ArcSinh[c*x])]
```

Rubi [A] time = 0.501433, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5739, 5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 14}

$$2bc^2d^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-b^2c^2d^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)+\frac{1}{2}bc^3d^2x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (b^2*c^4*d^2*x^2)/4 + (b*c^3*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
2 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/x + (c^2*d^2*(a + b*
ArcSinh[c*x])^2)/4 + c^2*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2 - (d^2*(1
 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (2*c^2*d^2*(a + b*ArcSinh[c
*x])^3)/(3*b) + 2*c^2*d^2*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcSinh[c*x])
] + b^2*c^2*d^2*Log[x] + 2*b*c^2*d^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*A
rcSinh[c*x])] - b^2*c^2*d^2*PolyLog[3, E^(2*ArcSinh[c*x])]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
```

```
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \left(2c^2d\right)\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(cx)\right.}{x}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x} + c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2 -$$

$$= \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{x}$$

$$= \frac{1}{4}b^2c^4d^2x^2 + \frac{1}{2}bc^3d^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) - \frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\right.}{x}$$

Mathematica [A] time = 0.879375, size = 305, normalized size = 1.12

$$\frac{1}{2}d^2\left(4abc^2\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+2\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\right)-\frac{2}{3}b^2c^2\left(-6\sinh^{-1}(cx)\text{Poly}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^2*(-(a^2/x^2) + a^2*c^4*x^2 - (2*a*b*(c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*
x]))/x^2 + a*b*c^2*(-(c*x*Sqrt[1 + c^2*x^2]) + (1 + 2*c^2*x^2)*ArcSinh[c*x]
) + 4*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh
[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 + 4*a*b*c^2*(ArcSinh[c*x]*(ArcSinh[c*x]
+ 2*Log[1 - E^(-2*ArcSinh[c*x])]) - PolyLog[2, E^(-2*ArcSinh[c*x])]) - (2*b
^2*c^2*(2*ArcSinh[c*x]^2*(ArcSinh[c*x] - 3*Log[1 - E^(2*ArcSinh[c*x])]) - 6
*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + 3*PolyLog[3, E^(2*ArcSinh[c*
x])]))/3 + (b^2*c^2*((1 + 2*ArcSinh[c*x]^2)*Cosh[2*ArcSinh[c*x]] - 2*ArcSin
h[c*x]*Sinh[2*ArcSinh[c*x]]))/4))/2
```

Maple [B] time = 0.342, size = 719, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] d^2*a*b*c^2+4*c^2*d^2*b^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*
c^2*d^2*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-d^2*a*b*arcsinh(c*x)
/x^2-2*c^2*d^2*a*b*arcsinh(c*x)^2+1/2*c^2*d^2*a*b*arcsinh(c*x)+4*c^2*d^2*b^
2*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+1/4*b^2*c^4*d^2*x^2-1/2*d^2
*a^2/x^2+1/8*d^2*b^2*c^2-c*d^2*b^2*arcsinh(c*x)/x*(c^2*x^2+1)^(1/2)-1/2*c^3
*d^2*a*b*x*(c^2*x^2+1)^(1/2)+c^4*d^2*a*b*arcsinh(c*x)*x^2-1/2*c^3*d^2*b^2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)*x+4*c^2*d^2*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^
2+1)^(1/2))+4*c^2*d^2*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-c*d^2*a*
b/x*(c^2*x^2+1)^(1/2)-2*c^2*d^2*b^2*ln(c*x+(c^2*x^2+1)^(1/2))+c^2*d^2*b^2*l
n(1+c*x+(c^2*x^2+1)^(1/2))+c^2*d^2*b^2*ln(c*x+(c^2*x^2+1)^(1/2)-1)+2*c^2*d^
2*a^2*ln(c*x)+1/4*c^2*d^2*b^2*arcsinh(c*x)^2-2/3*c^2*d^2*b^2*arcsinh(c*x)^3
-4*c^2*d^2*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-1/2*d^2*b^2*arcsinh(c*x)^2/
x^2+1/2*c^4*d^2*a^2*x^2-4*c^2*d^2*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+c^2
*d^2*b^2*arcsinh(c*x)+4*c^2*d^2*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))+4*c^2*
d^2*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1/2*c^4*d^2*b^2*arcsinh(c*x)^2*x^
2+2*c^2*d^2*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2c^4d^2x^2 + 2a^2c^2d^2\log(x) - abd^2\left(\frac{\sqrt{c^2x^2+1}c}{x} + \frac{\operatorname{arsinh}(cx)}{x^2}\right) - \frac{a^2d^2}{2x^2} + \int b^2c^4d^2x\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2abc^4d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^4*d^2*x^2 + 2*a^2*c^2*d^2*log(x) - a*b*d^2*(sqrt(c^2*x^2 + 1)*c/x
 + arcsinh(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(b^2*c^4*d^2*x*log(c*x +
sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^4*d^2*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*b^2
*c^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 4*a*b*c^2*d^2*log(c*x + sqrt(c^
```

```
2*x^2 + 1))/x + b^2*d^2*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{a^2}{x^3}\,dx + \int\frac{2a^2c^2}{x}\,dx + \int a^2c^4x\,dx + \int\frac{b^2\,\text{asinh}^2\,(cx)}{x^3}\,dx + \int\frac{2ab\,\text{asinh}\,(cx)}{x^3}\,dx + \int\frac{2b^2c^2\,\text{asinh}^2\,(cx)}{x}\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c**2/x, x) + Integral(a**2*c
**4*x, x) + Integral(b**2*asinh(c*x)**2/x**3, x) + Integral(2*a*b*asinh(c*x
)/x**3, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x, x) + Integral(b**2*c**4*
x*asinh(c*x)**2, x) + Integral(4*a*b*c**2*asinh(c*x)/x, x) + Integral(2*a*b
*c**4*x*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2dx^2 + d\right)^2(b\,\text{arsinh}\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")

[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2/x^3, x)
```

3.215 $\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=248

$$-\frac{11}{3}b^2c^3d^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{11}{3}b^2c^3d^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{5}{3}bc^3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{4c^2d^2(}{}$$

```
[Out] -(b^2*c^2*d^2)/(3*x) + 2*b^2*c^4*d^2*x - (5*b*c^3*d^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*x^2) + (8*c^4*d^2*x*(a + b*ArcSinh[c*x])^2)/3 - (4*c^2*d^2*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x])^2)/(3*x) - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/(3*x^3) - (22*b*c^3*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 -
 (11*b^2*c^3*d^2*PolyLog[2, -E^ArcSinh[c*x]])/3 + (11*b^2*c^3*d^2*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rubi [A] time = 0.706998, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5739, 5653, 5717, 8, 5742, 5760, 4182, 2279, 2391, 14}

$$-\frac{11}{3}b^2c^3d^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{11}{3}b^2c^3d^2\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)-\frac{5}{3}bc^3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)-\frac{4c^2d^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^2)/(3*x) + 2*b^2*c^4*d^2*x - (5*b*c^3*d^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*x^2) + (8*c^4*d^2*x*(a + b*ArcSinh[c*x])^2)/3 - (4*c^2*d^2*(1 + c^2*x^2)*
(a + b*ArcSinh[c*x])^2)/(3*x) - (d^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2
)/(3*x^3) - (22*b*c^3*d^2*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 -
 (11*b^2*c^3*d^2*PolyLog[2, -E^ArcSinh[c*x]])/3 + (11*b^2*c^3*d^2*PolyLog[2
, E^ArcSinh[c*x]])/3
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
```

```
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^2\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(4c^2d\right)\int \frac{\left(d+c^2dx^2\right)\left(a+b\sinh^{-1}(c\right.}{x^2}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{4c^2d^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3x}$$

$$= \frac{11}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}+$$

$$= -\frac{b^2c^2d^2}{3x}-\frac{10}{3}b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)}{}$$

$$= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3}}{}$$

$$= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3}}{}$$

$$= -\frac{b^2c^2d^2}{3x}+2b^2c^4d^2x-\frac{5}{3}bc^3d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^2\left(1+c^2x^2\right)^{3}}{}$$

Mathematica [A] time = 0.90395, size = 357, normalized size = 1.44

$$\frac{d^2\left(11b^2c^3x^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-11b^2c^3x^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+3a^2c^4x^4-6a^2c^2x^2-a^2-6abc^3x^3\sqrt{c^2x^2+1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^2*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-a^2 - 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 + 6*b^2*c^4*x^4 -
a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh
[c*x] - 12*a*b*c^2*x^2*ArcSinh[c*x] + 6*a*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x] - b^2*ArcSinh[c*x]^2 - 6*b^2*c^2*x^2*ArcSinh[c*x]^2 + 3*b^2*c^4*x^4*ArcS
inh[c*x]^2 - 11*a*b*c^3*x^3*ArcTanh[Sqrt[1 + c^2*x^2]] + 11*b^2*c^3*x^3*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 11*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 +
E^(-ArcSinh[c*x])] + 11*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 11*b^2
*c^3*x^3*PolyLog[2, E^(-ArcSinh[c*x])]))/(3*x^3)
```

Maple [A] time = 0.281, size = 408, normalized size = 1.7

$$c^4d^2a^2x-2\,\frac{c^2d^2a^2}{x}-\frac{d^2a^2}{3\,x^3}+c^4d^2b^2\left(\mathrm{Arcsinh}\left(cx\right)\right)^2x-2\,c^3d^2b^2\mathrm{Arcsinh}\left(cx\right)\sqrt{c^2x^2+1}+2\,b^2c^4d^2x-2\,\frac{d^2b^2c^2\left(\mathrm{Arcsinh}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] c^4*d^2*a^2*x-2*c^2*d^2*a^2/x-1/3*d^2*a^2/x^3+c^4*d^2*b^2*arcsinh(c*x)^2*x-
2*c^3*d^2*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*b^2*c^4*d^2*x-2*c^2*d^2*b^2*
arcsinh(c*x)^2/x-1/3*c*d^2*b^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-1/3*d^2*b
^2/x^3*arcsinh(c*x)^2-1/3*b^2*c^2*d^2/x-11/3*c^3*d^2*b^2*arcsinh(c*x)*ln(1+
c*x+(c^2*x^2+1)^(1/2))-11/3*b^2*c^3*d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1
1/3*c^3*d^2*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+11/3*b^2*c^3*d^2*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))+2*c^4*d^2*a*b*arcsinh(c*x)*x-4*c^2*d^2*a*b*
arcsinh(c*x)/x-2/3*d^2*a*b*arcsinh(c*x)/x^3-2*c^3*d^2*a*b*(c^2*x^2+1)^(1/2)
-11/3*c^3*d^2*a*b*arctanh(1/(c^2*x^2+1)^(1/2))-1/3*c*d^2*a*b/x^2*(c^2*x^2+1
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2c^4d^2x\,\mathrm{arsinh}\,(cx)^2 + 2\,b^2c^4d^2\left(x - \frac{\sqrt{c^2x^2+1}\,\mathrm{arsinh}\,(cx)}{c}\right) + a^2c^4d^2x + 2\left(cx\,\mathrm{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)abc^3d^2 - 4\left(c\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*c^4*d^2*x*arcsinh(c*x)^2 + 2*b^2*c^4*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh
(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3
*d^2 - 4*(c*arcsinh(1/(sqrt(c^2)*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^2 + 1
/3*((c^2*arcsinh(1/(sqrt(c^2)*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsi
nh(c*x)/x^3)*a*b*d^2 - 2*a^2*c^2*d^2/x - 1/3*a^2*d^2/x^3 - 1/3*(6*b^2*c^2*d
^2*x^2 + b^2*d^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 + integrate(2/3*(6*b^2
*c^5*d^2*x^4 + 7*b^2*c^3*d^2*x^2 + b^2*c*d^2 + (6*b^2*c^4*d^2*x^3 + b^2*c^2
*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 +
(c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\mathrm{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2c^4\,dx + \int\frac{a^2}{x^4}\,dx + \int\frac{2a^2c^2}{x^2}\,dx + \int b^2c^4\,\mathrm{asinh}^2\,(cx)\,dx + \int\frac{b^2\,\mathrm{asinh}^2\,(cx)}{x^4}\,dx + \int 2abc^4\,\mathrm{asinh}\,(cx)\,dx + \int\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(2*a**2*c**
2/x**2, x) + Integral(b**2*c**4*asinh(c*x)**2, x) + Integral(b**2*asinh(c*x
)**2/x**4, x) + Integral(2*a*b*c**4*asinh(c*x), x) + Integral(2*a*b*asinh(c
*x)/x**4, x) + Integral(2*b**2*c**2*asinh(c*x)**2/x**2, x) + Integral(4*a*b
*c**2*asinh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2 (b \operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*(b*arcsinh(c*x) + a)^2/x^4, x)
```

3.216 $\int x^4 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=465

$$\frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{2}{33} d^3 x^5 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{8}{231} d^3 x^5 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx\right.$$

```
[Out] (100976*b^2*d^3*x)/(4002075*c^4) - (50488*b^2*d^3*x^3)/(12006225*c^2) + (12
622*b^2*d^3*x^5)/6670125 + (9410*b^2*c^2*d^3*x^7)/1120581 + (182*b^2*c^4*d^
3*x^9)/29403 + (2*b^2*c^6*d^3*x^11)/1331 - (256*b*d^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(17325*c^5) + (128*b*d^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(17325*c^3) - (32*b*d^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(5775*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(693*c
^5) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(1155*c^5) - (2*b*
d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(1617*c^5) + (8*b*d^3*(1 + c^
2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(297*c^5) - (2*b*d^3*(1 + c^2*x^2)^(11/2
)*(a + b*ArcSinh[c*x]))/(121*c^5) + (16*d^3*x^5*(a + b*ArcSinh[c*x])^2)/115
5 + (8*d^3*x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/231 + (2*d^3*x^5*(1 +
c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/33 + (d^3*x^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x])^2)/11
```

Rubi [A] time = 1.05765, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 1153}

$$\frac{1}{11} d^3 x^5 \left(c^2 x^2+1\right)^3 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{2}{33} d^3 x^5 \left(c^2 x^2+1\right)^2 \left(a+b \sinh^{-1}(cx)\right)^2+\frac{8}{231} d^3 x^5 \left(c^2 x^2+1\right) \left(a+b \sinh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (100976*b^2*d^3*x)/(4002075*c^4) - (50488*b^2*d^3*x^3)/(12006225*c^2) + (12
622*b^2*d^3*x^5)/6670125 + (9410*b^2*c^2*d^3*x^7)/1120581 + (182*b^2*c^4*d^
3*x^9)/29403 + (2*b^2*c^6*d^3*x^11)/1331 - (256*b*d^3*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(17325*c^5) + (128*b*d^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/(17325*c^3) - (32*b*d^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(5775*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(693*c
^5) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(1155*c^5) - (2*b*
d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(1617*c^5) + (8*b*d^3*(1 + c^
2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(297*c^5) - (2*b*d^3*(1 + c^2*x^2)^(11/2
```

```
)*(a + b*ArcSinh[c*x]))/(121*c^5) + (16*d^3*x^5*(a + b*ArcSinh[c*x])^2)/115
5 + (8*d^3*x^5*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/231 + (2*d^3*x^5*(1 +
c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/33 + (d^3*x^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x])^2)/11
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^4 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{11}d^3x^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{11}(6d)\int x^4\left(d+c^2dx^2\right)^2\left(a+b\sin\right.$$

$$= -\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{77c^5}+\frac{4bd^3\left(1+c^2x^2\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right.}{99c^5}$$

$$= -\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{165c^5}+\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right.}{231c^5}$$

$$= -\frac{16bd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{693c^5}+\frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{1155c^5}$$

$$= \frac{16b^2d^3x}{7623c^4}-\frac{8b^2d^3x^3}{22869c^2}+\frac{2b^2d^3x^5}{12705}+\frac{226b^2c^2d^3x^7}{53361}+\frac{46b^2c^4d^3x^9}{9801}+\frac{2b^2c^6d^3x^{11}}{1331}$$

$$= \frac{8368b^2d^3x}{800415c^4}-\frac{4184b^2d^3x^3}{2401245c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{182b^2c^4d^3x^9}{29403}+$$

$$= \frac{8368b^2d^3x}{800415c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{182b^2c^4d^3x^9}{29403}$$

$$= \frac{100976b^2d^3x}{4002075c^4}-\frac{50488b^2d^3x^3}{12006225c^2}+\frac{12622b^2d^3x^5}{6670125}+\frac{9410b^2c^2d^3x^7}{1120581}+\frac{182b^2c^4d^3x}{29403}$$

<u>Mathematica</u> [A] time = 0.461173, size = 299, normalized size = 0.64

$$d^3\Big(12006225a^2c^5x^5\left(105c^6x^6+385c^4x^4+495c^2x^2+231\right)-6930ab\sqrt{c^2x^2+1}\left(33075c^{10}x^{10}+111475c^8x^8+117625c^6x^6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(12006225*a^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)
- 6930*a*b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^4 + 11762
5*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10) + 2*b^2*c*x*(174940920 - 2915
6820*c^2*x^2 + 13120569*c^4*x^4 + 58224375*c^6*x^6 + 42917875*c^8*x^8 + 104
18625*c^10*x^10) - 6930*b*(-3465*a*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4
 + 105*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(50488 - 25244*c^2*x^2 + 18933*c^4*x^
4 + 117625*c^6*x^6 + 111475*c^8*x^8 + 33075*c^10*x^10))*ArcSinh[c*x] + 1200
6225*b^2*c^5*x^5*(231 + 495*c^2*x^2 + 385*c^4*x^4 + 105*c^6*x^6)*ArcSinh[c*
x]^2))/(13867189875*c^5)
```

Maple [A] time = 0.092, size = 544, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^5*(d^3*a^2*(1/11*c^11*x^11+1/3*c^9*x^9+3/7*c^7*x^7+1/5*c^5*x^5)+d^3*b^2
*(1/11*arcsinh(c*x)^2*c^3*x^3*(c^2*x^2+1)^4-1/33*arcsinh(c*x)^2*c*x*(c^2*x^
2+1)^4+16/1155*arcsinh(c*x)^2*c*x+1/231*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+2/
385*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2+8/1155*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-
5487704/4622396625*c*x*(c^2*x^2+1)^2-100976/4002075*arcsinh(c*x)*(c^2*x^2+1
)^(1/2)-606416/13867189875*c*x*(c^2*x^2+1)-428/323433*c*x*(c^2*x^2+1)^4-148
174/110937519*c*x*(c^2*x^2+1)^3+382986368/13867189875*c*x+34/3267*arcsinh(c
*x)*c^2*x^2*(c^2*x^2+1)^(7/2)+1468/160083*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^
(5/2)+2/1331*c*x*(c^2*x^2+1)^5-2/121*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(9/2)
+28384/4002075*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(3/2)+9904/4002075*arcsinh(
c*x)*c^2*x^2*(c^2*x^2+1)^(1/2))+2*d^3*a*b*(1/11*arcsinh(c*x)*c^11*x^11+1/3*
arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x)*c^7*x^7+1/5*arcsinh(c*x)*c^5*x^5-1/12
1*c^10*x^10*(c^2*x^2+1)^(1/2)-91/3267*c^8*x^8*(c^2*x^2+1)^(1/2)-4705/160083
*c^6*x^6*(c^2*x^2+1)^(1/2)-6311/1334025*c^4*x^4*(c^2*x^2+1)^(1/2)+25244/400
2075*c^2*x^2*(c^2*x^2+1)^(1/2)-50488/4002075*(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.24128, size = 1497, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*c^6*d^3*x^11*arcsinh(c*x)^2 + 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4*
d^3*x^9*arcsinh(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 + 3/7*b^2*c^2*d^3*x^7*arcsinh(
c*x)^2 + 3/7*a^2*c^2*d^3*x^7 + 2/7623*(693*x^11*arcsinh(c*x) - (63*sqrt(c^2
*x^2 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^
6/c^6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256
*sqrt(c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 - 2/26413695*(3465*(63*sqrt(c^2*x^2
 + 1)*x^10/c^2 - 70*sqrt(c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(c^2*x^2 + 1)*x^6/c^
6 - 96*sqrt(c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(c^2*x^2 + 1)*x^2/c^10 - 256*sqr
```

```
t(c^2*x^2 + 1)/c^12)*c*arcsinh(c*x) - (19845*c^10*x^11 - 26950*c^8*x^9 + 39
600*c^6*x^7 - 66528*c^4*x^5 + 147840*c^2*x^3 - 887040*x)/c^10)*b^2*c^6*d^3
+ 1/5*b^2*d^3*x^5*arcsinh(c*x)^2 + 2/945*(315*x^9*arcsinh(c*x) - (35*sqrt(c
^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x
^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*
c^4*d^3 - 2/297675*(315*(35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1
)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 1
28*sqrt(c^2*x^2 + 1)/c^10)*c*arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 +
3024*c^4*x^5 - 6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^3 + 1/5*a^2*d^3*x^5 +
 6/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2
 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*
a*b*c^2*d^3 - 2/8575*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1
)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsi
nh(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^2*d^
3 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x
^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d^3 - 2/1125*(15*(3*sqrt(
c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^
6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d^3
```

Fricas [A] time = 2.87466, size = 1135, normalized size = 2.44

$$\frac{10418625\left(121\,a^2+2\,b^2\right)c^{11}d^3x^{11}+471625\left(9801\,a^2+182\,b^2\right)c^9d^3x^9+12375\left(480249\,a^2+9410\,b^2\right)c^7d^3x^7+2079\left(1334\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/13867189875*(10418625*(121*a^2 + 2*b^2)*c^11*d^3*x^11 + 471625*(9801*a^2
+ 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 + 9410*b^2)*c^7*d^3*x^7 + 2079*(
1334025*a^2 + 12622*b^2)*c^5*d^3*x^5 - 58313640*b^2*c^3*d^3*x^3 + 349881840
*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 + 385*b^2*c^9*d^3*x^9 + 495*
b^2*c^7*d^3*x^7 + 231*b^2*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 693
0*(363825*a*b*c^11*d^3*x^11 + 1334025*a*b*c^9*d^3*x^9 + 1715175*a*b*c^7*d^3
*x^7 + 800415*a*b*c^5*d^3*x^5 - (33075*b^2*c^10*d^3*x^10 + 111475*b^2*c^8*d
^3*x^8 + 117625*b^2*c^6*d^3*x^6 + 18933*b^2*c^4*d^3*x^4 - 25244*b^2*c^2*d^3
*x^2 + 50488*b^2*d^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 693
0*(33075*a*b*c^10*d^3*x^10 + 111475*a*b*c^8*d^3*x^8 + 117625*a*b*c^6*d^3*x^
6 + 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3*x^2 + 50488*a*b*d^3)*sqrt(c^2
*x^2 + 1))/c^5
```

Sympy [A] time = 115.345, size = 702, normalized size = 1.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 + 3*a**2*c**2*d*
*3*x**7/7 + a**2*d**3*x**5/5 + 2*a*b*c**6*d**3*x**11*asinh(c*x)/11 - 2*a*b*
c**5*d**3*x**10*sqrt(c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*asinh(c*x)/3
 - 182*a*b*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)/3267 + 6*a*b*c**2*d**3*x**7*a
sinh(c*x)/7 - 9410*a*b*c*d**3*x**6*sqrt(c**2*x**2 + 1)/160083 + 2*a*b*d**3*
x**5*asinh(c*x)/5 - 12622*a*b*d**3*x**4*sqrt(c**2*x**2 + 1)/(1334025*c) + 5
0488*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(4002075*c**3) - 100976*a*b*d**3*sqr
t(c**2*x**2 + 1)/(4002075*c**5) + b**2*c**6*d**3*x**11*asinh(c*x)**2/11 + 2
*b**2*c**6*d**3*x**11/1331 - 2*b**2*c**5*d**3*x**10*sqrt(c**2*x**2 + 1)*asi
nh(c*x)/121 + b**2*c**4*d**3*x**9*asinh(c*x)**2/3 + 182*b**2*c**4*d**3*x**9
/29403 - 182*b**2*c**3*d**3*x**8*sqrt(c**2*x**2 + 1)*asinh(c*x)/3267 + 3*b*
*2*c**2*d**3*x**7*asinh(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/1120581 - 9410
*b**2*c*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/160083 + b**2*d**3*x**5*as
inh(c*x)**2/5 + 12622*b**2*d**3*x**5/6670125 - 12622*b**2*d**3*x**4*sqrt(c*
*2*x**2 + 1)*asinh(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(12006225*c**2)
+ 50488*b**2*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(4002075*c**3) + 1009
76*b**2*d**3*x/(4002075*c**4) - 100976*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(4002075*c**5), Ne(c, 0)), (a**2*d**3*x**5/5, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.217 $\int x^3 \left(d+c^2 d x^2\right)^3 \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=376

$$-\frac{1}{50} b c d^3 x^5 \left(c^2x^2+1\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32} b c d^3 x^5 \left(c^2x^2+1\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)-\frac{31}{960} b c d^3 x^5 \sqrt{c^2x^2+1} \left(a+b\sinh\right.$$

```
[Out] (-79*b^2*d^3*x^2)/(5120*c^2) + (79*b^2*d^3*x^4)/15360 + (401*b^2*c^2*d^3*x^
6)/28800 + (57*b^2*c^4*d^3*x^8)/6400 + (b^2*c^6*d^3*x^10)/500 + (79*b*d^3*x
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2560*c^3) - (79*b*d^3*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(3840*c) - (31*b*c*d^3*x^5*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/960 - (b*c*d^3*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/32 - (b*c*d^3*x^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/50 -
 (79*d^3*(a + b*ArcSinh[c*x])^2)/(5120*c^4) + (d^3*x^4*(a + b*ArcSinh[c*x])
^2)/40 + (d^3*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/20 + (3*d^3*x^4*(1
+ c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/40 + (d^3*x^4*(1 + c^2*x^2)^3*(a + b*A
rcSinh[c*x])^2)/10
```

Rubi [A] time = 1.65703, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5744, 5661, 5758, 5675, 30, 5742, 14, 266, 43}

$$-\frac{1}{50} b c d^3 x^5 \left(c^2x^2+1\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32} b c d^3 x^5 \left(c^2x^2+1\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)-\frac{31}{960} b c d^3 x^5 \sqrt{c^2x^2+1} \left(a+b\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-79*b^2*d^3*x^2)/(5120*c^2) + (79*b^2*d^3*x^4)/15360 + (401*b^2*c^2*d^3*x^
6)/28800 + (57*b^2*c^4*d^3*x^8)/6400 + (b^2*c^6*d^3*x^10)/500 + (79*b*d^3*x
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2560*c^3) - (79*b*d^3*x^3*Sqrt[1
+ c^2*x^2]*(a + b*ArcSinh[c*x]))/(3840*c) - (31*b*c*d^3*x^5*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/960 - (b*c*d^3*x^5*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x]))/32 - (b*c*d^3*x^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/50 -
 (79*d^3*(a + b*ArcSinh[c*x])^2)/(5120*c^4) + (d^3*x^4*(a + b*ArcSinh[c*x])
^2)/40 + (d^3*x^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/20 + (3*d^3*x^4*(1
+ c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/40 + (d^3*x^4*(1 + c^2*x^2)^3*(a + b*A
rcSinh[c*x])^2)/10
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
```

```
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{10}d^3x^4\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{5}(3d)\int x^3\left(d+c^2dx^2\right)^2\left(a+b\sinh\right.$$

$$= -\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{40}d^3x^4\left(1+c^2x^2\right)^2\left(a+b\sinh\right.$$

$$= -\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{50}bcd^3x^5\left(1+c^2x^2\right)^{5/2}\left(a+b\,\mathrm{si}\right.$$

$$= -\frac{31}{960}bcd^3x^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{32}bcd^3x^5\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.$$

$$= \frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}-\frac{79bd^3x^3\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{3840c}$$

$$= \frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}+\frac{79bd^3x\sqrt{1+c^2x^2}}{25}$$

$$= -\frac{79b^2d^3x^2}{5120c^2}+\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}+\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}+\frac{79b}{}$$

Mathematica [A] time = 0.460209, size = 285, normalized size = 0.76

$$d^3\left(cx\left(28800a^2c^3x^3\left(4c^6x^6+15c^4x^4+20c^2x^2+10\right)-30ab\sqrt{c^2x^2+1}\left(768c^8x^8+2736c^6x^6+3208c^4x^4+790c^2x^2-11\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(28800*a^2*c^3*x^3*(10 + 20*c^2*x^2 + 15*c^4*x^4 + 4*c^6*x^6) - 3
0*a*b*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^2 + 3208*c^4*x^4 + 2736*c^6*x^6
+ 768*c^8*x^8) + b^2*c*x*(-17775 + 5925*c^2*x^2 + 16040*c^4*x^4 + 10260*c^6
*x^6 + 2304*c^8*x^8)) + 30*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(-1185 + 790*c^2*x^
2 + 3208*c^4*x^4 + 2736*c^6*x^6 + 768*c^8*x^8)) + 15*a*(-79 + 1280*c^4*x^4
+ 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10))*ArcSinh[c*x] + 225*b^2*(-79
 + 1280*c^4*x^4 + 2560*c^6*x^6 + 1920*c^8*x^8 + 512*c^10*x^10)*ArcSinh[c*x]
^2))/(1152000*c^4)
```

Maple [A] time = 0.086, size = 508, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^4*(d^3*a^2*(1/10*c^10*x^10+3/8*c^8*x^8+1/2*c^6*x^6+1/4*c^4*x^4)+d^3*b^2
*(1/10*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^4-1/40*arcsinh(c*x)^2*c^2*x^2*(c^
2*x^2+1)^3-1/40*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^2-1/40*arcsinh(c*x)^2*c^
2*x^2*(c^2*x^2+1)-1/40*arcsinh(c*x)^2*(c^2*x^2+1)-1/50*arcsinh(c*x)*c*x*(c^
2*x^2+1)^(9/2)+7/800*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7/2)+49/4800*arcsinh(c*x
)*c*x*(c^2*x^2+1)^(5/2)+49/3840*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)+49/2560*
arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x+49/5120*arcsinh(c*x)^2+1/500*c^2*x^2*(c^
2*x^2+1)^4+29/32000*c^2*x^2*(c^2*x^2+1)^3-229/288000*c^2*x^2*(c^2*x^2+1)^2-
4591/1152000*c^2*x^2*(c^2*x^2+1)-61/4500*c^2*x^2-61/4500)+2*d^3*a*b*(1/10*a
rcsinh(c*x)*c^10*x^10+3/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+1/4
*arcsinh(c*x)*c^4*x^4-1/100*c^9*x^9*(c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(c^2*
x^2+1)^(1/2)-401/9600*c^5*x^5*(c^2*x^2+1)^(1/2)-79/7680*c^3*x^3*(c^2*x^2+1)
^(1/2)+79/5120*c*x*(c^2*x^2+1)^(1/2)-79/5120*arcsinh(c*x)))
```

Maxima [B] time = 1.44086, size = 1669, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*c^6*d^3*x^10*arcsinh(c*x)^2 + 1/10*a^2*c^6*d^3*x^10 + 3/8*b^2*c^4*
d^3*x^8*arcsinh(c*x)^2 + 3/8*a^2*c^4*d^3*x^8 + 1/2*b^2*c^2*d^3*x^6*arcsinh(
c*x)^2 + 1/2*a^2*c^2*d^3*x^6 + 1/6400*(1280*x^10*arcsinh(c*x) - (128*sqrt(c
^2*x^2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)
*x^5/c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 3
15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^10))*c)*a*b*c^6*d^3 + 1/64000*((12
8*x^10/c^2 - 180*x^8/c^4 + 280*x^6/c^6 - 525*x^4/c^8 + 1575*x^2/c^10 - 1575
*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^12)*c^2 - 10*(128*sqrt(c^2*x^
2 + 1)*x^9/c^2 - 144*sqrt(c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(c^2*x^2 + 1)*x^5/
c^6 - 210*sqrt(c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(c^2*x^2 + 1)*x/c^10 - 315*ar
csinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^10))*c*arcsinh(c*x))*b^2*c^6*d^3 + 1/4*
b^2*d^3*x^4*arcsinh(c*x)^2 + 1/512*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2
 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6
 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^
8))*c)*a*b*c^4*d^3 + 1/3072*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 - 315*x
^2/c^8 + 315*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*s
qrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 +
 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(s
qrt(c^2)*c^8))*c*arcsinh(c*x))*b^2*c^4*d^3 + 1/4*a^2*d^3*x^4 + 1/48*(48*x^6
*arcsinh(c*x) - (8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4
 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6)
)*c)*a*b*c^2*d^3 + 1/288*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c^2
*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c
^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh
(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/16*(8*x^
4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 +
 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*a*b*d^3 + 1/32*((x^4/c^2 -
3*x^2/c^4 + 3*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^6)*c^2 - 2*(2*sq
rt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(
c^2))/(sqrt(c^2)*c^4))*c*arcsinh(c*x))*b^2*d^3
```

Fricas [A] time = 2.78327, size = 996, normalized size = 2.65

$$\frac{2304\left(50\,a^2+b^2\right)c^{10}d^3x^{10}+540\left(800\,a^2+19\,b^2\right)c^8d^3x^8+40\left(14400\,a^2+401\,b^2\right)c^6d^3x^6+75\left(3840\,a^2+79\,b^2\right)c^4d^3x^4-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1152000*(2304*(50*a^2 + b^2)*c^10*d^3*x^10 + 540*(800*a^2 + 19*b^2)*c^8*d
^3*x^8 + 40*(14400*a^2 + 401*b^2)*c^6*d^3*x^6 + 75*(3840*a^2 + 79*b^2)*c^4*
d^3*x^4 - 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 + 1920*b^2*c^8
*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 + 1280*b^2*c^4*d^3*x^4 - 79*b^2*d^3)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 30*(7680*a*b*c^10*d^3*x^10 + 28800*a*b*c^8*d^3*x
^8 + 38400*a*b*c^6*d^3*x^6 + 19200*a*b*c^4*d^3*x^4 - 1185*a*b*d^3 - (768*b^
2*c^9*d^3*x^9 + 2736*b^2*c^7*d^3*x^7 + 3208*b^2*c^5*d^3*x^5 + 790*b^2*c^3*d
^3*x^3 - 1185*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
- 30*(768*a*b*c^9*d^3*x^9 + 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^3*x^5 + 7
90*a*b*c^3*d^3*x^3 - 1185*a*b*c*d^3*x)*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A] time = 82.1906, size = 654, normalized size = 1.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 + a**2*c**2*d*
*3*x**6/2 + a**2*d**3*x**4/4 + a*b*c**6*d**3*x**10*asinh(c*x)/5 - a*b*c**5*
d**3*x**9*sqrt(c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*asinh(c*x)/4 - 57*a
*b*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)/800 + a*b*c**2*d**3*x**6*asinh(c*x) -
 401*a*b*c*d**3*x**5*sqrt(c**2*x**2 + 1)/4800 + a*b*d**3*x**4*asinh(c*x)/2
- 79*a*b*d**3*x**3*sqrt(c**2*x**2 + 1)/(3840*c) + 79*a*b*d**3*x*sqrt(c**2*x
**2 + 1)/(2560*c**3) - 79*a*b*d**3*asinh(c*x)/(2560*c**4) + b**2*c**6*d**3*
x**10*asinh(c*x)**2/10 + b**2*c**6*d**3*x**10/500 - b**2*c**5*d**3*x**9*sqr
t(c**2*x**2 + 1)*asinh(c*x)/50 + 3*b**2*c**4*d**3*x**8*asinh(c*x)**2/8 + 57
*b**2*c**4*d**3*x**8/6400 - 57*b**2*c**3*d**3*x**7*sqrt(c**2*x**2 + 1)*asin
h(c*x)/800 + b**2*c**2*d**3*x**6*asinh(c*x)**2/2 + 401*b**2*c**2*d**3*x**6/
28800 - 401*b**2*c*d**3*x**5*sqrt(c**2*x**2 + 1)*asinh(c*x)/4800 + b**2*d**
3*x**4*asinh(c*x)**2/4 + 79*b**2*d**3*x**4/15360 - 79*b**2*d**3*x**3*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c**2) + 79*b**2
*d**3*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2560*c**3) - 79*b**2*d**3*asinh(c*x
)**2/(5120*c**4), Ne(c, 0)), (a**2*d**3*x**4/4, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^3 (b\operatorname{arsinh}(cx)+a)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)^2*x^3, x)
```

3.218 $\int x^2 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=382

$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{21}d^3x^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{105}d^3x^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)$$

```
[Out] (-10516*b^2*d^3*x)/(99225*c^2) + (5258*b^2*d^3*x^3)/297675 + (4198*b^2*c^2*
d^3*x^5)/165375 + (374*b^2*c^4*d^3*x^7)/27783 + (2*b^2*c^6*d^3*x^9)/729 + (
64*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c^3) - (32*b*d^3*x^2*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c) + (16*b*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(315*c^3) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*
ArcSinh[c*x]))/(525*c^3) + (2*b*d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(441*c^3) - (2*b*d^3*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^3)
+ (16*d^3*x^3*(a + b*ArcSinh[c*x])^2)/315 + (8*d^3*x^3*(1 + c^2*x^2)*(a + b
*ArcSinh[c*x])^2)/105 + (2*d^3*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/
21 + (d^3*x^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/9
```

Rubi [A] time = 0.861356, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5744, 5661, 5758, 5717, 8, 30, 266, 43, 5732, 12, 373}

$$\frac{1}{9}d^3x^3\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{2}{21}d^3x^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{105}d^3x^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-10516*b^2*d^3*x)/(99225*c^2) + (5258*b^2*d^3*x^3)/297675 + (4198*b^2*c^2*
d^3*x^5)/165375 + (374*b^2*c^4*d^3*x^7)/27783 + (2*b^2*c^6*d^3*x^9)/729 + (
64*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c^3) - (32*b*d^3*x^2*
Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(945*c) + (16*b*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/(315*c^3) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*
ArcSinh[c*x]))/(525*c^3) + (2*b*d^3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]
))/(441*c^3) - (2*b*d^3*(1 + c^2*x^2)^(9/2)*(a + b*ArcSinh[c*x]))/(81*c^3)
+ (16*d^3*x^3*(a + b*ArcSinh[c*x])^2)/315 + (8*d^3*x^3*(1 + c^2*x^2)*(a + b
*ArcSinh[c*x])^2)/105 + (2*d^3*x^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/
21 + (d^3*x^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/9
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{9}d^3x^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{3}(2d)\int x^2\left(d+c^2dx^2\right)^2\left(a+b\sinh\right.$$

$$= \frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{63c^3} - \frac{2bd^3\left(1+c^2x^2\right)^{9/2}\left(a+b\sinh^{-1}(cx)\right)}{81c^3}$$

$$= \frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{105c^3} + \frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{441c^3}$$

$$= \frac{16bd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{315c^3} + \frac{4bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right.}{525c^3}$$

$$= -\frac{4b^2d^3x}{567c^2} + \frac{2b^2d^3x^3}{1701} + \frac{2}{189}b^2c^2d^3x^5 + \frac{38b^2c^4d^3x^7}{3969} + \frac{2}{729}b^2c^6d^3x^9 - \frac{32bd^3x^2}{}$$

$$= -\frac{3796b^2d^3x}{99225c^2} + \frac{5258b^2d^3x^3}{297675} + \frac{4198b^2c^2d^3x^5}{165375} + \frac{374b^2c^4d^3x^7}{27783} + \frac{2}{729}b^2c^6d^3x^9$$

$$= -\frac{10516b^2d^3x}{99225c^2} + \frac{5258b^2d^3x^3}{297675} + \frac{4198b^2c^2d^3x^5}{165375} + \frac{374b^2c^4d^3x^7}{27783} + \frac{2}{729}b^2c^6d^3x^9$$

Mathematica [A] time = 0.484018, size = 275, normalized size = 0.72

$$\frac{d^3\left(99225a^2c^3x^3\left(35c^6x^6+135c^4x^4+189c^2x^2+105\right)-630ab\sqrt{c^2x^2+1}\left(1225c^8x^8+4675c^6x^6+6297c^4x^4+2629c^2x^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(99225*a^2*c^3*x^3*(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6) - 63
0*a*b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 6297*c^4*x^4 + 4675*c^6*x^6
 + 1225*c^8*x^8) + b^2*(-3312540*c*x + 552090*c^3*x^3 + 793422*c^5*x^5 + 42
0750*c^7*x^7 + 85750*c^9*x^9) - 630*b*(-315*a*c^3*x^3*(105 + 189*c^2*x^2 +
135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(-5258 + 2629*c^2*x^2 + 629
7*c^4*x^4 + 4675*c^6*x^6 + 1225*c^8*x^8))*ArcSinh[c*x] + 99225*b^2*c^3*x^3*
(105 + 189*c^2*x^2 + 135*c^4*x^4 + 35*c^6*x^6)*ArcSinh[c*x]^2))/(31255875*c
^3)
```

Maple [A] time = 0.05, size = 462, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^3*(d^3*a^2*(1/9*c^9*x^9+3/7*c^7*x^7+3/5*c^5*x^5+1/3*c^3*x^3)+d^3*b^2*(1
/9*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^4-16/315*arcsinh(c*x)^2*c*x-1/63*arcsinh(
c*x)^2*c*x*(c^2*x^2+1)^3-2/105*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^2-8/315*arcsi
nh(c*x)^2*c*x*(c^2*x^2+1)-2/81*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(7/2)-80/39
69*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(5/2)-1244/99225*arcsinh(c*x)*c^2*x^2*(
c^2*x^2+1)^(3/2)+436/99225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+10516/992
25*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/729*c*x*(c^2*x^2+1)^4-3406208/31255875*
c*x+622/250047*c*x*(c^2*x^2+1)^3+15224/10418625*c*x*(c^2*x^2+1)^2-115504/31
255875*c*x*(c^2*x^2+1))+2*d^3*a*b*(1/9*arcsinh(c*x)*c^9*x^9+3/7*arcsinh(c*x
)*c^7*x^7+3/5*arcsinh(c*x)*c^5*x^5+1/3*arcsinh(c*x)*c^3*x^3-1/81*c^8*x^8*(c
^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(c^2*x^2+1)^(1/2)-2099/33075*c^4*x^4*(c^2*
x^2+1)^(1/2)-2629/99225*c^2*x^2*(c^2*x^2+1)^(1/2)+5258/99225*(c^2*x^2+1)^(1
/2)))
```

Maxima [B] time = 1.33379, size = 1245, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*c^6*d^3*x^9*arcsinh(c*x)^2 + 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3*
x^7*arcsinh(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 + 3/5*b^2*c^2*d^3*x^5*arcsinh(c*x)
^2 + 2/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt
(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 - 2/893025*(315*(35*s
qrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 +
 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c*
arcsinh(c*x) - (1225*c^8*x^9 - 1800*c^6*x^7 + 3024*c^4*x^5 - 6720*c^2*x^3 +
 40320*x)/c^8)*b^2*c^6*d^3 + 3/5*a^2*c^2*d^3*x^5 + 6/245*(35*x^7*arcsinh(c*
x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^
2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^3 - 2/8575*(105
*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^
2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 1
26*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^4*d^3 + 1/3*b^2*d^3*x^3*arcsi
nh(c*x)^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sq
rt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^3 - 2/375*(
```

```
15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*
x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^
2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^3 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^
2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d^
3
```

Fricas [A] time = 2.75059, size = 963, normalized size = 2.52

$$\frac{42875\left(81\,a^2+2\,b^2\right)c^9d^3x^9+1125\left(11907\,a^2+374\,b^2\right)c^7d^3x^7+189\left(99225\,a^2+4198\,b^2\right)c^5d^3x^5+105\left(99225\,a^2+5258\,b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/31255875*(42875*(81*a^2 + 2*b^2)*c^9*d^3*x^9 + 1125*(11907*a^2 + 374*b^2)
*c^7*d^3*x^7 + 189*(99225*a^2 + 4198*b^2)*c^5*d^3*x^5 + 105*(99225*a^2 + 52
58*b^2)*c^3*d^3*x^3 - 3312540*b^2*c*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 + 135
*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 + 105*b^2*c^3*d^3*x^3)*log(c*x + sqr
t(c^2*x^2 + 1))^2 + 630*(11025*a*b*c^9*d^3*x^9 + 42525*a*b*c^7*d^3*x^7 + 59
535*a*b*c^5*d^3*x^5 + 33075*a*b*c^3*d^3*x^3 - (1225*b^2*c^8*d^3*x^8 + 4675*
b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4 + 2629*b^2*c^2*d^3*x^2 - 5258*b^2*d^
3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 630*(1225*a*b*c^8*d^3*
x^8 + 4675*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 + 2629*a*b*c^2*d^3*x^2 -
5258*a*b*d^3)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 49.5627, size = 626, normalized size = 1.64

$$\begin{cases} \frac{a^2c^6d^3x^9}{9}+\frac{3a^2c^4d^3x^7}{7}+\frac{3a^2c^2d^3x^5}{5}+\frac{a^2d^3x^3}{3}+\frac{2abc^6d^3x^9\operatorname{asinh}(cx)}{9}-\frac{2abc^5d^3x^8\sqrt{c^2x^2+1}}{81}+\frac{6abc^4d^3x^7\operatorname{asinh}(cx)}{7}-\frac{374abc^3d^3x^6\sqrt{c^2x^2+1}}{3969}+\frac{6abc^2\cdots}{\cdots} \\ \frac{a^2d^3x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 + 3*a**2*c**2*d*
*3*x**5/5 + a**2*d**3*x**3/3 + 2*a*b*c**6*d**3*x**9*asinh(c*x)/9 - 2*a*b*c*
```

```
*5*d**3*x**8*sqrt(c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*asinh(c*x)/7 - 3
74*a*b*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)/3969 + 6*a*b*c**2*d**3*x**5*asinh
(c*x)/5 - 4198*a*b*c*d**3*x**4*sqrt(c**2*x**2 + 1)/33075 + 2*a*b*d**3*x**3*
asinh(c*x)/3 - 5258*a*b*d**3*x**2*sqrt(c**2*x**2 + 1)/(99225*c) + 10516*a*b
*d**3*sqrt(c**2*x**2 + 1)/(99225*c**3) + b**2*c**6*d**3*x**9*asinh(c*x)**2/
9 + 2*b**2*c**6*d**3*x**9/729 - 2*b**2*c**5*d**3*x**8*sqrt(c**2*x**2 + 1)*a
sinh(c*x)/81 + 3*b**2*c**4*d**3*x**7*asinh(c*x)**2/7 + 374*b**2*c**4*d**3*x
**7/27783 - 374*b**2*c**3*d**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/3969 + 3
*b**2*c**2*d**3*x**5*asinh(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/165375 - 41
98*b**2*c*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/33075 + b**2*d**3*x**3*a
sinh(c*x)**2/3 + 5258*b**2*d**3*x**3/297675 - 5258*b**2*d**3*x**2*sqrt(c**2
*x**2 + 1)*asinh(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c**2) + 10516*b*
*2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(99225*c**3), Ne(c, 0)), (a**2*d**3*
x**3/3, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.219 $\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=261

$$-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{768c}$$

```
[Out] (175*b^2*d^3*x^2)/3072 + (35*b^2*c^2*d^3*x^4)/3072 + (7*b^2*d^3*(1 + c^2*x^
2)^3)/(1152*c^2) + (b^2*d^3*(1 + c^2*x^2)^4)/(256*c^2) - (35*b*d^3*x*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(512*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2)
*(a + b*ArcSinh[c*x]))/(768*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x]))/(192*c) - (b*d^3*x*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(32
*c) - (35*d^3*(a + b*ArcSinh[c*x])^2)/(1024*c^2) + (d^3*(1 + c^2*x^2)^4*(a
+ b*ArcSinh[c*x])^2)/(8*c^2)
```

Rubi [A] time = 0.256826, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5717, 5684, 5682, 5675, 30, 14, 261}

$$-\frac{bd^3x\left(c^2x^2+1\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}-\frac{7bd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{35bd^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{768c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```



```
[Out] (175*b^2*d^3*x^2)/3072 + (35*b^2*c^2*d^3*x^4)/3072 + (7*b^2*d^3*(1 + c^2*x^
2)^3)/(1152*c^2) + (b^2*d^3*(1 + c^2*x^2)^4)/(256*c^2) - (35*b*d^3*x*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(512*c) - (35*b*d^3*x*(1 + c^2*x^2)^(3/2)
*(a + b*ArcSinh[c*x]))/(768*c) - (7*b*d^3*x*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x]))/(192*c) - (b*d^3*x*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(32
*c) - (35*d^3*(a + b*ArcSinh[c*x])^2)/(1024*c^2) + (d^3*(1 + c^2*x^2)^4*(a
+ b*ArcSinh[c*x])^2)/(8*c^2)
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}-\frac{\left(bd^3\right)\int\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{4c}$$

$$=-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{32c}+\frac{d^3\left(1+c^2x^2\right)^4\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2}+$$

$$=\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{7bd^3x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{192c}-\frac{bd^3x\left(1+c^2x^2\right)^{7/2}}{3}$$

$$=\frac{7b^2d^3\left(1+c^2x^2\right)^3}{1152c^2}+\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{35bd^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right.}{768c}$$

$$=\frac{7b^2d^3\left(1+c^2x^2\right)^3}{1152c^2}+\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{35bd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{512c}-$$

$$=\frac{175b^2d^3x^2}{3072}+\frac{35b^2c^2d^3x^4}{3072}+\frac{7b^2d^3\left(1+c^2x^2\right)^3}{1152c^2}+\frac{b^2d^3\left(1+c^2x^2\right)^4}{256c^2}-\frac{35bd^3x\sqrt{1}}{}$$

Mathematica [A] time = 0.659267, size = 256, normalized size = 0.98

$$\frac{d^3\left(cx\left(1152a^2cx\left(c^6x^6+4c^4x^4+6c^2x^2+4\right)-6ab\sqrt{c^2x^2+1}\left(48c^6x^6+200c^4x^4+326c^2x^2+279\right)+b^2cx\left(36c^6x^6+200c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(c*x*(1152*a^2*c*x*(4 + 6*c^2*x^2 + 4*c^4*x^4 + c^6*x^6) + b^2*c*x*(83
7 + 489*c^2*x^2 + 200*c^4*x^4 + 36*c^6*x^6) - 6*a*b*Sqrt[1 + c^2*x^2]*(279
+ 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*(-(b*c*x*Sqrt[1 + c^2*x^2]
*(279 + 326*c^2*x^2 + 200*c^4*x^4 + 48*c^6*x^6)) + 3*a*(93 + 512*c^2*x^2 +
768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8))*ArcSinh[c*x] + 9*b^2*(93 + 512*c^
2*x^2 + 768*c^4*x^4 + 512*c^6*x^6 + 128*c^8*x^8)*ArcSinh[c*x]^2))/(9216*c^2
)
```

Maple [A] time = 0.043, size = 426, normalized size = 1.6

$$\frac{1}{c^2}\left(d^3a^2\left(\frac{c^8x^8}{8}+\frac{c^6x^6}{2}+\frac{3c^4x^4}{4}+\frac{c^2x^2}{2}\right)+d^3b^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2c^2x^2\left(c^2x^2+1\right)^3}{8}+\frac{(\mathrm{Arcsinh}\,(cx))^2c^2x^2\left(c^2x^2+1\right)^2}{8}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c^2*(d^3*a^2*(1/8*c^8*x^8+1/2*c^6*x^6+3/4*c^4*x^4+1/2*c^2*x^2)+d^3*b^2*(1
/8*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)^3+1/8*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2
+1)^2+1/8*arcsinh(c*x)^2*c^2*x^2*(c^2*x^2+1)+1/8*arcsinh(c*x)^2*(c^2*x^2+1)
-1/32*arcsinh(c*x)*c*x*(c^2*x^2+1)^(7/2)-7/192*arcsinh(c*x)*c*x*(c^2*x^2+1)
^(5/2)-35/768*arcsinh(c*x)*c*x*(c^2*x^2+1)^(3/2)-35/512*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*c*x-35/1024*arcsinh(c*x)^2+1/256*c^2*x^2*(c^2*x^2+1)^3+23/2304*
c^2*x^2*(c^2*x^2+1)^2+197/9216*c^2*x^2*(c^2*x^2+1)+1/18*c^2*x^2+1/18)+2*d^3
*a*b*(1/8*arcsinh(c*x)*c^8*x^8+1/2*arcsinh(c*x)*c^6*x^6+3/4*arcsinh(c*x)*c^
4*x^4+1/2*arcsinh(c*x)*c^2*x^2-1/64*c^7*x^7*(c^2*x^2+1)^(1/2)-25/384*c^5*x^
5*(c^2*x^2+1)^(1/2)-163/1536*c^3*x^3*(c^2*x^2+1)^(1/2)-93/1024*c*x*(c^2*x^2
+1)^(1/2)+93/1024*arcsinh(c*x)))
```

Maxima [B] time = 1.47431, size = 1416, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*c^6*d^3*x^8*arcsinh(c*x)^2 + 1/8*a^2*c^6*d^3*x^8 + 1/2*b^2*c^4*d^3*
x^6*arcsinh(c*x)^2 + 1/2*a^2*c^4*d^3*x^6 + 3/4*b^2*c^2*d^3*x^4*arcsinh(c*x)
^2 + 1/1536*(384*x^8*arcsinh(c*x) - (48*sqrt(c^2*x^2 + 1)*x^7/c^2 - 56*sqrt
(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(c^2*x^2 + 1
)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^8))*c)*a*b*c^6*d^3 + 1/
9216*((36*x^8/c^2 - 56*x^6/c^4 + 105*x^4/c^6 - 315*x^2/c^8 + 315*log(c^2*x/
sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^10)*c^2 - 6*(48*sqrt(c^2*x^2 + 1)*x^7/c^
2 - 56*sqrt(c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(c^2*x^2 + 1)*x^3/c^6 - 105*sqrt(
c^2*x^2 + 1)*x/c^8 + 105*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^8))*c*arcsin
h(c*x))*b^2*c^6*d^3 + 3/4*a^2*c^2*d^3*x^4 + 1/48*(48*x^6*arcsinh(c*x) - (8*
sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2
+ 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^6))*c)*a*b*c^4*d^3 +
1/288*((8*x^6/c^2 - 15*x^4/c^4 + 45*x^2/c^6 - 45*log(c^2*x/sqrt(c^2) + sqrt
(c^2*x^2 + 1))^2/c^8)*c^2 - 6*(8*sqrt(c^2*x^2 + 1)*x^5/c^2 - 10*sqrt(c^2*x^
2 + 1)*x^3/c^4 + 15*sqrt(c^2*x^2 + 1)*x/c^6 - 15*arcsinh(c^2*x/sqrt(c^2))/(
sqrt(c^2)*c^6))*c*arcsinh(c*x))*b^2*c^4*d^3 + 1/2*b^2*d^3*x^2*arcsinh(c*x)^
2 + 3/16*(8*x^4*arcsinh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^
2 + 1)*x/c^4 + 3*arcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c)*a*b*c^2*d^3 +
```

```
3/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c^2*x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/
c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*a
rcsinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^4))*c*arcsinh(c*x))*b^2*c^2*d^3 + 1/2*
a^2*d^3*x^2 + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsin
h(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2)))*a*b*d^3 + 1/4*(c^2*(x^2/c^2 - log(c^2*
x/sqrt(c^2) + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - ar
csinh(c^2*x/sqrt(c^2))/(sqrt(c^2)*c^2))*arcsinh(c*x))*b^2*d^3
```

Fricas [A] time = 2.82054, size = 855, normalized size = 3.28

$$\frac{36\left(32\,a^2+b^2\right)c^8d^3x^8+8\left(576\,a^2+25\,b^2\right)c^6d^3x^6+3\left(2304\,a^2+163\,b^2\right)c^4d^3x^4+9\left(512\,a^2+93\,b^2\right)c^2d^3x^2+9\left(128\,b^2c^8d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(36*(32*a^2 + b^2)*c^8*d^3*x^8 + 8*(576*a^2 + 25*b^2)*c^6*d^3*x^6 +
3*(2304*a^2 + 163*b^2)*c^4*d^3*x^4 + 9*(512*a^2 + 93*b^2)*c^2*d^3*x^2 + 9*(
128*b^2*c^8*d^3*x^8 + 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 + 512*b^2*c
^2*d^3*x^2 + 93*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*(384*a*b*c^8*d^
3*x^8 + 1536*a*b*c^6*d^3*x^6 + 2304*a*b*c^4*d^3*x^4 + 1536*a*b*c^2*d^3*x^2
+ 279*a*b*d^3 - (48*b^2*c^7*d^3*x^7 + 200*b^2*c^5*d^3*x^5 + 326*b^2*c^3*d^3
*x^3 + 279*b^2*c*d^3*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6
*(48*a*b*c^7*d^3*x^7 + 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d^3*x^3 + 279*a*b*
c*d^3*x)*sqrt(c^2*x^2 + 1))/c^2
```

Sympy [A] time = 35.2351, size = 573, normalized size = 2.2

$$\begin{cases} \frac{a^2c^6d^3x^8}{8} + \frac{a^2c^4d^3x^6}{2} + \frac{3a^2c^2d^3x^4}{4} + \frac{a^2d^3x^2}{2} + \frac{abc^6d^3x^8\operatorname{asinh}(cx)}{4} - \frac{abc^5d^3x^7\sqrt{c^2x^2+1}}{32} + abc^4d^3x^6\operatorname{asinh}(cx) - \frac{25abc^3d^3x^5\sqrt{c^2x^2+1}}{192} + \frac{3abc^2 \cdots}{} \\ \frac{a^2d^3x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 + 3*a**2*c**2*d**3
*x**4/4 + a**2*d**3*x**2/2 + a*b*c**6*d**3*x**8*asinh(c*x)/4 - a*b*c**5*d**
```

```
3*x**7*sqrt(c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*asinh(c*x) - 25*a*b*c**3
*d**3*x**5*sqrt(c**2*x**2 + 1)/192 + 3*a*b*c**2*d**3*x**4*asinh(c*x)/2 - 16
3*a*b*c*d**3*x**3*sqrt(c**2*x**2 + 1)/768 + a*b*d**3*x**2*asinh(c*x) - 93*a
*b*d**3*x*sqrt(c**2*x**2 + 1)/(512*c) + 93*a*b*d**3*asinh(c*x)/(512*c**2) +
 b**2*c**6*d**3*x**8*asinh(c*x)**2/8 + b**2*c**6*d**3*x**8/256 - b**2*c**5*
d**3*x**7*sqrt(c**2*x**2 + 1)*asinh(c*x)/32 + b**2*c**4*d**3*x**6*asinh(c*x
)**2/2 + 25*b**2*c**4*d**3*x**6/1152 - 25*b**2*c**3*d**3*x**5*sqrt(c**2*x**
2 + 1)*asinh(c*x)/192 + 3*b**2*c**2*d**3*x**4*asinh(c*x)**2/4 + 163*b**2*c*
*2*d**3*x**4/3072 - 163*b**2*c*d**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/768
 + b**2*d**3*x**2*asinh(c*x)**2/2 + 93*b**2*d**3*x**2/1024 - 93*b**2*d**3*x
*sqrt(c**2*x**2 + 1)*asinh(c*x)/(512*c) + 93*b**2*d**3*asinh(c*x)**2/(1024*
c**2), Ne(c, 0)), (a**2*d**3*x**2/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^3 (b\,\mathrm{arsinh}\,(cx)+a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)^2*x, x)
```

3.220 $\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=291

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{6}{35}d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{35}d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{}$$

```
[Out] (4322*b^2*d^3*x)/3675 + (1514*b^2*c^2*d^3*x^3)/11025 + (234*b^2*c^4*d^3*x^5
)/6125 + (2*b^2*c^6*d^3*x^7)/343 - (32*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]))/(35*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105
*c) - (12*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d^
3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c) + (16*d^3*x*(a + b*ArcSi
nh[c*x])^2)/35 + (8*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (6*d^3
*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/35 + (d^3*x*(1 + c^2*x^2)^3*(a +
 b*ArcSinh[c*x])^2)/7
```

Rubi [A] time = 0.403062, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5684, 5653, 5717, 8, 194}

$$\frac{1}{7}d^3x\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+\frac{6}{35}d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{35}d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (4322*b^2*d^3*x)/3675 + (1514*b^2*c^2*d^3*x^3)/11025 + (234*b^2*c^4*d^3*x^5
)/6125 + (2*b^2*c^6*d^3*x^7)/343 - (32*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x]))/(35*c) - (16*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(105
*c) - (12*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(175*c) - (2*b*d^
3*(1 + c^2*x^2)^(7/2)*(a + b*ArcSinh[c*x]))/(49*c) + (16*d^3*x*(a + b*ArcSi
nh[c*x])^2)/35 + (8*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/35 + (6*d^3
*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/35 + (d^3*x*(1 + c^2*x^2)^3*(a +
 b*ArcSinh[c*x])^2)/7
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{7}d^3x\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{7}(6d)\int \left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c} + \frac{6}{35}d^3x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= -\frac{12bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{175c} - \frac{2bd^3\left(1+c^2x^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)}{49c}$$

$$= \frac{2}{49}b^2d^3x + \frac{2}{49}b^2c^2d^3x^3 + \frac{6}{245}b^2c^4d^3x^5 + \frac{2}{343}b^2c^6d^3x^7 - \frac{16bd^3\left(1+c^2x^2\right)^{3/2}(a}{105c}$$

$$= \frac{962b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 - \frac{32bd^3\sqrt{1+c^2x^2}\,(a}{35}$$

$$= \frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} + \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 - \frac{32bd^3\sqrt{1+c^2x^2}\,(}{35}$$

Mathematica [A] time = 0.535683, size = 239, normalized size = 0.82

$$\frac{d^3\left(11025a^2cx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right) - 210ab\sqrt{c^2x^2+1}\left(75c^6x^6+351c^4x^4+757c^2x^2+2161\right) - 210b\sinh^{-1}(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^3*(11025*a^2*c*x*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) - 210*a*b*Sq
rt[1 + c^2*x^2]*(2161 + 757*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6) + 2*b^2*c*x
*(226905 + 26495*c^2*x^2 + 7371*c^4*x^4 + 1125*c^6*x^6) - 210*b*(-105*a*c*x
*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 + c^2*x^2]*(2161 + 7
57*c^2*x^2 + 351*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*c*x*(35 +
35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6)*ArcSinh[c*x]^2))/(385875*c)
```

Maple [A] time = 0.041, size = 372, normalized size = 1.3

$$\frac{1}{c}\left(d^3a^2\left(\frac{c^7x^7}{7}+\frac{3\,c^5x^5}{5}+c^3x^3+cx\right)+d^3b^2\left(\frac{16\,(\mathrm{Arcsinh}\,(cx))^2\,cx}{35}+\frac{(\mathrm{Arcsinh}\,(cx))^2\,cx\left(c^2x^2+1\right)^3}{7}+\frac{6\,(\mathrm{Arcsinh}\,(cx))^2}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(d^3*a^2*(1/7*c^7*x^7+3/5*c^5*x^5+c^3*x^3+c*x)+d^3*b^2*(16/35*arcsinh(c
*x)^2*c*x+1/7*arcsinh(c*x)^2*c*x*(c^2*x^2+1)^3+6/35*arcsinh(c*x)^2*c*x*(c^2
*x^2+1)^2+8/35*arcsinh(c*x)^2*c*x*(c^2*x^2+1)-4322/3675*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)+413312/385875*c*x-2/49*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(5/2)-1
34/1225*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(3/2)-962/3675*arcsinh(c*x)*c^2*x^
2*(c^2*x^2+1)^(1/2)+2/343*c*x*(c^2*x^2+1)^3+888/42875*c*x*(c^2*x^2+1)^2+302
56/385875*c*x*(c^2*x^2+1))+2*d^3*a*b*(1/7*arcsinh(c*x)*c^7*x^7+3/5*arcsinh(
c*x)*c^5*x^5+arcsinh(c*x)*c^3*x^3+arcsinh(c*x)*c*x-1/49*c^6*x^6*(c^2*x^2+1)
^(1/2)-117/1225*c^4*x^4*(c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(c^2*x^2+1)^(1/2
)-2161/3675*(c^2*x^2+1)^(1/2)))
```

Maxima [B] time = 1.29658, size = 961, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*c^6*d^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3*
x^5*arcsinh(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*arcsinh(c*x) - (5*
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*(5*sqr
t(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*
x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsinh(c*x) - (75*c^6*x^7 - 126*c^4*
x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*c^6*d^3 + b^2*c^2*d^3*x^3*arcsinh(c*x)
^2 + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*
x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 - 2/375*(15*(3*s
qrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1
)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*c^4*d^3 +
 a^2*c^2*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*c^2*d^
3 + b^2*d^3*x*arcsinh(c*x)^2 + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x
)/c) + a^2*d^3*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Fricas [A] time = 2.78674, size = 818, normalized size = 2.81

$$\frac{1125\left(49\,a^2+2\,b^2\right)c^7d^3x^7+189\left(1225\,a^2+78\,b^2\right)c^5d^3x^5+35\left(11025\,a^2+1514\,b^2\right)c^3d^3x^3+105\left(3675\,a^2+4322\,b^2\right)cd^3x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*d^3*x^7 + 189*(1225*a^2 + 78*b^2)*c^5*d
^3*x^5 + 35*(11025*a^2 + 1514*b^2)*c^3*d^3*x^3 + 105*(3675*a^2 + 4322*b^2)*
c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 + 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^3*x^
3 + 35*b^2*c*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 210*(525*a*b*c^7*d^3*x
^7 + 2205*a*b*c^5*d^3*x^5 + 3675*a*b*c^3*d^3*x^3 + 3675*a*b*c*d^3*x - (75*b
^2*c^6*d^3*x^6 + 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 + 2161*b^2*d^3)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 210*(75*a*b*c^6*d^3*x^6 +
 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 + 2161*a*b*d^3)*sqrt(c^2*x^2 + 1
))/c
```

Sympy [A] time = 18.6535, size = 524, normalized size = 1.8

$$\begin{cases} \frac{a^2c^6d^3x^7}{7}+\frac{3a^2c^4d^3x^5}{5}+a^2c^2d^3x^3+a^2d^3x+\frac{2abc^6d^3x^7\operatorname{asinh}(cx)}{7}-\frac{2abc^5d^3x^6\sqrt{c^2x^2+1}}{49}+\frac{6abc^4d^3x^5\operatorname{asinh}(cx)}{5}-\frac{234abc^3d^3x^4\sqrt{c^2x^2+1}}{1225}+2ab \\ a^2d^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 + a**2*c**2*d**3
*x**3 + a**2*d**3*x + 2*a*b*c**6*d**3*x**7*asinh(c*x)/7 - 2*a*b*c**5*d**3*x
**6*sqrt(c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asinh(c*x)/5 - 234*a*b*c*
*3*d**3*x**4*sqrt(c**2*x**2 + 1)/1225 + 2*a*b*c**2*d**3*x**3*asinh(c*x) - 1
514*a*b*c*d**3*x**2*sqrt(c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asinh(c*x) - 43
22*a*b*d**3*sqrt(c**2*x**2 + 1)/(3675*c) + b**2*c**6*d**3*x**7*asinh(c*x)**
2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(c**2*x**2 + 1)
*asinh(c*x)/49 + 3*b**2*c**4*d**3*x**5*asinh(c*x)**2/5 + 234*b**2*c**4*d**3
*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/1225 +
b**2*c**2*d**3*x**3*asinh(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025 - 1514*b
**2*c*d**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/3675 + b**2*d**3*x*asinh(c*x
)**2 + 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(c**2*x**2 + 1)*asinh(c*x
```

```
)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

Giac [B] time = 2.98611, size = 1025, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/7*a^2*c^6*d^3*x^7 + 3/5*a^2*c^4*d^3*x^5 + 2/245*(35*x^7*log(c*x + sqrt(c^
2*x^2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2
 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*a*b*c^6*d^3 + 1/25725*(3675*x^7*lo
g(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3
 - 1680*x)/c^7 - 105*(5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(
c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^8
))*b^2*c^6*d^3 + 2/25*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 +
1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b*c^4*d^3
+ 1/375*(225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 - 20*c^2*
x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*c^4*d^3 + a^2*c^2
*d^3*x^3 + 2/3*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) -
 3*sqrt(c^2*x^2 + 1))/c^3)*a*b*c^2*d^3 + 1/9*(9*x^3*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^
2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*c^2*d^3 + 2*(x*log(c*x + sqr
t(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b*d^3 + (x*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))/c^2))*b
^2*d^3 + a^2*d^3*x
```

3.221 $\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=337

$$-bd^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d^3\text{PolyLog}\left(3,e^{-2\sinh^{-1}(cx)}\right)-\frac{1}{18}bcd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}\right.$$

```
[Out] (71*b^2*c^2*d^3*x^2)/144 + (7*b^2*c^4*d^3*x^4)/144 + (b^2*d^3*(1 + c^2*x^2)
^3)/108 - (19*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/24 - (7*b*c
*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/36 - (b*c*d^3*x*(1 + c^2*x
^2)^(5/2)*(a + b*ArcSinh[c*x]))/18 - (19*d^3*(a + b*ArcSinh[c*x])^2)/48 + (
d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (d^3*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/6 + (d^3*(
a + b*ArcSinh[c*x])^3)/(3*b) + d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*Arc
Sinh[c*x])] - b*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*ArcSinh[c*x])] -
(b^2*d^3*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.68407, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 5684, 14, 261}

$$bd^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}b^2d^3\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)-\frac{1}{18}bcd^3x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (71*b^2*c^2*d^3*x^2)/144 + (7*b^2*c^4*d^3*x^4)/144 + (b^2*d^3*(1 + c^2*x^2)
^3)/108 - (19*b*c*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/24 - (7*b*c
*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/36 - (b*c*d^3*x*(1 + c^2*x
^2)^(5/2)*(a + b*ArcSinh[c*x]))/18 - (19*d^3*(a + b*ArcSinh[c*x])^2)/48 + (
d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/2 + (d^3*(1 + c^2*x^2)^2*(a + b*A
rcSinh[c*x])^2)/4 + (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/6 - (d^3*(
a + b*ArcSinh[c*x])^3)/(3*b) + d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcS
inh[c*x])] + b*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])] - (b
^2*d^3*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{6}d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2+d\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{1}{18}bcd^3x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right.$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{7}{36}bcd^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{18}bcd^3x\left(1\right.$$

$$= \frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-\frac{7}{36}bcd^3x\left(1+c\right.$$

$$= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+\right.$$

$$= \frac{71}{144}b^2c^2d^3x^2+\frac{7}{144}b^2c^4d^3x^4+\frac{1}{108}b^2d^3\left(1+c^2x^2\right)^3-\frac{19}{24}bcd^3x\sqrt{1+c^2x^2}\left(a+\right.$$

Mathematica [A] time = 0.658456, size = 416, normalized size = 1.23

$$\frac{d^3\left(-3456ab\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+3456b^2\sinh^{-1}(cx)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-1728b^2\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^3*(5184*a^2*c^2*x^2 + 2592*a^2*c^4*x^4 + 576*a^2*c^6*x^6 - 3600*a*b*c*x*
Sqrt[1 + c^2*x^2] - 1056*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 192*a*b*c^5*x^5*Sq
rt[1 + c^2*x^2] + 3600*a*b*ArcSinh[c*x] + 10368*a*b*c^2*x^2*ArcSinh[c*x] +
5184*a*b*c^4*x^4*ArcSinh[c*x] + 1152*a*b*c^6*x^6*ArcSinh[c*x] + 3456*a*b*Ar
cSinh[c*x]^2 - 1152*b^2*ArcSinh[c*x]^3 + 783*b^2*Cosh[2*ArcSinh[c*x]] + 156
6*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + 27*b^2*Cosh[4*ArcSinh[c*x]] + 2
16*b^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + b^2*Cosh[6*ArcSinh[c*x]] + 18*
b^2*ArcSinh[c*x]^2*Cosh[6*ArcSinh[c*x]] + 6912*a*b*ArcSinh[c*x]*Log[1 - E^(
-2*ArcSinh[c*x])] + 3456*b^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3
456*a^2*Log[c*x] - 3456*a*b*PolyLog[2, E^(-2*ArcSinh[c*x])] + 3456*b^2*ArcS
inh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] - 1728*b^2*PolyLog[3, E^(2*ArcSinh[
c*x])] - 1566*b^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 108*b^2*ArcSinh[c*x]*
Sinh[4*ArcSinh[c*x]] - 6*b^2*ArcSinh[c*x]*Sinh[6*ArcSinh[c*x]]))/3456
```

Maple [B] time = 0.199, size = 706, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 811/3456*d^3*b^2+1/3*d^3*a*b*arcsinh(c*x)*c^6*x^6-11/36*d^3*b^2*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)*c^3*x^3-25/24*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*
x+3/2*d^3*a*b*arcsinh(c*x)*c^4*x^4+3*d^3*a*b*arcsinh(c*x)*c^2*x^2-1/18*d^3*
a*b*c^5*x^5*(c^2*x^2+1)^(1/2)-11/36*d^3*a*b*c^3*x^3*(c^2*x^2+1)^(1/2)-25/24
*d^3*a*b*c*x*(c^2*x^2+1)^(1/2)-1/18*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*
c^5*x^5+d^3*a^2*ln(c*x)+25/48*d^3*b^2*arcsinh(c*x)^2-2*d^3*b^2*polylog(3,c*
x+(c^2*x^2+1)^(1/2))-2*d^3*b^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-1/3*d^3*b^
2*arcsinh(c*x)^3+25/48*b^2*c^2*d^3*x^2+11/144*b^2*c^4*d^3*x^4+1/6*d^3*b^2*a
rcsinh(c*x)^2*c^6*x^6+3/4*d^3*b^2*arcsinh(c*x)^2*c^4*x^4+1/6*d^3*a^2*c^6*x^
6+3/4*d^3*a^2*c^4*x^4+3/2*d^3*a^2*c^2*x^2+2*d^3*b^2*arcsinh(c*x)*polylog(2,
c*x+(c^2*x^2+1)^(1/2))+d^3*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+d
^3*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*d^3*b^2*arcsinh(c*x)*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))+1/108*d^3*b^2*c^6*x^6+2*d^3*a*b*polylog(2,c
*x+(c^2*x^2+1)^(1/2))+2*d^3*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-d^3*a*b*a
rcsinh(c*x)^2+25/24*d^3*a*b*arcsinh(c*x)+3/2*d^3*b^2*arcsinh(c*x)^2*c^2*x^2
+2*d^3*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*d^3*a*b*arcsinh(c*x)*
ln(1+c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}a^2c^6d^3x^6+\frac{3}{4}a^2c^4d^3x^4+\frac{3}{2}a^2c^2d^3x^2+a^2d^3\log(x)+\int b^2c^6d^3x^5\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,abc^6d^3x^5\log\left(cx+\sqrt{c^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 + 3/2*a^2*c^2*d^3*x^2 + a^2*d^3*l
og(x) + integrate(b^2*c^6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^
6*d^3*x^5*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x^3*log(c*x + sqrt(c
^2*x^2 + 1))^2 + 6*a*b*c^4*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2
*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b*c^2*d^3*x*log(c*x + sqrt(c^2*
x^2 + 1)) + b^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))^2/x + 2*a*b*d^3*log(c*x +
sqrt(c^2*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^6d^3x^6+3\,a^2c^4d^3x^4+3\,a^2c^2d^3x^2+a^2d^3+\left(b^2c^6d^3x^6+3\,b^2c^4d^3x^4+3\,b^2c^2d^3x^2+b^2d^3\right)\text{arsinh}\,(cx)^2+2\,(a}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int\frac{a^2}{x}\,dx+\int 3a^2c^2x\,dx+\int 3a^2c^4x^3\,dx+\int a^2c^6x^5\,dx+\int\frac{b^2\,\text{asinh}^2\,(cx)}{x}\,dx+\int\frac{2ab\,\text{asinh}\,(cx)}{x}\,dx+\int 3b^2c^2x\,\text{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(3*a**2*c*
*4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(b**2*asinh(c*x)**2/x,
x) + Integral(2*a*b*asinh(c*x)/x, x) + Integral(3*b**2*c**2*x*asinh(c*x)**2
, x) + Integral(3*b**2*c**4*x**3*asinh(c*x)**2, x) + Integral(b**2*c**6*x**
5*asinh(c*x)**2, x) + Integral(6*a*b*c**2*x*asinh(c*x), x) + Integral(6*a*b
*c**4*x**3*asinh(c*x), x) + Integral(2*a*b*c**6*x**5*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^3 (b\,\text{arsinh}\,(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)^2/x, x)
```

3.222 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=307

$$-2b^2cd^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd^3\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{6}{5}c^2d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{5}c^2d^3x\,($$

```
[Out] (122*b^2*c^2*d^3*x)/25 + (14*b^2*c^4*d^3*x^3)/75 + (2*b^2*c^6*d^3*x^5)/125
- (22*b*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x]))/25 + (16*c^2*d^3*x*(a + b*ArcSinh[c*x])^2)/5 + (8*c^2*d^3
*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 + (6*c^2*d^3*x*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x])^2)/5 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x -
 4*b*c*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^3*PolyL
og[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^3*PolyLog[2, E^ArcSinh[c*x]]
```

Rubi [A] time = 0.739251, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5739, 5684, 5653, 5717, 8, 194, 5744, 5742, 5760, 4182, 2279, 2391}

$$-2b^2cd^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+2b^2cd^3\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{6}{5}c^2d^3x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2+\frac{8}{5}c^2d^3x\,($$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```


```
[Out] (122*b^2*c^2*d^3*x)/25 + (14*b^2*c^4*d^3*x^3)/75 + (2*b^2*c^6*d^3*x^5)/125
- (22*b*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c
^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/5 - (2*b*c*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x]))/25 + (16*c^2*d^3*x*(a + b*ArcSinh[c*x])^2)/5 + (8*c^2*d^3
*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/5 + (6*c^2*d^3*x*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x])^2)/5 - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/x -
 4*b*c*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]] - 2*b^2*c*d^3*PolyL
og[2, -E^ArcSinh[c*x]] + 2*b^2*c*d^3*PolyLog[2, E^ArcSinh[c*x]]
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
```

```
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(6c^2d\right)\int\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)$$

$$= \frac{2}{5}bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{6}{5}c^2d^3x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= \frac{2}{3}bcd^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right) - \frac{2}{25}bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{16}{15}b^2c^2d^3x - \frac{22}{45}b^2c^4d^3x^3 - \frac{2}{25}b^2c^6d^3x^5 + 2bcd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right) -$$

$$= -\frac{38}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.$$

$$= \frac{122}{25}b^2c^2d^3x + \frac{14}{75}b^2c^4d^3x^3 + \frac{2}{125}b^2c^6d^3x^5 - \frac{22}{5}bcd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.$$

Mathematica [A] time = 1.41425, size = 466, normalized size = 1.52

$$\frac{1}{720}d^3\left(1440b^2c\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right) - 1440b^2c\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right) + 144a^2c^6x^5 + 720a^2c^4x^3 + 2160a^2c^2x - \frac{7}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^3*((-720*a^2)/x + 2160*a^2*c^2*x + 3460*b^2*c^2*x + 720*a^2*c^4*x^3 + 14
4*a^2*c^6*x^5 - (17568*a*b*c*Sqrt[1 + c^2*x^2])/5 - (2016*a*b*c^3*x^2*Sqrt[
1 + c^2*x^2])/5 - (288*a*b*c^5*x^4*Sqrt[1 + c^2*x^2])/5 - (1440*a*b*ArcSinh
[c*x])/x + 4320*a*b*c^2*x*ArcSinh[c*x] + 1440*a*b*c^4*x^3*ArcSinh[c*x] + 28
8*a*b*c^6*x^5*ArcSinh[c*x] - 3420*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - (7
20*b^2*ArcSinh[c*x]^2)/x + 1890*b^2*c^2*x*ArcSinh[c*x]^2 - 1440*a*b*c*ArcTa
nh[Sqrt[1 + c^2*x^2]] + 80*b^2*c^2*x*Cosh[2*ArcSinh[c*x]] + 360*b^2*c^2*x*A
rcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 90*b^2*c*ArcSinh[c*x]*Cosh[3*ArcSinh[c
*x]] - (18*b^2*c*ArcSinh[c*x]*Cosh[5*ArcSinh[c*x]])/5 + 1440*b^2*c*ArcSinh[
c*x]*Log[1 - E^(-ArcSinh[c*x])] - 1440*b^2*c*ArcSinh[c*x]*Log[1 + E^(-ArcSi
nh[c*x])] + 1440*b^2*c*PolyLog[2, -E^(-ArcSinh[c*x])] - 1440*b^2*c*PolyLog[
```

```
2, E^(-ArcSinh[c*x])] - 10*b^2*c*Sinh[3*ArcSinh[c*x]] - 45*b^2*c*ArcSinh[c*
x]^2*Sinh[3*ArcSinh[c*x]] + (18*b^2*c*Sinh[5*ArcSinh[c*x]])/25 + 9*b^2*c*Ar
cSinh[c*x]^2*Sinh[5*ArcSinh[c*x]]))/720
```

Maple [A] time = 0.163, size = 516, normalized size = 1.7

$$-2\,c{d}^{3}ab{\rm Artanh} \left({\frac{1}{\sqrt{{c}^{2}{x}^{2}+1}}}\right)-{\frac{2\,{d}^{3}ab{c}^{5}{x}^{4}}{25}\sqrt{{c}^{2}{x}^{2}+1}}-{\frac{14\,{d}^{3}ab{c}^{3}{x}^{2}}{25}\sqrt{{c}^{2}{x}^{2}+1}}-{\frac{14\,{d}^{3}{b}^{2}{\rm Arcsinh} \left(cx\right){c}^{3}{x}^{2}}{25}\sqrt{{c}^{2}{x}^{2}+1}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] -14/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^2+2/5*d^3*a*b*arcsinh(c
*x)*c^6*x^5+2*d^3*a*b*arcsinh(c*x)*c^4*x^3+6*d^3*a*b*arcsinh(c*x)*c^2*x-2/2
5*d^3*a*b*c^5*x^4*(c^2*x^2+1)^(1/2)-14/25*d^3*a*b*c^3*x^2*(c^2*x^2+1)^(1/2)
-2/25*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5*x^4-d^3*a^2/x+122/25*b^2*c
^2*d^3*x+14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-2*c*d^3*b^2*arcsinh(c*
x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*c*d^3*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1
)^(1/2))+3*d^3*b^2*arcsinh(c*x)^2*c^2*x+1/5*d^3*b^2*arcsinh(c*x)^2*c^6*x^5+
d^3*b^2*arcsinh(c*x)^2*c^4*x^3-122/25*c*d^3*a*b*(c^2*x^2+1)^(1/2)-2*c*d^3*a
*b*arctanh(1/(c^2*x^2+1)^(1/2))-122/25*c*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)-2*d^3*a*b*arcsinh(c*x)/x+1/5*d^3*a^2*c^6*x^5+d^3*a^2*c^4*x^3+3*d^3*a^2
*c^2*x-d^3*b^2*arcsinh(c*x)^2/x-2*b^2*c*d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2
))+2*b^2*c*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{5}\,a^{2}c^{6}d^{3}x^{5}+\frac{2}{75}\left(15\,x^{5}\operatorname{arsinh}\left(cx\right)-\left(\frac{3\,\sqrt{c^{2}x^{2}+1}x^{4}}{c^{2}}-\frac{4\,\sqrt{c^{2}x^{2}+1}x^{2}}{c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}+1}}{c^{6}}\right)c\right)abc^{6}d^{3}+a^{2}c^{4}d^{3}x^{3}+\frac{2}{3}\left(3\,x^{3}\operatorname{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^6*d^3*x^5 + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/
c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*c^6*d^3
 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2
```

```
 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 + 3*b^2*c^2*d^3*x*arcsinh(c*x)^2 +
 6*b^2*c^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^2*d^3*x + 6
*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c*d^3 - 2*(c*arcsinh(1/(sqrt(c^
2)*abs(x))) + arcsinh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/5*(b^2*c^6*d^3*x^6 +
5*b^2*c^4*d^3*x^4 - 5*b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2/x - integrate
(2/5*(b^2*c^9*d^3*x^8 + 6*b^2*c^7*d^3*x^6 + 5*b^2*c^5*d^3*x^4 - 5*b^2*c^3*d
^3*x^2 - 5*b^2*c*d^3 + (b^2*c^8*d^3*x^7 + 5*b^2*c^6*d^3*x^5 - 5*b^2*c^2*d^3
*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4 + c*x^2 + (c^2
*x^3 + x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\operatorname{arsinh}(cx)^2 + 2\left(abc\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int 3a^2c^4x^2\,dx + \int a^2c^6x^4\,dx + \int 3b^2c^2\operatorname{asinh}^2(cx)\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^2}\,dx + \int 6abc^2\operatorname{as}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*asinh
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**2, x) + Integral(3*b**2*c**4*x**2*
asinh(c*x)**2, x) + Integral(b**2*c**6*x**4*asinh(c*x)**2, x) + Integral(6*
```

```
a*b*c**4*x**2*asinh(c*x), x) + Integral(2*a*b*c**6*x**4*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^3(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)^2/x^2, x)
```

3.223 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=354

$$-3bc^2d^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{3}{2}b^2c^2d^3\text{PolyLog}\left(3,e^{-2\sinh^{-1}(cx)}\right)+\frac{7}{8}bc^3d^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\,\text{s}\right.$$

```
[Out] (21*b^2*c^4*d^3*x^2)/32 + (b^2*c^6*d^3*x^4)/32 - (3*b*c^3*d^3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/16 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b
*ArcSinh[c*x]))/8 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x -
(3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/32 + (3*c^2*d^3*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/2 + (3*c^2*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(2*x^2) + (c^2*d^3*(a + b*ArcSin
h[c*x])^3)/b + 3*c^2*d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(-2*ArcSinh[c*x])
] + b^2*c^2*d^3*Log[x] - 3*b*c^2*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(-2*
ArcSinh[c*x])] - (3*b^2*c^2*d^3*PolyLog[3, E^(-2*ArcSinh[c*x])])/2
```

Rubi [A] time = 0.749885, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5739, 5744, 5659, 3716, 2190, 2531, 2282, 6589, 5682, 5675, 30, 5684, 14, 266, 43}

$$3bc^2d^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)-\frac{3}{2}b^2c^2d^3\text{PolyLog}\left(3,e^{2\sinh^{-1}(cx)}\right)+\frac{7}{8}bc^3d^3x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (21*b^2*c^4*d^3*x^2)/32 + (b^2*c^6*d^3*x^4)/32 - (3*b*c^3*d^3*x*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/16 + (7*b*c^3*d^3*x*(1 + c^2*x^2)^(3/2)*(a + b
*ArcSinh[c*x]))/8 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/x -
(3*c^2*d^3*(a + b*ArcSinh[c*x])^2)/32 + (3*c^2*d^3*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/2 + (3*c^2*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/4 - (d
^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*d^3*(a + b*ArcSin
h[c*x])^3)/b + 3*c^2*d^3*(a + b*ArcSinh[c*x])^2*Log[1 - E^(2*ArcSinh[c*x])]
 + b^2*c^2*d^3*Log[x] + 3*b*c^2*d^3*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*Ar
cSinh[c*x])] - (3*b^2*c^2*d^3*PolyLog[3, E^(2*ArcSinh[c*x])])/2
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2}+\left(3c^2d\right)\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -\frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}+\frac{3}{4}c^2d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= \frac{7}{8}bc^3d^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{7}{8}bc^3d^3x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(c\right.$$

$$= \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{7}{8}bc^3d^3x$$

$$= \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{7}{8}bc^3d^3x$$

$$= \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{7}{8}bc^3d^3x$$

$$= \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{7}{8}bc^3d^3x$$

$$= \frac{21}{32}b^2c^4d^3x^2+\frac{1}{32}b^2c^6d^3x^4-\frac{3}{16}bc^3d^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{7}{8}bc^3d^3x$$

Mathematica [A] time = 1.17451, size = 459, normalized size = 1.3

$$\frac{1}{256}d^3\left(-768abc^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+768b^2c^2\sinh^{-1}(cx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)-384b^2c^2\text{PolyLog}\left(3,e^{2\sinh^{-1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*((-128*a^2)/x^2 + 384*a^2*c^4*x^2 + 64*a^2*c^6*x^4 - (256*a*b*c*Sqrt[1
 + c^2*x^2])/x - 336*a*b*c^3*x*Sqrt[1 + c^2*x^2] - 32*a*b*c^5*x^3*Sqrt[1 +
c^2*x^2] + 336*a*b*c^2*ArcSinh[c*x] - (256*a*b*ArcSinh[c*x])/x^2 + 768*a*b*
c^4*x^2*ArcSinh[c*x] + 128*a*b*c^6*x^4*ArcSinh[c*x] - (256*b^2*c*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x])/x + 768*a*b*c^2*ArcSinh[c*x]^2 - (128*b^2*ArcSinh[c*x
]^2)/x^2 - 256*b^2*c^2*ArcSinh[c*x]^3 + 80*b^2*c^2*Cosh[2*ArcSinh[c*x]] + 1
60*b^2*c^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] + b^2*c^2*Cosh[4*ArcSinh[c*x
```

```
]] + 8*b^2*c^2*ArcSinh[c*x]^2*Cosh[4*ArcSinh[c*x]] + 1536*a*b*c^2*ArcSinh[c
*x]*Log[1 - E^(-2*ArcSinh[c*x])] + 768*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*
ArcSinh[c*x])] + 768*a^2*c^2*Log[x] + 256*b^2*c^2*Log[c*x] - 768*a*b*c^2*Po
lyLog[2, E^(-2*ArcSinh[c*x])] + 768*b^2*c^2*ArcSinh[c*x]*PolyLog[2, E^(2*Ar
cSinh[c*x])] - 384*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])] - 160*b^2*c^2*Arc
Sinh[c*x]*Sinh[2*ArcSinh[c*x]] - 4*b^2*c^2*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]
]))/256
```

Maple [B] time = 0.411, size = 838, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] d^3*a*b*c^2-1/2*d^3*a^2/x^2+81/256*d^3*b^2*c^2+21/32*b^2*c^4*d^3*x^2+1/32*b
^2*c^6*d^3*x^4+1/4*c^6*d^3*a^2*x^4+3/2*c^4*d^3*a^2*x^2+c^2*d^3*b^2*ln(1+c*x
+(c^2*x^2+1)^(1/2))+c^2*d^3*b^2*ln(c*x+(c^2*x^2+1)^(1/2)-1)-2*c^2*d^3*b^2*l
n(c*x+(c^2*x^2+1)^(1/2))+3*c^2*d^3*a^2*ln(c*x)+21/32*c^2*d^3*b^2*arcsinh(c*
x)^2-6*c^2*d^3*b^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-6*c^2*d^3*b^2*polylog(3
,-c*x-(c^2*x^2+1)^(1/2))-c^2*d^3*b^2*arcsinh(c*x)^3+c^2*d^3*b^2*arcsinh(c*x
)-1/2*d^3*b^2*arcsinh(c*x)^2/x^2+1/2*c^6*d^3*a*b*arcsinh(c*x)*x^4+3*c^4*d^3
*a*b*arcsinh(c*x)*x^2-1/8*c^5*d^3*a*b*x^3*(c^2*x^2+1)^(1/2)-21/16*c^3*d^3*a
*b*x*(c^2*x^2+1)^(1/2)-c*d^3*a*b/x*(c^2*x^2+1)^(1/2)-c*d^3*b^2*arcsinh(c*x)
/x*(c^2*x^2+1)^(1/2)+6*c^2*d^3*a*b*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))
+6*c^2*d^3*a*b*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/8*c^5*d^3*b^2*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-21/16*c^3*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^
(1/2)*x+6*c^2*d^3*a*b*polylog(2,c*x+(c^2*x^2+1)^(1/2))-d^3*a*b*arcsinh(c*x)
/x^2+6*c^2*d^3*a*b*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-3*c^2*d^3*a*b*arcsinh(
c*x)^2+21/16*c^2*d^3*a*b*arcsinh(c*x)+6*c^2*d^3*b^2*arcsinh(c*x)*polylog(2,
c*x+(c^2*x^2+1)^(1/2))+3*c^2*d^3*b^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1
/2))+3*c^2*d^3*b^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))+6*c^2*d^3*b^2
*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+1/4*c^6*d^3*b^2*arcsinh(c*x
)^2*x^4+3/2*c^4*d^3*b^2*arcsinh(c*x)^2*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,a^2c^6d^3x^4+\frac{3}{2}\,a^2c^4d^3x^2+3\,a^2c^2d^3\log\left(x\right)-abd^3\left(\frac{\sqrt{c^2x^2+1}c}{x}+\frac{\operatorname{arsinh}\left(cx\right)}{x^2}\right)-\frac{a^2d^3}{2\,x^2}+\int b^2c^6d^3x^3\log\left(cx+\sqrt{c^2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 + 3*a^2*c^2*d^3*log(x) - a*b*d^3*
(sqrt(c^2*x^2 + 1)*c/x + arcsinh(c*x)/x^2) - 1/2*a^2*d^3/x^2 + integrate(b^
2*c^6*d^3*x^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*a*b*c^6*d^3*x^3*log(c*x +
sqrt(c^2*x^2 + 1)) + 3*b^2*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + 6*a*b
*c^4*d^3*x*log(c*x + sqrt(c^2*x^2 + 1)) + 3*b^2*c^2*d^3*log(c*x + sqrt(c^2*
x^2 + 1))^2/x + 6*a*b*c^2*d^3*log(c*x + sqrt(c^2*x^2 + 1))/x + b^2*d^3*log(
c*x + sqrt(c^2*x^2 + 1))^2/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\operatorname{arsinh}(cx)^2 + 2\left(abc\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x^3}\,dx + \int \frac{3a^2c^2}{x}\,dx + \int 3a^2c^4x\,dx + \int a^2c^6x^3\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{x^3}\,dx + \int \frac{3b^2c^2\operatorname{asinh}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(3*a**2
*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(b**2*asinh(c*x)**2/x**
3, x) + Integral(2*a*b*asinh(c*x)/x**3, x) + Integral(3*b**2*c**2*asinh(c*x
```

```
)**2/x, x) + Integral(3*b**2*c**4*x*asinh(c*x)**2, x) + Integral(b**2*c**6*
x**3*asinh(c*x)**2, x) + Integral(6*a*b*c**2*asinh(c*x)/x, x) + Integral(6*
a*b*c**4*x*asinh(c*x), x) + Integral(2*a*b*c**6*x**3*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^3 (b \operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)^2/x^3, x)
```

3.224 $\int \frac{(d+c^2dx^2)^3(a+b\sinh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=326

$$-\frac{17}{3}b^2c^3d^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{17}{3}b^2c^3d^3\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{8}{3}c^4d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}bc^3d^3$$

```
[Out] -(b^2*c^2*d^3)/(3*x) + (50*b^2*c^4*d^3*x)/9 + (2*b^2*c^6*d^3*x^3)/27 - 5*b*
c^3*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + (b*c^3*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/9 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/(3*x^2) + (16*c^4*d^3*x*(a + b*ArcSinh[c*x])^2)/3 + (8*c^4*d^3*x*(1
+ c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*c^2*d^3*(1 + c^2*x^2)^2*(a + b*Ar
cSinh[c*x])^2)/x - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(3*x^3) - (
34*b*c^3*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 - (17*b^2*c^3*
d^3*PolyLog[2, -E^ArcSinh[c*x]])/3 + (17*b^2*c^3*d^3*PolyLog[2, E^ArcSinh[c
*x]])/3
```

Rubi [A] time = 1.02137, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5739, 5684, 5653, 5717, 8, 5744, 5742, 5760, 4182, 2279, 2391, 270}

$$-\frac{17}{3}b^2c^3d^3\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)+\frac{17}{3}b^2c^3d^3\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)+\frac{8}{3}c^4d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}bc^3d^3$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```


```
[Out] -(b^2*c^2*d^3)/(3*x) + (50*b^2*c^4*d^3*x)/9 + (2*b^2*c^6*d^3*x^3)/27 - 5*b*
c^3*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]) + (b*c^3*d^3*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x]))/9 - (b*c*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x]))/(3*x^2) + (16*c^4*d^3*x*(a + b*ArcSinh[c*x])^2)/3 + (8*c^4*d^3*x*(1
+ c^2*x^2)*(a + b*ArcSinh[c*x])^2)/3 - (2*c^2*d^3*(1 + c^2*x^2)^2*(a + b*Ar
cSinh[c*x])^2)/x - (d^3*(1 + c^2*x^2)^3*(a + b*ArcSinh[c*x])^2)/(3*x^3) - (
34*b*c^3*d^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/3 - (17*b^2*c^3*
d^3*PolyLog[2, -E^ArcSinh[c*x]])/3 + (17*b^2*c^3*d^3*PolyLog[2, E^ArcSinh[c
*x]])/3
```


Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
```

```
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^3\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\left(2c^2d\right)\int \frac{\left(d+c^2dx^2\right)^2\left(a+b\sinh^{-1}(cx\right.}{x^2}$$

$$= -\frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{2c^2d^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{x}$$

$$= \frac{17}{9}bc^3d^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx\right.}{3x^2}$$

$$= -\frac{b^2c^2d^3}{3x}-\frac{11}{9}b^2c^4d^3x-\frac{14}{27}b^2c^6d^3x^3+\frac{17}{3}bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{b^2c^2d^3}{3x}-\frac{46}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{b^2c^2d^3}{3x}+\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3-5bc^3d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+$$

Mathematica [A] time = 1.11144, size = 461, normalized size = 1.41

$$d^3\left(153b^2c^3x^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-153b^2c^3x^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+9a^2c^6x^6+81a^2c^4x^4-81a^2c^2x^2-9a^2-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^3*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-9*a^2 - 81*a^2*c^2*x^2 - 9*b^2*c^2*x^2 + 81*a^2*c^4*x^4 + 150*b^2*c^
4*x^4 + 9*a^2*c^6*x^6 + 2*b^2*c^6*x^6 - 9*a*b*c*x*Sqrt[1 + c^2*x^2] - 150*a
*b*c^3*x^3*Sqrt[1 + c^2*x^2] - 6*a*b*c^5*x^5*Sqrt[1 + c^2*x^2] - 18*a*b*Arc
Sinh[c*x] - 162*a*b*c^2*x^2*ArcSinh[c*x] + 162*a*b*c^4*x^4*ArcSinh[c*x] + 1
8*a*b*c^6*x^6*ArcSinh[c*x] - 9*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 150
*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^5*x^5*Sqrt[1 + c^2*x^
```

```
2]*ArcSinh[c*x] - 9*b^2*ArcSinh[c*x]^2 - 81*b^2*c^2*x^2*ArcSinh[c*x]^2 + 81
*b^2*c^4*x^4*ArcSinh[c*x]^2 + 9*b^2*c^6*x^6*ArcSinh[c*x]^2 - 153*a*b*c^3*x^
3*ArcTanh[Sqrt[1 + c^2*x^2]] + 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 - E^(-Arc
Sinh[c*x])] - 153*b^2*c^3*x^3*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 153
*b^2*c^3*x^3*PolyLog[2, -E^(-ArcSinh[c*x])] - 153*b^2*c^3*x^3*PolyLog[2, E^
(-ArcSinh[c*x])]))/(27*x^3)
```

Maple [A] time = 0.331, size = 528, normalized size = 1.6

$$-\frac{17\,c^3d^3ab}{3}\mathrm{Artanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)-\frac{2\,d^3ab\mathrm{Arcsinh}\,(cx)}{3\,x^3}+\frac{c^6d^3b^2\,(\mathrm{Arcsinh}\,(cx))^2\,x^3}{3}+3\,c^4d^3b^2\,(\mathrm{Arcsinh}\,(cx))^2\,x-3\,\frac{d^3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -2/3*d^3*a*b*arcsinh(c*x)/x^3-1/3*b^2*c^2*d^3/x+50/9*b^2*c^4*d^3*x-2/9*c^5*
d^3*a*b*x^2*(c^2*x^2+1)^(1/2)-1/3*c*d^3*a*b/x^2*(c^2*x^2+1)^(1/2)-1/3*c*d^3
*b^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-2/9*c^5*d^3*b^2*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*x^2+2/3*c^6*d^3*a*b*arcsinh(c*x)*x^3+6*c^4*d^3*a*b*arcsinh(c*x)
*x-6*c^2*d^3*a*b*arcsinh(c*x)/x+2/27*b^2*c^6*d^3*x^3-17/3*b^2*c^3*d^3*polyl
og(2,-c*x-(c^2*x^2+1)^(1/2))+17/3*b^2*c^3*d^3*polylog(2,c*x+(c^2*x^2+1)^(1/
2))+1/3*c^6*d^3*b^2*arcsinh(c*x)^2*x^3+3*c^4*d^3*b^2*arcsinh(c*x)^2*x-3*c^2
*d^3*b^2*arcsinh(c*x)^2/x-50/9*c^3*d^3*a*b*(c^2*x^2+1)^(1/2)-17/3*c^3*d^3*a
*b*arctanh(1/(c^2*x^2+1)^(1/2))-50/9*c^3*d^3*b^2*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)-17/3*c^3*d^3*b^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+17/3*c^3*d^3
*b^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/3*d^3*a^2/x^3+1/3*c^6*d^3*a
^2*x^3+3*c^4*d^3*a^2*x-3*c^2*d^3*a^2/x-1/3*d^3*b^2/x^3*arcsinh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a^2c^6d^3x^3+\frac{2}{9}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)abc^6d^3+3\,b^2c^4d^3x\,\mathrm{arsinh}\,(cx)^2+6\,b^2c^4d^3\left(x-\frac{\sqrt{c^2x^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^6*d^3*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^
2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arcsinh(c*x)^2
+ 6*b^2*c^4*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + 3*a^2*c^4*d^3*x +
6*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*c^3*d^3 - 6*(c*arcsinh(1/(sqrt
(c^2)*abs(x))) + arcsinh(c*x)/x)*a*b*c^2*d^3 + 1/3*((c^2*arcsinh(1/(sqrt(c^
2)*abs(x))) - sqrt(c^2*x^2 + 1)/x^2)*c - 2*arcsinh(c*x)/x^3)*a*b*d^3 - 3*a^
2*c^2*d^3/x - 1/3*a^2*d^3/x^3 + 1/3*(b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 -
b^2*d^3)*log(c*x + sqrt(c^2*x^2 + 1))^2/x^3 - integrate(2/3*(b^2*c^9*d^3*x^
8 + b^2*c^7*d^3*x^6 - 9*b^2*c^5*d^3*x^4 - 10*b^2*c^3*d^3*x^2 - b^2*c*d^3 +
(b^2*c^8*d^3*x^7 - 9*b^2*c^4*d^3*x^3 - b^2*c^2*d^3*x)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^6d^3x^6 + 3\,a^2c^4d^3x^4 + 3\,a^2c^2d^3x^2 + a^2d^3 + \left(b^2c^6d^3x^6 + 3\,b^2c^4d^3x^4 + 3\,b^2c^2d^3x^2 + b^2d^3\right)\text{arsinh}\left(cx\right)^2 + 2\left(a\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^6*d^3*x^6 + 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 + a^2*d^3
 + (b^2*c^6*d^3*x^6 + 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*c^6*d^3*x^6 + 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 + a
*b*d^3)*arcsinh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3a^2c^4\,dx + \int \frac{a^2}{x^4}\,dx + \int \frac{3a^2c^2}{x^2}\,dx + \int a^2c^6x^2\,dx + \int 3b^2c^4\,\text{asinh}^2\left(cx\right)dx + \int \frac{b^2\,\text{asinh}^2\left(cx\right)}{x^4}\,dx + \int 6abc^4\,\text{asi}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**3*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(3*a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(3*a**2*c
**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(3*b**2*c**4*asinh(c*x
)**2, x) + Integral(b**2*asinh(c*x)**2/x**4, x) + Integral(6*a*b*c**4*asinh
```

```
(c*x), x) + Integral(2*a*b*asinh(c*x)/x**4, x) + Integral(3*b**2*c**2*asinh
(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*asinh(c*x)**2, x) + Integral(6*
a*b*c**2*asinh(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*asinh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^3 (b \operatorname{arsinh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^3*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^3*(b*arcsinh(c*x) + a)^2/x^4, x)
```

3.225 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=277

$$-\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib^2\text{PolyLog}\left(3,-\right.}{c^5d}$$

```
[Out] (-22*b^2*x)/(9*c^4*d) + (2*b^2*x^3)/(27*c^2*d) + (22*b*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(9*c^5*d) - (2*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(9*c^3*d) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d) + (x^3*(a + b*ArcSinh[
c*x])^2)/(3*c^2*d) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5
*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d
) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d) + (
(2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d) - ((2*I)*b^2*PolyLog[3,
I*E^ArcSinh[c*x]])/(c^5*d)
```

Rubi [A] time = 0.553391, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5767, 5693, 4180, 2531, 2282, 6589, 5717, 8, 5758, 30}

$$-\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d}+\frac{2ib^2\text{PolyLog}\left(3,-\right.}{c^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```


```
[Out] (-22*b^2*x)/(9*c^4*d) + (2*b^2*x^3)/(27*c^2*d) + (22*b*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(9*c^5*d) - (2*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x]))/(9*c^3*d) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d) + (x^3*(a + b*ArcSinh[
c*x])^2)/(3*c^2*d) + (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5
*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d
) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d) + (
(2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d) - ((2*I)*b^2*PolyLog[3,
I*E^ArcSinh[c*x]])/(c^5*d)
```


Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
```

```
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} - \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{3cd}$$

$$= -\frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} + \int$$

$$= \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d} - \frac{x\left(a+b\right.}{}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d}$$

$$= -\frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} + \frac{22b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^5d} - \frac{2bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d}$$

Mathematica [A] time = 1.29032, size = 365, normalized size = 1.32

$$-\frac{2}{3}ab\left(9i\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) - 9i\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) + c^2x^2\sqrt{c^2x^2+1} - 11\sqrt{c^2x^2+1} - 3c^3x^3\sinh^{-1}(cx) + 9cx\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-3*a^2*c*x + a^2*c^3*x^3 + 3*a^2*ArcTan[c*x] - (2*a*b*(-11*Sqrt[1 + c^2*x^
2] + c^2*x^2*Sqrt[1 + c^2*x^2] + 9*c*x*ArcSinh[c*x] - 3*c^3*x^3*ArcSinh[c*x
] - (9*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (9*I)*ArcSinh[c*x]*Log[1
 + I*E^ArcSinh[c*x]] + (9*I)*PolyLog[2, (-I)*E^ArcSinh[c*x]] - (9*I)*PolyLo
g[2, I*E^ArcSinh[c*x]]))/3 + 3*b^2*((5*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/2 -
(5*c*x*(2 + ArcSinh[c*x]^2))/4 - (ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]])/18 + I
*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^ArcSinh[c*x]]))
 - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSinh
[c*x]]) - 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] + 2*PolyLog[3, I/E^ArcSinh[c*x]
]) + ((2 + 9*ArcSinh[c*x]^2)*Sinh[3*ArcSinh[c*x]])/108))/(3*c^5*d)
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b\text{Arcsinh}(cx))^2}{c^2dx^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2\left(\frac{c^2x^3 - 3x}{c^4d} + \frac{3\arctan(cx)}{c^5d}\right) + \int \frac{b^2x^4\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^2dx^2 + d} + \frac{2abx^4\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^2dx^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*((c^2*x^3 - 3*x)/(c^4*d) + 3*arctan(c*x)/(c^5*d)) + integrate(b^2*x
^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4 \operatorname{arsinh}(cx)^2 + 2abx^4 \operatorname{arsinh}(cx) + a^2x^4}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^2*d
*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^4}{c^2x^2+1}\,dx + \int \frac{b^2x^4\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^4\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**4/(c**2*x**2 + 1), x) + Integral(b**2*x**4*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^4}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d), x)
```

3.226 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=199

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx\sqrt{c^2x^2+}}{}$$

```
[Out] (b^2*x^2)/(4*c^2*d) - (b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^3*d
) + (a + b*ArcSinh[c*x])^2/(4*c^4*d) + (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*
d) + (a + b*ArcSinh[c*x])^3/(3*b*c^4*d) - ((a + b*ArcSinh[c*x])^2*Log[1 + E
^(2*ArcSinh[c*x])])/(c^4*d) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c^4*d) + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d)
```

Rubi [A] time = 0.407942, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5767, 5714, 3718, 2190, 2531, 2282, 6589, 5758, 5675, 30}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d}+\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx\sqrt{c^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```



```
[Out] (b^2*x^2)/(4*c^2*d) - (b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c^3*d
) + (a + b*ArcSinh[c*x])^2/(4*c^4*d) + (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*
d) + (a + b*ArcSinh[c*x])^3/(3*b*c^4*d) - ((a + b*ArcSinh[c*x])^2*Log[1 + E
^(2*ArcSinh[c*x])])/(c^4*d) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c^4*d) + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d)
```


Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
```

```
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{b\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x\right.}{c^4d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}$$

$$= \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d} + \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}$$

Mathematica [C] time = 0.457684, size = 279, normalized size = 1.4

$$\frac{-48ab\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-48ab\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+24b^2\sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+12b^2\text{Poly}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (12*a^2*c^2*x^2 - 12*a*b*c*x*Sqrt[1 + c^2*x^2] + 12*a*b*ArcSinh[c*x] + 24*a
*b*c^2*x^2*ArcSinh[c*x] + 24*a*b*ArcSinh[c*x]^2 - 8*b^2*ArcSinh[c*x]^3 + 3*
b^2*Cosh[2*ArcSinh[c*x]] + 6*b^2*ArcSinh[c*x]^2*Cosh[2*ArcSinh[c*x]] - 24*b
^2*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] - 48*a*b*ArcSinh[c*x]*Log[1
- I*E^ArcSinh[c*x]] - 48*a*b*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - 12*a^
2*Log[1 + c^2*x^2] + 24*b^2*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] -
 48*a*b*PolyLog[2, (-I)*E^ArcSinh[c*x]] - 48*a*b*PolyLog[2, I*E^ArcSinh[c*x
]] + 12*b^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 6*b^2*ArcSinh[c*x]*Sinh[2*Ar
cSinh[c*x]])/(24*c^4*d)
```

Maple [A] time = 0.114, size = 380, normalized size = 1.9

$$\frac{a^2x^2}{2c^2d}-\frac{a^2\ln\left(c^2x^2+1\right)}{2dc^4}+\frac{b^2\left(\text{Arcsinh}\left(cx\right)\right)^3}{3dc^4}+\frac{b^2\left(\text{Arcsinh}\left(cx\right)\right)^2x^2}{2c^2d}-\frac{b^2\text{Arcsinh}\left(cx\right)x}{2c^3d}\sqrt{c^2x^2+1}+\frac{b^2\left(\text{Arcsinh}\left(cx\right)\right)}{4dc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a^2/d*x^2-1/2/c^4*a^2/d*ln(c^2*x^2+1)+1/3/c^4*b^2/d*arcsinh(c*x)^3+
1/2/c^2*b^2/d*arcsinh(c*x)^2*x^2-1/2/c^3*b^2/d*arcsinh(c*x)*(c^2*x^2+1)^(1/
2)*x+1/4/c^4*b^2/d*arcsinh(c*x)^2+1/4*b^2*x^2/c^2/d+1/8/c^4*b^2/d-1/c^4*b^2
/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^4*b^2/d*arcsinh(c*x)*
polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(
1/2))^2)/c^4/d+1/c^4*a*b/d*arcsinh(c*x)^2+1/c^2*a*b/d*arcsinh(c*x)*x^2-1/2/
c^3*a*b/d*x*(c^2*x^2+1)^(1/2)+1/2/c^4*a*b/d*arcsinh(c*x)-2/c^4*a*b/d*arcsin
h(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^4*a*b/d*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{x^2}{c^2d}-\frac{\log\left(c^2x^2+1\right)}{c^4d}\right)+\frac{\left(b^2c^2x^2-b^2\log\left(c^2x^2+1\right)\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\,c^4d}+\int-\frac{\left(b^2c^2x^2-\left(2\,abc^4-b^2c^4\right)x^4-\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x^2/(c^2*d) - log(c^2*x^2 + 1)/(c^4*d)) + 1/2*(b^2*c^2*x^2 - b^2*l
og(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d) + integrate(-(b^2*c
^2*x^2 - (2*a*b*c^4 - b^2*c^4)*x^4 - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) -
 (b^2*c*x*log(c^2*x^2 + 1) + (2*a*b*c^3 - b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d*x^3 + c^4*d*x + (c^5*d*x^2 + c^3*d)*sqr
t(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^3\,\text{arsinh}\,(cx)^2+2\,abx^3\,\text{arsinh}\,(cx)+a^2x^3}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^2*d
*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2x^3}{c^2x^2+1}\,dx+\int\frac{b^2x^3\,\text{asinh}^2\,(cx)}{c^2x^2+1}\,dx+\int\frac{2abx^3\,\text{asinh}\,(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**3/(c**2*x**2 + 1), x) + Integral(b**2*x**3*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)^2 x^3}{c^2 dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^3/(c^2*d*x^2 + d), x)
```

3.227 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=198

$$\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib^2\text{PolyLog}\left(3,-i\right.}{c^3d}$$

```
[Out] (2*b^2*x)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^3*d) +
(x*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^Arc
Sinh[c*x]])/(c^3*d) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(c^3*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d) - ((2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^3*d) + ((2*I)
*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rubi [A] time = 0.293411, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5767, 5693, 4180, 2531, 2282, 6589, 5717, 8}

$$\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d}-\frac{2ib^2\text{PolyLog}\left(3,-i\right.}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (2*b^2*x)/(c^2*d) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^3*d) +
(x*(a + b*ArcSinh[c*x])^2)/(c^2*d) - (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^Arc
Sinh[c*x]])/(c^3*d) + ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSi
nh[c*x]])/(c^3*d) - ((2*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^3*d) - ((2*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^3*d) + ((2*I)
*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d)
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
```

```
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2} - \frac{(2b)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{cd}$$

$$= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\text{Subst}\left(\int (a+bx)^2\,\mathbf{sech}(x)\,a\right.}{c^3d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d}$$

$$= \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d}$$

Mathematica [A] time = 0.570927, size = 293, normalized size = 1.48

$$\frac{2ab\left(i\left(\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)-\sqrt{c^2x^2+1}+cx\sinh^{-1}(cx)+i\sinh^{-1}(cx)\left(\log\left(1-ie^{\cdots}\right.\right.\right.}{c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (a^2*x)/(c^2*d) - (a^2*ArcTan[c*x])/(c^3*d) + (2*a*b*(-Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E^
ArcSinh[c*x]]) + I*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLog[2, I/E^ArcSin
h[c*x]])))/(c^3*d) + (b^2*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*(2 + Arc
```

```
Sinh[c*x]^2) - I*(-(ArcSinh[c*x]^2*(Log[1 - I/E^ArcSinh[c*x]] - Log[1 + I/E
^ArcSinh[c*x]])) - 2*ArcSinh[c*x]*(PolyLog[2, (-I)/E^ArcSinh[c*x]] - PolyLo
g[2, I/E^ArcSinh[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSinh[c*x]] - PolyLog[3, I
/E^ArcSinh[c*x]]))))/(c^3*d)
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b\text{Arcsinh}(cx))^2}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{x}{c^2d} - \frac{\arctan(cx)}{c^3d}\right) + \int \frac{b^2x^2\log\left(cx + \sqrt{c^2x^2+1}\right)^2}{c^2dx^2 + d} + \frac{2\,abx^2\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/(c^2*d) - arctan(c*x)/(c^3*d)) + integrate(b^2*x^2*log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^2*d*x^2 + d) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^2
*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\,\text{arsinh}(cx)^2 + 2\,abx^2\,\text{arsinh}(cx) + a^2x^2}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^2*d
*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^2}{c^2x^2+1}\,dx + \int \frac{b^2x^2\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx^2\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x**2/(c**2*x**2 + 1), x) + Integral(b**2*x**2*asinh(c*x)**2/
(c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2x^2}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d), x)
```

3.228 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=105

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+\right.}{}$$

```
[Out] -(a + b*ArcSinh[c*x])^3/(3*b*c^2*d) + ((a + b*ArcSinh[c*x])^2*Log[1 + E^(2*
ArcSinh[c*x])])/(c^2*d) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[
c*x])])/(c^2*d) - (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rubi [A] time = 0.183706, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5714, 3718, 2190, 2531, 2282, 6589}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^2d}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d}+\frac{\log\left(e^{2\sinh^{-1}(cx)}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] -(a + b*ArcSinh[c*x])^3/(3*b*c^2*d) + ((a + b*ArcSinh[c*x])^2*Log[1 + E^(2*
ArcSinh[c*x])])/(c^2*d) + (b*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[
c*x])])/(c^2*d) - (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*c^2*d)
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} - \frac{(2b)\,\text{Subst}\left(\int (a+bx)\,\text{l}\right.}{}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2}{c^2d}$$
$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^2d} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+e^{2\sinh^{-1}(cx)}\right)}{c^2d} + \frac{b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2}{c^2d}$$

Mathematica [B] time = 0.238463, size = 281, normalized size = 2.68

$$\frac{12b\text{PolyLog}\left(2, \frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(cx)\right) + 12b\text{PolyLog}\left(2, \frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)\left(a+b\sinh^{-1}(cx)\right) - 12b^2\text{PolyLog}\left(3, \frac{ce^{\text{si}}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2),x]
```

```
[Out] (-6*a*b*ArcSinh[c*x]^2 - 2*b^2*ArcSinh[c*x]^3 + 12*a*b*ArcSinh[c*x]*Log[1 +
 (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh
[c*x])/Sqrt[-c^2]] + 12*a*b*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] + 6*b^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*a^2*
Log[1 + c^2*x^2] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/
Sqrt[-c^2]] + 12*b*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*
x])/c] - 12*b^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^2*PolyLog[
3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c])/(6*c^2*d)
```

Maple [A] time = 0.049, size = 223, normalized size = 2.1

$$\frac{a^2\ln\left(c^2x^2+1\right)}{2c^2d} - \frac{b^2\left(\text{Arcsinh}(cx)\right)^3}{3c^2d} + \frac{b^2\left(\text{Arcsinh}(cx)\right)^2}{c^2d}\ln\left(1+\left(cx+\sqrt{c^2x^2+1}\right)^2\right) + \frac{b^2\text{Arcsinh}(cx)}{c^2d}\text{polylog}\left(2,-\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] 1/2/c^2*a^2/d*ln(c^2*x^2+1)-1/3/c^2*b^2/d*arcsinh(c*x)^3+1/c^2*b^2/d*arcsin
h(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^2*b^2/d*arcsinh(c*x)*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/
c^2/d-1/c^2*a*b/d*arcsinh(c*x)^2+2/c^2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^
2+1)^(1/2))^2)+1/c^2*a*b/d*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2\log\left(c^2x^2+1\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\,c^2d}+\frac{a^2\log\left(c^2dx^2+d\right)}{2\,c^2d}-\int-\frac{\left(2\,abc^2x^2-\left(b^2c^2x^2+b^2\right)\log\left(c^2x^2+1\right)-\left(b^2cx\log\right.\right.}{c^4dx^3+c^2dx+\left(c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d) + 1/2*a^2*l
og(c^2*d*x^2 + d)/(c^2*d) - integrate(-(2*a*b*c^2*x^2 - (b^2*c^2*x^2 + b^2)
*log(c^2*x^2 + 1) - (b^2*c*x*log(c^2*x^2 + 1) - 2*a*b*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d*x^3 + c^2*d*x + (c^3*d*x^2 + c*d)*sq
rt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x\,\mathrm{arsinh}\,(cx)^2+2\,abx\,\mathrm{arsinh}\,(cx)+a^2x}{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a^2*x)/(c^2*d*x^2 +
 d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x}{c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2*x/(c**2*x**2 + 1), x) + Integral(b**2*x*asinh(c*x)**2/(c**2*
x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2 x}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d), x)
```

3.229 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib^2\text{PolyLog}\left(3,-\right.}{cd}$$

```
[Out] (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c*d) - ((2*I)*b*(a + b*A
rcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b^2*PolyLog[3, (-I)*E
^ArcSinh[c*x]])/(c*d) - ((2*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d)
```

Rubi [A] time = 0.123686, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5693, 4180, 2531, 2282, 6589}

$$-\frac{2ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd}+\frac{2ib^2\text{PolyLog}\left(3,-\right.}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c*d) - ((2*I)*b*(a + b*A
rcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d) + ((2*I)*b^2*PolyLog[3, (-I)*E
^ArcSinh[c*x]])/(c*d) - ((2*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d)
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{cd}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{(2ib)\,\text{Subst}\left(\int (a+bx)\log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(c\right.}{cd}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{2ib\left(a\right.}{}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{2ib\left(a\right.}{}$$

$$= \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd} - \frac{2ib\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2\left(-ie^{\sinh^{-1}(cx)}\right)}{cd} + \frac{2ib\left(a\right.}{}$$

Mathematica [A] time = 0.240249, size = 274, normalized size = 1.99

$$-\frac{c\left(2bc\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(cx)\right)-2bc\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)\left(a+b\sinh^{-1}(cx)\right)-2b^2c\text{PolyLog}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2),x]
```

```
[Out] -((c*(a^2*Sqrt[-c^2]*ArcTan[c*x] - 2*a*b*c*ArcSinh[c*x]*Log[1 + (c*E^ArcSin
h[c*x])/Sqrt[-c^2]] - b^2*c*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 2*a*b*c*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + b^2*
c*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b*c*(a + b*ArcS
inh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 2*b*c*(a + b*ArcSinh[
c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 2*b^2*c*PolyLog[3, (c*E^A
rcSinh[c*x])/Sqrt[-c^2]] + 2*b^2*c*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c
]))/((-c^2)^(3/2)*d))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{(a+b\text{Arcsinh}(cx))^2}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\arctan(cx)}{cd}+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^2+d}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] a^2*arctan(c*x)/(c*d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d
*x^2 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^2 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^2 + d), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^2+1}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^2+1}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^2+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**2
 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2 + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d), x)
```

3.230 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=116

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right.}{2d}$$

```
[Out] (-2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*(a + b*ArcSi
nh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d + (b*(a + b*ArcSinh[c*x])*PolyL
og[2, E^(2*ArcSinh[c*x])])/d + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d)
- (b^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d)
```

Rubi [A] time = 0.207527, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5720, 5461, 4182, 2531, 2282, 6589}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] (-2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d - (b*(a + b*ArcSi
nh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d + (b*(a + b*ArcSinh[c*x])*PolyL
og[2, E^(2*ArcSinh[c*x])])/d + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d)
- (b^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx = \frac{\text{Subst}\left(\int (a+bx)^2\mathbf{csch}(x)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int (a+bx)^2\mathbf{csch}(2x)\,dx, x, \sinh^{-1}(cx)\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{(2b)\,\text{Subst}\left(\int (a+bx)\log\left(1-e^{2x}\right)\,dx, x, \sin\right.}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{b\left(\right.}{}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{b\left(\right.}{}$$
$$= -\frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right)}{d} - \frac{b\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2\left(-e^{2\sinh^{-1}(cx)}\right)}{d} + \frac{b\left(\right.}{}$$

Mathematica [B] time = 0.366319, size = 400, normalized size = 3.45

$$-\frac{12b^2\text{PolyLog}\left(2, \frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(cx)\right) + 12b^2\text{PolyLog}\left(2, \frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)\left(a+b\sinh^{-1}(cx)\right) - 6ab^2\text{PolyLog}\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)),x]
```

```
[Out] -(2*a^3 + 6*a^2*b*ArcSinh[c*x] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh
[c*x])/Sqrt[-c^2]] + 6*b^3*ArcSinh[c*x]^2*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-
c^2]] + 12*a*b^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 6*b^
3*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a^2*b*Log[1 - E
^(2*ArcSinh[c*x])] - 12*a*b^2*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] - 6*
b^3*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 3*a^2*b*Log[1 + c^2*x^2] +
 12*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 12
*b^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] - 6*a*b
^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 6*b^3*ArcSinh[c*x]*PolyLog[2, E^(2*ArcS
inh[c*x])] - 12*b^3*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt[-c^2]] - 12*b^3*Poly
Log[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 3*b^3*PolyLog[3, E^(2*ArcSinh[c*x])
])/(6*b*d)
```

Maple [B] time = 0.074, size = 354, normalized size = 3.1

$$\frac{a^2\ln(cx)}{d}-\frac{a^2\ln\left(c^2x^2+1\right)}{2d}+\frac{b^2(\operatorname{Arcsinh}(cx))^2}{d}\ln\left(1+cx+\sqrt{c^2x^2+1}\right)+2\frac{b^2\operatorname{Arcsinh}(cx)\operatorname{polylog}\left(2,-cx-\sqrt{c^2x^2+}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x)
```

```
[Out] a^2/d*ln(c*x)-1/2*a^2/d*ln(c^2*x^2+1)+b^2/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^
2+1)^(1/2))+2*b^2/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*b^2/d*
polylog(3,-c*x-(c^2*x^2+1)^(1/2))-b^2/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1
)^(1/2))^2)-b^2/d*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b^
2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d+b^2/d*arcsinh(c*x)^2*ln(1-c*x-(c^
2*x^2+1)^(1/2))+2*b^2/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2
/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*a*b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))
^2)-1/2*a*b/d*dilog(1/(c*x+(c^2*x^2+1)^(1/2))^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a^2\left(\frac{\log\left(c^2x^2+1\right)}{d}-\frac{2\log(x)}{d}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^3+dx}+\frac{2ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(log(c^2*x^2 + 1)/d - 2*log(x)/d) + integrate(b^2*log(c*x + sqrt(c
^2*x^2 + 1))^2/(c^2*d*x^3 + d*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*
d*x^3 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2+2ab\operatorname{arsinh}(cx)+a^2}{c^2dx^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^3 + d*x),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^3+x}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^3+x}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^2x^3+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)**2/(c**2*x**3
 + x), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**3 + x), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x), x)
```

3.231 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{2ibc\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2ibc\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2b^2c\mathrm{PolyLog}\left(2,-\right.}{d}$$

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x)) - (2*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/d - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d - (2*
b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d + ((2*I)*b*c*(a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]])/d + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d - ((2*I)*b^2
*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d + ((2*I)*b^2*c*PolyLog[3, I*E^ArcSinh
[c*x]])/d
```

Rubi [A] time = 0.335271, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5747, 5693, 4180, 2531, 2282, 6589, 5760, 4182, 2279, 2391}

$$\frac{2ibc\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2ibc\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{2b^2c\mathrm{PolyLog}\left(2,-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x)) - (2*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/d - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d - (2*
b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d + ((2*I)*b*c*(a + b*ArcSinh[c*x])*Poly
Log[2, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2,
 I*E^ArcSinh[c*x]])/d + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d - ((2*I)*b^2
*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d + ((2*I)*b^2*c*PolyLog[3, I*E^ArcSinh
[c*x]])/d
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
```

```
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{c\,\text{Subst}\left(\int (a+bx)^2\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{d} + \frac{(2bc)\,\text{Subst}\left(\int (a+\cdots\right.}{\cdots}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}(cx)\right)\text{ta}\cdots}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}(cx)\right)\text{ta}\cdots}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}(cx)\right)\text{ta}\cdots}{d}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{2c\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d} - \frac{4bc\left(a+b\sinh^{-1}(cx)\right)\text{ta}\cdots}{d}$$

Mathematica [A] time = 1.06068, size = 363, normalized size = 1.78

$$-\frac{\frac{1}{2}iabc\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)-\frac{1}{2}iabc\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)),x]
```

```
[Out] -((a^2/x + (2*a*b*ArcSinh[c*x])/x + a^2*c*ArcTan[c*x] + 2*a*b*c*ArcTanh[Sqr
t[1 + c^2*x^2]] + (I/2)*a*b*c*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^A
rcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - (I/2)*a*b*c*(ArcSinh[c
*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh
[c*x]]) - b^2*c*(-(ArcSinh[c*x]^2/(c*x)) + 2*ArcSinh[c*x]*Log[1 - E^(-ArcSi
nh[c*x])] + I*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - I*ArcSinh[c*x]^2*L
og[1 + I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 2*Po
lyLog[2, -E^(-ArcSinh[c*x])] + (2*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - (2*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*PolyLog[2, E^(
-ArcSinh[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (2*I)*PolyLog[3,
I/E^ArcSinh[c*x]]))/d)
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{(a+b\text{Arcsinh}\,(cx))^2}{x^2\left(c^2dx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^2\left(\frac{c\arctan(cx)}{d}+\frac{1}{dx}\right)+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^4+dx^2}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^4+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -a^2*(c*arctan(c*x)/d + 1/(d*x)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^4 + d*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*x^4 +
d*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^2dx^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^4 + d*x^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^4+x^2}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^4+x^2}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^4+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + x**2), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**4 + x**2), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^2), x)
```

3.232 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=194

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,-e\right.}{2d}$$

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) - (a + b*ArcSinh[c*x]
)^2/(2*d*x^2) + (2*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/
d + (b^2*c^2*Log[x])/d + (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/d - (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/
d - (b^2*c^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d) + (b^2*c^2*PolyLog[3, E
^(2*ArcSinh[c*x])])/(2*d)
```

Rubi [A] time = 0.388449, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5747, 5720, 5461, 4182, 2531, 2282, 6589, 5723, 29}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,-e\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x)) - (a + b*ArcSinh[c*x]
)^2/(2*d*x^2) + (2*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/
d + (b^2*c^2*Log[x])/d + (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSi
nh[c*x])])/d - (b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/
d - (b^2*c^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d) + (b^2*c^2*PolyLog[3, E
^(2*ArcSinh[c*x])])/(2*d)
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
```

```
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\sqrt{1+c^2x^2}}\,dx}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{c^2\,\text{Subst}\left(\int (a+bx)^2\mathbf{csch}(x)\mathbf{sech}\ldots\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{b^2c^2\log(x)}{d} - \frac{\left(2c^2\right)\text{Subst}\left(\int (a+\ldots\right.}{\ldots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\ldots}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\ldots}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\ldots}\right.}{d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\ldots}\right.}{d}$$

Mathematica [C] time = 1.06281, size = 419, normalized size = 2.16

$$-\frac{-4abc^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right) - 4abc^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right) + 2abc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right) + 2b^2c^2\sinh^{-1}(cx)\text{P}\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)),x]
```

```
[Out] -((I/12)*b^2*c^2*Pi^3 + a^2/x^2 + (2*a*b*c*Sqrt[1 + c^2*x^2])/x + (2*a*b*Ar
cSinh[c*x])/x^2 + (2*b^2*c*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/x + (b^2*ArcSinh
[c*x]^2)/x^2 - (4*b^2*c^2*ArcSinh[c*x]^3)/3 - 2*b^2*c^2*ArcSinh[c*x]^2*Log[
1 + E^(-2*ArcSinh[c*x])] - 4*a*b*c^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]]
 - 4*a*b*c^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] + 4*a*b*c^2*ArcSinh[c*x
]*Log[1 - E^(2*ArcSinh[c*x])] + 2*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSi
nh[c*x])] + 2*a^2*c^2*Log[x] - 2*b^2*c^2*Log[c*x] - a^2*c^2*Log[1 + c^2*x^2
] + 2*b^2*c^2*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - 4*a*b*c^2*Pol
yLog[2, (-I)*E^ArcSinh[c*x]] - 4*a*b*c^2*PolyLog[2, I*E^ArcSinh[c*x]] + 2*a
*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*c^2*ArcSinh[c*x]*PolyLog[2, E
^(2*ArcSinh[c*x])] + b^2*c^2*PolyLog[3, -E^(-2*ArcSinh[c*x])] - b^2*c^2*Pol
yLog[3, E^(2*ArcSinh[c*x])])/(2*d)
```

Maple [B] time = 0.148, size = 719, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x)
```

```
[Out] -2*c^2*a*b/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*c^2*a*b/d*arcsinh(c
*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*c^2*a*b/d*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2
+1)^(1/2))^2)-c*a*b/d/x*(c^2*x^2+1)^(1/2)-c*b^2/d*arcsinh(c*x)/x*(c^2*x^2+1
)^(1/2)-1/2*a^2/d/x^2+c^2*a*b/d-c^2*a^2/d*ln(c*x)+1/2*c^2*a^2/d*ln(c^2*x^2+
1)+2*c^2*b^2/d*polylog(3,-c*x-(c^2*x^2+1)^(1/2))+2*c^2*b^2/d*polylog(3,c*x+
(c^2*x^2+1)^(1/2))+c^2*b^2/d*arcsinh(c*x)+c^2*b^2/d*ln(c*x+(c^2*x^2+1)^(1/2
)-1)-2*c^2*b^2/d*ln(c*x+(c^2*x^2+1)^(1/2))+c^2*b^2/d*ln(1+c*x+(c^2*x^2+1)^(
1/2))-1/2*b^2/d*arcsinh(c*x)^2/x^2-1/2*b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^
(1/2))^2)/d+c^2*a*b/d*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-2*c^2*a*b/d*pol
ylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*c^2*a*b/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))
+c^2*b^2/d*arcsinh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+c^2*b^2/d*arcsinh
(c*x)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-c^2*b^2/d*arcsinh(c*x)^2*ln(1+c
*x+(c^2*x^2+1)^(1/2))-2*c^2*b^2/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))-c^2*b^2/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-2*c^2*b^2/d*arcs
inh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-a*b/d*arcsinh(c*x)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(\frac{c^2\log\left(c^2x^2+1\right)}{d}-\frac{2\,c^2\log\left(x\right)}{d}-\frac{1}{dx^2}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^5+dx^3}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^5+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log(c^2*x^2 + 1)/d - 2*c^2*log(x)/d - 1/(d*x^2))*a^2 + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^5 + d*x^3) + 2*a*b*log(c*x + sqr
t(c^2*x^2 + 1))/(c^2*d*x^5 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\left(cx\right)^2+2\,ab\,\text{arsinh}\left(cx\right)+a^2}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^5 + d*x^3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^2x^5+x^3}\,dx+\int\frac{b^2\,\text{asinh}^2\left(cx\right)}{c^2x^5+x^3}\,dx+\int\frac{2ab\,\text{asinh}\left(cx\right)}{c^2x^5+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**5 + x**3), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**5 + x**3), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^3), x)
```

3.233 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)}\,dx$

Optimal. Leaf size=297

$$-\frac{2ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{2ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{7b^2c^3}{}$$

```
[Out] -(b^2*c^2)/(3*d*x) - (b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^2)
 - (a + b*ArcSinh[c*x])^2/(3*d*x^3) + (c^2*(a + b*ArcSinh[c*x])^2)/(d*x) +
(2*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d + (14*b*c^3*(a + b*
ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d) + (7*b^2*c^3*PolyLog[2, -E^Arc
Sinh[c*x]])/(3*d) - ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/d + ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/d - (7*b^2*c^3*PolyLog[2, E^ArcSinh[c*x]])/(3*d) + ((2*I)*b^2*c^3*Poly
Log[3, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]
])/d
```

Rubi [A] time = 0.638874, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5747, 5693, 4180, 2531, 2282, 6589, 5760, 4182, 2279, 2391, 30}

$$-\frac{2ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{2ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d}+\frac{7b^2c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -(b^2*c^2)/(3*d*x) - (b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*d*x^2)
 - (a + b*ArcSinh[c*x])^2/(3*d*x^3) + (c^2*(a + b*ArcSinh[c*x])^2)/(d*x) +
(2*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d + (14*b*c^3*(a + b*
ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d) + (7*b^2*c^3*PolyLog[2, -E^Arc
Sinh[c*x]])/(3*d) - ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/d + ((2*I)*b*c^3*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*
x]])/d - (7*b^2*c^3*PolyLog[2, E^ArcSinh[c*x]])/(3*d) + ((2*I)*b^2*c^3*Poly
Log[3, (-I)*E^ArcSinh[c*x]])/d - ((2*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]
])/d
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} - c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\sqrt{1+c^2x^2}}\,dx}{3d}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + c^4\int \frac{(a}{}$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx} +$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx} +$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx} +$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx} +$$

$$= -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx} +$$

Mathematica [B] time = 8.01868, size = 602, normalized size = 2.03

$$\frac{2ab\left(-\frac{1}{2}ic^4\left(\frac{2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)}{c}\right) + \frac{1}{2}ic^4\left(\frac{2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\text{s}}{}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)),x]
```

```
[Out] -a^2/(3*d*x^3) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcTan[c*x])/d + (2*a*b*(-(c*Sq
rt[1 + c^2*x^2])/(6*x^2) - ArcSinh[c*x]/(3*x^3) + (c^3*ArcTanh[Sqrt[1 + c^2
*x^2]])/6 - c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) - (I/2)*
c^4*(-ArcSinh[c*x]^2/(2*c) + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c +
 (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + (I/2)*c^4*(-ArcSinh[c*x]^2/(2*c)
+ (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[
c*x]])/c)))/d + (b^2*c^3*(-4*Coth[ArcSinh[c*x]/2] + 14*ArcSinh[c*x]^2*Coth[
ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*x]
^2*Csch[ArcSinh[c*x]/2]^4)/2 - 56*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] -
 (24*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]^2*Lo
```

```
g[1 + I/E^ArcSinh[c*x]] + 56*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 56*P
olyLog[2, -E^(-ArcSinh[c*x])] - (48*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSi
nh[c*x]] + (48*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 56*PolyLog[2,
 E^(-ArcSinh[c*x])] - (48*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (48*I)*PolyL
og[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (8*ArcSin
h[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2] - 14*Ar
cSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d)
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \frac{(a+b\text{Arcsinh}\,(cx))^2}{x^4\left(c^2dx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(\frac{3\,c^3\arctan\,(cx)}{d}+\frac{3\,c^2x^2-1}{dx^3}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^2dx^6+dx^4}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2dx^6+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^3*arctan(c*x)/d + (3*c^2*x^2 - 1)/(d*x^3))*a^2 + integrate(b^2*log
(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^6 + d*x^4) + 2*a*b*log(c*x + sqrt(c^2*
x^2 + 1))/(c^2*d*x^6 + d*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2}{c^2dx^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^2*d*x^6 + d*x^4
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^6+x^4}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^2x^6+x^4}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^2x^6+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d),x)
```

```
[Out] (Integral(a**2/(c**2*x**6 + x**4), x) + Integral(b**2*asinh(c*x)**2/(c**2*x
**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**6 + x**4), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)*x^4), x)
```

3.234 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=279

$$\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib^2\text{PolyLog}\left(3,-ie^{s}\right.}{c^5d^2}$$

```
[Out] (2*b^2*x)/(c^4*d^2) + (b*(a + b*ArcSinh[c*x]))/(c^5*d^2*Sqrt[1 + c^2*x^2])
- (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d^2) + (3*x*(a + b*ArcS
inh[c*x])^2)/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2
*x^2)) - (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) - (b^2
*ArcTan[c*x])/(c^5*d^2) + ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/(c^5*d^2) - ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcS
inh[c*x]])/(c^5*d^2) - ((3*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^2
) + ((3*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rubi [A] time = 0.531413, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5751, 5767, 5693, 4180, 2531, 2282, 6589, 5717, 8, 266, 43, 5732, 388, 205}

$$\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}-\frac{3ib^2\text{PolyLog}\left(3,-ie^{s}\right.}{c^5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```


```
[Out] (2*b^2*x)/(c^4*d^2) + (b*(a + b*ArcSinh[c*x]))/(c^5*d^2*Sqrt[1 + c^2*x^2])
- (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d^2) + (3*x*(a + b*ArcS
inh[c*x])^2)/(2*c^4*d^2) - (x^3*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2
*x^2)) - (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(c^5*d^2) - (b^2
*ArcTan[c*x])/(c^5*d^2) + ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^A
rcSinh[c*x]])/(c^5*d^2) - ((3*I)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcS
inh[c*x]])/(c^5*d^2) - ((3*I)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^2
) + ((3*I)*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^2)
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
```

```
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx=-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2c^2d}$$

$$=\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}}+\frac{b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}-\frac{x^3\left(a-\right.}{2c^2}$$

$$=-\frac{b^2x}{c^4d^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)}{2c^4d^2}$$

$$=\frac{2b^2x}{c^4d^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}$$

$$=\frac{2b^2x}{c^4d^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}$$

$$=\frac{2b^2x}{c^4d^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}$$

$$=\frac{2b^2x}{c^4d^2}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2\sqrt{1+c^2x^2}}-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}$$

Mathematica [A] time = 2.19116, size = 482, normalized size = 1.73

$$-\frac{2ab\left(-3i(c^2x^2+1)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+3i(c^2x^2+1)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}-2c^3x^3\sinh^{-1}(cx)+3ic^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right.\right.}{c^7x^2+c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((2*a^2*x)/c^4 + (a^2*x)/(c^4 + c^6*x^2) - (3*a^2*ArcTan[c*x])/c^5 - (2*a*b
*(Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^2] - 3*c*x*ArcSinh[c*x] - 2*
c^3*x^3*ArcSinh[c*x] + (3*I)*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + (3*I)
*c^2*x^2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]*Log[1
+ I*E^ArcSinh[c*x]] - (3*I)*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]]
- (3*I)*(1 + c^2*x^2)*PolyLog[2, (-I)*E^ArcSinh[c*x]] + (3*I)*(1 + c^2*x^2)
*PolyLog[2, I*E^ArcSinh[c*x]]))/(c^5 + c^7*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt
[1 + c^2*x^2] - 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + (c*x*ArcSinh[c*x]^2)/(2
+ 2*c^2*x^2) + c*x*(2 + ArcSinh[c*x]^2) + (I/2)*((4*I)*ArcTan[Tanh[ArcSinh[
c*x]/2]] + 3*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Lo
g[1 + I/E^ArcSinh[c*x]] + 6*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] -
6*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 6*PolyLog[3, (-I)/E^ArcSinh[c
*x]] - 6*PolyLog[3, I/E^ArcSinh[c*x]])))/c^5)/(2*d^2)
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a+b\text{Arcsinh}\left(cx\right)\right)^2}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^2\left(\frac{x}{c^6d^2x^2+c^4d^2}+\frac{2\,x}{c^4d^2}-\frac{3\,\arctan\left(cx\right)}{c^5d^2}\right)+\int \frac{b^2x^4\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^4+2\,c^2d^2x^2+d^2}+\frac{2\,abx^4\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^6*d^2*x^2 + c^4*d^2) + 2*x/(c^4*d^2) - 3*arctan(c*x)/(c^5*d^2
)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*
d^2*x^2 + d^2) + 2*a*b*x^4*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^
2*d^2*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4\operatorname{arsinh}(cx)^2 + 2\,abx^4\operatorname{arsinh}(cx) + a^2x^4}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^4}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^4\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^4\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**4*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2x^4}{\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^2, x)
```

3.235 $\int \frac{x^3(a+b\sinh^{-1}(cx))^2}{(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=213

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{bx\left(a+b\sinh\right.}{c^3d^2\sqrt{c^2x^2}}$$

```
[Out] -((b*x*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[
c*x])^2/(2*c^4*d^2) - (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)
) - (a + b*ArcSinh[c*x])^3/(3*b*c^4*d^2) + ((a + b*ArcSinh[c*x])^2*Log[1 +
E^(2*ArcSinh[c*x])])/(c^4*d^2) + (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (b*(a
 + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^4*d^2) - (b^2*PolyLo
g[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```

Rubi [A] time = 0.404791, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5751, 5714, 3718, 2190, 2531, 2282, 6589, 5675, 260}

$$\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^4d^2}-\frac{b^2\text{PolyLog}\left(3,-e^{2\sinh^{-1}(cx)}\right)}{2c^4d^2}-\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{bx\left(a+b\sinh\right.}{c^3d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```


```
[Out] -((b*x*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[
c*x])^2/(2*c^4*d^2) - (x^2*(a + b*ArcSinh[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)
) - (a + b*ArcSinh[c*x])^3/(3*b*c^4*d^2) + ((a + b*ArcSinh[c*x])^2*Log[1 +
E^(2*ArcSinh[c*x])])/(c^4*d^2) + (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (b*(a
 + b*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^4*d^2) - (b^2*PolyLo
g[3, -E^(2*ArcSinh[c*x])])/(2*c^4*d^2)
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
```

```
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{b\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2} + \frac{\int\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int(a+bx)^2\tanh(x)\,dx, x, \sin\ldots}{c^4d^2}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(\ldots\right.}{3bc^4d^2}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(\ldots\right.}{3bc^4d^2}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(\ldots\right.}{3bc^4d^2}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(\ldots\right.}{3bc^4d^2}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(\ldots\right.}{3bc^4d^2}$$

Mathematica [C] time = 1.06315, size = 320, normalized size = 1.5

$$\frac{4ab\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+4ab\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+2b^2\left(-\sinh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - a*b*ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - a*b*ArcSinh[c*x]*(ArcSinh[c*x
] - 4*Log[1 + I*E^ArcSinh[c*x]]) + a^2*Log[1 + c^2*x^2] + 4*a*b*PolyLog[2,
(-I)*E^ArcSinh[c*x]] + 4*a*b*PolyLog[2, I*E^ArcSinh[c*x]] + 2*b^2*(-((c*x*A
rcSinh[c*x])/Sqrt[1 + c^2*x^2]) + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) + ArcSinh[
c*x]^3/3 + ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + Log[1 + c^2*x^2]/2
 - ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] - PolyLog[3, -E^(-2*ArcSin
h[c*x])]/2))/(2*c^4*d^2)
```

Maple [B] time = 0.197, size = 499, normalized size = 2.3

$$\frac{a^2}{2\,d^2c^4\left(c^2x^2+1\right)}+\frac{a^2\ln\left(c^2x^2+1\right)}{2\,d^2c^4}-\frac{b^2\left(\text{Arcsinh}\left(cx\right)\right)^3}{3\,d^2c^4}-\frac{b^2\text{Arcsinh}\left(cx\right)x}{c^3d^2}\frac{1}{\sqrt{c^2x^2+1}}+\frac{b^2\text{Arcsinh}\left(cx\right)x^2}{c^2d^2\left(c^2x^2+1\right)}+\frac{b^2\left(\text{Arcs}\right.}{2\,d^2c^4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] 1/2/c^4*a^2/d^2/(c^2*x^2+1)+1/2/c^4*a^2/d^2*ln(c^2*x^2+1)-1/3/c^4*b^2/d^2*a
rcsinh(c*x)^3-1/c^3*b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x+1/c^2*b^2/d^2*
arcsinh(c*x)/(c^2*x^2+1)*x^2+1/2/c^4*b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+1/c
^4*b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-2/c^4*b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2))
+1/c^4*b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^4*b^2/d^2*arcsinh(c*x)^2
*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^4*b^2/d^2*arcsinh(c*x)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))^2)-1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/c^4/d^
2-1/c^4*a*b/d^2*arcsinh(c*x)^2-1/c^3*a*b/d^2/(c^2*x^2+1)^(1/2)*x+1/c^2*a*b/
d^2*x^2/(c^2*x^2+1)+1/c^4*a*b/d^2*arcsinh(c*x)/(c^2*x^2+1)+1/c^4*a*b/d^2/(c
^2*x^2+1)+2/c^4*a*b/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/c^4*
a*b/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^2\left(\frac{1}{c^6d^2x^2+c^4d^2}+\frac{\log\left(c^2x^2+1\right)}{c^4d^2}\right)+\frac{\left(b^2+\left(b^2c^2x^2+b^2\right)\log\left(c^2x^2+1\right)\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\left(c^6d^2x^2+c^4d^2\right)}-\int-\frac{\left(2\,abc^4x^4-b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^6*d^2*x^2 + c^4*d^2) + log(c^2*x^2 + 1)/(c^4*d^2)) + 1/2*(b^2
 + (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^
6*d^2*x^2 + c^4*d^2) - integrate(-(2*a*b*c^4*x^4 - b^2*c^2*x^2 - b^2 - (b^2
*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + (2*a*b*c^3*x^3 - b^2*c*x
 - (b^2*c^3*x^3 + b^2*c*x)*log(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1))/(c^8*d^2*x^5 + 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*d^2*x^4 +
 2*c^5*d^2*x^2 + c^3*d^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2x^3\operatorname{arsinh}(cx)^2+2\,abx^3\operatorname{arsinh}(cx)+a^2x^3}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2x^3}{c^4x^4+2c^2x^2+1}\,dx+\int\frac{b^2x^3\operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx+\int\frac{2abx^3\operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**3*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^3}{(c^2 d x^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^3/(c^2*d*x^2 + d)^2, x)
```

3.236 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib^2\text{PolyLog}\left(3,-ie^{\text{si}}\right.}{c^3d^2}$$

```
[Out] -((b*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) - (x*(a + b*ArcSinh
[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/(c^3*d^2) + (b^2*ArcTan[c*x])/(c^3*d^2) - (I*b*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[
c*x]])/(c^3*d^2) - (I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d^2)
```

Rubi [A] time = 0.298199, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5751, 5693, 4180, 2531, 2282, 6589, 5717, 203}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2}+\frac{ib^2\text{PolyLog}\left(3,-ie^{\text{si}}\right.}{c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```


```
[Out] -((b*(a + b*ArcSinh[c*x]))/(c^3*d^2*Sqrt[1 + c^2*x^2])) - (x*(a + b*ArcSinh
[c*x])^2)/(2*c^2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcS
inh[c*x]])/(c^3*d^2) + (b^2*ArcTan[c*x])/(c^3*d^2) - (I*b*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[
c*x]])/(c^3*d^2) - (I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c^3*d^2)
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
```

```
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2}+\frac{\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2c^2d}$$
$$=-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\text{Subst}\left(\int(a+bx)^2\textbf{sech}(x)\,dx,x,\sinh^{-1}\right.}{2c^3d^2}$$
$$=-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}+$$
$$=-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}+$$
$$=-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}+$$
$$=-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{c^3d^2\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^3d^2}+$$

Mathematica [A] time = 1.80203, size = 385, normalized size = 1.81

$$-\frac{-\frac{1}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)+\frac{1}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -((a^2*c*x)/(1 + c^2*x^2) + (2*b^2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (b^2*c
*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + (a*b*((-I)*Sqrt[1 + c^2*x^2] + ArcSinh[c
*x]))/(-I + c*x) + (a*b*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x) - a
^2*ArcTan[c*x] - (I/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcS
inh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + (I/2)*a*b*(ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]
]) + I*b^2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSinh[c*x]^2*Log[1 - I/E
^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 2*ArcSinh[c*x]*
PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x
]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3, I/E^ArcSinh[c*x]]))/(
2*c^3*d^2)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(a + b\operatorname{Arcsinh}\left(cx\right)\right)^{2}}{\left(c^{2}dx^{2} + d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\, a^{2}\left(\frac{x}{c^{4}d^{2}x^{2} + c^{2}d^{2}} - \frac{\arctan\left(cx\right)}{c^{3}d^{2}}\right) + \int \frac{b^{2}x^{2}\log\left(cx + \sqrt{c^{2}x^{2} + 1}\right)^{2}}{c^{4}d^{2}x^{4} + 2\,c^{2}d^{2}x^{2} + d^{2}} + \frac{2\,abx^{2}\log\left(cx + \sqrt{c^{2}x^{2} + 1}\right)}{c^{4}d^{2}x^{4} + 2\,c^{2}d^{2}x^{2} + d^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(x/(c^4*d^2*x^2 + c^2*d^2) - arctan(c*x)/(c^3*d^2)) + integrate(b^
2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) +
2*a*b*x^2*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2),
```

```
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\,\text{arsinh}\,(cx)^2 + 2\,abx^2\,\text{arsinh}\,(cx) + a^2x^2}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^4*d
^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x^2\,\text{asinh}^2\,(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx^2\,\text{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x**2*
asinh(c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asinh
(c*x)/(c**4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^2}{(c^2dx^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^2, x)
```

3.237 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}$$

```
[Out] (b*x*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])
^2/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d^2)
```

Rubi [A] time = 0.106848, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5717, 5687, 260}

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] (b*x*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])
^2/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d^2)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
```

```
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx &= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}+\frac{b\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{cd^2}\\ &= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\int \frac{x}{1+c^2x^2}\,dx}{d^2}\\ &= \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d^2\left(1+c^2x^2\right)}-\frac{b^2\log\left(1+c^2x^2\right)}{2c^2d^2}\end{aligned}$$

Mathematica [A] time = 0.218641, size = 145, normalized size = 1.71

$$-\frac{a^2}{2c^2d^2\left(c^2x^2+1\right)}+\frac{abx}{cd^2\sqrt{c^2x^2+1}}+\frac{b\sinh^{-1}(cx)\left(bcx\sqrt{c^2x^2+1}-a\right)}{c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^2d^2}-\frac{b^2\sinh^{-1}(cx)^2}{2c^2d^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^2,x]
```

```
[Out] -a^2/(2*c^2*d^2*(1 + c^2*x^2)) + (a*b*x)/(c*d^2*Sqrt[1 + c^2*x^2]) + (b*(-a
 + b*c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x])/(c^2*d^2*(1 + c^2*x^2)) - (b^2*Ar
cSinh[c*x]^2)/(2*c^2*d^2*(1 + c^2*x^2)) - (b^2*Log[1 + c^2*x^2])/(2*c^2*d^2
)
```

Maple [B] time = 0.063, size = 222, normalized size = 2.6

$$-\frac{a^2}{2\,c^2d^2\left(c^2x^2+1\right)}+2\,\frac{b^2\mathrm{Arcsinh}\left(cx\right)}{c^2d^2}+\frac{b^2\mathrm{Arcsinh}\left(cx\right)x}{cd^2}\frac{1}{\sqrt{c^2x^2+1}}-\frac{b^2\mathrm{Arcsinh}\left(cx\right)x^2}{d^2\left(c^2x^2+1\right)}-\frac{b^2\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{2\,c^2d^2\left(c^2x^2+1\right)}-\frac{b^2\mathrm{A}}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2/c^2*a^2/d^2/(c^2*x^2+1)+2/c^2*b^2/d^2*arcsinh(c*x)+1/c*b^2/d^2*arcsinh
(c*x)/(c^2*x^2+1)^(1/2)*x-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)*x^2-1/2/c^2*b^2/
d^2*arcsinh(c*x)^2/(c^2*x^2+1)-1/c^2*b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-1/c^2
*b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^2*a*b/d^2/(c^2*x^2+1)*arcsinh(
c*x)+1/c*a*b/d^2*x/(c^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{2\left(c^4d^2x^2+c^2d^2\right)}-\frac{a^2}{2\left(c^4d^2x^2+c^2d^2\right)}+\int\frac{\left(\left(2\,abc^2+b^2c^2\right)x^2+\sqrt{c^2x^2+1}\left(2\,abc+b^2c\right)x+b^2\right)\log\left(cx+\sqrt{c^2x^2\right.}}{c^6d^2x^5+2\,c^4d^2x^3+c^2d^2x+\left(c^5d^2x^4+2\,c^3d^2x^2+cd^2\right)\sqrt{c^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^2 + c^2*d^2) - 1/2*a^2/(
c^4*d^2*x^2 + c^2*d^2) + integrate(((2*a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^
2 + 1)*(2*a*b*c + b^2*c)*x + b^2)*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^2*x^5
 + 2*c^4*d^2*x^3 + c^2*d^2*x + (c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)*sqrt(c
^2*x^2 + 1)), x)
```

Fricas [B] time = 2.6841, size = 397, normalized size = 4.67

$$\frac{2\,abc^2x^2+2\,\sqrt{c^2x^2+1}abcx-b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-a^2+2\,ab-\left(b^2c^2x^2+b^2\right)\log\left(c^2x^2+1\right)+2\left(abc^2x^2+\sqrt{c^2x^2+1}\right.}{2\left(c^4d^2x^2+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*x^2 + 2*sqrt(c^2*x^2 + 1)*a*b*c*x - b^2*log(c*x + sqrt(c^2*x
^2 + 1))^2 - a^2 + 2*a*b - (b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(a*b*c^
2*x^2 + sqrt(c^2*x^2 + 1)*b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^
```

```
2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^4*d^2*x^2 + c^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2x\,\mathrm{asinh}^2\,(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2abx\,\mathrm{asinh}\,(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2*x/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*x*asinh(
c*x)**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*asinh(c*x)/(c*
*4*x**4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx) + a)^2x}{(c^2dx^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^2, x)
```

3.238 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2} dx$

Optimal. Leaf size=210

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib^2\text{PolyLog}\left(3,-ie^{\sin}\right.}{cd^2}$$

```
[Out] (b*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]
)^2)/(2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]]
)/(c*d^2) - (b^2*ArcTan[c*x])/(c*d^2) - (I*b*(a + b*ArcSinh[c*x])*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(c*d^2) + (I*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^
ArcSinh[c*x]])/(c*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c*d^2) -
(I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d^2)
```

Rubi [A] time = 0.245082, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5690, 5693, 4180, 2531, 2282, 6589, 5717, 203}

$$-\frac{ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{cd^2}+\frac{ib^2\text{PolyLog}\left(3,-ie^{\sin}\right.}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```


```
[Out] (b*(a + b*ArcSinh[c*x]))/(c*d^2*Sqrt[1 + c^2*x^2]) + (x*(a + b*ArcSinh[c*x]
)^2)/(2*d^2*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]]
)/(c*d^2) - (b^2*ArcTan[c*x])/(c*d^2) - (I*b*(a + b*ArcSinh[c*x])*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(c*d^2) + (I*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^
ArcSinh[c*x]])/(c*d^2) + (I*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c*d^2) -
(I*b^2*PolyLog[3, I*E^ArcSinh[c*x]])/(c*d^2)
```


Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
```

```
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+c^2dx^2}\,dx}{2d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{b^2\int \frac{1}{1+c^2x^2}\,dx}{d^2} + \frac{\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,d\ldots\right.}{2cd^2}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{b^2\,\text{ta}\ldots}{}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{b^2\,\text{ta}\ldots}{}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{b^2\,\text{ta}\ldots}{}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{cd^2\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{cd^2} - \frac{b^2\,\text{ta}\ldots}{}$$

Mathematica [A] time = 1.35834, size = 403, normalized size = 1.92

$$\frac{2ab\left(-i\left(c^2x^2+1\right)\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)+i\left(c^2x^2+1\right)\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}+ic^2x^2\sinh^{-1}(cx)\log\left(1-ie^{\sinh^{-1}(cx)}\right)-ic^2x^2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}\ldots}\right.\right.}{c^3x^2+c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^2,x]
```

```
[Out] ((a^2*x)/(1 + c^2*x^2) + (a^2*ArcTan[c*x])/c + (2*a*b*(Sqrt[1 + c^2*x^2] +
c*x*ArcSinh[c*x] + I*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + I*c^2*x^2*Arc
Sinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] - I*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*
x]] - I*c^2*x^2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]] - I*(1 + c^2*x^2)*Po
lyLog[2, (-I)*E^ArcSinh[c*x]] + I*(1 + c^2*x^2)*PolyLog[2, I*E^ArcSinh[c*x]
]))/(c + c^3*x^2) + (2*b^2*(ArcSinh[c*x]/Sqrt[1 + c^2*x^2] + (c*x*ArcSinh[c
*x]^2)/(2 + 2*c^2*x^2) - (I/2)*((-4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + ArcSi
nh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*
x]] + 2*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*PolyL
og[2, I/E^ArcSinh[c*x]] + 2*PolyLog[3, (-I)/E^ArcSinh[c*x]] - 2*PolyLog[3,
I/E^ArcSinh[c*x]])))/c)/(2*d^2)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{Arcsinh}\,(cx))^2}{\left(c^2dx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^2\left(\frac{x}{c^2d^2x^2+d^2}+\frac{\arctan(cx)}{cd^2}\right)+\int \frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^4+2\,c^2d^2x^2+d^2}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(x/(c^2*d^2*x^2 + d^2) + arctan(c*x)/(c*d^2)) + integrate(b^2*log(c
*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2) + 2*a*b*log(c
*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^4 + 2*c
^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^4x^4+2c^2x^2+1}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^4x^4+2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**4 + 2*c**2*x**2 + 1), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
4 + 2*c**2*x**2 + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^2, x)
```

3.239 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=193

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sin}\right.}{2d^2}$$

```
[Out] -((b*c*x*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c*
x])^2/(2*d^2*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSin
h[c*x])])/d^2 + (b^2*Log[1 + c^2*x^2])/(2*d^2) - (b*(a + b*ArcSinh[c*x])*Po
lyLog[2, -E^(2*ArcSinh[c*x])])/d^2 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])])/d^2 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^2) - (b^
2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sin}\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```


```
[Out] -((b*c*x*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) + (a + b*ArcSinh[c*
x])^2/(2*d^2*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSin
h[c*x])])/d^2 + (b^2*Log[1 + c^2*x^2])/(2*d^2) - (b*(a + b*ArcSinh[c*x])*Po
lyLog[2, -E^(2*ArcSinh[c*x])])/d^2 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(
2*ArcSinh[c*x])])/d^2 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^2) - (b^
2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```


Rule *5755*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
```

```
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)}\,dx}{d}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{\text{Subst}\left(\int (a+bx)^2\mathbf{csch}(x)\mathbf{sech}(x)\,dx, x, s\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{b^2\log\left(1+c^2x^2\right)}{2d^2} + \frac{2\,\text{Subst}\left(\int (a+bx)^2 c\right.}{}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right.}{d^2}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{2\sinh^{-1}(cx)}\right.}{d^2}$$

Mathematica [C] time = 1.80625, size = 428, normalized size = 2.22

$$\frac{2ab\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+ab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)+ab\left(\text{s}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^2),x]
```

```
[Out] (a^2/(1 + c^2*x^2) - (a*b*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c*x) -
 (a*b*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 2*a*b*ArcSinh[c*x]
^2 + 4*a*b*ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + 2*a^2*Log[c*x] - a^2*
Log[1 + c^2*x^2] + a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[
c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + a*b*(ArcSinh[c*x]*(ArcSinh[c*
x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + 2*a*b
*PolyLog[2, E^(2*ArcSinh[c*x])] + 2*b^2*((I/24)*Pi^3 - (c*x*ArcSinh[c*x])/S
qrt[1 + c^2*x^2] + ArcSinh[c*x]^2/(2 + 2*c^2*x^2) - (2*ArcSinh[c*x]^3)/3 -
ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] + ArcSinh[c*x]^2*Log[1 - E^(2*A
rcSinh[c*x])] + Log[1 + c^2*x^2]/2 + ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh
[c*x])] + ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + PolyLog[3, -E^(-2*A
rcSinh[c*x])]/2 - PolyLog[3, E^(2*ArcSinh[c*x])]/2))/(2*d^2)
```

Maple [B] time = 0.128, size = 724, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x)
```

```
[Out] a*b/d^2/(c^2*x^2+1)-a*b/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*b^2/d
^2*arcsinh(c*x)^2/(c^2*x^2+1)+b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)-b^2/d^2*arcs
inh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d^2*arcsinh(c*x)*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)+b^2/d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2
))+2*b^2/d^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+b^2/d^2*arcsinh
(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2/d^2*arcsinh(c*x)*polylog(2,c*x+(c
^2*x^2+1)^(1/2))+2*a*b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d^2*poly
log(2,c*x+(c^2*x^2+1)^(1/2))-b^2/d^2*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*c*x+b^2
/d^2*arcsinh(c*x)/(c^2*x^2+1)*c^2*x^2-a*b/d^2/(c^2*x^2+1)^(1/2)*c*x+a*b/d^2
*c^2*x^2/(c^2*x^2+1)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^2+2*a*
b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))+2*a*b/d^2*arcsinh(c*x)*ln(1-
```

```
c*x-(c^2*x^2+1)^(1/2))+a^2/d^2*ln(c*x)-2*b^2/d^2*ln(c*x+(c^2*x^2+1)^(1/2))+
b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+1/2*a^2/d^2/(c^2*x^2+1)-1/2*a^2/d^2
*ln(c^2*x^2+1)-2*b^2/d^2*polylog(3,-c*x-(c^2*x^2+1)^(1/2))-2*b^2/d^2*polylo
g(3,c*x+(c^2*x^2+1)^(1/2))+a*b/d^2*arcsinh(c*x)/(c^2*x^2+1)-2*a*b/d^2*arcsi
nh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^2\left(\frac{1}{c^2d^2x^2+d^2}-\frac{\log\left(c^2x^2+1\right)}{d^2}+\frac{2\,\log\left(x\right)}{d^2}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^5+2\,c^2d^2x^3+d^2x}+\frac{2\,ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^5+2\,c^2d^2x^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(1/(c^2*d^2*x^2 + d^2) - log(c^2*x^2 + 1)/d^2 + 2*log(x)/d^2) + int
egrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^
2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{arsinh}\left(cx\right)^2+2\,ab\,\mathrm{arsinh}\left(cx\right)+a^2}{c^4d^2x^5+2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^5 + 2*c
^2*d^2*x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^4x^5+2c^2x^3+x}\,dx+\int\frac{b^2\,\mathrm{asinh}^2\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx+\int\frac{2ab\,\mathrm{asinh}\left(cx\right)}{c^4x^5+2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(b**2*asinh(c*x)
**2/(c**4*x**5 + 2*c**2*x**3 + x), x) + Integral(2*a*b*asinh(c*x)/(c**4*x**
5 + 2*c**2*x**3 + x), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{\left(c^2dx^2+d\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x), x)
```

3.240 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=287

$$\frac{3ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{3ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2b^2c\text{PolyLog}\left(2,\right.}{d^2}$$

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) - (a + b*ArcSinh[c*x]
)^2/(d^2*x*(1 + c^2*x^2)) - (3*c^2*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^
2*x^2)) - (3*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 + (b^2*c*
ArcTan[c*x])/d^2 - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d^2
 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^2 + ((3*I)*b*c*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - ((3*I)*b*c*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/d^2 + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d^
2 - ((3*I)*b^2*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d^2 + ((3*I)*b^2*c*PolyLo
g[3, I*E^ArcSinh[c*x]])/d^2
```

Rubi [A] time = 0.543779, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 5755, 5760, 4182, 2279, 2391}

$$\frac{3ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{3ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2b^2c\text{PolyLog}\left(2,\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```


```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*Sqrt[1 + c^2*x^2])) - (a + b*ArcSinh[c*x]
)^2/(d^2*x*(1 + c^2*x^2)) - (3*c^2*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^
2*x^2)) - (3*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 + (b^2*c*
ArcTan[c*x])/d^2 - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/d^2
 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^2 + ((3*I)*b*c*(a + b*ArcSinh[c*
x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 - ((3*I)*b*c*(a + b*ArcSinh[c*x])*
PolyLog[2, I*E^ArcSinh[c*x]])/d^2 + (2*b^2*c*PolyLog[2, E^ArcSinh[c*x]])/d^
2 - ((3*I)*b^2*c*PolyLog[3, (-I)*E^ArcSinh[c*x]])/d^2 + ((3*I)*b^2*c*PolyLo
```

```
g[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2}$$

$$= \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} + \frac{(2bc)\int \frac{a+b\sinh}{x\sqrt{1+c^2}}}{d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{2b^2c\tan^{-1}(cx)}{d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-}\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-}\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-}\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2x\left(1+c^2x^2\right)} - \frac{3c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2\left(1+c^2x^2\right)} - \frac{3c\left(a+b\sinh^{-}\right.}{}$$

Mathematica [A] time = 7.19479, size = 549, normalized size = 1.91

$$\frac{2abc\left(\frac{3}{4}i\left(2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-\frac{1}{2}\sinh^{-1}(cx)^2+2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-\frac{3}{4}i\left(2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^2),x]
```

```
[Out] -(a^2/(d^2*x)) - (a^2*c^2*x)/(2*d^2*(1 + c^2*x^2)) - (3*a^2*c*ArcTan[c*x])/
(2*d^2) + (2*a*b*c*((Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x])/(4*(-1 - I*c*x)) -
 ArcSinh[c*x]/(c*x) - (I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(4*(I + c*x)) -
ArcTanh[Sqrt[1 + c^2*x^2]] + ((3*I)/4)*(-ArcSinh[c*x]^2/2 + 2*ArcSinh[c*x]*
Log[1 + I*E^ArcSinh[c*x]] + 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((3*I)/4)*
(-ArcSinh[c*x]^2/2 + 2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2
, I*E^ArcSinh[c*x]])))/d^2 + (b^2*c*((-2*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
```

```
(c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 4*ArcTan[Tanh[ArcSinh[c*x]/2]] - ArcSi
nh[c*x]^2*Coth[ArcSinh[c*x]/2] + 4*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])]
+ (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (3*I)*ArcSinh[c*x]^2*Log
[1 + I/E^ArcSinh[c*x]] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*Poly
Log[2, -E^(-ArcSinh[c*x])] + (6*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^ArcSinh[c
*x]] - (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - 4*PolyLog[2, E^(-A
rcSinh[c*x])] + (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[3, I/
E^ArcSinh[c*x]] + ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(2*d^2)
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{x^2\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a^2\left(\frac{3\,c^2x^2 + 2}{c^2d^2x^3 + d^2x} + \frac{3\,c\arctan\,(cx)}{d^2}\right) + \int \frac{b^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^4d^2x^6 + 2\,c^2d^2x^4 + d^2x^2} + \frac{2\,ab\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^4d^2x^6 + 2\,c^2d^2x^4 + d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((3*c^2*x^2 + 2)/(c^2*d^2*x^3 + d^2*x) + 3*c*arctan(c*x)/d^2) + in
tegrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d
^2*x^2) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +
 d^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2 + 2\,ab\,\text{arsinh}\,(cx) + a^2}{c^4d^2x^6 + 2\,c^2d^2x^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{b^2\operatorname{asinh}^2(cx)}{c^4x^6+2c^2x^4+x^2}\,dx + \int \frac{2ab\operatorname{asinh}(cx)}{c^4x^6+2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**6 + 2*c**2*x**4 + x**2), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**6 + 2*c**2*x**4 + x**2), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{(c^2dx^2+d)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^2), x)
```

3.241 $\int \frac{(a+b\sinh^{-1}(cx))^2}{x^3(d+c^2dx^2)^2}\,dx$

Optimal. Leaf size=253

$$\frac{2bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{b^2c^2\text{PolyLog}\left(3,\right.}{d^2}$$

```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*x*Sqrt[1 + c^2*x^2])) - (c^2*(a + b*ArcSi
nh[c*x])^2)/(d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2*d^2*x^2*(1 + c^
2*x^2)) + (4*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 +
(b^2*c^2*Log[x])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + (2*b*c^2*(a + b
*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^2 - (2*b*c^2*(a + b*ArcSi
nh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2 - (b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/d^2 + (b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/d^2
```

Rubi [A] time = 0.580781, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5747, 5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260, 271, 191, 5732, 446, 72}

$$\frac{2bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{2bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}-\frac{b^2c^2\text{PolyLog}\left(3,\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```


```
[Out] -((b*c*(a + b*ArcSinh[c*x]))/(d^2*x*Sqrt[1 + c^2*x^2])) - (c^2*(a + b*ArcSi
nh[c*x])^2)/(d^2*(1 + c^2*x^2)) - (a + b*ArcSinh[c*x])^2/(2*d^2*x^2*(1 + c^
2*x^2)) + (4*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^2 +
(b^2*c^2*Log[x])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + (2*b*c^2*(a + b
*ArcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^2 - (2*b*c^2*(a + b*ArcSi
nh[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^2 - (b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/d^2 + (b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/d^2
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(\right.}{2d^2x^2\left(1+c^2\right.}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \frac{\left(2c^2\right)\text{Subst}\left(\int(a+\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} - \frac{b^2c^2\log\left(1+c^2x^2\right)}{d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(c\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(c\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(c\right.}{}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}} - \frac{c^2\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\left(1+c^2x^2\right)} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^2x^2\left(1+c^2x^2\right)} + \frac{4c^2\left(a+b\sinh^{-1}(c\right.}{}$$

Mathematica [B] time = 0.925183, size = 594, normalized size = 2.35

$$\frac{-4abc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right) + 8bc^2\text{PolyLog}\left(2,\frac{ce^{\sinh^{-1}(cx)}}{\sqrt{-c^2}}\right)\left(a+b\sinh^{-1}(cx)\right) + 8bc^2\text{PolyLog}\left(2,\frac{\sqrt{-c^2}e^{\sinh^{-1}(cx)}}{c}\right)\left(a+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^2),x]
```

```
[Out] ((-2*a^2)/x^2 - (2*a*b*c)/(x*Sqrt[1 + c^2*x^2]) + a^2/(x^2 + c^2*x^4) + 4*a
^2*c^2*ArcSinh[c*x] - (4*a*b*ArcSinh[c*x])/x^2 - (2*b^2*c*ArcSinh[c*x])/(x*
Sqrt[1 + c^2*x^2]) + (2*a*b*ArcSinh[c*x])/(x^2 + c^2*x^4) - (2*b^2*ArcSinh[
```

```
c*x]^2)/x^2 + (b^2*ArcSinh[c*x]^2)/(x^2 + c^2*x^4) + 8*a*b*c^2*ArcSinh[c*x]
*Log[1 + (c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 +
(c*E^ArcSinh[c*x])/Sqrt[-c^2]] + 8*a*b*c^2*ArcSinh[c*x]*Log[1 + (Sqrt[-c^2]
*E^ArcSinh[c*x])/c] + 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 + (Sqrt[-c^2]*E^ArcSin
h[c*x])/c] - 4*a^2*c^2*Log[1 - E^(2*ArcSinh[c*x])] - 8*a*b*c^2*ArcSinh[c*x]
*Log[1 - E^(2*ArcSinh[c*x])] - 4*b^2*c^2*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSin
h[c*x])] + 2*b^2*c^2*Log[x] + 2*a^2*c^2*Log[1 + c^2*x^2] - b^2*c^2*Log[1 +
c^2*x^2] + 8*b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, (c*E^ArcSinh[c*x])/Sqrt[
-c^2]] + 8*b*c^2*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[-c^2]*E^ArcSinh[c*x]
)/c] - 4*a*b*c^2*PolyLog[2, E^(2*ArcSinh[c*x])] - 4*b^2*c^2*ArcSinh[c*x]*Po
lyLog[2, E^(2*ArcSinh[c*x])] - 8*b^2*c^2*PolyLog[3, (c*E^ArcSinh[c*x])/Sqrt
[-c^2]] - 8*b^2*c^2*PolyLog[3, (Sqrt[-c^2]*E^ArcSinh[c*x])/c] + 2*b^2*c^2*P
olyLog[3, E^(2*ArcSinh[c*x])])/(2*d^2)
```

Maple [B] time = 0.157, size = 799, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2*a^2/d^2/x^2-1/2*c^2*a^2/d^2/(c^2*x^2+1)+c^2*b^2/d^2*ln(1+c*x+(c^2*x^2+
1)^(1/2))+c^2*a^2/d^2*ln(c^2*x^2+1)+4*c^2*b^2/d^2*polylog(3,-c*x-(c^2*x^2+1
)^(1/2))+4*c^2*b^2/d^2*polylog(3,c*x+(c^2*x^2+1)^(1/2))-2*c^2*a^2/d^2*ln(c*
x)-c^2*b^2/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+c^2*b^2/d^2*ln(c*x+(c^2*x^2+
1)^(1/2)-1)-c^2*b^2/d^2*arcsinh(c*x)^2/(c^2*x^2+1)+2*c^2*b^2/d^2*arcsinh(c*
x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2*c^2*b^2/d^2*arcsinh(c*x)*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))^2)-2*c^2*b^2/d^2*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1
)^(1/2))-4*c^2*b^2/d^2*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-2*c^2
*b^2/d^2*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-4*c^2*b^2/d^2*arcsinh(c
*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+2*c^2*a*b/d^2*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)-4*c^2*a*b/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-4*c^2*a*b/d^2
*polylog(2,c*x+(c^2*x^2+1)^(1/2))-b^2*c^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2)
)^2)/d^2-c*b^2/d^2*arcsinh(c*x)/x/(c^2*x^2+1)^(1/2)-c*a*b/d^2/x/(c^2*x^2+1)
^(1/2)-2*c^2*a*b/d^2*arcsinh(c*x)/(c^2*x^2+1)+4*c^2*a*b/d^2*arcsinh(c*x)*ln
(1+(c*x+(c^2*x^2+1)^(1/2))^2)-4*c^2*a*b/d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+
1)^(1/2))-4*c^2*a*b/d^2*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-a*b/d^2*ar
csinh(c*x)/x^2/(c^2*x^2+1)-1/2*b^2/d^2*arcsinh(c*x)^2/x^2/(c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{2c^2\log\left(c^2x^2+1\right)}{d^2}-\frac{4c^2\log\left(x\right)}{d^2}-\frac{2c^2x^2+1}{c^2d^2x^4+d^2x^2}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4d^2x^7+2c^2d^2x^5+d^2x^3}+\frac{2ab\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^4d^2x^7+2c^2d^2x^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*c^2*log(c^2*x^2 + 1)/d^2 - 4*c^2*log(x)/d^2 - (2*c^2*x^2 + 1)/(c
^2*d^2*x^4 + d^2*x^2)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*
d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^
4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^7 + 2*c
^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^4x^7+2c^2x^5+x^3}\,dx+\int\frac{b^2\operatorname{asinh}^2\left(cx\right)}{c^4x^7+2c^2x^5+x^3}\,dx+\int\frac{2ab\operatorname{asinh}\left(cx\right)}{c^4x^7+2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**7 + 2*c**2*x**5 + x**3), x) + Integral(2*a*b*asinh(c*x)/(c*
```

```
*4*x**7 + 2*c**2*x**5 + x**3), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^2}{\left(c^2dx^2+d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^3), x)
```

3.242 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=401

$$-\frac{5ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{13b^2c^3\text{PolyLog}}{3}$$

```
[Out] -(b^2*c^2)/(3*d^2*x) + (2*b*c^3*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[1 + c^2*x
^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*Ar
cSinh[c*x])^2/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])^2)/(3
*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*x^
2)) + (5*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 - (b^2*c^3*
ArcTan[c*x])/d^2 + (26*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/
(3*d^2) + (13*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^2) - ((5*I)*b*c^3*(
a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + ((5*I)*b*c^3*(a
+ b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^2 - (13*b^2*c^3*PolyLog[2
, E^ArcSinh[c*x]])/(3*d^2) + ((5*I)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c*x]]
)/d^2 - ((5*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^2
```

Rubi [A] time = 0.963293, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 5755, 5760, 4182, 2279, 2391, 325}

$$-\frac{5ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{5ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2}+\frac{13b^2c^3\text{PolyLog}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```


```
[Out] -(b^2*c^2)/(3*d^2*x) + (2*b*c^3*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt[1 + c^2*x
^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]) - (a + b*Ar
cSinh[c*x])^2/(3*d^2*x^3*(1 + c^2*x^2)) + (5*c^2*(a + b*ArcSinh[c*x])^2)/(3
*d^2*x*(1 + c^2*x^2)) + (5*c^4*x*(a + b*ArcSinh[c*x])^2)/(2*d^2*(1 + c^2*x^
2)) + (5*c^3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/d^2 - (b^2*c^3*
ArcTan[c*x])/d^2 + (26*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/
```

```
(3*d^2) + (13*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^2) - ((5*I)*b*c^3*(
a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^2 + ((5*I)*b*c^3*(a
+ b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^2 - (13*b^2*c^3*PolyLog[2
, E^ArcSinh[c*x]])/(3*d^2) + ((5*I)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c*x]]
)/d^2 - ((5*I)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^2
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
```

)

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} - \frac{1}{3}\left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^2}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x\left(1+c^2x^2\right)} + \left(5c^4\right)\int \frac{\left(a+b\sin\right.}{\left(d+c\right.}$$

$$= -\frac{b^2c^2}{3d^2x} - \frac{13bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+\right.}{3d^2x}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\right.}{3d^2x\left(\right.}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\right.}{3d^2x\left(\right.}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\right.}{3d^2x\left(\right.}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\right.}{3d^2x\left(\right.}$$

$$= -\frac{b^2c^2}{3d^2x} + \frac{2bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2x^3\left(1+c^2x^2\right)} + \frac{5c^2\left(a+b\right.}{3d^2x\left(\right.}$$

Mathematica [A] time = 8.73125, size = 764, normalized size = 1.91

$$2ab\left(-\frac{5}{4}ic^4\left(\frac{2\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)}{c}\right) + \frac{5}{4}ic^4\left(\frac{2\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sin}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^2),x]
```

```
[Out] -a^2/(3*d^2*x^3) + (2*a^2*c^2)/(d^2*x) + (a^2*c^4*x)/(2*d^2*(1 + c^2*x^2))
+ (5*a^2*c^3*ArcTan[c*x])/(2*d^2) + (2*a*b*(-(c*Sqrt[1 + c^2*x^2])/(6*x^2)
```

```
- (c^3*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(4*(-1 - I*c*x)) - ArcSinh[c*x
]/(3*x^3) + (c^4*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(4*(I*c + c^2*x)) +
(c^3*ArcTanh[Sqrt[1 + c^2*x^2]])/6 - 2*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[S
qrt[1 + c^2*x^2]]) - ((5*I)/4)*c^4*(-ArcSinh[c*x]^2/(2*c) + (2*ArcSinh[c*x]
*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + ((
5*I)/4)*c^4*(-ArcSinh[c*x]^2/(2*c) + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*
x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c)))/d^2 + (b^2*c^3*((24*ArcSinh[
c*x])/Sqrt[1 + c^2*x^2] + (12*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 48*ArcTan
[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 26*ArcSinh[c*x]^2*Coth[Ar
cSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*ArcSinh[c*x]^2
*Csch[ArcSinh[c*x]/2]^4)/2 - 104*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] -
(60*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (60*I)*ArcSinh[c*x]^2*Log
[1 + I/E^ArcSinh[c*x]] + 104*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 104*
PolyLog[2, -E^(-ArcSinh[c*x])] - (120*I)*ArcSinh[c*x]*PolyLog[2, (-I)/E^Arc
Sinh[c*x]] + (120*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] + 104*PolyLo
g[2, E^(-ArcSinh[c*x])] - (120*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (120*I)
*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 - (8*
ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[ArcSinh[c*x]/2] -
 26*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^2)
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{x^4\left(c^2dx^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\left(\frac{15\,c^3\arctan\,(cx)}{d^2} + \frac{15\,c^4x^4 + 10\,c^2x^2 - 2}{c^2d^2x^5 + d^2x^3}\right)a^2 + \int \frac{b^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4} + \frac{2\,ab\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^3*arctan(c*x)/d^2 + (15*c^4*x^4 + 10*c^2*x^2 - 2)/(c^2*d^2*x^5 +
d^2*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^4*d^2*x^8 +
 2*c^2*d^2*x^6 + d^2*x^4) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^4*d^2*x^8
 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^4d^2x^8 + 2\,c^2d^2x^6 + d^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^4*d^2*x^8 + 2*c
^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^4x^8+2c^2x^6+x^4}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^4x^8+2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(b**2*asinh(c
*x)**2/(c**4*x**8 + 2*c**2*x**6 + x**4), x) + Integral(2*a*b*asinh(c*x)/(c*
*4*x**8 + 2*c**2*x**6 + x**4), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^2*x^4), x)
```

3.243 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib^2\text{PolyLog}\left(3,-i\right.}{4c^5d^3}$$

```
[Out] -(b^2*x)/(12*c^4*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c^5*d^3*(
1 + c^2*x^2)^(3/2)) - (5*b*(a + b*ArcSinh[c*x]))/(4*c^5*d^3*Sqrt[1 + c^2*x^
2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a +
b*ArcSinh[c*x])^2)/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*Ar
cTan[E^ArcSinh[c*x]])/(4*c^5*d^3) + (7*b^2*ArcTan[c*x])/(6*c^5*d^3) - (((3*
I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) + (
((3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3) +
(((3*I)/4)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) - (((3*I)/4)*b^2*
PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rubi [A] time = 0.545126, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5751, 5693, 4180, 2531, 2282, 6589, 5717, 203, 266, 43, 5732, 12, 385}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3}+\frac{3ib^2\text{PolyLog}\left(3,-i\right.}{4c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```


```
[Out] -(b^2*x)/(12*c^4*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c^5*d^3*(
1 + c^2*x^2)^(3/2)) - (5*b*(a + b*ArcSinh[c*x]))/(4*c^5*d^3*Sqrt[1 + c^2*x^
2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) - (3*x*(a +
b*ArcSinh[c*x])^2)/(8*c^4*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*Ar
cTan[E^ArcSinh[c*x]])/(4*c^5*d^3) + (7*b^2*ArcTan[c*x])/(6*c^5*d^3) - (((3*
I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) + (
((3*I)/4)*b*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^5*d^3) +
(((3*I)/4)*b^2*PolyLog[3, (-I)*E^ArcSinh[c*x]])/(c^5*d^3) - (((3*I)/4)*b^2*
PolyLog[3, I*E^ArcSinh[c*x]])/(c^5*d^3)
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}+\frac{3\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{2c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}(\right.}{8c^4d^3\left(1+c^2x\right.}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{3x\left(a+b\sinh^{-1}\right.}{8c^4d^3\left(1+c^2\right.}$$

$$= -\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx\right.}{4c^2d^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx\right.}{4c^2d^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx\right.}{4c^2d^3\left(1+c^2x^2\right)}$$

$$= -\frac{b^2x}{12c^4d^3\left(1+c^2x^2\right)}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^5d^3\left(1+c^2x^2\right)^{3/2}}-\frac{5b\left(a+b\sinh^{-1}(cx)\right)}{4c^5d^3\sqrt{1+c^2x^2}}-\frac{x^3\left(a+b\sinh^{-1}(cx\right.}{4c^2d^3\left(1+c^2x^2\right)}$$

Mathematica [A] time = 3.19315, size = 552, normalized size = 1.72

$$\frac{\frac{9}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)-\frac{9}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*c*x)/(1 + c^2*x^2)^2 - (15*a^2*c*x)/(1 + c^2*x^2) + (15*a*b*(Sqrt[1
 + c^2*x^2] - I*ArcSinh[c*x]))/(-1 + I*c*x) + (15*a*b*(Sqrt[1 + c^2*x^2] +
I*ArcSinh[c*x]))/(-1 - I*c*x) - (I*a*b*((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*
ArcSinh[c*x]))/(-I + c*x)^2 + (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*Arc
Sinh[c*x]))/(I + c*x)^2 + 9*a^2*ArcTan[c*x] + ((9*I)/2)*a*b*(ArcSinh[c*x]*(
ArcSinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c
*x]]) - ((9*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c
*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((-2*c*x)/(1 + c^2*x^2) + (4*
ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (30*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (
6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (15*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^
2) + 56*ArcTan[Tanh[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^Arc
Sinh[c*x]] + (9*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSin
h[c*x]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E
^ArcSinh[c*x]] - (18*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3,
 I/E^ArcSinh[c*x]]))/(24*c^5*d^3)
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b\mathrm{Arcsinh}(cx))^2}{\left(c^2 d x^2 + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\, a^2\left(\frac{5\, c^2 x^3 + 3\, x}{c^8 d^3 x^4 + 2\, c^6 d^3 x^2 + c^4 d^3} - \frac{3\, \arctan(cx)}{c^5 d^3}\right) + \int \frac{b^2 x^4 \log\left(cx + \sqrt{c^2 x^2 + 1}\right)^2}{c^6 d^3 x^6 + 3\, c^4 d^3 x^4 + 3\, c^2 d^3 x^2 + d^3} + \frac{2\, abx^4 \log\left(cx + \sqrt{c^2 x^2 +\right.}{c^6 d^3 x^6 + 3\, c^4 d^3 x^4 + 3\, c^2 d^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((5*c^2*x^3 + 3*x)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 3*arc
tan(c*x)/(c^5*d^3)) + integrate(b^2*x^4*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6
*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^4*log(c*x + sqrt(
c^2*x^2 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4\,\text{arsinh}\,(cx)^2 + 2\,abx^4\,\text{arsinh}\,(cx) + a^2x^4}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^4}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^4\,\text{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^4\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**4/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**4*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**4*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^4}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^3, x)
```

3.244 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=167

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}-\frac{b^2}{12c^4d^3\left(c^2x^2+1\right)}-$$

```
[Out] -b^2/(12*c^4*d^3*(1 + c^2*x^2)) + (b*x^3*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1
+ c^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(2*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (a + b*ArcSinh[c*x])^2/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x])^2)/(4*d^
3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/(3*c^4*d^3)
```

Rubi [A] time = 0.336759, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5723, 5751, 5675, 260, 266, 43}

$$\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(c^2x^2+1\right)^2}+\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{c^2x^2+1}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3}-\frac{b^2}{12c^4d^3\left(c^2x^2+1\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -b^2/(12*c^4*d^3*(1 + c^2*x^2)) + (b*x^3*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1
+ c^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(2*c^3*d^3*Sqrt[1 + c^2*x^2]
) - (a + b*ArcSinh[c*x])^2/(4*c^4*d^3) + (x^4*(a + b*ArcSinh[c*x])^2)/(4*d^
3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/(3*c^4*d^3)
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3 \left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3}$$

$$= \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{b^2\int \frac{x^3}{\left(1+c^2x^2\right)^2}\,dx}{6d^3} - \frac{b\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}}{2cd^3}$$

$$= \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}} + \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{b^2\,\mathrm{Subst}\left(\int\right.}{1}$$

$$= \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^4d^3} + \frac{x^4\left(a+b\sinh^{-1}\right.}{4d^3\left(1+c^2x\right.}$$

$$= -\frac{b^2}{12c^4d^3\left(1+c^2x^2\right)} + \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{2c^3d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx\right.}{4c^4d^3}$$

Mathematica [A] time = 0.39542, size = 186, normalized size = 1.11

$$-\frac{6a^2c^2x^2+3a^2-8abc^3x^3\sqrt{c^2x^2+1}-6abcx\sqrt{c^2x^2+1}+2b\sinh^{-1}(cx)\left(a\left(6c^2x^2+3\right)-bcx\sqrt{c^2x^2+1}\left(4c^2x^2+3\right)\right)+b^2}{12c^4d^3\left(c^2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] -(3*a^2 + b^2 + 6*a^2*c^2*x^2 + b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 + c^2*x^2] -
 8*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] + 2*b*(-(b*c*x*Sqrt[1 + c^2*x^2]*(3 + 4*c^
2*x^2)) + a*(3 + 6*c^2*x^2))*ArcSinh[c*x] + 3*b^2*(1 + 2*c^2*x^2)*ArcSinh[c
*x]^2 + 4*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(12*c^4*d^3*(1 + c^2*x^2)^2)
```

Maple [B] time = 0.248, size = 523, normalized size = 3.1

$$-\frac{a^2}{2c^4d^3\left(c^2x^2+1\right)} + \frac{a^2}{4c^4d^3\left(c^2x^2+1\right)^2} + \frac{4b^2\mathrm{Arcsinh}\,(cx)}{3c^4d^3} + \frac{2b^2\mathrm{Arcsinh}\,(cx)\,x^3}{3cd^3\left(c^4x^4+2c^2x^2+1\right)}\sqrt{c^2x^2+1} - \frac{2b^2\mathrm{Arcsinh}\,(cx)}{3d^3\left(c^4x^4+2c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/2/c^4*a^2/d^3/(c^2*x^2+1)+1/4/c^4*a^2/d^3/(c^2*x^2+1)^2+4/3/c^4*b^2/d^3*
arcsinh(c*x)+2/3/c*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)*x^3-2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^4-1/2/c^2*b^2/d^3
/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2*x^2+1/2/c^3*b^2/d^3/(c^4*x^4+2*c^2*x^
2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-4/3/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)
*arcsinh(c*x)*x^2-1/4/c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2-1/12
/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*x^2-2/3/c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1
)*arcsinh(c*x)-1/12/c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-2/3/c^4*b^2/d^3*ln(1+
(c*x+(c^2*x^2+1)^(1/2))^2)-1/c^4*a*b/d^3/(c^2*x^2+1)*arcsinh(c*x)+1/2/c^4*a
*b/d^3/(c^2*x^2+1)^2*arcsinh(c*x)-1/6/c^3*a*b/d^3/(c^2*x^2+1)^(3/2)*x+2/3/c
^3*a*b/d^3*x/(c^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,c^2x^2+1\right)a^2}{4\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}-\frac{\left(2\,b^2c^2x^2+b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{4\left(c^8d^3x^4+2\,c^6d^3x^2+c^4d^3\right)}+\int\frac{\left(3\,b^2c^2x^2+2\left(2\,abc^4+b^2c^4\right)x^4+b^2+\left(b^2cx\right.\right.}{2\left(c^{10}d^3x^7+3\,c^8d^3x^5+3\,c^6d^3x^3+c^4d^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*x^2 + 1)*a^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - 1/4*(2*b
^2*c^2*x^2 + b^2)*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^8*d^3*x^4 + 2*c^6*d^3*x
^2 + c^4*d^3) + integrate(1/2*(3*b^2*c^2*x^2 + 2*(2*a*b*c^4 + b^2*c^4)*x^4
+ b^2 + (b^2*c*x + 2*(2*a*b*c^3 + b^2*c^3)*x^3)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1))/(c^10*d^3*x^7 + 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 + c^4*d^
3*x + (c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [A] time = 2.8493, size = 605, normalized size = 3.62

$$\frac{8\,abc^4x^4-\left(6\,a^2-16\,ab+b^2\right)c^2x^2-3\left(2\,b^2c^2x^2+b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+8\,ab-b^2-4\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(8*a*b*c^4*x^4 - (6*a^2 - 16*a*b + b^2)*c^2*x^2 - 3*(2*b^2*c^2*x^2 + b
^2)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 3*a^2 + 8*a*b - b^2 - 4*(b^2*c^4*x^4 +
 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + (4*b^2*c^3*x^3
+ 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x
^4 + 2*a*b*c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(4*a*b*c^3*x^3
+ 3*a*b*c*x)*sqrt(c^2*x^2 + 1))/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^3}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^3\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^3\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**3*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**3*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2x^3}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^3/(c^2*d*x^2 + d)^3, x)
```

3.245 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=318

$$-\frac{ib\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib^2\mathrm{PolyLog}\left(3,-ie^{\sinh^{-1}(cx)}\right)}{4c^3d^3}$$

```
[Out] (b^2*x)/(12*c^2*d^3*(1 + c^2*x^2)) - (b*(a + b*ArcSinh[c*x]))/(6*c^3*d^3*(1
 + c^2*x^2)^(3/2)) + (b*(a + b*ArcSinh[c*x]))/(4*c^3*d^3*Sqrt[1 + c^2*x^2])
 - (x*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSi
nh[c*x])^2)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^Ar
cSinh[c*x]])/(4*c^3*d^3) - (b^2*ArcTan[c*x])/(6*c^3*d^3) - ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b^2*PolyLog[
3, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) - ((I/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]
])/(c^3*d^3)
```

Rubi [A] time = 0.417253, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5751, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199}

$$-\frac{ib\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3}+\frac{ib^2\mathrm{PolyLog}\left(3,-ie^{\sinh^{-1}(cx)}\right)}{4c^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```


```
[Out] (b^2*x)/(12*c^2*d^3*(1 + c^2*x^2)) - (b*(a + b*ArcSinh[c*x]))/(6*c^3*d^3*(1
 + c^2*x^2)^(3/2)) + (b*(a + b*ArcSinh[c*x]))/(4*c^3*d^3*Sqrt[1 + c^2*x^2])
 - (x*(a + b*ArcSinh[c*x])^2)/(4*c^2*d^3*(1 + c^2*x^2)^2) + (x*(a + b*ArcSi
nh[c*x])^2)/(8*c^2*d^3*(1 + c^2*x^2)) + ((a + b*ArcSinh[c*x])^2*ArcTan[E^Ar
cSinh[c*x]])/(4*c^3*d^3) - (b^2*ArcTan[c*x])/(6*c^3*d^3) - ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b*(a + b*
ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c^3*d^3) + ((I/4)*b^2*PolyLog[
3, (-I)*E^ArcSinh[c*x]])/(c^3*d^3) - ((I/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]
])/(c^3*d^3)
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = -\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}dx}{2cd^3}+\frac{\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}dx}{4c^2d}$$

$$= -\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2d^3\left(1+c^2x^2\right)}+\frac{b^2\int\frac{1}{\left(1+c^2x^2\right)^2}dx}{6c^2d^3}$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}$$

$$= \frac{b^2x}{12c^2d^3\left(1+c^2x^2\right)}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{6c^3d^3\left(1+c^2x^2\right)^{3/2}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{4c^3d^3\sqrt{1+c^2x^2}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}$$

Mathematica [A] time = 2.39617, size = 550, normalized size = 1.73

$$\frac{\frac{3}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)-\frac{3}{2}iab\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((-6*a^2*c*x)/(1 + c^2*x^2)^2 + (3*a^2*c*x)/(1 + c^2*x^2) + (a*b*((2 + I*c*
x)*Sqrt[1 + c^2*x^2] + (3*I)*ArcSinh[c*x]))/(-I + c*x)^2 + (3*a*b*((-I)*Sqr
t[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (3*a*b*(I*Sqrt[1 + c^2*x^2] +
ArcSinh[c*x]))/(I + c*x) - (I*a*b*((2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSin
h[c*x]))/(I + c*x)^2 + 3*a^2*ArcTan[c*x] + ((3*I)/2)*a*b*(ArcSinh[c*x]*(Arc
```

```
Sinh[c*x] - 4*Log[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]
]) - ((3*I)/2)*a*b*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]
]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]) + b^2*((2*c*x)/(1 + c^2*x^2) - (4*ArcS
inh[c*x])/(1 + c^2*x^2)^(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x
*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (3*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 8
*ArcTan[Tanh[ArcSinh[c*x]/2]] - (3*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*
x]] + (3*I)*ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (6*I)*ArcSinh[c*x]*P
olyLog[2, (-I)/E^ArcSinh[c*x]] + (6*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[
c*x]] - (6*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (6*I)*PolyLog[3, I/E^ArcSin
h[c*x]]))/(24*c^3*d^3)
```

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b\text{Arcsinh}(cx))^2}{\left(c^2dx^2 + d\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}a^2\left(\frac{c^2x^3 - x}{c^6d^3x^4 + 2c^4d^3x^2 + c^2d^3} + \frac{\arctan(cx)}{c^3d^3}\right) + \int \frac{b^2x^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^6d^3x^6 + 3c^4d^3x^4 + 3c^2d^3x^2 + d^3} + \frac{2abx^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^6d^3x^6 + 3c^4d^3x^4 + 3c^2d^3x^2 + d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((c^2*x^3 - x)/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + arctan(c*x
)/(c^3*d^3)) + integrate(b^2*x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^
6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*x^2*log(c*x + sqrt(c^2*x^2
 + 1))/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\,\text{arsinh}\,(cx)^2 + 2\,abx^2\,\text{arsinh}\,(cx) + a^2x^2}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/(c^6*d
^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x^2\,\text{asinh}^2\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx^2\,\text{asinh}\,(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integ
ral(b**2*x**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x)
 + Integral(2*a*b*x**2*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 +
1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^2}{(c^2dx^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^3, x)
```

3.246 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=145

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{c^2x^2+1}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b^2}{12c^2d^3\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{6c^2d^3}$$

```
[Out] b^2/(12*c^2*d^3*(1 + c^2*x^2)) + (b*x*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c
^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])^2/(4*c^2*d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/
(6*c^2*d^3)
```

Rubi [A] time = 0.145654, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5717, 5690, 5687, 260, 261}

$$\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{c^2x^2+1}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(c^2x^2+1\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{b^2}{12c^2d^3\left(c^2x^2+1\right)}-\frac{b^2\log\left(c^2x^2+1\right)}{6c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] b^2/(12*c^2*d^3*(1 + c^2*x^2)) + (b*x*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c
^2*x^2)^(3/2)) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^3*Sqrt[1 + c^2*x^2]) - (
a + b*ArcSinh[c*x])^2/(4*c^2*d^3*(1 + c^2*x^2)^2) - (b^2*Log[1 + c^2*x^2])/
(6*c^2*d^3)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx=-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}+\frac{b\int\frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2cd^3}$$

$$=\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-\frac{b^2\int\frac{x}{\left(1+c^2x^2\right)^2}\,dx}{6d^3}+\frac{b\int\frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3cd^3}$$

$$=\frac{b^2}{12c^2d^3\left(1+c^2x^2\right)}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-$$

$$=\frac{b^2}{12c^2d^3\left(1+c^2x^2\right)}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^3\sqrt{1+c^2x^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d^3\left(1+c^2x^2\right)^2}-$$

Mathematica [A] time = 0.24088, size = 152, normalized size = 1.05

$$\frac{-3a^2+4abc^3x^3\sqrt{c^2x^2+1}+6abcx\sqrt{c^2x^2+1}+2b\sinh^{-1}(cx)\left(bcx\sqrt{c^2x^2+1}\left(2c^2x^2+3\right)-3a\right)+b^2c^2x^2-3b^2\sinh^{-1}(cx)^2}{12d^3\left(c^3x^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^3,x]
```

```
[Out] (-3*a^2 + b^2 + b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 + c^2*x^2] + 4*a*b*c^3*x^3*S
qrt[1 + c^2*x^2] + 2*b*(-3*a + b*c*x*Sqrt[1 + c^2*x^2]*(3 + 2*c^2*x^2))*Arc
Sinh[c*x] - 3*b^2*ArcSinh[c*x]^2 - 2*(b + b*c^2*x^2)^2*Log[1 + c^2*x^2])/(1
2*d^3*(c + c^3*x^2)^2)
```

Maple [B] time = 0.095, size = 432, normalized size = 3.

$$-\frac{a^2}{4c^2d^3\left(c^2x^2+1\right)^2}+\frac{2b^2\mathrm{Arcsinh}(cx)}{3c^2d^3}+\frac{cb^2\mathrm{Arcsinh}(cx)\,x^3}{3d^3\left(c^4x^4+2c^2x^2+1\right)}\sqrt{c^2x^2+1}-\frac{c^2b^2\mathrm{Arcsinh}(cx)\,x^4}{3d^3\left(c^4x^4+2c^2x^2+1\right)}+\frac{b^2\mathrm{Arcsinh}}{2cd^3\left(c^4x^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] -1/4/c^2*a^2/d^3/(c^2*x^2+1)^2+2/3/c^2*b^2/d^3*arcsinh(c*x)+1/3*c*b^2/d^3/(
c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-1/3*c^2*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^4+1/2/c*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arc
sinh(c*x)*(c^2*x^2+1)^(1/2)*x-2/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x
)*x^2-1/4/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2+1/12*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*x^2-1/3/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+1
/12/c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)-1/3/c^2*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)
^(1/2))^2)-1/2/c^2*a*b/d^3/(c^2*x^2+1)^2*arcsinh(c*x)+1/6/c*a*b/d^3/(c^2*x^
2+1)^(3/2)*x+1/3/c*a*b/d^3*x/(c^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{4\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}-\frac{a^2}{4\left(c^6d^3x^4+2\,c^4d^3x^2+c^2d^3\right)}+\int\frac{\left(\left(4\,abc^2+b^2c^2\right)x^2+\sqrt{c^2x^2+1}\left(4\,abc+b^2c\right.\right.}{2\left(c^8d^3x^7+3\,c^6d^3x^5+3\,c^4d^3x^3+c^2d^3x+\left(c^7d^3x^6+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*
d^3) - 1/4*a^2/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + integrate(1/2*((4*
a*b*c^2 + b^2*c^2)*x^2 + sqrt(c^2*x^2 + 1)*(4*a*b*c + b^2*c)*x + b^2)*log(c
*x + sqrt(c^2*x^2 + 1))/(c^8*d^3*x^7 + 3*c^6*d^3*x^5 + 3*c^4*d^3*x^3 + c^2*
d^3*x + (c^7*d^3*x^6 + 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 + c*d^3)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [B] time = 2.83139, size = 590, normalized size = 4.07

$$\frac{4\,abc^4x^4+\left(8\,ab+b^2\right)c^2x^2-3\,b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-3\,a^2+4\,ab+b^2-2\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\log\left(c^2x^2+1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*b*c^4*x^4 + (8*a*b + b^2)*c^2*x^2 - 3*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 - 3*a^2 + 4*a*b + b^2 - 2*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*log(c
```

```
^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + (2*b^2*c^3*x^3 + 3*b^2*c*x
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(a*b*c^4*x^4 + 2*a*b*
c^2*x^2 + a*b)*log(-c*x + sqrt(c^2*x^2 + 1)) + 2*(2*a*b*c^3*x^3 + 3*a*b*c*x
)*sqrt(c^2*x^2 + 1))/(c^6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2x\operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2abx\operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral
(b**2*x*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Int
egral(2*a*b*x*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d
**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2x}{(c^2dx^2+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^3, x)
```

3.247 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=309

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib^2\text{PolyLog}\left(3,-\right.}{4cd^3}$$

```
[Out] -(b^2*x)/(12*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c^
2*x^2)^(3/2)) + (3*b*(a + b*ArcSinh[c*x]))/(4*c*d^3*Sqrt[1 + c^2*x^2]) + (x
*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]
)^2)/(8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x
]])/(4*c*d^3) - (5*b^2*ArcTan[c*x])/(6*c*d^3) - (((3*I)/4)*b*(a + b*ArcSinh
[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b^2*PolyLog[3,
(-I)*E^ArcSinh[c*x]])/(c*d^3) - (((3*I)/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]]
)/(c*d^3)
```

Rubi [A] time = 0.364411, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199}

$$-\frac{3ib\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4cd^3}+\frac{3ib^2\text{PolyLog}\left(3,-\right.}{4cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```


```
[Out] -(b^2*x)/(12*d^3*(1 + c^2*x^2)) + (b*(a + b*ArcSinh[c*x]))/(6*c*d^3*(1 + c^
2*x^2)^(3/2)) + (3*b*(a + b*ArcSinh[c*x]))/(4*c*d^3*Sqrt[1 + c^2*x^2]) + (x
*(a + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) + (3*x*(a + b*ArcSinh[c*x]
)^2)/(8*d^3*(1 + c^2*x^2)) + (3*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x
]])/(4*c*d^3) - (5*b^2*ArcTan[c*x])/(6*c*d^3) - (((3*I)/4)*b*(a + b*ArcSinh
[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b*(a + b*ArcSi
nh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/(c*d^3) + (((3*I)/4)*b^2*PolyLog[3,
(-I)*E^ArcSinh[c*x]])/(c*d^3) - (((3*I)/4)*b^2*PolyLog[3, I*E^ArcSinh[c*x]]
)/(c*d^3)
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{3\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{3x\left(a+b\sinh^{-1}(cx)\right)^2}{8d^3\left(1+c^2x^2\right)} - \frac{b^2\int \frac{1}{\left(1+c^2x^2\right)^2}\,dx}{6d^3} - \cdots$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \cdots$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \cdots$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \cdots$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \cdots$$

$$= -\frac{b^2x}{12d^3\left(1+c^2x^2\right)} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{6cd^3\left(1+c^2x^2\right)^{3/2}} + \frac{3b\left(a+b\sinh^{-1}(cx)\right)}{4cd^3\sqrt{1+c^2x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \cdots$$

Mathematica [A] time = 2.23292, size = 546, normalized size = 1.77

$$\frac{\frac{ab\left(\frac{9}{2}i\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)-\frac{9}{2}i\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1-ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\cdots\right.\right.\right.}{c}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^3,x]
```

```
[Out] ((6*a^2*x)/(1 + c^2*x^2)^2 + (9*a^2*x)/(1 + c^2*x^2) + (9*a^2*ArcTan[c*x])/
c + (a*b*((9*((-I)*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(-I + c*x) + (9*(I*Sq
rt[1 + c^2*x^2] + ArcSinh[c*x]))/(I + c*x) - (I*((-2*I + c*x)*Sqrt[1 + c^2*
x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 + (I*((2*I + c*x)*Sqrt[1 + c^2*x^2] +
```

```
3*ArcSinh[c*x]))/(I + c*x)^2 + ((9*I)/2)*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Lo
g[1 + I*E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((9*I)/2)*(
ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*
E^ArcSinh[c*x]])))/c + (b^2*((-2*c*x)/(1 + c^2*x^2) + (4*ArcSinh[c*x])/(1 +
 c^2*x^2)^(3/2) + (18*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (6*c*x*ArcSinh[c*x]
^2)/(1 + c^2*x^2)^2 + (9*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) - 40*ArcTan[Tanh
[ArcSinh[c*x]/2]] - (9*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (9*I)*
ArcSinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - (18*I)*ArcSinh[c*x]*PolyLog[2, (
-I)/E^ArcSinh[c*x]] + (18*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] - (1
8*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]] + (18*I)*PolyLog[3, I/E^ArcSinh[c*x]])
)/c)/(24*d^3)
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{(c^2dx^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,a^2\left(\frac{3\,c^2x^3 + 5\,x}{c^4d^3x^4 + 2\,c^2d^3x^2 + d^3} + \frac{3\,\arctan\,(cx)}{cd^3}\right) + \int \frac{b^2\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3} + \frac{2\,ab\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*((3*c^2*x^3 + 5*x)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) + 3*arctan(c
*x)/(c*d^3)) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^6 +
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(
c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^6 + 3*c
^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^6x^6+3c^4x^4+3c^2x^2+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**6 + 3*c**4*x**4 + 3*c**2*x**2 + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^3, x)
```

3.248 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=275

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sin\cdots}\right.}{2d^3}$$

```
[Out] -b^2/(12*d^3*(1 + c^2*x^2)) - (b*c*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*
x^2)^(3/2)) - (4*b*c*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) + (a
 + b*ArcSinh[c*x])^2/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])^2/(2*d^
3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d
^3 + (2*b^2*Log[1 + c^2*x^2])/(3*d^3) - (b*(a + b*ArcSinh[c*x])*PolyLog[2,
-E^(2*ArcSinh[c*x])])/d^3 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh
[c*x])])/d^3 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^3) - (b^2*PolyLog
[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rubi [A] time = 0.506935, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260, 5690, 261}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}+\frac{b^2\text{PolyLog}\left(3,-e^{2\sin\cdots}\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] -b^2/(12*d^3*(1 + c^2*x^2)) - (b*c*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*
x^2)^(3/2)) - (4*b*c*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) + (a
 + b*ArcSinh[c*x])^2/(4*d^3*(1 + c^2*x^2)^2) + (a + b*ArcSinh[c*x])^2/(2*d^
3*(1 + c^2*x^2)) - (2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d
^3 + (2*b^2*Log[1 + c^2*x^2])/(3*d^3) - (b*(a + b*ArcSinh[c*x])*PolyLog[2,
-E^(2*ArcSinh[c*x])])/d^3 + (b*(a + b*ArcSinh[c*x])*PolyLog[2, E^(2*ArcSinh
[c*x])])/d^3 + (b^2*PolyLog[3, -E^(2*ArcSinh[c*x])])/(2*d^3) - (b^2*PolyLog
[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule *5755*

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{2d^3} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^2}\,dx}{d}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3\left(1+c^2x^2\right)} - \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,d}{3d^3}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}$$

$$= -\frac{b^2}{12d^3\left(1+c^2x^2\right)} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{4bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2}$$

Mathematica [C] time = 3.67633, size = 560, normalized size = 2.04

$$\frac{ab\left(24\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)+12\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-4\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-4\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\right)+12\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^3),x]
```

```
[Out] ((6*a^2)/(1 + c^2*x^2)^2 + (12*a^2)/(1 + c^2*x^2) + 24*a^2*Log[c*x] - 12*a^
2*Log[1 + c^2*x^2] + a*b*((-15*(Sqrt[1 + c^2*x^2] - I*ArcSinh[c*x]))/(I + c
```

```
*x) - (15*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-I + c*x) - 24*ArcSinh[c*x
]^2 - ((-2*I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(-I + c*x)^2 - ((2*
I + c*x)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x])/(I + c*x)^2 + 48*ArcSinh[c*x]*
Log[1 - E^(2*ArcSinh[c*x])] + 12*(ArcSinh[c*x]*(ArcSinh[c*x] - 4*Log[1 + I*
E^ArcSinh[c*x]]) - 4*PolyLog[2, (-I)*E^ArcSinh[c*x]]) + 12*(ArcSinh[c*x]*(A
rcSinh[c*x] - 4*Log[1 - I*E^ArcSinh[c*x]]) - 4*PolyLog[2, I*E^ArcSinh[c*x]]
) + 24*PolyLog[2, E^(2*ArcSinh[c*x])]) + b^2*(I*Pi^3 - 2/(1 + c^2*x^2) - (4
*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) - (32*c*x*ArcSinh[c*x])/Sqrt[1 + c^2
*x^2] + (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (12*ArcSinh[c*x]^2)/(1 + c^2*x
^2) - 16*ArcSinh[c*x]^3 - 24*ArcSinh[c*x]^2*Log[1 + E^(-2*ArcSinh[c*x])] +
24*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 16*Log[1 + c^2*x^2] + 24*Ar
cSinh[c*x]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 24*ArcSinh[c*x]*PolyLog[2, E^
(2*ArcSinh[c*x])] + 12*PolyLog[3, -E^(-2*ArcSinh[c*x])] - 12*PolyLog[3, E^(
2*ArcSinh[c*x])]))/(24*d^3)
```

Maple [B] time = 0.237, size = 1129, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x)
```

```
[Out] -4/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3*x^3-3
/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c*x-4/3*a*b
/d^3/(c^4*x^4+2*c^2*x^2+1)*c^3*x^3*(c^2*x^2+1)^(1/2)+a*b/d^3/(c^4*x^4+2*c^2
*x^2+1)*arcsinh(c*x)*c^2*x^2-3/2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c*x*(c^2*x^2
+1)^(1/2)+1/2*b^2*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))^2)/d^3+1/4*a^2/d^3/(c^
2*x^2+1)^2+1/2*a^2/d^3/(c^2*x^2+1)+a^2/d^3*ln(c*x)-1/2*a^2/d^3*ln(c^2*x^2+1
)-8/3*b^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,-c*x-(c^2*x^2+1
)^(1/2))-2*b^2/d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))+4/3*b^2/d^3*ln(1+(c*x+(
c^2*x^2+1)^(1/2))^2)-1/12*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)+4/3*b^2/d^3/(c^4*x^
4+2*c^2*x^2+1)*arcsinh(c*x)*c^4*x^4+1/2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsi
nh(c*x)^2*c^2*x^2+8/3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*c^2*x^2+4/
3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c^4*x^4+8/3*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*c
^2*x^2+3/2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)-2*a*b/d^3*arcsinh(c*x
)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+
1)^(1/2))+2*a*b/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))-1/12*b^2/d^3/(
c^4*x^4+2*c^2*x^2+1)*c^2*x^2+b^2/d^3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1
/2))+2*b^2/d^3*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))-b^2/d^3*arcsi
nh(c*x)^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2/d^3*arcsinh(c*x)*polylog(2,-(
c*x+(c^2*x^2+1)^(1/2))^2)+b^2/d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2)
```

```
)+2*b^2/d^3*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))+4/3*a*b/d^3/(c^4*
x^4+2*c^2*x^2+1)-a*b/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/d^3*po
lylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*a*b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))+
3/4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2+4/3*b^2/d^3/(c^4*x^4+2*c^2
*x^2+1)*arcsinh(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a^2\left(\frac{2c^2x^2+3}{c^4d^3x^4+2c^2d^3x^2+d^3}-\frac{2\log\left(c^2x^2+1\right)}{d^3}+\frac{4\log(x)}{d^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^7+3c^4d^3x^5+3c^2d^3x^3+d^3x}+\frac{2ab\log\left(cx\right.}{c^6d^3x^7+3c^4d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*((2*c^2*x^2 + 3)/(c^4*d^3*x^4 + 2*c^2*d^3*x^2 + d^3) - 2*log(c^2*x^
2 + 1)/d^3 + 4*log(x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x) + 2*a*b*log(c*x + sqrt
(c^2*x^2 + 1))/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2+2ab\operatorname{arsinh}(cx)+a^2}{c^6d^3x^7+3c^4d^3x^5+3c^2d^3x^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^7 + 3*c
^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int\frac{b^2\operatorname{asinh}^2(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx+\int\frac{2ab\operatorname{asinh}(cx)}{c^6x^7+3c^4x^5+3c^2x^3+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integral(b
**2*asinh(c*x)**2/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x) + Integra
l(2*a*b*asinh(c*x)/(c**6*x**7 + 3*c**4*x**5 + 3*c**2*x**3 + x), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^3 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x), x)
```

3.249 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=389

$$\frac{15ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{15ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{2b^2c\text{PolyLog}\left(2,\right.}{d^3}$$

```
[Out] (b^2*c^2*x)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(6*d^3*(1 +
 c^2*x^2)^(3/2)) - (7*b*c*(a + b*ArcSinh[c*x]))/(4*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])^2/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c
*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*
(1 + c^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^
3) + (11*b^2*c*ArcTan[c*x])/(6*d^3) - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E
^ArcSinh[c*x]])/d^3 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^3 + (((15*I)/
4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 - (((15*I)
/4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 + (2*b^2*c*P
olyLog[2, E^ArcSinh[c*x]])/d^3 - (((15*I)/4)*b^2*c*PolyLog[3, (-I)*E^ArcSin
h[c*x]])/d^3 + (((15*I)/4)*b^2*c*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rubi [A] time = 0.766648, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199, 5755, 5760, 4182, 2279, 2391}

$$\frac{15ibc\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{15ibc\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}-\frac{2b^2c\text{PolyLog}\left(2,\right.}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```



```
[Out] (b^2*c^2*x)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(6*d^3*(1 +
 c^2*x^2)^(3/2)) - (7*b*c*(a + b*ArcSinh[c*x]))/(4*d^3*Sqrt[1 + c^2*x^2]) -
 (a + b*ArcSinh[c*x])^2/(d^3*x*(1 + c^2*x^2)^2) - (5*c^2*x*(a + b*ArcSinh[c
*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (15*c^2*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*
(1 + c^2*x^2)) - (15*c*(a + b*ArcSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^
3) + (11*b^2*c*ArcTan[c*x])/(6*d^3) - (4*b*c*(a + b*ArcSinh[c*x])*ArcTanh[E
```

```
^ArcSinh[c*x]])/d^3 - (2*b^2*c*PolyLog[2, -E^ArcSinh[c*x]])/d^3 + (((15*I)/
4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 - (((15*I)
/4)*b*c*(a + b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 + (2*b^2*c*P
olyLog[2, E^ArcSinh[c*x]])/d^3 - (((15*I)/4)*b^2*c*PolyLog[3, (-I)*E^ArcSin
h[c*x]])/d^3 + (((15*I)/4)*b^2*c*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \frac{5c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{4d^3\left(1+c^2x^2\right)^2} + \frac{(2bc)\int \frac{a+b\sinh^{-1}(\cdots}{x\left(1+c^2x^2\right)^{\cdots}}}{d^3}\cdots$$

$$= -\frac{b^2c^2x}{3d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)}{d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \cdots$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \cdots$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \cdots$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \cdots$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \cdots$$

$$= \frac{b^2c^2x}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{7bc\left(a+b\sinh^{-1}(cx)\right)}{4d^3\sqrt{1+c^2x^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^3x\left(1+c^2x^2\right)^2} - \cdots$$

Mathematica [A] time = 7.61938, size = 716, normalized size = 1.84

$$\frac{2abc\left(\frac{15}{16}i\left(2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)-\frac{1}{2}\sinh^{-1}(cx)^2+2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)\right)-\frac{15}{16}i\left(2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\cdots\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^3),x]
```

```
[Out] -(a^2/(d^3*x)) - (a^2*c^2*x)/(4*d^3*(1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d^3
*(1 + c^2*x^2)) - (15*a^2*c*ArcTan[c*x])/(8*d^3) + (2*a*b*c*((7*(Sqrt[1 + c
^2*x^2] + I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(c*x) - (7*(I*S
qrt[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I + c*x)) + ((I/48)*((-2*I + c*x)*Sq
rt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(-I + c*x)^2 - ((I/48)*((2*I + c*x)*Sqrt
[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 - ArcTanh[Sqrt[1 + c^2*x^2]] +
 ((15*I)/16)*(-ArcSinh[c*x]^2/2 + 2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]]
+ 2*PolyLog[2, (-I)*E^ArcSinh[c*x]]) - ((15*I)/16)*(-ArcSinh[c*x]^2/2 + 2*A
rcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]] + 2*PolyLog[2, I*E^ArcSinh[c*x]])))/d
^3 + (b^2*c*((2*c*x)/(1 + c^2*x^2) - (4*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) -
 (42*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (6*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)
^2 - (21*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2) + 88*ArcTan[Tanh[ArcSinh[c*x]/2]
] - 12*ArcSinh[c*x]^2*Coth[ArcSinh[c*x]/2] + 48*ArcSinh[c*x]*Log[1 - E^(-Ar
cSinh[c*x])] + (45*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] - (45*I)*Arc
Sinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] - 48*ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + 48*PolyLog[2, -E^(-ArcSinh[c*x])] + (90*I)*ArcSinh[c*x]*PolyLog[2
, (-I)/E^ArcSinh[c*x]] - (90*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x]] -
 48*PolyLog[2, E^(-ArcSinh[c*x])] + (90*I)*PolyLog[3, (-I)/E^ArcSinh[c*x]]
- (90*I)*PolyLog[3, I/E^ArcSinh[c*x]] + 12*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]
/2]))/(24*d^3)
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{Arcsinh}\,(cx))^2}{x^2\left(c^2dx^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\,a^2\left(\frac{15\,c^4x^4+25\,c^2x^2+8}{c^4d^3x^5+2\,c^2d^3x^3+d^3x}+\frac{15\,c\arctan(cx)}{d^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2}+\frac{2\,ab\log\left(cx+\sqrt{c^2}\right.}{c^6d^3x^8+3\,c^4d^3x^6+3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*((15*c^4*x^4 + 25*c^2*x^2 + 8)/(c^4*d^3*x^5 + 2*c^2*d^3*x^3 + d^3*
x) + 15*c*arctan(c*x)/d^3) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(
c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2) + 2*a*b*log(c*x + sq
rt(c^2*x^2 + 1))/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{c^6d^3x^8 + 3\,c^4d^3x^6 + 3\,c^2d^3x^4 + d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^8 + 3*c
^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^6x^8+3c^4x^6+3c^2x^4+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**3,x)
```

```
[Out] (Integral(a**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + Integra
l(b**2*asinh(c*x)**2/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x) + I
ntegral(2*a*b*asinh(c*x)/(c**6*x**8 + 3*c**4*x**6 + 3*c**2*x**4 + x**2), x)
)/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(c^2dx^2 + d)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^2), x)
```

3.250 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=381

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3b^2c^2\text{PolyLog}\left(3\right.}{2d^3}$$

```
[Out] (b^2*c^2)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(d^3*x*(1 + c
^2*x^2)^(3/2)) - (5*b*c^3*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) + (4*b*c^3*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a
 + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])^2/(2*d
^3*x^2*(1 + c^2*x^2)^2) - (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^3*(1 + c^2*x^
2)) + (6*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (b^2
*c^2*Log[x])/d^3 - (7*b^2*c^2*Log[1 + c^2*x^2])/(6*d^3) + (3*b*c^2*(a + b*A
rcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^3 - (3*b*c^2*(a + b*ArcSinh
[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^3 - (3*b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/(2*d^3) + (3*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rubi [A] time = 0.802988, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 19, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.731, Rules used = {5747, 5755, 5720, 5461, 4182, 2531, 2282, 6589, 5687, 260, 5690, 261, 271, 192, 191, 5732, 12, 1251, 893}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^3}-\frac{3b^2c^2\text{PolyLog}\left(3\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```


```
[Out] (b^2*c^2)/(12*d^3*(1 + c^2*x^2)) - (b*c*(a + b*ArcSinh[c*x]))/(d^3*x*(1 + c
^2*x^2)^(3/2)) - (5*b*c^3*x*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) + (4*b*c^3*x*(a + b*ArcSinh[c*x]))/(3*d^3*Sqrt[1 + c^2*x^2]) - (3*c^2*(a
 + b*ArcSinh[c*x])^2)/(4*d^3*(1 + c^2*x^2)^2) - (a + b*ArcSinh[c*x])^2/(2*d
^3*x^2*(1 + c^2*x^2)^2) - (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^3*(1 + c^2*x^
2)) + (6*c^2*(a + b*ArcSinh[c*x])^2*ArcTanh[E^(2*ArcSinh[c*x])])/d^3 + (b^2
*c^2*Log[x])/d^3 - (7*b^2*c^2*Log[1 + c^2*x^2])/(6*d^3) + (3*b*c^2*(a + b*A
```

```
rcSinh[c*x])*PolyLog[2, -E^(2*ArcSinh[c*x])])/d^3 - (3*b*c^2*(a + b*ArcSinh
[c*x])*PolyLog[2, E^(2*ArcSinh[c*x])])/d^3 - (3*b^2*c^2*PolyLog[3, -E^(2*Ar
cSinh[c*x])])/(2*d^3) + (3*b^2*c^2*PolyLog[3, E^(2*ArcSinh[c*x])])/(2*d^3)
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

```
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5732

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(1 + c^2*x^2)^p, x]}, Dist[d^p*(a + b
*ArcSinh[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegerQ
[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -2^
(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2d^3x^2\left(1+c^2x^2\right)^2} - \left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^3}\,dx + \frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)^{5/2}}\,dx}{d^3}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{4bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\left(1+c^2x^2\right)^{3/2}} - \frac{8bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\right.}{4d^3\left(1\right.}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{8bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^3\sqrt{1+c^2x^2}} - \frac{3c^2\left(a+b\right.}{4d^3\left(1\right.}$$

$$= \frac{b^2c^2}{4d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\sinh^{-1}(c\right.}{3d^3\sqrt{1+c^2x^2}}$$

$$= \frac{b^2c^2}{4d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\sinh^{-1}(c\right.}{3d^3\sqrt{1+c^2x^2}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\sinh^{-1}(\right.}{3d^3\sqrt{1+c^2x^2}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\sinh^{-1}(\right.}{3d^3\sqrt{1+c^2x^2}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\sinh^{-1}(\right.}{3d^3\sqrt{1+c^2x^2}}$$

$$= \frac{b^2c^2}{12d^3\left(1+c^2x^2\right)} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^3x\left(1+c^2x^2\right)^{3/2}} - \frac{5bc^3x\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} + \frac{4bc^3x\left(a+b\sinh^{-1}(\right.}{3d^3\sqrt{1+c^2x^2}}$$

<u>Mathematica</u> [C] time = 9.57152, size = 759, normalized size = 1.99

$$2ab\left(\frac{3}{2}c^3\left(\frac{2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sinh^{-1}(cx)\log\left(1+ie^{\sinh^{-1}(cx)}\right)}{c}\right) + \frac{3}{2}c^3\left(\frac{2\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c} - \frac{\sinh^{-1}(cx)^2}{2c} + \frac{2\sinh\ldots}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^3),x]
```

```
[Out] -a^2/(2*d^3*x^2) - (a^2*c^2)/(4*d^3*(1 + c^2*x^2)^2) - (a^2*c^2)/(d^3*(1 +
c^2*x^2)) - (3*a^2*c^2*Log[x])/d^3 + (3*a^2*c^2*Log[1 + c^2*x^2])/(2*d^3) +
 (2*a*b*(-(c^2*((2*I - c*x)*Sqrt[1 + c^2*x^2] - 3*ArcSinh[c*x]))/(48*(-I +
c*x)^2) - (((9*I)/16)*c^2*(Sqrt[1 + c^2*x^2] + I*ArcSinh[c*x]))/(-1 - I*c*x
) - (((9*I)/16)*c^3*(I*Sqrt[1 + c^2*x^2] + ArcSinh[c*x]))/(I*c + c^2*x) - (
c*x*Sqrt[1 + c^2*x^2] + ArcSinh[c*x])/(2*x^2) + (c^2*((2*I + c*x)*Sqrt[1 +
c^2*x^2] + 3*ArcSinh[c*x]))/(48*(I + c*x)^2) + (3*c^3*(-ArcSinh[c*x]^2/(2*c
) + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/c))/2 + (3*c^3*(-ArcSinh[c*x]^2/(2*c) + (2*ArcSinh[c*x]*Log[1 -
 I*E^ArcSinh[c*x]])/c + (2*PolyLog[2, I*E^ArcSinh[c*x]])/c))/2 - 3*c^2*(-Ar
cSinh[c*x]^2/2 + ArcSinh[c*x]*Log[1 - E^(2*ArcSinh[c*x])] + PolyLog[2, E^(2
*ArcSinh[c*x])]/2)))/d^3 + (b^2*c^2*(-3*ArcSinh[c*x]*PolyLog[2, -E^(-2*ArcS
inh[c*x])] - 3*ArcSinh[c*x]*PolyLog[2, E^(2*ArcSinh[c*x])] + ((-3*I)*Pi^3 +
 2/(1 + c^2*x^2) + (4*c*x*ArcSinh[c*x])/(1 + c^2*x^2)^(3/2) + (56*c*x*ArcSi
nh[c*x])/Sqrt[1 + c^2*x^2] - (24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*x) - (1
2*ArcSinh[c*x]^2)/(c^2*x^2) - (6*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 - (24*ArcS
inh[c*x]^2)/(1 + c^2*x^2) + 48*ArcSinh[c*x]^3 + 72*ArcSinh[c*x]^2*Log[1 + E
^(-2*ArcSinh[c*x])] - 72*ArcSinh[c*x]^2*Log[1 - E^(2*ArcSinh[c*x])] + 24*Lo
g[c*x] - 56*Log[Sqrt[1 + c^2*x^2]] - 36*PolyLog[3, -E^(-2*ArcSinh[c*x])] +
36*PolyLog[3, E^(2*ArcSinh[c*x])]))/24))/d^3
```

Maple [B] time = 0.289, size = 1436, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x)
```

```
[Out] -4/3*c^2*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+3*c^2*a*b/d^3*polylog(2
,-(c*x+(c^2*x^2+1)^(1/2))^2)-6*c^2*a*b/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2)
)-6*c^2*a*b/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))-1/2*b^2/d^3/(c^4*x^4+2*c^2
*x^2+1)/x^2*arcsinh(c*x)^2+1/12*c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*x^2-3*c^2
*b^2/d^3*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*c^2*b^2/d^3*arcsinh(c
*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+3*c^2*b^2/d^3*arcsinh(c*x)^2*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)+3*c^2*b^2/d^3*arcsinh(c*x)*polylog(2,-(c*x+(c^2*x^2
+1)^(1/2))^2)-3*c^2*b^2/d^3*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))-6*c^
2*b^2/d^3*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-9/4*c^2*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*arcsinh(c*x)^2-4/3*c^2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)-4/3
*c^6*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^4-8/3*c^4*a*b/d^3/(c^4*x^4+2*c^2*x^2+1
```

```
)*x^2-8/3*c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^2-6*c^2*a*b/d^3*
arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-6*c^2*a*b/d^3*arcsinh(c*x)*ln(1-c*
x-(c^2*x^2+1)^(1/2))-9/2*c^2*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)+6*c
^2*a*b/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-4/3*c^6*b^2/d^3/(c^
4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^4-3/2*c^4*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*a
rcsinh(c*x)^2*x^2-a*b/d^3/(c^4*x^4+2*c^2*x^2+1)/x^2*arcsinh(c*x)-1/2*a^2/d^
3/x^2-1/4*c^2*a^2/d^3/(c^2*x^2+1)^2-c^2*a^2/d^3/(c^2*x^2+1)+1/12*c^2*b^2/d^
3/(c^4*x^4+2*c^2*x^2+1)+c^2*b^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-1)+c^2*b^2/d^3
*ln(1+c*x+(c^2*x^2+1)^(1/2))-3*c^2*a^2/d^3*ln(c*x)+3/2*c^2*a^2/d^3*ln(c^2*x
^2+1)+8/3*c^2*b^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2))+6*c^2*b^2/d^3*polylog(3,-c*
x-(c^2*x^2+1)^(1/2))+6*c^2*b^2/d^3*polylog(3,c*x+(c^2*x^2+1)^(1/2))-7/3*c^2
*b^2/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-3/2*b^2*c^2*polylog(3,-(c*x+(c^2*x
^2+1)^(1/2))^2)/d^3+1/2*c^3*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*x+4/3*c^5*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*x^3*(c^2*x^2+1)^(1/2)
-3*c^4*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*x^2+1/2*c^3*a*b/d^3/(c^4*
x^4+2*c^2*x^2+1)*x*(c^2*x^2+1)^(1/2)-c*a*b/d^3/(c^4*x^4+2*c^2*x^2+1)/x*(c^2
*x^2+1)^(1/2)-c*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)/x*arcsinh(c*x)*(c^2*x^2+1)^(1
/2)+4/3*c^5*b^2/d^3/(c^4*x^4+2*c^2*x^2+1)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a^2\left(\frac{6c^4x^4+9c^2x^2+2}{c^4d^3x^6+2c^2d^3x^4+d^3x^2}-\frac{6c^2\log\left(c^2x^2+1\right)}{d^3}+\frac{12c^2\log(x)}{d^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^9+3c^4d^3x^7+3c^2d^3x^5+d^3x^3}+\frac{}{c^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*((6*c^4*x^4 + 9*c^2*x^2 + 2)/(c^4*d^3*x^6 + 2*c^2*d^3*x^4 + d^3*x^
2) - 6*c^2*log(c^2*x^2 + 1)/d^3 + 12*c^2*log(x)/d^3) + integrate(b^2*log(c*
x + sqrt(c^2*x^2 + 1))^2/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3
*x^3) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3
*c^2*d^3*x^5 + d^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{arsinh}(cx)^2+2ab\operatorname{arsinh}(cx)+a^2}{c^6d^3x^9+3c^4d^3x^7+3c^2d^3x^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^9 + 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^3 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^3), x)
```

3.251 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=529

$$-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{35ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{19b^2c^3\text{Poly}}{}$$

```
[Out] -(b^2*c^2)/(2*d^3*x) + (b^2*c^2)/(6*d^3*x*(1 + c^2*x^2)) + (b^2*c^4*x)/(12*
d^3*(1 + c^2*x^2)) - (b*c^3*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^3*x^2*(1 + c^2*x^2)^(3/2)) + (29*b*c^3
*(a + b*ArcSinh[c*x]))/(12*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/
(3*d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x])^2)/(3*d^3*x*(1 +
c^2*x^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(12*d^3*(1 + c^2*x^2)^2) +
(35*c^4*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*Ar
cSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (17*b^2*c^3*ArcTan[c*x])/(6
*d^3) + (38*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d^3) + (
19*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^3) - (((35*I)/4)*b*c^3*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/4)*b*c^3*(a +
 b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 - (19*b^2*c^3*PolyLog[2,
 E^ArcSinh[c*x]])/(3*d^3) + (((35*I)/4)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c
*x]])/d^3 - (((35*I)/4)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rubi [A] time = 1.30742, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 17, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.654, Rules used = {5747, 5690, 5693, 4180, 2531, 2282, 6589, 5717, 203, 199, 5755, 5760, 4182, 2279, 2391, 290, 325}

$$-\frac{35ibc^3\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{35ibc^3\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{4d^3}+\frac{19b^2c^3\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```


```
[Out] -(b^2*c^2)/(2*d^3*x) + (b^2*c^2)/(6*d^3*x*(1 + c^2*x^2)) + (b^2*c^4*x)/(12*
d^3*(1 + c^2*x^2)) - (b*c^3*(a + b*ArcSinh[c*x]))/(6*d^3*(1 + c^2*x^2)^(3/2
)) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^3*x^2*(1 + c^2*x^2)^(3/2)) + (29*b*c^3
```

```
*(a + b*ArcSinh[c*x]))/(12*d^3*Sqrt[1 + c^2*x^2]) - (a + b*ArcSinh[c*x])^2/
(3*d^3*x^3*(1 + c^2*x^2)^2) + (7*c^2*(a + b*ArcSinh[c*x])^2)/(3*d^3*x*(1 +
c^2*x^2)^2) + (35*c^4*x*(a + b*ArcSinh[c*x])^2)/(12*d^3*(1 + c^2*x^2)^2) +
(35*c^4*x*(a + b*ArcSinh[c*x])^2)/(8*d^3*(1 + c^2*x^2)) + (35*c^3*(a + b*Ar
cSinh[c*x])^2*ArcTan[E^ArcSinh[c*x]])/(4*d^3) - (17*b^2*c^3*ArcTan[c*x])/(6
*d^3) + (38*b*c^3*(a + b*ArcSinh[c*x])*ArcTanh[E^ArcSinh[c*x]])/(3*d^3) + (
19*b^2*c^3*PolyLog[2, -E^ArcSinh[c*x]])/(3*d^3) - (((35*I)/4)*b*c^3*(a + b*
ArcSinh[c*x])*PolyLog[2, (-I)*E^ArcSinh[c*x]])/d^3 + (((35*I)/4)*b*c^3*(a +
 b*ArcSinh[c*x])*PolyLog[2, I*E^ArcSinh[c*x]])/d^3 - (19*b^2*c^3*PolyLog[2,
 E^ArcSinh[c*x]])/(3*d^3) + (((35*I)/4)*b^2*c^3*PolyLog[3, (-I)*E^ArcSinh[c
*x]])/d^3 - (((35*I)/4)*b^2*c^3*PolyLog[3, I*E^ArcSinh[c*x]])/d^3
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
```

```
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} - \frac{1}{3}\left(7c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^3}\,dx + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(1+c^2x^2\right)^{5/2}}\,dx}{3d^3}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} + \frac{7c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x\left(1+c^2x^2\right)^2} + \frac{1}{3}\left(35c^4\right)\int \frac{\left(a+b\right.}{\left(d\right.}$$

$$= \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} - \frac{19bc^3\left(a+b\sinh^{-1}(cx)\right)}{9d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d^3x^3\left(1+c^2x^2\right)^2} +$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{19b^2c^4x}{18d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}$$

$$= -\frac{b^2c^2}{2d^3x} + \frac{b^2c^2}{6d^3x\left(1+c^2x^2\right)} + \frac{b^2c^4x}{12d^3\left(1+c^2x^2\right)} - \frac{bc^3\left(a+b\sinh^{-1}(cx)\right)}{6d^3\left(1+c^2x^2\right)^{3/2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^3x^2\left(1+c^2x^2\right)^{3/2}}$$

Mathematica [A] time = 10.1588, size = 937, normalized size = 1.77

$$\frac{11a^2xc^4}{8d^3\left(c^2x^2+1\right)} + \frac{a^2xc^4}{4d^3\left(c^2x^2+1\right)^2} + \frac{35a^2\tan^{-1}(cx)c^3}{8d^3} + \frac{b^2\left(-\frac{1}{2}cx\sinh^{-1}(cx)^2\mathbf{csch}^4\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 2\sinh^{-1}(cx)\mathbf{csch}^2\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^3),x]
```

```
[Out] -a^2/(3*d^3*x^3) + (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(1 + c^2*x^2)^2
) + (11*a^2*c^4*x)/(8*d^3*(1 + c^2*x^2)) + (35*a^2*c^3*ArcTan[c*x])/(8*d^3)
 + (2*a*b*(-(c*Sqrt[1 + c^2*x^2])/(6*x^2) + ((I/48)*c^3*((2*I - c*x)*Sqrt[1
 + c^2*x^2] - 3*ArcSinh[c*x]))/(-I + c*x)^2 - (11*c^3*(Sqrt[1 + c^2*x^2] +
I*ArcSinh[c*x]))/(16*(-1 - I*c*x)) - ArcSinh[c*x]/(3*x^3) + (11*c^4*(I*Sqrt
[1 + c^2*x^2] + ArcSinh[c*x]))/(16*(I*c + c^2*x)) + ((I/48)*c^3*((2*I + c*x
)*Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]))/(I + c*x)^2 + (c^3*ArcTanh[Sqrt[1 +
c^2*x^2]])/6 - 3*c^2*(-(ArcSinh[c*x]/x) - c*ArcTanh[Sqrt[1 + c^2*x^2]]) - (
(35*I)/16)*c^4*(-ArcSinh[c*x]^2/(2*c) + (2*ArcSinh[c*x]*Log[1 + I*E^ArcSinh
[c*x]])/c + (2*PolyLog[2, (-I)*E^ArcSinh[c*x]])/c) + ((35*I)/16)*c^4*(-ArcS
inh[c*x]^2/(2*c) + (2*ArcSinh[c*x]*Log[1 - I*E^ArcSinh[c*x]])/c + (2*PolyLo
g[2, I*E^ArcSinh[c*x]])/c)))/d^3 + (b^2*c^3*((-2*c*x)/(1 + c^2*x^2) + (4*Ar
cSinh[c*x])/(1 + c^2*x^2)^(3/2) + (66*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + (6*
c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^2 + (33*c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)
 - 136*ArcTan[Tanh[ArcSinh[c*x]/2]] - 4*Coth[ArcSinh[c*x]/2] + 38*ArcSinh[c
*x]^2*Coth[ArcSinh[c*x]/2] - 2*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - (c*x*A
rcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^4)/2 - 152*ArcSinh[c*x]*Log[1 - E^(-ArcS
inh[c*x])] - (105*I)*ArcSinh[c*x]^2*Log[1 - I/E^ArcSinh[c*x]] + (105*I)*Arc
Sinh[c*x]^2*Log[1 + I/E^ArcSinh[c*x]] + 152*ArcSinh[c*x]*Log[1 + E^(-ArcSin
h[c*x])] - 152*PolyLog[2, -E^(-ArcSinh[c*x])] - (210*I)*ArcSinh[c*x]*PolyLo
g[2, (-I)/E^ArcSinh[c*x]] + (210*I)*ArcSinh[c*x]*PolyLog[2, I/E^ArcSinh[c*x
]] + 152*PolyLog[2, E^(-ArcSinh[c*x])] - (210*I)*PolyLog[3, (-I)/E^ArcSinh[
c*x]] + (210*I)*PolyLog[3, I/E^ArcSinh[c*x]] - 2*ArcSinh[c*x]*Sech[ArcSinh[
c*x]/2]^2 - (8*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]^4)/(c^3*x^3) + 4*Tanh[Ar
cSinh[c*x]/2] - 38*ArcSinh[c*x]^2*Tanh[ArcSinh[c*x]/2]))/(24*d^3)
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{x^4\left(c^2dx^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}a^2\left(\frac{105\,c^3\arctan(cx)}{d^3}+\frac{105\,c^6x^6+175\,c^4x^4+56\,c^2x^2-8}{c^4d^3x^7+2\,c^2d^3x^5+d^3x^3}\right)+\int\frac{b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4}+\frac{2\,ab\log}{c^6d^3x^{10}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(105*c^3*arctan(c*x)/d^3 + (105*c^6*x^6 + 175*c^4*x^4 + 56*c^2*x^2
 - 8)/(c^4*d^3*x^7 + 2*c^2*d^3*x^5 + d^3*x^3)) + integrate(b^2*log(c*x + sq
rt(c^2*x^2 + 1))^2/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4)
 + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2
*d^3*x^6 + d^3*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}(cx)^2+2\,ab\,\text{arsinh}(cx)+a^2}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(c^6*d^3*x^10 + 3*
c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^3 x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^3*x^4), x)
```

3.252 $\int \left(\pi + c^2\pi x^2\right)^{5/2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=300

$$-\frac{\pi^{5/2}b\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{18c}-\frac{5\pi^{5/2}b\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{48c}+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+$$

```
[Out] (245*b^2*Pi^(5/2)*x*Sqrt[1 + c^2*x^2])/1152 + (65*b^2*Pi^(5/2)*x*(1 + c^2*x
^2)^(3/2))/1728 + (b^2*Pi^(5/2)*x*(1 + c^2*x^2)^(5/2))/108 - (115*b^2*Pi^(5
/2)*ArcSinh[c*x])/(1152*c) - (5*b*c*Pi^(5/2)*x^2*(a + b*ArcSinh[c*x]))/16 -
 (5*b*Pi^(5/2)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(48*c) - (b*Pi^(5/2)*(
1 + c^2*x^2)^3*(a + b*ArcSinh[c*x]))/(18*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x])^2)/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5
*Pi^(5/2)*(a + b*ArcSinh[c*x])^3)/(48*b*c)
```

Rubi [A] time = 0.381804, antiderivative size = 420, normalized size of antiderivative = 1.4, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{5\pi^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{1}{6}x\left(\pi c^2x^2+\pi\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{5}{24}\pi x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2])/1152 + (65*b^2*Pi^2*x*(1 + c^2*x^2)*
Sqrt[Pi + c^2*Pi*x^2])/1728 + (b^2*Pi^2*x*(1 + c^2*x^2)^2*Sqrt[Pi + c^2*Pi*
x^2])/108 - (115*b^2*Pi^2*Sqrt[Pi + c^2*Pi*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[
1 + c^2*x^2]) - (5*b*c*Pi^2*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))
/(16*Sqrt[1 + c^2*x^2]) - (5*b*Pi^2*(1 + c^2*x^2)^(3/2)*Sqrt[Pi + c^2*Pi*x^
2]*(a + b*ArcSinh[c*x]))/(48*c) - (b*Pi^2*(1 + c^2*x^2)^(5/2)*Sqrt[Pi + c^2
*Pi*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (5*Pi^2*x*Sqrt[Pi + c^2*Pi*x^2]*(a
+ b*ArcSinh[c*x])^2)/16 + (5*Pi*x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/24 + (x*(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*Pi^2*
Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(\pi+c^2\pi x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{6}x\left(\pi+c^2\pi x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{6}(5\pi)\int \left(\pi+c^2\pi x^2\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

$$= -\frac{b\pi^2\left(1+c^2x^2\right)^{5/2}\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{5}{24}\pi x\left(\pi+c^2\pi x^2\right)^{3/2}\left(a\right.$$

$$= \frac{1}{108}b^2\pi^2x\left(1+c^2x^2\right)^2\sqrt{\pi+c^2\pi x^2}-\frac{5b\pi^2\left(1+c^2x^2\right)^{3/2}\sqrt{\pi+c^2\pi x^2}\left(a+b\sin\right.}{48c}$$

$$= \frac{65b^2\pi^2x\left(1+c^2x^2\right)\sqrt{\pi+c^2\pi x^2}}{1728}+\frac{1}{108}b^2\pi^2x\left(1+c^2x^2\right)^2\sqrt{\pi+c^2\pi x^2}-\frac{5bc\pi}{}$$

$$= \frac{245b^2\pi^2x\sqrt{\pi+c^2\pi x^2}}{1152}+\frac{65b^2\pi^2x\left(1+c^2x^2\right)\sqrt{\pi+c^2\pi x^2}}{1728}+\frac{1}{108}b^2\pi^2x\left(1+c\right.$$

$$= \frac{245b^2\pi^2x\sqrt{\pi+c^2\pi x^2}}{1152}+\frac{65b^2\pi^2x\left(1+c^2x^2\right)\sqrt{\pi+c^2\pi x^2}}{1728}+\frac{1}{108}b^2\pi^2x\left(1+c\right.$$

Mathematica [A] time = 0.95247, size = 284, normalized size = 0.95

$$\frac{\pi^{5/2}\left(12\sinh^{-1}(cx)\left(360a^2+540ab\sinh\left(2\sinh^{-1}(cx)\right)+108ab\sinh\left(4\sinh^{-1}(cx)\right)+12ab\sinh\left(6\sinh^{-1}(cx)\right)-270b^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(5/2)*(9504*a^2*c*x*Sqrt[1 + c^2*x^2] + 7488*a^2*c^3*x^3*Sqrt[1 + c^2*x
^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2] + 1440*b^2*ArcSinh[c*x]^3 - 3240*a
*b*Cosh[2*ArcSinh[c*x]] - 324*a*b*Cosh[4*ArcSinh[c*x]] - 24*a*b*Cosh[6*ArcS
inh[c*x]] + 1620*b^2*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sinh[4*ArcSinh[c*x]] + 4
*b^2*Sinh[6*ArcSinh[c*x]] + 72*b*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh
[c*x]] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]) + 12*ArcSinh[c*
x]*(360*a^2 - 270*b^2*Cosh[2*ArcSinh[c*x]] - 27*b^2*Cosh[4*ArcSinh[c*x]] -
2*b^2*Cosh[6*ArcSinh[c*x]] + 540*a*b*Sinh[2*ArcSinh[c*x]] + 108*a*b*Sinh[4*
ArcSinh[c*x]] + 12*a*b*Sinh[6*ArcSinh[c*x]])))/(13824*c)
```

Maple [A] time = 0.103, size = 486, normalized size = 1.6

$$\frac{a^2x}{6}\left(\pi c^2x^2+\pi\right)^{\frac{5}{2}}+\frac{5a^2\pi x}{24}\left(\pi c^2x^2+\pi\right)^{\frac{3}{2}}+\frac{5a^2\pi^2x}{16}\sqrt{\pi c^2x^2+\pi}+\frac{5a^2\pi^3}{16}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/6*a^2*x*(Pi*c^2*x^2+Pi)^(5/2)+5/24*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(3/2)+5/16*a^
2*Pi^2*x*(Pi*c^2*x^2+Pi)^(1/2)+5/16*a^2*Pi^3*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi
*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/6*b^2*Pi^(5/2)*c^4*arcsinh(c*x)^2*(c^2
*x^2+1)^(1/2)*x^5-1/18*b^2*Pi^(5/2)*c^5*arcsinh(c*x)*x^6+1/108*b^2*Pi^(5/2)
*c^4*x^5*(c^2*x^2+1)^(1/2)+13/24*b^2*Pi^(5/2)*c^2*arcsinh(c*x)^2*(c^2*x^2+1
)^(1/2)*x^3-13/48*b^2*Pi^(5/2)*c^3*arcsinh(c*x)*x^4+97/1728*b^2*Pi^(5/2)*c^
2*x^3*(c^2*x^2+1)^(1/2)+11/16*b^2*Pi^(5/2)*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)
*x-11/16*b^2*Pi^(5/2)*c*arcsinh(c*x)*x^2+5/48*b^2*Pi^(5/2)/c*arcsinh(c*x)^3
+299/1152*b^2*Pi^(5/2)*x*(c^2*x^2+1)^(1/2)-299/1152*b^2*Pi^(5/2)*arcsinh(c*
x)/c+1/3*a*b*Pi^(5/2)*c^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^5-1/18*a*b*Pi^(5
/2)*c^5*x^6+13/12*a*b*Pi^(5/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^3-13/48
*a*b*Pi^(5/2)*c^3*x^4+11/8*a*b*Pi^(5/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-11
/16*a*b*Pi^(5/2)*c*x^2+5/16*a*b*Pi^(5/2)/c*arcsinh(c*x)^2-17/36*a*b*Pi^(5/2
)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\pi + \pi c^2 x^2}\left(\pi^2 a^2 c^4 x^4 + 2\,\pi^2 a^2 c^2 x^2 + \pi^2 a^2 + \left(\pi^2 b^2 c^4 x^4 + 2\,\pi^2 b^2 c^2 x^2 + \pi^2 b^2\right)\text{arsinh}\,(cx)^2 + 2\left(\pi^2 abc^4 x^4 + 2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi^2*a^2*c^4*x^4 + 2*pi^2*a^2*c^2*x^2 + pi^
2*a^2 + (pi^2*b^2*c^4*x^4 + 2*pi^2*b^2*c^2*x^2 + pi^2*b^2)*arcsinh(c*x)^2 +
 2*(pi^2*a*b*c^4*x^4 + 2*pi^2*a*b*c^2*x^2 + pi^2*a*b)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}} (b\,\text{arsinh}\,(cx) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(5/2)*(b*arcsinh(c*x) + a)^2, x)
```

3.253 $\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=210

$$\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{\pi^{3/2}b\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{8c}-\frac{3}{8}$$

```
[Out] (15*b^2*Pi^(3/2)*x*Sqrt[1 + c^2*x^2])/64 + (b^2*Pi^(3/2)*x*(1 + c^2*x^2)^(3
/2))/32 - (9*b^2*Pi^(3/2)*ArcSinh[c*x])/(64*c) - (3*b*c*Pi^(3/2)*x^2*(a + b
*ArcSinh[c*x]))/8 - (b*Pi^(3/2)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(8*c)
 + (3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(Pi + c^2*P
i*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (Pi^(3/2)*(a + b*ArcSinh[c*x])^3)/
(8*b*c)
```

Rubi [A] time = 0.227046, antiderivative size = 294, normalized size of antiderivative = 1.4, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{\pi\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(\pi c^2x^2+\pi\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}\pi x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{\pi b(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*Pi*x*Sqrt[Pi + c^2*Pi*x^2])/64 + (b^2*Pi*x*(1 + c^2*x^2)*Sqrt[Pi +
c^2*Pi*x^2])/32 - (9*b^2*Pi*Sqrt[Pi + c^2*Pi*x^2]*ArcSinh[c*x])/(64*c*Sqrt[
1 + c^2*x^2]) - (3*b*c*Pi*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]))/(
8*Sqrt[1 + c^2*x^2]) - (b*Pi*(1 + c^2*x^2)^(3/2)*Sqrt[Pi + c^2*Pi*x^2]*(a +
 b*ArcSinh[c*x]))/(8*c) + (3*Pi*x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x]
)^2)/8 + (x*(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (Pi*Sqrt[Pi
 + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(\pi + c^2\pi x^2\right)^{3/2} \left(a + b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{4}x\left(\pi + c^2\pi x^2\right)^{3/2}\left(a + b\sinh^{-1}(cx)\right)^2 + \frac{1}{4}(3\pi)\int \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(\right.$$

$$= -\frac{b\pi\left(1 + c^2x^2\right)^{3/2}\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{8c} + \frac{3}{8}\pi x\sqrt{\pi + c^2\pi x^2}\left(a + b\,\mathrm{si}\right.$$

$$= \frac{1}{32}b^2\pi x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2} - \frac{3bc\pi x^2\sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)}{8\sqrt{1 + c^2x^2}} - \frac{b\pi}{}$$

$$= \frac{15}{64}b^2\pi x\sqrt{\pi + c^2\pi x^2} + \frac{1}{32}b^2\pi x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2} - \frac{3bc\pi x^2\sqrt{\pi + c^2\pi x^2}}{8\sqrt{1 +}}$$

$$= \frac{15}{64}b^2\pi x\sqrt{\pi + c^2\pi x^2} + \frac{1}{32}b^2\pi x\left(1 + c^2x^2\right)\sqrt{\pi + c^2\pi x^2} - \frac{9b^2\pi\sqrt{\pi + c^2\pi x^2}\,\mathrm{si}}{64c\sqrt{1 + c^2x}}$$

Mathematica [A] time = 0.621436, size = 202, normalized size = 0.96

$$\frac{\pi^{3/2}\left(64a^2c^3x^3\sqrt{c^2x^2+1} + 160a^2cx\sqrt{c^2x^2+1} + 4\sinh^{-1}(cx)\left(4a\left(6a + 8b\sinh\left(2\sinh^{-1}(cx)\right) + b\sinh\left(4\sinh^{-1}(cx)\right)\right) - \right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi + c^2*Pi*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Pi^(3/2)*(160*a^2*c*x*Sqrt[1 + c^2*x^2] + 64*a^2*c^3*x^3*Sqrt[1 + c^2*x^2]
 + 32*b^2*ArcSinh[c*x]^3 - 64*a*b*Cosh[2*ArcSinh[c*x]] - 4*a*b*Cosh[4*ArcSi
nh[c*x]] + 32*b^2*Sinh[2*ArcSinh[c*x]] + b^2*Sinh[4*ArcSinh[c*x]] + 8*b*Arc
Sinh[c*x]^2*(12*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) + 4*
```

```
ArcSinh[c*x]*(-16*b^2*Cosh[2*ArcSinh[c*x]] - b^2*Cosh[4*ArcSinh[c*x]] + 4*a
*(6*a + 8*b*Sinh[2*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]))))/(256*c)
```

Maple [B] time = 0.081, size = 350, normalized size = 1.7

$$\frac{a^2x}{4}\left(\pi\,c^2x^2+\pi\right)^{\frac{3}{2}}+\frac{3\,a^2\pi\,x}{8}\sqrt{\pi\,c^2x^2+\pi}+\frac{3\,a^2\pi^2}{8}\ln\left(\pi\,c^2x\frac{1}{\sqrt{\pi\,c^2}}+\sqrt{\pi\,c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi\,c^2}}+\frac{b^2\pi^{\frac{3}{2}}c^2\left(\mathrm{Arcsinh}\,(cx)\right)^2x^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/4*a^2*x*(Pi*c^2*x^2+Pi)^(3/2)+3/8*a^2*Pi*x*(Pi*c^2*x^2+Pi)^(1/2)+3/8*a^2*
Pi^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/4*b
^2*Pi^(3/2)*c^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^3-1/8*b^2*Pi^(3/2)*c^3*a
rcsinh(c*x)*x^4+1/32*b^2*Pi^(3/2)*c^2*x^3*(c^2*x^2+1)^(1/2)+5/8*b^2*Pi^(3/2
)*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x-5/8*b^2*Pi^(3/2)*c*arcsinh(c*x)*x^2+1/
8*b^2*Pi^(3/2)/c*arcsinh(c*x)^3+17/64*b^2*Pi^(3/2)*x*(c^2*x^2+1)^(1/2)-17/6
4*b^2*Pi^(3/2)*arcsinh(c*x)/c+1/2*a*b*Pi^(3/2)*c^2*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*x^3-1/8*a*b*Pi^(3/2)*c^3*x^4+5/4*a*b*Pi^(3/2)*arcsinh(c*x)*(c^2*x^2+
1)^(1/2)*x-5/8*a*b*Pi^(3/2)*c*x^2+3/8*a*b*Pi^(3/2)/c*arcsinh(c*x)^2-1/2*a*b
*Pi^(3/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(\pi a^2c^2x^2+\pi a^2+\left(\pi b^2c^2x^2+\pi b^2\right)\mathrm{arsinh}\,(cx)^2+2\left(\pi abc^2x^2+\pi ab\right)\mathrm{arsinh}\,(cx)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(pi*a^2*c^2*x^2 + pi*a^2 + (pi*b^2*c^2*x^2 +
 pi*b^2)*arcsinh(c*x)^2 + 2*(pi*a*b*c^2*x^2 + pi*a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 151.738, size = 405, normalized size = 1.93

$$\begin{cases} \frac{\pi^{\frac{3}{2}} a^2 c^2 x^3 \sqrt{c^2 x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a^2 x \sqrt{c^2 x^2+1}}{8} + \frac{3\pi^{\frac{3}{2}} a^2 \operatorname{asinh}(cx)}{8c} - \frac{\pi^{\frac{3}{2}} abc^3 x^4}{8} + \frac{\pi^{\frac{3}{2}} abc^2 x^3 \sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{2} - \frac{5\pi^{\frac{3}{2}} abcx^2}{8} + \frac{5\pi^{\frac{3}{2}} abx\sqrt{c^2x^2+1} \operatorname{asinh}(cx)}{4} \\ \pi^{\frac{3}{2}} a^2 x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((pi**(3/2)*a**2*c**2*x**3*sqrt(c**2*x**2 + 1)/4 + 5*pi**(3/2)*a**
2*x*sqrt(c**2*x**2 + 1)/8 + 3*pi**(3/2)*a**2*asinh(c*x)/(8*c) - pi**(3/2)*a
*b*c**3*x**4/8 + pi**(3/2)*a*b*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/2 -
 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(c**2*x**2 + 1)*asinh(c*x
)/4 + 3*pi**(3/2)*a*b*asinh(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*asinh(
c*x)/8 + pi**(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/4 + pi*
*(3/2)*b**2*c**2*x**3*sqrt(c**2*x**2 + 1)/32 - 5*pi**(3/2)*b**2*c*x**2*asin
h(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(c**2*x**2 + 1)*asinh(c*x)**2/8 + 17*pi**
(3/2)*b**2*x*sqrt(c**2*x**2 + 1)/64 + pi**(3/2)*b**2*asinh(c*x)**3/(8*c) -
17*pi**(3/2)*b**2*asinh(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*a**2*x, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\pi + \pi c^2 x^2\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((pi + pi*c^2*x^2)^(3/2)*(b*arcsinh(c*x) + a)^2, x)
```

3.254 $\int \sqrt{\pi + c^2\pi x^2}\left(a + b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{1}{2}\sqrt{\pi}bcx^2\left(a+b\sinh^{-1}(cx)\right)+\frac{\sqrt{\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc}+\frac{1}{4}\sqrt{\pi}b^2x\sqrt{c^2x^2+1}-\sqrt{\pi}$$

```
[Out] (b^2*Sqrt[Pi]*x*Sqrt[1 + c^2*x^2])/4 - (b^2*Sqrt[Pi]*ArcSinh[c*x])/(4*c) -
(b*c*Sqrt[Pi]*x^2*(a + b*ArcSinh[c*x]))/2 + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b
*ArcSinh[c*x])^2)/2 + (Sqrt[Pi]*(a + b*ArcSinh[c*x])^3)/(6*b*c)
```

Rubi [A] time = 0.112915, antiderivative size = 184, normalized size of antiderivative = 1.51, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5682, 5675, 5661, 321, 215}

$$\frac{\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{\pi c^2x^2+\pi}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}b^2x\sqrt{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[Pi + c^2*Pi*x^2])/4 - (b^2*Sqrt[Pi + c^2*Pi*x^2]*ArcSinh[c*x])/
(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x
]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2
)/2 + (Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^
2])
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{\pi+c^2\pi x^2}\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{\pi\right.}$$

$$=-\frac{bcx^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{}}{}$$

$$=\frac{1}{4}b^2x\sqrt{\pi+c^2\pi x^2}-\frac{bcx^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{\pi+c^2\pi x^2}\left(a+b\right.$$

$$=\frac{1}{4}b^2x\sqrt{\pi+c^2\pi x^2}-\frac{b^2\sqrt{\pi+c^2\pi x^2}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{\pi+c^2\pi x^2}\left(a+b\sinh^{-1}\right.}{2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.344711, size = 124, normalized size = 1.02

$$\frac{\sqrt{\pi}\left(3\left(4a^2cx\sqrt{c^2x^2+1}-2ab\cosh\left(2\sinh^{-1}(cx)\right)+b^2\sinh\left(2\sinh^{-1}(cx)\right)\right)+6\sinh^{-1}(cx)\left(2a\left(a+b\sinh\left(2\sinh^{-1}(cx)\right)\right.\right.\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Pi + c^2*Pi*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[Pi]*(4*b^2*ArcSinh[c*x]^3 + 6*b*ArcSinh[c*x]^2*(2*a + b*Sinh[2*ArcSin
h[c*x]]) + 3*(4*a^2*c*x*Sqrt[1 + c^2*x^2] - 2*a*b*Cosh[2*ArcSinh[c*x]] + b^
2*Sinh[2*ArcSinh[c*x]]) + 6*ArcSinh[c*x]*(-(b^2*Cosh[2*ArcSinh[c*x]]) + 2*a
*(a + b*Sinh[2*ArcSinh[c*x]]))))/(24*c)
```

Maple [B] time = 0.071, size = 213, normalized size = 1.8

$$\frac{a^2x}{2}\sqrt{\pi c^2x^2+\pi}+\frac{a^2\pi}{2}\ln\left(\pi c^2x\frac{1}{\sqrt{\pi c^2}}+\sqrt{\pi c^2x^2+\pi}\right)\frac{1}{\sqrt{\pi c^2}}+\frac{b^2\sqrt{\pi}\left(\mathrm{Arcsinh}\left(cx\right)\right)^2x}{2}\sqrt{c^2x^2+1}-\frac{b^2\sqrt{\pi}c\mathrm{Arcsinh}\left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*c^2*x^2+Pi)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/2*a^2*x*(Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c
^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/2*b^2*Pi^(1/2)*arcsinh(c*x)^2*(c^2*x^2+1
)^(1/2)*x-1/2*b^2*Pi^(1/2)*c*arcsinh(c*x)*x^2+1/6*b^2*Pi^(1/2)/c*arcsinh(c*
x)^3+1/4*b^2*x*Pi^(1/2)*(c^2*x^2+1)^(1/2)-1/4*b^2*arcsinh(c*x)*Pi^(1/2)/c+a
*b*Pi^(1/2)*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x-1/2*a*b*Pi^(1/2)*c*x^2+1/2*a*b
*Pi^(1/2)/c*arcsinh(c*x)^2-1/2*a*b*Pi^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\pi+\pi c^2x^2}\left(b^2\,\mathrm{arsinh}\left(cx\right)^2+2\,ab\,\mathrm{arsinh}\left(cx\right)+a^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\pi}\left(\int a^2\sqrt{c^2x^2+1}\,dx+\int b^2\sqrt{c^2x^2+1}\operatorname{asinh}^2(cx)\,dx+\int 2ab\sqrt{c^2x^2+1}\operatorname{asinh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c**2*x**2+pi)**(1/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] sqrt(pi)*(Integral(a**2*sqrt(c**2*x**2 + 1), x) + Integral(b**2*sqrt(c**2*x
**2 + 1)*asinh(c*x)**2, x) + Integral(2*a*b*sqrt(c**2*x**2 + 1)*asinh(c*x),
 x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\pi+\pi c^2x^2}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi*c^2*x^2+pi)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(pi + pi*c^2*x^2)*(b*arcsinh(c*x) + a)^2, x)
```

3.255 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{\pi+c^2\pi x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}bc}$$

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Rubi [A] time = 0.0502889, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {5675}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{\pi+c^2\pi x^2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{\pi}}$$

Mathematica [A] time = 0.0240984, size = 25, normalized size = 1.

$$\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3\sqrt{\pi}bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[Pi + c^2*Pi*x^2],x]
```

```
[Out] (a + b*ArcSinh[c*x])^3/(3*b*c*Sqrt[Pi])
```

Maple [B] time = 0.063, size = 72, normalized size = 2.9

$$a^2 \ln\left(\pi c^2 x \frac{1}{\sqrt{\pi c^2}} + \sqrt{\pi c^2 x^2 + \pi}\right) \frac{1}{\sqrt{\pi c^2}} + \frac{b^2 (\mathrm{Arcsinh}(cx))^3}{3c\sqrt{\pi}} + \frac{ab (\mathrm{Arcsinh}(cx))^2}{c\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(1/2),x)
```

```
[Out] a^2*ln(Pi*x*c^2/(Pi*c^2)^(1/2)+(Pi*c^2*x^2+Pi)^(1/2))/(Pi*c^2)^(1/2)+1/3*b^
2/c/Pi^(1/2)*arcsinh(c*x)^3+a*b*arcsinh(c*x)^2/c/Pi^(1/2)
```

Maxima [B] time = 1.363, size = 230, normalized size = 9.2

$$\frac{b^2 \operatorname{arsinh}\left(\frac{c^2 x}{\sqrt{c^2}}\right) \operatorname{arsinh}(cx)^2}{\sqrt{\pi c^2}} + \frac{2\,ab \operatorname{arsinh}\left(\frac{c^2 x}{\sqrt{c^2}}\right) \operatorname{arsinh}(cx)}{\sqrt{\pi c^2}} + \frac{1}{3}\left(\frac{\operatorname{arsinh}\left(\frac{c^2 x}{\sqrt{c^2}}\right)^3}{\sqrt{\pi c^2}} - \frac{3\sqrt{c^2} \operatorname{arsinh}\left(\frac{c^2 x}{\sqrt{c^2}}\right)^2 \operatorname{arsinh}(cx)}{\sqrt{\pi c^2}\,c}\right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arcsinh(c^2*x/sqrt(c^2))*arcsinh(c*x)^2/sqrt(pi*c^2) + 2*a*b*arcsinh(c^
2*x/sqrt(c^2))*arcsinh(c*x)/sqrt(pi*c^2) + 1/3*(arcsinh(c^2*x/sqrt(c^2))^3/
sqrt(pi*c^2) - 3*sqrt(c^2)*arcsinh(c^2*x/sqrt(c^2))^2*arcsinh(c*x)/(sqrt(pi
*c^2)*c))*b^2 - a*b*sqrt(c^2)*arcsinh(c^2*x/sqrt(c^2))^2/(sqrt(pi*c^2)*c) +
 a^2*arcsinh(c^2*x/sqrt(c^2))/sqrt(pi*c^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{\sqrt{\pi + \pi c^2 x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(pi + pi*c^2*x
^2), x)
```

Sympy [A] time = 3.12885, size = 88, normalized size = 3.52

$$\begin{cases} a^2 \left(\begin{cases} \frac{\sqrt{-\frac{1}{c^2}}\operatorname{asin}\left(x\sqrt{-c^2}\right)}{\sqrt{\pi}} & \text{for}\ \pi c^2 < 0 \\ \frac{\sqrt{\frac{1}{c^2}}\operatorname{asinh}\left(x\sqrt{c^2}\right)}{\sqrt{\pi}} & \text{for}\ \pi c^2 > 0 \end{cases}\right) & \text{for}\ b = 0 \\ \frac{a^2x}{\sqrt{\pi}} & \text{for}\ c = 0 \\ \frac{(a+b\operatorname{asinh}(cx))^3}{3\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((a**2*Piecewise((sqrt(-1/c**2)*asin(x*sqrt(-c**2))/sqrt(pi), pi*c
**2 < 0), (sqrt(c**(-2))*asinh(x*sqrt(c**2))/sqrt(pi), pi*c**2 > 0)), Eq(b,
 0)), (a**2*x/sqrt(pi), Eq(c, 0)), ((a + b*asinh(c*x))**3/(3*sqrt(pi)*b*c),
 True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\sqrt{\pi + \pi c^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(pi + pi*c^2*x^2), x)
```

3.256 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{b^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{\pi^{3/2}c}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\pi^{3/2}c}-\frac{2b\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi^{3/2}c}$$

```
[Out] (a + b*ArcSinh[c*x])^2/(c*Pi^(3/2)) + (x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[P
i + c^2*Pi*x^2]) - (2*b*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(
c*Pi^(3/2)) - (b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*Pi^(3/2))
```

Rubi [A] time = 0.176843, antiderivative size = 179, normalized size of antiderivative = 1.72, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{\pi c\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi c^2x^2+\pi}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{\pi c\sqrt{\pi c^2x^2+\pi}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}+1\right)\left(a+b\sinh^{-1}(cx)\right)}{\pi c\sqrt{\pi c^2x^2+\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(Pi*Sqrt[Pi + c^2*Pi*x^2]) + (Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x])^2)/(c*Pi*Sqrt[Pi + c^2*Pi*x^2]) - (2*b*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*Pi*Sqrt[Pi + c^2*Pi
*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*Pi*Sqrt
[Pi + c^2*Pi*x^2])
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int\frac{e^{2x}(a+bx)}{1+e^{2x}}\,d\right.}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\pi\sqrt{\pi+c^2\pi x^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c\pi\sqrt{\pi+c^2\pi x^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{c\pi\sqrt{\pi+c^2\pi x^2}}$$

Mathematica [A] time = 0.387552, size = 153, normalized size = 1.47

$$\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+a\left(acx-b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)\right)+2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\log\left(e^{-2}\right.\right.}{\pi^{3/2}c\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*Pi^(3/2)*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.129, size = 306, normalized size = 2.9

$$\frac{a^2x}{\pi}\frac{1}{\sqrt{\pi c^2x^2+\pi}} - \frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2cx^2}{\pi^{\frac{3}{2}}\left(c^2x^2+1\right)} + \frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2x}{\pi^{\frac{3}{2}}}\frac{1}{\sqrt{c^2x^2+1}} - \frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2}{\pi^{\frac{3}{2}}c\left(c^2x^2+1\right)} + 2\frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2}{\pi^{3/2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(3/2),x)
```

```
[Out] a^2/Pi*x/(Pi*c^2*x^2+Pi)^(1/2)-b^2/Pi^(3/2)*arcsinh(c*x)^2*c/(c^2*x^2+1)*x^
2+b^2/Pi^(3/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)*x-b^2/Pi^(3/2)*arcsinh(c*x)
^2/c/(c^2*x^2+1)+2*b^2/c/Pi^(3/2)*arcsinh(c*x)^2-2*b^2/c/Pi^(3/2)*arcsinh(c
*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-b^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^
2)/c/Pi^(3/2)+4*a*b/c/Pi^(3/2)*arcsinh(c*x)-2*a*b/Pi^(3/2)*arcsinh(c*x)*c/(
c^2*x^2+1)*x^2+2*a*b/Pi^(3/2)*arcsinh(c*x)/(c^2*x^2+1)^(1/2)*x-2*a*b/Pi^(3/
2)*arcsinh(c*x)/c/(c^2*x^2+1)-2*a*b/c/Pi^(3/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc\sqrt{\frac{1}{\pi c^4}}\log\left(x^2+\frac{1}{c^2}\right)}{\pi}+b^2\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx+\frac{2\,abx\,\operatorname{arsinh}\left(cx\right)}{\pi\sqrt{\pi+\pi c^2x^2}}+\frac{a^2x}{\pi\sqrt{\pi+\pi c^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*b*c*sqrt(1/(pi*c^4))*log(x^2 + 1/c^2)/pi + b^2*integrate(log(c*x + sqrt(
c^2*x^2 + 1))^2/(pi + pi*c^2*x^2)^(3/2), x) + 2*a*b*x*arcsinh(c*x)/(pi*sqrt
(pi + pi*c^2*x^2)) + a^2*x/(pi*sqrt(pi + pi*c^2*x^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi+\pi c^2x^2}\left(b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2\right)}{\pi^2c^4x^4+2\,\pi^2c^2x^2+\pi^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^2*c^4*x^4 + 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{asinh}^2(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx+\int \frac{2ab\operatorname{asinh}(cx)}{c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(3/2),x)
```

```
[Out] (Integral(a**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x) +
Integral(b**2*asinh(c*x)**2/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2
 + 1)), x) + Integral(2*a*b*asinh(c*x)/(c**2*x**2*sqrt(c**2*x**2 + 1) + sqr
t(c**2*x**2 + 1)), x))/pi**(3/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(3/2), x)
```

3.257 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2b^2\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3\pi^{5/2}c}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3\pi^{5/2}c\left(c^2x^2+1\right)}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi c^2x^2+\pi\right)^{3/2}}+\frac{2\left(a+b\sinh\right.}{3\pi^{5/}}$$

```
[Out] -(b^2*x)/(3*Pi^(5/2)*Sqrt[1 + c^2*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*Pi^
(5/2)*(1 + c^2*x^2)) + (2*(a + b*ArcSinh[c*x])^2)/(3*c*Pi^(5/2)) + (x*(a +
b*ArcSinh[c*x])^2)/(3*Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x
])^2)/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (4*b*(a + b*ArcSinh[c*x])*Log[1 + E^
(2*ArcSinh[c*x])])/(3*c*Pi^(5/2)) - (2*b^2*PolyLog[2, -E^(2*ArcSinh[c*x])])
/(3*c*Pi^(5/2))
```

Rubi [A] time = 0.281537, antiderivative size = 292, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191}

$$-\frac{2b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3\pi^2c\sqrt{\pi c^2x^2+\pi}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3\pi^2c\sqrt{c^2x^2+1}\sqrt{\pi c^2x^2+\pi}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi c^2x^2+\pi}}+\frac{2\sqrt{c^2x^2+1}\left(a+b\right.}{3\pi^2c\sqrt{\pi c^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*Pi^
2*Sqrt[1 + c^2*x^2]*Sqrt[Pi + c^2*Pi*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*
Pi*(Pi + c^2*Pi*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*Pi^2*Sqrt[Pi
+ c^2*Pi*x^2]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*Pi^2*Sqr
t[Pi + c^2*Pi*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E
^(2*ArcSinh[c*x])])/(3*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2]) - (2*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*Pi^2*Sqrt[Pi + c^2*Pi*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
```

```
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx}{3\pi} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}\,dx}{3\pi^2\sqrt{\pi+c^2\pi x^2}}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi^2\sqrt{\pi+c^2\pi x^2}} - \frac{\left(b^2\sqrt{1+c\right.}{3\pi^2}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh\right.}{3\pi^2\sqrt{\pi+c}}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh\right.}{3\pi^2\sqrt{\pi+c}}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh\right.}{3\pi^2\sqrt{\pi+c}}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh\right.}{3\pi^2\sqrt{\pi+c}}$$

$$= -\frac{b^2x}{3\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3c\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3\pi\left(\pi+c^2\pi x^2\right)^{3/2}} + \frac{2x\left(a+b\sinh\right.}{3\pi^2\sqrt{\pi+c}}$$

Mathematica [A] time = 0.623747, size = 293, normalized size = 1.44

$$\frac{2b^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+2a^2c^3x^3+3a^2cx+ab\sqrt{c^2x^2+1}-2abc^2x^2\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)-2ab\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Pi + c^2*Pi*x^2)^(5/2),x]
```

```
[Out] (3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 - b^2*c^3*x^3 + a*b*Sqrt[1 + c^2*x^2]
- b^2*(-3*c*x - 2*c^3*x^3 + 2*Sqrt[1 + c^2*x^2] + 2*c^2*x^2*Sqrt[1 + c^2*x^
2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-6*a*c*x - 4*a*c^3*x^3 - b*Sqrt[1 + c^
2*x^2] + 4*b*(1 + c^2*x^2)^(3/2)*Log[1 + E^(-2*ArcSinh[c*x])]) - 2*a*b*Sqrt
[1 + c^2*x^2]*Log[1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^
2*x^2] + 2*b^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])])/(3*c*P
i^(5/2)*(1 + c^2*x^2)^(3/2))
```

Maple [B] time = 0.224, size = 1730, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(Pi*c^2*x^2+Pi)^(5/2),x)
```

```
[Out] -3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsinh(c*x)*x^2+8*a*b/Pi^(5/2)
/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x-4*a*b/Pi^(5/2)*c^3/(3*c^2*x
^2+4)/(c^2*x^2+1)*x^4-4/3*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6+8*
b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4-22/3*b^2/Pi^(
5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2*x^2+16/3*b^2/Pi^(5/2)*c/(
3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+4/3*b^2/Pi^(5/2)*c^7/(3*c^2*x^2
+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^8-2*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^
2+1)^2*arcsinh(c*x)^2*x^6+16/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2
*arcsinh(c*x)*x^6-20/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh
(c*x)^2*x^4-4*a*b/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^6
+34/3*a*b/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^3-40/
3*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^4+4*a*b/Pi^(5
/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)*x^5-44/3*a*b/Pi^(5/2)*
c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)*x^2+6*b^2/Pi^(5/2)*c^3/(3*c^2*x^
2+4)/(c^2*x^2+1)^2*x^4+14/3*b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2-
```

```
8/3*b^2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)^2+4/3*b^2/Pi^(5
/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)+2/3*b^2/Pi^(5/2)*c^7/(3*c^2*
x^2+4)/(c^2*x^2+1)^2*x^8+10/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*
x^6+4/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-7/3*b^2/Pi^(5/2)*c^2/(3*
c^2*x^2+4)/(c^2*x^2+1)^(3/2)*x^3-b^2/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)*x
^2+4*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c*x)^2*x-2/3*b^2/
Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)*x^6-b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(
c^2*x^2+1)^(3/2)*x^5-5/3*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*x^4+8/3
*a*b/c/Pi^(5/2)*arcsinh(c*x)-4/3*a*b/c/Pi^(5/2)*ln(1+(c*x+(c^2*x^2+1)^(1/2)
)^2)-4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)+4/3*b^
2/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2-3*a*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^
2*x^2+1)*x^2+4/3*a*b/Pi^(5/2)*c^7/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^8+16/3*a*b/
Pi^(5/2)*c^5/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^6+8*a*b/Pi^(5/2)*c^3/(3*c^2*x^2+
4)/(c^2*x^2+1)^2*x^4+16/3*a*b/Pi^(5/2)*c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*x^2-16
/3*a*b/Pi^(5/2)/c/(3*c^2*x^2+4)/(c^2*x^2+1)^2*arcsinh(c*x)-2/3*b^2*polylog(
2,-(c*x+(c^2*x^2+1)^(1/2))^2)/c/Pi^(5/2)-4/3*b^2/Pi^(5/2)*c^5/(3*c^2*x^2+4)
/(c^2*x^2+1)*arcsinh(c*x)*x^6+2*b^2/Pi^(5/2)*c^4/(3*c^2*x^2+4)/(c^2*x^2+1)^
(3/2)*arcsinh(c*x)^2*x^5-4*b^2/Pi^(5/2)*c^3/(3*c^2*x^2+4)/(c^2*x^2+1)*arcsi
nh(c*x)*x^4+17/3*b^2/Pi^(5/2)*c^2/(3*c^2*x^2+4)/(c^2*x^2+1)^(3/2)*arcsinh(c
*x)^2*x^3+4/3*b^2/c/Pi^(5/2)*arcsinh(c*x)^2+1/3*a^2/Pi*x/(Pi*c^2*x^2+Pi)^(3
/2)+2/3*a^2/Pi^2*x/(Pi*c^2*x^2+Pi)^(1/2)-4/3*b^2/Pi^(5/2)/(3*c^2*x^2+4)/(c^
2*x^2+1)^(3/2)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}abc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2+\pi^{\frac{5}{2}}c^2}-\frac{2\log\left(c^2x^2+1\right)}{\pi^{\frac{5}{2}}c^2}\right)+\frac{2}{3}ab\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right)^{\frac{3}{2}}}+\frac{2x}{\pi^2\sqrt{\pi+\pi c^2x^2}}\right)\operatorname{arsinh}(cx)+\frac{1}{3}a^2\left(\frac{x}{\pi\left(\pi+\pi c^2x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 + pi^(5/2)*c^2) - 2*log(c^2*x^2 + 1)/(pi^(5/
2)*c^2)) + 2/3*a*b*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi + pi
*c^2*x^2)))*arcsinh(c*x) + 1/3*a^2*(x/(pi*(pi + pi*c^2*x^2)^(3/2)) + 2*x/(p
i^2*sqrt(pi + pi*c^2*x^2))) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/
(pi + pi*c^2*x^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\pi + \pi c^2 x^2}\left(b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2\right)}{\pi^3 c^6 x^6 + 3\,\pi^3 c^4 x^4 + 3\,\pi^3 c^2 x^2 + \pi^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(pi + pi*c^2*x^2)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2)/(pi^3*c^6*x^6 + 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 + pi^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{b^2 \operatorname{asinh}^2(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx + \int \frac{2ab \operatorname{asinh}(cx)}{c^4x^4\sqrt{c^2x^2+1}+2c^2x^2\sqrt{c^2x^2+1}+\sqrt{c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(pi*c**2*x**2+pi)**(5/2),x)
```

```
[Out] (Integral(a**2/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2
+ 1) + sqrt(c**2*x**2 + 1)), x) + Integral(b**2*asinh(c*x)**2/(c**4*x**4*sq
rt(c**2*x**2 + 1) + 2*c**2*x**2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)),
 x) + Integral(2*a*b*asinh(c*x)/(c**4*x**4*sqrt(c**2*x**2 + 1) + 2*c**2*x**
2*sqrt(c**2*x**2 + 1) + sqrt(c**2*x**2 + 1)), x))/pi**(5/2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(\pi + \pi c^2 x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(pi + pi*c^2*x^2)^(5/2), x)
```

3.258 $\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=358

$$\frac{4abx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}-\frac{2bcx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bx^3\sqrt{c^2dx^2+d}\left(a+b\right.}{45c\sqrt{c^2x^2+}}$$

```
[Out] (-52*b^2*Sqrt[d + c^2*d*x^2])/(225*c^4) + (4*a*b*x*Sqrt[d + c^2*d*x^2])/(15
*c^3*Sqrt[1 + c^2*x^2]) - (26*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(675*c
^4) + (2*b^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c^4) + (4*b^2*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(15*c^3*Sqrt[1 + c^2*x^2]) - (2*b*x^3*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*c*Sqrt[1 + c^2*x^2]) - (2*b*c*x^5*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) - (2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4) + (x^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/5
```

Rubi [A] time = 0.478806, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5742, 5758, 5717, 5653, 261, 5661, 266, 43}

$$\frac{4abx\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}-\frac{2bcx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}+\frac{1}{5}x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{2bx^3\sqrt{c^2dx^2+d}\left(a+b\right.}{45c\sqrt{c^2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (-52*b^2*Sqrt[d + c^2*d*x^2])/(225*c^4) + (4*a*b*x*Sqrt[d + c^2*d*x^2])/(15
*c^3*Sqrt[1 + c^2*x^2]) - (26*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/(675*c
^4) + (2*b^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c^4) + (4*b^2*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(15*c^3*Sqrt[1 + c^2*x^2]) - (2*b*x^3*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*c*Sqrt[1 + c^2*x^2]) - (2*b*c*x^5*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) - (2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4) + (x^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/5
```


Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{5}x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{5\sqrt{1+c^2x^2}}-\frac{(2$$

$$=-\frac{2bcx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{1+c^2x^2}}+\frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^2}+$$

$$=-\frac{2bx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{1+c^2x^2}}-\frac{2bcx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{1+c^2x^2}}$$

$$=\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{2bx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45c\sqrt{1+c^2x^2}}-\frac{2bcx^5\sqrt{d+c^2dx^2}\left(a\right.}{25\sqrt{1+}}$$

$$=\frac{2b^2\sqrt{d+c^2dx^2}}{25c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{4b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{75c^4}+\frac{2b^2\left(1+c\right.}{}$$

$$=-\frac{52b^2\sqrt{d+c^2dx^2}}{225c^4}+\frac{4abx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{26b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{675c^4}+\frac{2b^2\left(1\right.}{}$$

Mathematica [A] time = 0.268427, size = 222, normalized size = 0.62

$$\frac{\sqrt{c^2dx^2+d}\left(-30abcx\sqrt{c^2x^2+1}\left(9c^4x^4+5c^2x^2-30\right)-30b\sinh^{-1}(cx)\left(bcx\sqrt{c^2x^2+1}\left(9c^4x^4+5c^2x^2-30\right)-15a\left(c^2x^2\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(225*(-2 + 3*c^2*x^2)*(a + a*c^2*x^2)^2 - 30*a*b*c*x*S
qrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(-428 - 439*c^2*x^2
+ 16*c^4*x^4 + 27*c^6*x^6) - 30*b*(-15*a*(1 + c^2*x^2)^2*(-2 + 3*c^2*x^2) +
 b*c*x*Sqrt[1 + c^2*x^2]*(-30 + 5*c^2*x^2 + 9*c^4*x^4))*ArcSinh[c*x] + 225*
(-2 + 3*c^2*x^2)*(b + b*c^2*x^2)^2*ArcSinh[c*x]^2))/(3375*c^4*(1 + c^2*x^2)
)
```

Maple [B] time = 0.349, size = 1162, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(1/5*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(c^2*d*x^2+d)^(3/2))+b^2*
(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c
^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(
25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2
+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)
+2)/c^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/
2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)-1/864*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)
^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4/(c^2*x^2+1)+1/4000*(d*(c^
2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*
x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x
)^2+10*arcsinh(c*x)+2)/c^4/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*
(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^
(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^4/(c^2*x^
2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/(c^2*x^2+1)-1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c
^4/(c^2*x^2+1)-1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)
*(1+arcsinh(c*x))/c^4/(c^2*x^2+1)-1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*
c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh
(c*x))/c^4/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(
```

```
c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(
c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))/c^4/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.90919, size = 694, normalized size = 1.94

$$\frac{225\left(3\,b^2c^6x^6+4\,b^2c^4x^4-b^2c^2x^2-2\,b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(45\,abc^6x^6+60\,abc^4x^4-15\,abc^2x^2-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 + 4*b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 + 60*a*b*c^4
*x^4 - 15*a*b*c^2*x^2 - 30*a*b - (9*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 30*b^2*c*
x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (2
7*(25*a^2 + 2*b^2)*c^6*x^6 + 4*(225*a^2 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b
^2)*c^2*x^2 - 450*a^2 - 856*b^2 - 30*(9*a*b*c^5*x^5 + 5*a*b*c^3*x^3 - 30*a*
b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.259 $\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=291

$$-\frac{bcx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2x^2+1}}+\frac{x\sqrt{c^2$$

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d + c^2*d*x^2])/32 - (
b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c^3*Sqrt[1 + c^2*x^2]) - (b*x^2*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/4 - (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(24
*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.372315, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5742, 5758, 5675, 5661, 321, 215}

$$-\frac{bcx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{1}{4}x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2x^2+1}}+\frac{x\sqrt{c^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d + c^2*d*x^2])/32 - (
b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c^3*Sqrt[1 + c^2*x^2]) - (b*x^2*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c*Sqrt[1 + c^2*x^2]) - (b*c*x^4
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8*c^2) + (x^3*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/4 - (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(24
*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
```

```
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{4}x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\int\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{4\sqrt{1+c^2x^2}} - \frac{(b\cdots}{\cdots}$$

$$= -\frac{bcx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{1+c^2x^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{8c^2} + \frac{1}{4}\cdots$$

$$= \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{bx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{1+c^2x^2}} - \frac{bcx^4\sqrt{d+c^2dx^2}\,(a\cdots}{8\sqrt{1+\cdots}}$$

$$= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{bx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{1+c^2x^2}} - \cdots$$

$$= \frac{b^2x\sqrt{d+c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d+c^2dx^2} - \frac{b^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{64c^3\sqrt{1+c^2x^2}} - \frac{bx^2\sqrt{d+\cdots}}{\cdots}$$

Mathematica [A] time = 1.3118, size = 207, normalized size = 0.71

$$-\frac{-96a^2cx\left(2c^2x^2+1\right)\sqrt{c^2dx^2+d} + 96a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) + \frac{12ab\sqrt{c^2dx^2+d}\left(8\sinh^{-1}(cx)^2-4\sinh\left(4\sinh^{-1}(cx)\right)\sinh^{-1}\cdots\right.}{\sqrt{c^2x^2+1}}\cdots}{768c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(-96*a^2*c*x*(1 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 96*a^2*Sqrt[d]*Log[c*d*
x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (12*a*b*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c
*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/Sqrt[1
 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]
*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*ArcSinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/Sqr
t[1 + c^2*x^2])/(768*c^3)
```

Maple [B] time = 0.241, size = 701, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1/8*a
^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b^
2*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^5+3/8*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x^3-1/8*b^2*(d*(c^2*x^2+1))^(1/2)*
c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-1/8*b^2*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*x^2+1/8*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2+1)*
arcsinh(c*x)^2*x-1/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)-1/24*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3
+1/32*b^2*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*x^5+3/64*b^2*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)*x^3+1/64*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2+1)*x+1/
2*a*b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/8*a*b*(d*(c^
2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*x^4+3/4*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)*arcsinh(c*x)*x^3-1/8*a*b*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)*x
^2+1/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/(c^2*x^2+1)*arcsinh(c*x)*x-1/64*a*b*(d
*(c^2*x^2+1))^(1/2)/c^3/(c^2*x^2+1)^(1/2)-1/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^
2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^2\,\text{arsinh}\,(cx)^2+2\,abx^2\,\text{arsinh}\,(cx)+a^2x^2\right)\sqrt{c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2+d}\,(b\operatorname{arsinh}\left(cx\right)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

3.260 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=180

$$-\frac{2bcx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{2bx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}+\frac{2b^2\left(c\right.}{}$$

```
[Out] (4*b^2*Sqrt[d + c^2*d*x^2])/(9*c^2) + (2*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x
^2])/(27*c^2) - (2*b*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c*Sqrt[
1 + c^2*x^2]) - (2*b*c*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqr
t[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rubi [A] time = 0.152233, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5717, 5679, 444, 43}

$$-\frac{2bcx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{2bx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{c^2x^2+1}}+\frac{\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}+\frac{2b^2\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (4*b^2*Sqrt[d + c^2*d*x^2])/(9*c^2) + (2*b^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x
^2])/(27*c^2) - (2*b*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c*Sqrt[
1 + c^2*x^2]) - (2*b*c*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqr
t[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{\left(2b\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)\left(a+b\sinh\right.}{3c\sqrt{1+c^2x^2}}$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{1+c^2x^2}}+$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{1+c^2x^2}}+$$

$$=-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3c\sqrt{1+c^2x^2}}-\frac{2bcx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{1+c^2x^2}}+$$

$$=\frac{4b^2\sqrt{d+c^2dx^2}}{9c^2}+\frac{2b^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{27c^2}-\frac{2bx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{3c\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.27076, size = 166, normalized size = 0.92

$$\frac{\sqrt{c^2dx^2+d}\left(-6abcx\sqrt{c^2x^2+1}\left(c^2x^2+3\right)+6b\sinh^{-1}(cx)\left(3a\left(c^2x^2+1\right)^2-bcx\sqrt{c^2x^2+1}\left(c^2x^2+3\right)\right)+9\left(ac^2x^2+a\right)^2+\right.}{27c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-6*a*b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2) + 9*(a + a
*c^2*x^2)^2 + 2*b^2*(7 + 8*c^2*x^2 + c^4*x^4) + 6*b*(3*a*(1 + c^2*x^2)^2 -
b*c*x*Sqrt[1 + c^2*x^2]*(3 + c^2*x^2))*ArcSinh[c*x] + 9*(b + b*c^2*x^2)^2*A
rcSinh[c*x]^2))/(27*c^2*(1 + c^2*x^2))
```

Maple [B] time = 0.177, size = 657, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/3*a^2/c^2/d*(c^2*d*x^2+d)^(3/2)+b^2*(1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x
^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcs
inh(c*x)^2-6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/(c^2*x^
2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c
*x)^2+2*arcsinh(c*x)+2)/c^2/(c^2*x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arc
sinh(c*x)^2+6*arcsinh(c*x)+2)/c^2/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^
(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1
/2)+1)*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x
^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x
^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/(c^2*x^
2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^
2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^2/(c^2*x^2+1))
```

Maxima [A] time = 1.26318, size = 247, normalized size = 1.37

$$\frac{2}{27}\,b^2\left(\frac{\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}x^2+\frac{7\sqrt{c^2x^2+1}\,d^{\frac{3}{2}}}{c^2}}{d}-\frac{3\left(c^2d^{\frac{3}{2}}x^3+3\,d^{\frac{3}{2}}x\right)\operatorname{arsinh}(cx)}{cd}\right)+\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}b^2\operatorname{arsinh}(cx)^2}{3\,c^2d}+\frac{2\left(c^2dx^2+d\right)^{\frac{3}{2}}a\cdots}{3\,c^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*b^2*((sqrt(c^2*x^2 + 1)*d^(3/2)*x^2 + 7*sqrt(c^2*x^2 + 1)*d^(3/2)/c^2)
/d - 3*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*arcsinh(c*x)/(c*d)) + 1/3*(c^2*d*x^2
 + d)^(3/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/3*(c^2*d*x^2 + d)^(3/2)*a*b*arcs
inh(c*x)/(c^2*d) - 2/9*(c^2*d^(3/2)*x^3 + 3*d^(3/2)*x)*a*b/(c*d) + 1/3*(c^2
*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Fricas [A] time = 2.71232, size = 531, normalized size = 2.95

$$\frac{9\left(b^2c^4x^4 + 2\,b^2c^2x^2 + b^2\right)\sqrt{c^2dx^2 + d}\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + 6\left(3\,abc^4x^4 + 6\,abc^2x^2 + 3\,ab - \left(b^2c^3x^3 + 3\,b^2cx\right)\sqrt{c^2x^2 +}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 + 6*a*b*c^2*x^2 + 3*a*b - (b^2*c^3*x
^3 + 3*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + 2*(9*a^2 + 8*b^2)*c^2*x^2 + 9*a^2 + 1
4*b^2 - 6*(a*b*c^3*x^3 + 3*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^4*x^2 + c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{d\left(c^2x^2 + 1\right)}\,(a + b\,\mathrm{asinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.261 $\int \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=184

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}b^2x\sqrt{c^2d}$$

```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/4 - (b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(4*c
*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.120337, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5682, 5675, 5661, 321, 215}

$$\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{4}b^2x\sqrt{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (b^2*x*Sqrt[d + c^2*d*x^2])/4 - (b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(4*c
*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[1 + c^2*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d+c^2dx^2}\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{2\sqrt{1+c^2x^2}}-\frac{\left(bc\sqrt{d+\right.}{}$$

$$=-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\sqrt{d}}{}$$

$$=\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+\frac{1}{2}x\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{si}\right.$$

$$=\frac{1}{4}b^2x\sqrt{d+c^2dx^2}-\frac{b^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}}-\frac{bcx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.}{2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.08672, size = 200, normalized size = 1.09

$$\frac{1}{24}\left(12a^2x\sqrt{c^2dx^2+d}+\frac{12a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)}{c}+\frac{6ab\sqrt{c^2dx^2+d}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}\right.\right.\right.}{c\sqrt{c^2x^2+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d + c^2*d*x^2] + (12*a^2*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d
+ c^2*d*x^2]])/c + (b^2*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c
*x]*Cosh[2*ArcSinh[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(c
*Sqrt[1 + c^2*x^2]) + (6*a*b*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2
*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])))/(c*Sqrt[1 + c^2*x^2])
)/24
```

Maple [B] time = 0.146, size = 482, normalized size = 2.6

$$\frac{xa^2}{2}\sqrt{c^2dx^2+d}+\frac{a^2d}{2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}-\frac{b^2c\text{Arcsinh}\,(cx)\,x^2}{2}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+\frac{b^2\,(\text{Arcsinh}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*a^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*x^2+1/6*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(
c*x)^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/2
*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x-1/4*b^2*(d*(c^2*x^2
+1))^(1/2)/c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c
^2/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*x+1/2*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2+a*b*(d*(c^2*x^2+1))^(1
/2)*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/2*a*b*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x
^2+1)^(1/2)*x^2+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)*x-1/4*a*
b*(d*(c^2*x^2+1))^(1/2)/c/(c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

3.262 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=338

$$-\frac{2b\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2b\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}$$

```
[Out] 2*b^2*Sqrt[d + c^2*d*x^2] - (2*a*b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^
2] - (2*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
 (2*b*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] - (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.342552, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5742, 5760, 4182, 2531, 2282, 6589, 5653, 261}

$$-\frac{2b\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2b\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```


```
[Out] 2*b^2*Sqrt[d + c^2*d*x^2] - (2*a*b*c*x*Sqrt[d + c^2*d*x^2])/Sqrt[1 + c^2*x^
2] - (2*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] +
 (2*b*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] - (2*b^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(2bc\sqrt{d+\right.}{}$$

$$= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\sqrt{d+c^2dx^2}\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2d}$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2d}$$

$$= 2b^2\sqrt{d+c^2dx^2} - \frac{2abcx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} + \sqrt{d+c^2d}$$

Mathematica [A] time = 1.16183, size = 352, normalized size = 1.04

$$\frac{2ab\sqrt{c^2dx^2+d}\left(\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\left(1\right.\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d + c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] + (2*a*b*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*
Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^
(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (b^2*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 +
 c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh
[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x
])] + 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLo
g[2, E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E
^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]
```

Maple [B] time = 0.277, size = 823, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)*a^2+a^2*(c^2*d*x^2+d)^(1
/2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)^2*x^2*c^2-2*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c+2*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)*c^2*x^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x
)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2
*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/
2))+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^
2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)*arcsinh(c*x)*x^2*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*c*x+2*
a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))+2*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1
/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\text{asinh}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x, x)
```

3.263 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=209

$$-\frac{b^2c\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2}...}{}$$

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x) + (c*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + (c*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2]) + (2*b*c*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] - (b^2*c*Sqr
t[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.254924, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5737, 5659, 3716, 2190, 2279, 2391, 5675}

$$\frac{b^2c\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}+\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}-\frac{c\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2x^2+1}}-\frac{\sqrt{c^2dx}...}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x) - (c*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2*x^2] + (c*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2]) + (2*b*c*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] + (b^2*c*Sqrt
[d + c^2*d*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
```

```
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x}\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(c^2\sqrt{\right.}{}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{1+c^2x^2}} + \frac{\left(2bc\sqrt{a\right.}{}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+}}{}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+}}{}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+}}{}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{c\sqrt{d+}}{}$$

Mathematica [A] time = 1.15233, size = 232, normalized size = 1.11

$$\frac{b^2c\sqrt{c^2dx^2+d}\left(\sinh^{-1}(cx)\left(\left(3-\frac{3\sqrt{c^2x^2+1}}{cx}\right)\sinh^{-1}(cx)+\sinh^{-1}(cx)^2+6\log\left(1-e^{-2\sinh^{-1}(cx)}\right)\right)-3\text{PolyLog}\left(2,e^{-2\sinh}\right.\right.}{3\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[d + c^2*d*x^2])/x) + (a*b*Sqrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2 + 2*c*x*Log[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + a^2*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (b^2*c*Sqrt
[d + c^2*d*x^2]*(ArcSinh[c*x]*((3 - (3*Sqrt[1 + c^2*x^2])/(c*x))*ArcSinh[c*
x] + ArcSinh[c*x]^2 + 6*Log[1 - E^(-2*ArcSinh[c*x])]) - 3*PolyLog[2, E^(-2*
ArcSinh[c*x])]))/(3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.241, size = 625, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(3/2)+a^2*c^2*x*(c^2*d*x^2+d)^(1/2)+a^2*c^2*d*ln(x*c
^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsi
nh(c*x)^2*x/(c^2*x^2+1)*c^2-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x
^2+1)^(1/2)*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/x/(c^2*x^2+1)+2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(
1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*l
n(1-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
polylog(2,c*x+(c^2*x^2+1)^(1/2))*c+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)^2*c-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*x/(c^2*x^2+1)*c^2-2*a*b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/x/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x^2, x)
```

3.264 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=358

$$-\frac{bc^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}$$

```
[Out] -((b*c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^
2*c^2*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (
b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.382681, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5737, 5661, 266, 63, 208, 5760, 4182, 2531, 2282, 6589}

$$-\frac{bc^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{bc^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```


```
[Out] -((b*c*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (c^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^
2*c^2*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (
b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (b*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*Sqrt[d + c^2*d*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bc\sqrt{d+c^2dx^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2}\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(c^2\sqrt{d\cdots\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(c^2\sqrt{d\cdots\right.}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} - \frac{c^2\sqrt{d+\cdots}}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} - \frac{c^2\sqrt{d+\cdots}}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} - \frac{c^2\sqrt{d+\cdots}}{}$$

$$= -\frac{bc\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} - \frac{c^2\sqrt{d+\cdots}}{}$$

Mathematica [A] time = 5.06922, size = 446, normalized size = 1.25

$$\frac{1}{8}\left(\frac{2abc^2\sqrt{c^2dx^2+d}\left(4\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-4\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-4\,\text{s}\cdots\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 + 4*a^2*c^2*Sqrt[d]*Log[x] - 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*Sqrt[d + c^2*d*x^2]
*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])
] + 4*PolyLog[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - Ar
cSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x
^2] + (b^2*c^2*Sqrt[d + c^2*d*x^2]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] -
ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSin
h[c*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh
[c*x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*
PolyLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLo
```

```
g[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh
[c*x]*Tanh[ArcSinh[c*x]/2]))/Sqrt[1 + c^2*x^2])/8
```

Maple [B] time = 0.318, size = 870, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(3/2)-1/2*a^2*d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d
*x^2+d)^(1/2))/x)*c^2+1/2*a^2*(c^2*d*x^2+d)^(1/2)*c^2-1/2*b^2*arcsinh(c*x)^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*c^2-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/x*c-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)/x^2-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*
ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2+1/2*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*
c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(
3,c*x+(c^2*x^2+1)^(1/2))*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*
arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)*ar
csinh(c*x)*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/x*c-a*b*arcsinh(
c*x)*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)/x^2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2+a*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c
^2+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1
/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\operatorname{asinh}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x^3, x)
```

3.265 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=294

$$-\frac{b^2c^3\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}+\frac{c^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(\right.}{3x^2}$$

```
[Out] -(b^2*c^2*Sqrt[d + c^2*d*x^2])/(3*x) + (b^2*c^3*Sqrt[d + c^2*d*x^2]*ArcSinh
[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(3*x^2) + (c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^2)/(3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(3*d*x^3) + (2*b*c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-
2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (b^2*c^3*Sqrt[d + c^2*d*x^2]*Poly
Log[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.284515, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5723, 5728, 277, 215, 5659, 3716, 2190, 2279, 2391}

$$\frac{b^2c^3\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}-\frac{c^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}-\frac{bc\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx\right.}{3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*Sqrt[d + c^2*d*x^2])/(3*x) + (b^2*c^3*Sqrt[d + c^2*d*x^2]*ArcSinh
[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/(3*x^2) - (c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^2)/(3*Sqrt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(3*d*x^3) + (2*b*c^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2
*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) + (b^2*c^3*Sqrt[d + c^2*d*x^2]*PolyL
og[2, E^(2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
```

```
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}+\frac{\left(2bc\sqrt{d+c^2dx^2}\right)\int\frac{\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{x^3}\,d}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{\left(d+c^2dx^2\right)^{3/2}}{}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\right.}{3x^2}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\right.}{3x^2}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\right.}{3x^2}$$

$$= -\frac{b^2c^2\sqrt{d+c^2dx^2}}{3x}+\frac{b^2c^3\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}}-\frac{bc\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\right.}{3x^2}$$

Mathematica [A] time = 0.778704, size = 240, normalized size = 0.82

$$-\frac{\sqrt{c^2dx^2+d}\left(b^2c^3x^3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+a^2c^2x^2\sqrt{c^2x^2+1}+a^2\sqrt{c^2x^2+1}-2abc^3x^3\log(cx)-b\sinh^{-1}(cx)\left(-2a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + a^2*c^2*x^2*Sqrt[1
 + c^2*x^2] + b^2*c^2*x^2*Sqrt[1 + c^2*x^2] + b^2*(-(c^3*x^3) + Sqrt[1 + c^
2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(-(b*c*
x) - 2*a*(1 + c^2*x^2)^(3/2) + 2*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]) -
2*a*b*c^3*x^3*Log[c*x] + b^2*c^3*x^3*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*x
^3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.298, size = 2557, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsin
h(c*x)*c^8-6*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2
+1)*arcsinh(c*x)*c^6-20/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)
*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-10/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3
*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)*c^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c
^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7+2*a*b*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+
2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^
(1/2))*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2
+1)*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^2*x^2+1)^(
1/2)*c^7+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^
(1/2)*c^5+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)^2*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-1/3*a^2/d/x^3*(c^2*d*x^2+d)^(3/2)+1/3*b
^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x*arcsinh(c*x)*c^4+1/3*b^2
*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3*arcsinh(c*x)*c^6+1/3*a*b
*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3*c^6+1/3*a*b*(d*(c^2*x^2+
```

```
1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x*c^4-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*c^3+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^
4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+2/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2
))*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1
)*c^8-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)
*c^6-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^
4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*c+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^4/(c^
2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^
2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5-a*b*(d*(c^2
*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+2/3*a*b*(d
*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*
c^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^2/(c^2*x^2+1)^(
1/2)*c-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1
)*c^8-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)
*c^6-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*c^
4-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x^3/(c^2*x^2+1)*arc
sinh(c*x)-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^5/(c^2*x^2+1)
*arcsinh(c*x)^2*c^8-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x
^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^
2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c^6-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^4*x^4+3*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-10/3*b^2*(d*(c^2*x^
2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-1/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*
c^4-5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/x/(c^2*x^2+1)*arc
sinh(c*x)^2*c^2+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c
*x+(c^2*x^2+1)^(1/2))*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)^2*c^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)/(c^
2*x^2+1)^(1/2)*c^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+3*c^2*x^2+1)*x^
3*c^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima
```

```
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}(cx)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}\left(b\operatorname{arsinh}(cx)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2/x^4, x)
```

3.266 $\int x^3 \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=482

$$\frac{4abdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}-\frac{2bc^3dx^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{16bcdx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)$$

```
[Out] (-304*b^2*d*Sqrt[d + c^2*d*x^2])/(3675*c^4) + (4*a*b*d*x*Sqrt[d + c^2*d*x^2
])/(35*c^3*Sqrt[1 + c^2*x^2]) - (152*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2
])/(11025*c^4) - (38*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(6125*c^4)
+ (2*b^2*d*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^4) + (4*b^2*d*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(35*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(105*c*Sqrt[1 + c^2*x^2]) - (16*b*c*d*
x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(175*Sqrt[1 + c^2*x^2]) - (2*
b*c^3*d*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(49*Sqrt[1 + c^2*x^2]
) - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^4) + (d*x^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^2) + (3*d*x^4*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])^2)/35 + (x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/7
```

Rubi [A] time = 0.81108, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5744, 5742, 5758, 5717, 5653, 261, 5661, 266, 43, 14, 5730, 12, 446, 77}

$$\frac{4abdx\sqrt{c^2dx^2+d}}{35c^3\sqrt{c^2x^2+1}}-\frac{2bc^3dx^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{16bcdx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{c^2x^2+1}}+\frac{1}{7}x^4\left(c^2dx^2+d\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-304*b^2*d*Sqrt[d + c^2*d*x^2])/(3675*c^4) + (4*a*b*d*x*Sqrt[d + c^2*d*x^2
])/(35*c^3*Sqrt[1 + c^2*x^2]) - (152*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2
])/(11025*c^4) - (38*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(6125*c^4)
+ (2*b^2*d*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^4) + (4*b^2*d*x*Sqrt
[d + c^2*d*x^2]*ArcSinh[c*x])/(35*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(105*c*Sqrt[1 + c^2*x^2]) - (16*b*c*d*
x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(175*Sqrt[1 + c^2*x^2]) - (2*
b*c^3*d*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(49*Sqrt[1 + c^2*x^2]
) - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^4) + (d*x^2*Sqrt
```

```
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(35*c^2) + (3*d*x^4*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])^2)/35 + (x^4*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/7
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
```

```
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{1}{7}x^4 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{7}(3d) \int x^3 \sqrt{d+c^2dx^2} \left(a+b\sin\right.$$

$$= -\frac{2bcdx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{49\sqrt{1+c^2x^2}}$$

$$= -\frac{16bcdx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{175\sqrt{1+c^2x^2}} - \frac{2bc^3dx^7\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{49\sqrt{1+c^2x^2}}$$

$$= -\frac{2bdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{1+c^2x^2}} - \frac{16bcdx^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{175\sqrt{1+c^2x^2}}$$

$$= \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{2bdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105c\sqrt{1+c^2x^2}} - \frac{16bcdx^5\sqrt{d+}}{1}$$

$$= \frac{62b^2d\sqrt{d+c^2dx^2}}{1225c^4} + \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{74b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3675c^4} - \frac{3}{}$$

$$= -\frac{304b^2d\sqrt{d+c^2dx^2}}{3675c^4} + \frac{4abdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{152b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{11025c^4}$$

Mathematica [A] time = 0.383811, size = 251, normalized size = 0.52

$$\frac{d\sqrt{c^2dx^2+d}\left(11025a^2\left(5c^2x^2-2\right)\left(c^2x^2+1\right)^3 - 210abcx\left(75c^6x^6+168c^4x^4+35c^2x^2-210\right)\sqrt{c^2x^2+1} - 210b\sinh^{-1}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(11025*a^2*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) - 210*a*
b*c*x*Sqrt[1 + c^2*x^2]*(-210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6) + 2*
b^2*(-18692 - 20371*c^2*x^2 + 499*c^4*x^4 + 3303*c^6*x^6 + 1125*c^8*x^8) -
210*b*(-105*a*(1 + c^2*x^2)^3*(-2 + 5*c^2*x^2) + b*c*x*Sqrt[1 + c^2*x^2]*(-
210 + 35*c^2*x^2 + 168*c^4*x^4 + 75*c^6*x^6))*ArcSinh[c*x] + 11025*b^2*(1 +
 c^2*x^2)^3*(-2 + 5*c^2*x^2)*ArcSinh[c*x]^2))/(385875*c^4*(1 + c^2*x^2))
```

Maple [B] time = 0.38, size = 1766, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] a^2*(1/7*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(c^2*d*x^2+d)^(5/2))+b^2*
(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144
*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(
1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*
x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^
5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*
x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*d/c^4/(c^2*x^2
+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c
^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d/c^4
/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*
(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(
1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d/
c^4/(c^2*x^2+1)-1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+
1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c
*x)+2)*d/c^4/(c^2*x^2+1)+1/16000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x
^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c
*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^4/(c^2*x^
2+1)+1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)
+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+
1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsin
h(c*x)+2)*d/c^4/(c^2*x^2+1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^
8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+10
4*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1
)*(-1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)
-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x
^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^4/(c^2*x^2+1)-3/128*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/
c^4/(c^2*x^2+1)-3/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+
1)*(1+arcsinh(c*x))*d/c^4/(c^2*x^2+1)-1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^
4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arc
sinh(c*x))*d/c^4/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^
5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-
5*c*x*(c^2*x^2+1)^(1/2)+1)*(1+5*arcsinh(c*x))*d/c^4/(c^2*x^2+1)+1/6272*(d*(
```

```
c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*
c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x
^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d/c^4/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.98113, size = 942, normalized size = 1.95

$$11025\left(5\,b^2c^8dx^8+13\,b^2c^6dx^6+9\,b^2c^4dx^4-b^2c^2dx^2-2\,b^2d\right)\sqrt{c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(525\,abc^8dx^8+13\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/385875*(11025*(5*b^2*c^8*d*x^8 + 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 - b^2
*c^2*d*x^2 - 2*b^2*d)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
210*(525*a*b*c^8*d*x^8 + 1365*a*b*c^6*d*x^6 + 945*a*b*c^4*d*x^4 - 105*a*b*c
^2*d*x^2 - 210*a*b*d - (75*b^2*c^7*d*x^7 + 168*b^2*c^5*d*x^5 + 35*b^2*c^3*d
*x^3 - 210*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 + 1)) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 + 9*(15925*a^2 + 734*b^2)
*c^6*d*x^6 + (99225*a^2 + 998*b^2)*c^4*d*x^4 - (11025*a^2 + 40742*b^2)*c^2*
d*x^2 - 2*(11025*a^2 + 18692*b^2)*d - 210*(75*a*b*c^7*d*x^7 + 168*a*b*c^5*d
*x^5 + 35*a*b*c^3*d*x^3 - 210*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2
+ d))/(c^6*x^2 + c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.267 $\int x^2 \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=405

$$-\frac{bc^3dx^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{18\sqrt{c^2x^2+1}}-\frac{7bcdx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2$$

```
[Out] (-7*b^2*d*x*Sqrt[d + c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d + c^2*d*
x^2])/1728 + (b^2*c^2*d*x^5*Sqrt[d + c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x])/(1152*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^2*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(48*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(18*Sqrt[1 + c^2*x^2]) + (d*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2)/8 + (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c^3*Sqrt[1
 + c^2*x^2])
```

Rubi [A] time = 0.663907, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5744, 5742, 5758, 5675, 5661, 321, 215, 14, 5730, 12, 459}

$$-\frac{bc^3dx^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{18\sqrt{c^2x^2+1}}-\frac{7bcdx^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{c^2x^2+1}}+\frac{1}{6}x^3\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-7*b^2*d*x*Sqrt[d + c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d + c^2*d*
x^2])/1728 + (b^2*c^2*d*x^5*Sqrt[d + c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d + c^
2*d*x^2]*ArcSinh[c*x])/(1152*c^3*Sqrt[1 + c^2*x^2]) - (b*d*x^2*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c*Sqrt[1 + c^2*x^2]) - (7*b*c*d*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(48*Sqrt[1 + c^2*x^2]) - (b*c^3*d*x^6*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(18*Sqrt[1 + c^2*x^2]) + (d*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x])^2)/8 + (x^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/6 - (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(48*b*c^3*Sqrt[1
 + c^2*x^2])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{6}x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{2}d\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.$$

$$=-\frac{bcdx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{12\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.}{18\sqrt{1+c^2x^2}}$$

$$=-\frac{7bcdx^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{48\sqrt{1+c^2x^2}}-\frac{bc^3dx^6\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{18\sqrt{1+c^2x^2}}$$

$$=\frac{1}{64}b^2dx^3\sqrt{d+c^2dx^2}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}-\frac{bdx^2\sqrt{d+c^2dx^2}\left(a+b\sin\right.}{16c\sqrt{1+c^2x^2}}$$

$$=\frac{b^2dx\sqrt{d+c^2dx^2}}{128c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}-\frac{bdx^2\sqrt{}}{}$$

$$=-\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}-\frac{b^2d}{}$$

$$=-\frac{7b^2dx\sqrt{d+c^2dx^2}}{1152c^2}+\frac{43b^2dx^3\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d+c^2dx^2}+\frac{7b^2}{}$$

Mathematica [A] time = 1.19683, size = 508, normalized size = 1.25

$$\frac{-864a^2d^{3/2}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+2304a^2c^5dx^5\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+4032a^2c^3dx^3\sqrt{c^2x^2+1}\sqrt{c^2dx^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (864*a^2*c*d*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 4032*a^2*c^3*d*x^3*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*d*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] - 288*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 216*a
*b*d*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 108*a*b*d*Sqrt[d + c^2*d*x^
2]*Cosh[4*ArcSinh[c*x]] - 24*a*b*d*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]]
 - 864*a^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2
]] - 108*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 27*b^2*d*Sqrt[d +
 c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSi
nh[c*x]] + 12*b*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(18*b*Cosh[2*ArcSinh[c*x
```

```
]] - 9*b*Cosh[4*ArcSinh[c*x]] - 2*b*Cosh[6*ArcSinh[c*x]] - 36*a*Sinh[2*ArcS
inh[c*x]] + 36*a*Sinh[4*ArcSinh[c*x]] + 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-12*a - 3*b*Sinh[2*ArcSinh[c*x]] + 3*b
*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c^3*Sqrt[1 + c^2*x^
2])
```

Maple [B] time = 0.358, size = 934, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^7-1/16*a^2/c
^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+65/3456*
b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*x^3+1/8*a*b*(d*(c^2*x^2+1))^(1/2)*d
/c^2/(c^2*x^2+1)*arcsinh(c*x)*x+11/12*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*
x^2+1)*arcsinh(c*x)*x^5-1/18*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1
/2)*x^6-1/18*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)
*x^6-7/48*b^2*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-
1/16*b^2*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-7/48*
a*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^4-1/16*a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d+17/24*a*b*(d*(c^2*x^2+1))^(1
/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/16*a*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*
x^2+1)^(1/2)*x^2+1/16*b^2*(d*(c^2*x^2+1))^(1/2)*d/c^2/(c^2*x^2+1)*arcsinh(c
*x)^2*x+1/6*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^7+
11/24*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^5-1/16*a
^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)+1/6*a^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a^2
/c^2*x*(c^2*d*x^2+d)^(3/2)+7/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d/c^3/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)+7/1152*a*b*(d*(c^2*x^2+1))^(1/2)*d/c^3/(c^2*x^2+1)^(1
/2)+1/108*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^4/(c^2*x^2+1)*x^7+59/1728*b^2*(d*(c
^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*x^5-7/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d/c
^2/(c^2*x^2+1)*x+17/48*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)
^2*x^3-1/48*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3*
d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^2dx^4 + a^2dx^2 + \left(b^2c^2dx^4 + b^2dx^2\right)\text{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^4 + abdx^2\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^4 + a^2*d*x^2 + (b^2*c^2*d*x^4 + b^2*d*x^2)*arcsinh(c
*x)^2 + 2*(a*b*c^2*d*x^4 + a*b*d*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right) + a\right)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

3.268 $\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=267

$$-\frac{2bc^3dx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}-\frac{4bcdx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{c^2x^2+1}}-\frac{2bdx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{c^2x^2+1}}+$$

```
[Out] (16*b^2*d*Sqrt[d + c^2*d*x^2])/(75*c^2) + (8*b^2*d*(1 + c^2*x^2)*Sqrt[d + c
^2*d*x^2])/(225*c^2) + (2*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c
^2) - (2*b*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c*Sqrt[1 + c^2*
x^2]) - (4*b*c*d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 +
 c^2*x^2]) - (2*b*c^3*d*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*S
qrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2*d
)
```

Rubi [A] time = 0.21852, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5717, 194, 5679, 12, 1247, 698}

$$-\frac{2bc^3dx^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{25\sqrt{c^2x^2+1}}-\frac{4bcdx^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{c^2x^2+1}}-\frac{2bdx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{c^2x^2+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (16*b^2*d*Sqrt[d + c^2*d*x^2])/(75*c^2) + (8*b^2*d*(1 + c^2*x^2)*Sqrt[d + c
^2*d*x^2])/(225*c^2) + (2*b^2*d*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(125*c
^2) - (2*b*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c*Sqrt[1 + c^2*
x^2]) - (4*b*c*d*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 +
 c^2*x^2]) - (2*b*c^3*d*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*S
qrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(5*c^2*d
)
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
```

```
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}-\frac{\left(2bd\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{si}\right.}{5c\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}}$$

$$=-\frac{2bdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5c\sqrt{1+c^2x^2}}-\frac{4bcdx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}}$$

$$=\frac{16b^2d\sqrt{d+c^2dx^2}}{75c^2}+\frac{8b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{225c^2}+\frac{2b^2d\left(1+c^2x^2\right)^2\sqrt{d+c^2}}{125c^2}$$

Mathematica [A] time = 0.36113, size = 198, normalized size = 0.74

$$\frac{d\sqrt{c^2dx^2+d}\left(225a^2\left(c^2x^2+1\right)^3-30abcx\left(3c^4x^4+10c^2x^2+15\right)\sqrt{c^2x^2+1}+30b\sinh^{-1}(cx)\left(15a\left(c^2x^2+1\right)^3-bcx\sqrt{c^2x^2}\right.\right.}{1125c^2\left(c^2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d*Sqrt[d + c^2*d*x^2]*(225*a^2*(1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[1 + c^2*x
^2]*(15 + 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 + 187*c^2*x^2 + 47*c^4*x^4 +
 9*c^6*x^6) + 30*b*(15*a*(1 + c^2*x^2)^3 - b*c*x*Sqrt[1 + c^2*x^2]*(15 + 10
*c^2*x^2 + 3*c^4*x^4))*ArcSinh[c*x] + 225*b^2*(1 + c^2*x^2)^3*ArcSinh[c*x]^
2))/(1125*c^2*(1 + c^2*x^2))
```

Maple [B] time = 0.266, size = 1149, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/5*a^2/c^2/d*(c^2*d*x^2+d)^(5/2)+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6
*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+1
3*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x)+2)*
d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2
+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(
c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+
1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh
(c*x)+2)*d/c^2/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3
*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6
*arcsinh(c*x)+2)*d/c^2/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6
-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^
2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)*d/c^
2/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c
^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c
^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1
))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)
^(1/2)+1)*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*
(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))*d/
c^2/(c^2*x^2+1)+1/96*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)
^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d/c^2/(c^2*x
^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+
28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+
1)*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

<u>Maxima</u> [A] time = 1.28852, size = 311, normalized size = 1.16

$$\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}b^2\operatorname{arsinh}(cx)^2}{5\,c^2d}+\frac{2}{1125}\,b^2\left(\frac{9\,\sqrt{c^2x^2+1}c^2d^{\frac{5}{2}}x^4+38\,\sqrt{c^2x^2+1}d^{\frac{5}{2}}x^2+\frac{149\,\sqrt{c^2x^2+1}d^{\frac{5}{2}}}{c^2}}{d}-\frac{15\left(3\,c^4d^{\frac{5}{2}}x^5+10\,c^2d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*d*x^2 + d)^(5/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqrt(
c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 38*sqrt(c^2*x^2 + 1)*d^(5/2)*x^2 + 149*sqrt(
c^2*x^2 + 1)*d^(5/2)/c^2)/d - 15*(3*c^4*d^(5/2)*x^5 + 10*c^2*d^(5/2)*x^3 +
```

```
15*d^(5/2)*x)*arcsinh(c*x)/(c*d)) + 2/5*(c^2*d*x^2 + d)^(5/2)*a*b*arcsinh(c
*x)/(c^2*d) + 1/5*(c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) - 2/75*(3*c^4*d^(5/2)*x
^5 + 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

Fricas [A] time = 3.05506, size = 743, normalized size = 2.78

$$\frac{225\left(b^2c^6dx^6+3\,b^2c^4dx^4+3\,b^2c^2dx^2+b^2d\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+30\left(15\,abc^6dx^6+45\,abc^4dx^4+45\,abc^2d\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*(b^2*c^6*d*x^6 + 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 + b^2*d)*sqr
t(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(15*a*b*c^6*d*x^6 + 45
*a*b*c^4*d*x^4 + 45*a*b*c^2*d*x^2 + 15*a*b*d - (3*b^2*c^5*d*x^5 + 10*b^2*c^
3*d*x^3 + 15*b^2*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 + 1)) + (9*(25*a^2 + 2*b^2)*c^6*d*x^6 + (675*a^2 + 94*b^2)*c^4*d
*x^4 + (675*a^2 + 374*b^2)*c^2*d*x^2 + (225*a^2 + 298*b^2)*d - 30*(3*a*b*c^
5*d*x^5 + 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^
2 + d))/(c^4*x^2 + c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.269 $\int \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=294

$$\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd\left(c^2x\right.}{}$$

```
[Out] (15*b^2*d*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*d*x*(1 + c^2*x^2)*Sqrt[d + c^2*d
*x^2])/32 - (9*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c*Sqrt[1 + c^2*x
^2]) - (3*b*c*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c
^2*x^2]) - (b*d*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(8*c) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (d*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.245864, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\sqrt{c^2x^2+1}}+\frac{1}{4}x\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd\left(c^2x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*d*x*(1 + c^2*x^2)*Sqrt[d + c^2*d
*x^2])/32 - (9*b^2*d*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64*c*Sqrt[1 + c^2*x
^2]) - (3*b*c*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c
^2*x^2]) - (b*d*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(8*c) + (3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/8 + (x*(d + c
^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (d*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^3)/(8*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
```

```
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
```

```
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{4}x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{4}(3d)\int\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right.$$

$$=-\frac{bd\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{3}{8}dx\sqrt{d+c^2dx^2}\left(a+b\sinh\right.$$

$$=\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{3bcdx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{1+c^2x^2}}-\frac{bd\left(1\right.}{}$$

$$=\frac{15}{64}b^2dx\sqrt{d+c^2dx^2}+\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{3bcdx^2\sqrt{d+c^2dx^2}\left(a+\right.}{8\sqrt{1+c^2x}}$$

$$=\frac{15}{64}b^2dx\sqrt{d+c^2dx^2}+\frac{1}{32}b^2dx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}-\frac{9b^2d\sqrt{d+c^2dx^2}\sinh^{-1}}{64c\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.93777, size = 329, normalized size = 1.12

$$\frac{288a^2d^{3/2}\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+96a^2cdx\sqrt{c^2x^2+1}\left(2c^2x^2+5\right)\sqrt{c^2dx^2+d}-192abd\sqrt{c^2dx^2+d}\left(\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (96*a^2*c*d*x*Sqrt[1 + c^2*x^2]*(5 + 2*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 288*a
^2*d^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 32*
b^2*d*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh
```

```
[c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 192*a*b*d*Sqrt[d +
c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*Ar
cSinh[c*x]])) - 12*a*b*d*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*Arc
Sinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - b^2*d*Sqrt[d + c^2*d*x^
2]*(32*ArcSinh[c*x]^3 + 12*ArcSinh[c*x]*Cosh[4*ArcSinh[c*x]] - 3*(1 + 8*Arc
Sinh[c*x]^2)*Sinh[4*ArcSinh[c*x]]))/(768*c*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.22, size = 709, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/4*x*(c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/8*b^2*(d*(c^2*
x^2+1))^(1/2)*d*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-5/8*b^2*(d*(c^2*x^2+
1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+1/8*b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^3*d+1/32*b^2*(d*(c^2*x^2+1))^(1/2)*d*c
^4/(c^2*x^2+1)*x^5+19/64*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*x^3+17
/64*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*x+1/4*b^2*(d*(c^2*x^2+1))^(1/2)
*d*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^5+7/8*b^2*(d*(c^2*x^2+1))^(1/2)*d*c^2/(
c^2*x^2+1)*arcsinh(c*x)^2*x^3+5/8*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*a
rcsinh(c*x)^2*x-17/64*b^2*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)+3/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d-
17/64*a*b*(d*(c^2*x^2+1))^(1/2)*d/c/(c^2*x^2+1)^(1/2)+1/2*a*b*(d*(c^2*x^2+1
))^(1/2)*d*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/8*a*b*(d*(c^2*x^2+1))^(1/2)*d
*c^3/(c^2*x^2+1)^(1/2)*x^4+7/4*a*b*(d*(c^2*x^2+1))^(1/2)*d*c^2/(c^2*x^2+1)*
arcsinh(c*x)*x^3-5/8*a*b*(d*(c^2*x^2+1))^(1/2)*d*c/(c^2*x^2+1)^(1/2)*x^2+5/
4*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}(cx)\right)\sqrt{c^2dx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}} (a + b \operatorname{asinh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.270 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=498

$$-\frac{2bd\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] (22*b^2*d*Sqrt[d + c^2*d*x^2])/9 - (2*a*b*c*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[1
 + c^2*x^2] + (2*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/27 - (2*b^2*c*d*x
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (2*b*c*d*x*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^3*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/3 - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*
b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(2*b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.594588, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5744, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 5679, 444, 43}

$$-\frac{2bd\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (22*b^2*d*Sqrt[d + c^2*d*x^2])/9 - (2*a*b*c*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[1
 + c^2*x^2] + (2*b^2*d*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/27 - (2*b^2*c*d*x
*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (2*b*c*d*x*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^3*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2]) + d*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*
x])^2)/3 - (2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/Sqrt[1 + c^2*x^2] - (2*b*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
```

```
)*PolyLog[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d*Sqrt[d + c^2*d*x^
2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*
b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] -
(2*b^2*d*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{3}\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{2bcdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right.}{9\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{2bc^3dx^3\sqrt{d+c^2}}{9}$$

$$= -\frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cdx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - \frac{2bcdx\sqrt{d+c^2dx^2}\left(a+\right.}{3\sqrt{1+c^2}}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{2b^2}{}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{2b^2}{}$$

$$= \frac{22}{9}b^2d\sqrt{d+c^2dx^2} - \frac{2abcdx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2d\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{2b^2}{}$$

Mathematica [A] time = 2.43784, size = 520, normalized size = 1.04

$$\frac{2abd\sqrt{c^2dx^2+d}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\log\left(1\right.\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (a^2*d*(4 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/3 - (2*a*b*d*Sqrt[d + c^2*d*x^2]*
(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*Sqrt[1 + c^2*x^2
]) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + (2*a*b*d*Sqrt[d + c^2*d*x^2]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + (b^2*d*Sqrt[d + c^2*d*x^2]*(2*Sqrt[1 + c^2*x^2] - 2*c*x
*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + ArcSinh[c*x]^2*(Log[1 -
E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]) + 2*ArcSinh[c*x]*(PolyLog[
2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]) + 2*(PolyLog[3, -E^
(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])])))/Sqrt[1 + c^2*x^2] + (b^
2*d*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9
*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcS
inh[c*x]])))/(108*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.291, size = 1053, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 10/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+1/3*(c^2*
d*x^2+d)^(3/2)*a^2+2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)
*x^4*c^4-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,c*x+(c^2*x
^2+1)^(1/2))*d+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x
-(c^2*x^2+1)^(1/2))*d+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c
*x)^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,
-c*x-(c^2*x^2+1)^(1/2))*d+8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsi
nh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x
^2+1)^(1/2))*d+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+
(c^2*x^2+1)^(1/2))*d+2/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*c^4*x^4-b
^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2
+1)^(1/2))*d+70/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*c^2*x^2+2*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1
)^(1/2))*d+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1-
c*x-(c^2*x^2+1)^(1/2))*d-a^2*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))
/x)+a^2*(c^2*d*x^2+d)^(1/2)*d+68/27*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)
-2/9*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-8/3
*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c-2/9*a*b*(d*
(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)*c^3*x^3-8/3*a*b*(d*(c^2*x^2+1))^(1/2
```

```
)*d/(c^2*x^2+1)^(1/2)*c*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsi
nh(c*x)^2*x^4*c^4+5/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)*arcsinh(c*x)^
2*x^2*c^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c
*x-(c^2*x^2+1)^(1/2))*d-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^2 + abd\right)\operatorname{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}\left(a + b\operatorname{asinh}\left(cx\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2/x, x)
```

3.271 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=398

$$-\frac{b^2cd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{3bc^3dx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(c\right.$$

```
[Out] (b^2*c^2*d*x*Sqrt[d + c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d + c^2*d*x^2]*ArcSin
h[c*x])/(4*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*
d*x^2]*(a + b*ArcSinh[c*x]) + (3*c^2*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/2 + (c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x + (c*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^
2*x^2] - (b^2*c*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt
[1 + c^2*x^2]
```

Rubi [A] time = 0.421631, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5739, 5682, 5675, 5661, 321, 215, 5726, 5659, 3716, 2190, 2279, 2391, 195}

$$\frac{b^2cd\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{3bc^3dx^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{3}{2}c^2dx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (b^2*c^2*d*x*Sqrt[d + c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d + c^2*d*x^2]*ArcSin
h[c*x])/(4*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*x^2*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*
d*x^2]*(a + b*ArcSinh[c*x]) + (3*c^2*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^2)/2 - (c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x + (c*d*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2
*x^2] + (b^2*c*d*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[1
```

```
+ c^2*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx=-\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}+\left(3c^2d\right)\int\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.$$

$$=bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{2}c^2dx\sqrt{d+c^2dx^2}\left(a+b\sinh\right.$$

$$=-\frac{1}{2}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}+bcd\sqrt{1+c^2}$$

$$=\frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{2\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\left(a\right.}{2\sqrt{1+c^2x}}$$

$$=\frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\left(a\right.}{2\sqrt{1+c^2}}$$

$$=\frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\left(a\right.}{2\sqrt{1+c^2}}$$

$$=\frac{1}{4}b^2c^2dx\sqrt{d+c^2dx^2}-\frac{5b^2cd\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{4\sqrt{1+c^2x^2}}-\frac{3bc^3dx^2\sqrt{d+c^2dx^2}\left(a\right.}{2\sqrt{1+c^2}}$$

Mathematica [A] time = 3.03117, size = 369, normalized size = 0.93

$$\frac{-8b^2d\sqrt{c^2dx^2+d}\left(3cx\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\left(3\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx\left(\sinh^{-1}(cx)+3\right)\sinh^{-1}(cx)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (12*a^2*d*(-2 + c^2*x^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 24*a*b*d*S
qrt[d + c^2*d*x^2]*(-2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + c*x*ArcSinh[c*x]^2
+ 2*c*x*Log[c*x]) + 36*a^2*c*d^(3/2)*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d
]*Sqrt[d + c^2*d*x^2]] - 8*b^2*d*Sqrt[d + c^2*d*x^2]*(ArcSinh[c*x]*(3*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]*(3 + ArcSinh[c*x]) - 6*c*x*Log
[1 - E^(-2*ArcSinh[c*x])]) + 3*c*x*PolyLog[2, E^(-2*ArcSinh[c*x])]) + b^2*c
*d*x*Sqrt[d + c^2*d*x^2]*(4*ArcSinh[c*x]^3 - 6*ArcSinh[c*x]*Cosh[2*ArcSinh[
c*x]] + (3 + 6*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]) - 6*a*b*c*d*x*Sqrt[d +
 c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(24*x*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.259, size = 954, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] -1/4*a*b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)+2*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c*d+1/2*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c*d-b^2*(d*(c^2*x^2+1))^(1
/2)*arcsinh(c*x)^2*d/x/(c^2*x^2+1)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2
*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*x+2*b^2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c*d-b^2*
(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2-1/4*b^2*(d*(c^2*
x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-a*b*(d*(c^2*x^2+1))^(1/2)*
c^2*d/(c^2*x^2+1)*arcsinh(c*x)*x+a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1
)*arcsinh(c*x)*x^3-1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)*x^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*c*d/(c^2*x^2+1)^(1/2)*arcsinh(c*
x)-1/2*a*b*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x^2+3/2*a*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c*d-2*a*b*(d*(c^2*x^2+1))^
(1/2)*arcsinh(c*x)*d/x/(c^2*x^2+1)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c*d+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c*d-1/2*b^2*(d*(c^2*x
^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)^2*x+1/2*b^2*(d*(c^2*x^2+1))^(1/
2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c*d+3/2*a^2*c^2*d*x*(c^
2*d*x^2+d)^(1/2)-a^2/d/x*(c^2*d*x^2+d)^(5/2)+a^2*c^2*x*(c^2*d*x^2+d)^(3/2)+
3/2*a^2*c^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\text{arsinh}\left(cx\right)^2 + 2\left(abc^2dx^2 + abd\right)\text{arsinh}\left(cx\right)\right)\sqrt{c^2dx^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}\left(a + b\,\text{asinh}\left(cx\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2/x^2, x)
```

3.272 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=541

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}\cdots\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] 2*b^2*c^2*d*Sqrt[d + c^2*d*x^2] - (3*a*b*c^3*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[
1 + c^2*x^2] - (3*b^2*c^3*d*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^
2*x^2] - (b*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + (b*c^3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^
2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((d + c^2*d*x
^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*
d*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (3*b*
c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Po
lyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (3*b^2*c^2*d*Sqrt[d + c^2*d*x
^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (3*b^2*c^2*d*Sqrt[d +
c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.654389, antiderivative size = 541, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5739, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 14, 5730, 446, 80, 63, 208}

$$-\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{3bc^2d\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}\cdots\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] 2*b^2*c^2*d*Sqrt[d + c^2*d*x^2] - (3*a*b*c^3*d*x*Sqrt[d + c^2*d*x^2])/Sqrt[
1 + c^2*x^2] - (3*b^2*c^3*d*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^
2*x^2] - (b*c*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x
^2]) + (b*c^3*d*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^
2] + (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 - ((d + c^2*d*x
^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (3*c^2*d*Sqrt[d + c^2*d*x^2]*(a
```

```
 + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*
d*Sqrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (3*b*
c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[1 + c^2*x^2] + (3*b*c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Po
lyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (3*b^2*c^2*d*Sqrt[d + c^2*d*x
^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (3*b^2*c^2*d*Sqrt[d +
c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
```

```
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{2}\left(3c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{x}$$

$$= -\frac{bcd\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} +$$

$$= -\frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{bc^3dx\sqrt{d+c^2d}}{\sqrt{}}$$

$$= -b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} -$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} -$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} -$$

$$= 2b^2c^2d\sqrt{d+c^2dx^2} - \frac{3abc^3dx\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{3b^2c^3dx\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} -$$

Mathematica [A] time = 7.87242, size = 771, normalized size = 1.43

$$\frac{2abc^2d\sqrt{d\left(c^2x^2+1\right)}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)-cx+\sinh^{-1}(cx)\right.}{\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (a^2*c^2*d - (a^2*d)/(2*x^2))*Sqrt[d*(1 + c^2*x^2)] + (3*a^2*c^2*d^(3/2)*Lo
g[x])/2 - (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/2 + (2
*a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Log[1 + E^(-ArcSinh
[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/
Sqrt[1 + c^2*x^2] + b^2*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(2 - (2*c*x*ArcSinh[c*x
])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[c*x]^2*(Log[1 - E^(-ArcSin
h[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]
```

```
*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[c*x])]))/Sqrt[1 +
 c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x
])]))/Sqrt[1 + c^2*x^2]) + (a*b*c^2*d*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 - E^
(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog[2,
 -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Sech[
ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^2*c
^2*d*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - ArcSinh[
c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])]
 - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*x]/2]
] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*PolyLog[
2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3, E^(
-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Ta
nh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.349, size = 1131, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] 2*a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)*x^2-3*a*b*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*
c^2*d+3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(
c^2*x^2+1)^(1/2))*c^2*d+1/2*a^2*c^2*(c^2*d*x^2+d)^(3/2)+2*b^2*(d*(c^2*x^2+1
))^(1/2)*c^2*d/(c^2*x^2+1)-3/2*a^2*c^2*d^(3/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2
+d)^(1/2))/x)+3/2*a^2*c^2*(c^2*d*x^2+d)^(1/2)*d-1/2*a^2/d/x^2*(c^2*d*x^2+d)
^(5/2)-2*a*b*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*x^2+1)^(1/2)*x-a*b*(d*(c^2*x^
2+1))^(1/2)*d/x/(c^2*x^2+1)^(1/2)*c-2*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d/(c^2*
x^2+1)^(1/2)*arcsinh(c*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/x/(c^2
*x^2+1)^(1/2)*c+a*b*(d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)-a*
b*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)-3*a*b*(d*(c^2*x^2+1)
)^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d+3*a*b*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d
+b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*arcsinh(c*x)^2*x^2+3*b^2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(
1/2))*c^2*d+3/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*
ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d-3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d-3/2*b^2*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
```

```
^2*d+3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+
1)^(1/2))*c^2*d+2*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d/(c^2*x^2+1)*x^2+1/2*b^2*(
d*(c^2*x^2+1))^(1/2)*c^2*d/(c^2*x^2+1)*arcsinh(c*x)^2-1/2*b^2*arcsinh(c*x)^
2*(d*(c^2*x^2+1))^(1/2)*d/x^2/(c^2*x^2+1)-3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*c^2*d-2*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2*x^2+1)^(1/2))*c^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2} c^{2} d x^{2} + a^{2} d + \left(b^{2} c^{2} d x^{2} + b^{2} d\right) \operatorname{arsinh}\left(c x\right)^{2} + 2 \left(a b c^{2} d x^{2} + a b d\right) \operatorname{arsinh}\left(c x\right)\right) \sqrt{c^{2} d x^{2} + d}}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2/x^3, x)
```

3.273 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=378

$$-\frac{4b^2c^3d\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}+\frac{c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}+\frac{4c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx\right.}{3\sqrt{c^2x^2+1}}$$

```
[Out] -(b^2*c^2*d*Sqrt[d + c^2*d*x^2])/(3*x) + (b^2*c^3*d*Sqrt[d + c^2*d*x^2]*Arc
Sinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) - (c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/x + (4*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
 (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2])
 + (8*b*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSin
h[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (4*b^2*c^3*d*Sqrt[d + c^2*d*x^2]*PolyLog[
2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.584296, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5739, 5737, 5659, 3716, 2190, 2279, 2391, 5675, 5728, 277, 215}

$$\frac{4b^2c^3d\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}+\frac{c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{3b\sqrt{c^2x^2+1}}-\frac{4c^3d\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d*Sqrt[d + c^2*d*x^2])/(3*x) + (b^2*c^3*d*Sqrt[d + c^2*d*x^2]*Arc
Sinh[c*x])/(3*Sqrt[1 + c^2*x^2]) - (b*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) - (c^2*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^2)/x - (4*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*Sq
rt[1 + c^2*x^2]) - ((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
 (c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*Sqrt[1 + c^2*x^2])
 + (8*b*c^3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh
[c*x])])/(3*Sqrt[1 + c^2*x^2]) + (4*b^2*c^3*d*Sqrt[d + c^2*d*x^2]*PolyLog[2
, E^(2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5737

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 1)*S
qrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x] -
 Dist[(c^2*Sqrt[d + e*x^2])/(f^2*(m + 1)*Sqrt[1 + c^2*x^2]), Int[((f*x)^(m
+ 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3} + \left(c^2d\right)\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}$$

$$= -\frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{c^2d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{x}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2} - \frac{c^2d\sqrt{d+}}{}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}{3x^2}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}{3x^2}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}{3x^2}$$

$$= -\frac{b^2c^2d\sqrt{d+c^2dx^2}}{3x} + \frac{b^2c^3d\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{3\sqrt{1+c^2x^2}} - \frac{bcd\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}{3x^2}$$

Mathematica [A] time = 1.37363, size = 458, normalized size = 1.21

$$\frac{-4b^2c^3dx^3\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+3a^2c^3d^{3/2}x^3\sqrt{c^2x^2+1}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)-4a^2c^2dx^2\sqrt{c^2x^2+1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d*x*Sqrt[d + c^2*d*x^2]) - a^2*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*
x^2] - 4*a^2*c^2*d*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - b^2*c^2*d*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + b*d*Sqrt[d + c^2*d*x^2]*(3*a*c^3*
x^3 - b*(-4*c^3*x^3 + Sqrt[1 + c^2*x^2] + 4*c^2*x^2*Sqrt[1 + c^2*x^2]))*Arc
Sinh[c*x]^2 + b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + b*d*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*(-(b*c*x) - 2*a*Sqrt[1 + c^2*x^2]*(1 + 4*c^2*x^2
) + 8*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])]) + 8*a*b*c^3*d*x^3*Sqrt[d + c^
2*d*x^2]*Log[c*x] + 3*a^2*c^3*d^(3/2)*x^3*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqr
t[d]*Sqrt[d + c^2*d*x^2]] - 4*b^2*c^3*d*x^3*Sqrt[d + c^2*d*x^2]*PolyLog[2,
```

```
E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.319, size = 2796, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -28/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*ar
csinh(c*x)*c^2-64*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/
(c^2*x^2+1)*arcsinh(c*x)*c^8-104*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*
c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6+64*a*b*(d*(c^2*x^2+1))^(1/2)*d/
(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7+24*a*b*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*c^5-146/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*
x^2+1)*arcsinh(c*x)*c^4-16/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*
x^2+1)*x^5/(c^2*x^2+1)*c^8-20/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c
^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-4/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9
*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9
*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)-52*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24
*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^2*c^6-20/3*b^2*(d*(c^2*x
^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-2/
3*a^2*c^2/d/x*(c^2*d*x^2+d)^(5/2)-8*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+32*b^2*(d*(c^2*x^2+1))
^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7-
32*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arc
sinh(c*x)^2*c^8-73/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x
/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4
+9*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-14/3*b^2*(d*(c^2*x^2+1))^(1/2)
*d/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*arcsinh(c*x)^2*c^2-16/3*b^2*(d*(c
^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^
8+8/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*c^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)
/x^2/(c^2*x^2+1)^(1/2)*c-8*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^
2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+
9*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c+12*b^2*(d*(c^2*x^2+1))^(1
/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^5-1/3
*a^2/d/x^3*(c^2*d*x^2+d)^(5/2)+2/3*a^2*c^4*x*(c^2*d*x^2+d)^(3/2)-3*a*b*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3+3*b^2*(d
*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+4/
```

```
3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*ar
csinh(c*x)^2*c^3-3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^
2*x^2+1)^(1/2)*arcsinh(c*x)*c^3+8*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9
*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*c^7+16/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*
c^4*x^4+9*c^2*x^2+1)*x^3*c^6+4/3*a*b*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*
c^2*x^2+1)*x*c^4+4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x
*arcsinh(c*x)*c^4+16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)
*x^3*arcsinh(c*x)*c^6+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c
*x)^2*c^3*d-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c
^3*d+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1
/2))^2-1)*c^3*d+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x
)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3*d+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+
1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3*d-20/3*b^2*(d*(c^2*x^
2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-29/3*b^2*(d*(c^2
*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-10/3*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(
c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2
-4/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)*x^3*c^6+8/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^
3*d-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d+8/
3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/
2))*c^3*d+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c^
3*d+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d/(24*c^4*x^4+9*c^2*x^2+1)/(c^2*x^2+1)^(1
/2)*c^3+a^2*c^4*d*x*(c^2*d*x^2+d)^(1/2)+a^2*c^4*d^2*ln(x*c^2*d/(c^2*d)^(1/2
)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^2dx^2 + a^2d + \left(b^2c^2dx^2 + b^2d\right)\text{arsinh}\,(cx)^2 + 2\left(abc^2dx^2 + abd\right)\text{arsinh}\,(cx)\right)\sqrt{c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}(a + b\,\text{asinh}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2/x^4, x)
```

3.274 $\int x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=625

$$\frac{4abd^2x\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}} - \frac{2bc^5d^2x^9\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{81\sqrt{c^2x^2+1}} - \frac{38bc^3d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{441\sqrt{c^2x^2+1}} - \frac{2bcd^2x^5\sqrt{c^2d}}{2}$$

```
[Out] (-160*b^2*d^2*Sqrt[d + c^2*d*x^2])/(3969*c^4) + (4*a*b*d^2*x*Sqrt[d + c^2*d
*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (80*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*
d*x^2])/(11907*c^4) - (4*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(1323
*c^4) - (50*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(27783*c^4) + (2*b
^2*d^2*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2])/(729*c^4) + (4*b^2*d^2*x*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x])/(63*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d^2*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(189*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(21*Sqrt[1 + c^2*x^2]) - (38
*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(441*Sqrt[1 + c^2*
x^2]) - (2*b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(81*Sqrt
[1 + c^2*x^2]) - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^4
) + (d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^2) + (d^2*x^
4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/21 + (5*d*x^4*(d + c^2*d*x^2)
^(3/2)*(a + b*ArcSinh[c*x])^2)/63 + (x^4*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])^2)/9
```

Rubi [A] time = 1.23754, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5744, 5742, 5758, 5717, 5653, 261, 5661, 266, 43, 14, 5730, 12, 446, 77, 270, 1251, 897, 1153}

$$\frac{4abd^2x\sqrt{c^2dx^2+d}}{63c^3\sqrt{c^2x^2+1}} - \frac{2bc^5d^2x^9\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{81\sqrt{c^2x^2+1}} - \frac{38bc^3d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{441\sqrt{c^2x^2+1}} - \frac{2bcd^2x^5\sqrt{c^2d}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (-160*b^2*d^2*Sqrt[d + c^2*d*x^2])/(3969*c^4) + (4*a*b*d^2*x*Sqrt[d + c^2*d
*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (80*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*
d*x^2])/(11907*c^4) - (4*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/(1323
*c^4) - (50*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(27783*c^4) + (2*b
^2*d^2*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2])/(729*c^4) + (4*b^2*d^2*x*Sqrt[d
```

```
 + c^2*d*x^2]*ArcSinh[c*x])/(63*c^3*Sqrt[1 + c^2*x^2]) - (2*b*d^2*x^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(189*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(21*Sqrt[1 + c^2*x^2]) - (38
*b*c^3*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(441*Sqrt[1 + c^2*
x^2]) - (2*b*c^5*d^2*x^9*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(81*Sqrt
[1 + c^2*x^2]) - (2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^4
) + (d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(63*c^2) + (d^2*x^
4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/21 + (5*d*x^4*(d + c^2*d*x^2)
^(3/2)*(a + b*ArcSinh[c*x])^2)/63 + (x^4*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])^2)/9
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
```

```
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
```

```
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^3\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{9}x^4\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{9}(5d)\int x^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\,\mathrm{si}\right.$$

$$=-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{45\sqrt{1+c^2x^2}}-\frac{4bc^3d^2x^7\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{sinh}\right.}{63\sqrt{1+c^2x^2}}$$

$$=-\frac{8bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{105\sqrt{1+c^2x^2}}-\frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{sinh}\right.}{441\sqrt{1+c^2x^2}}$$

$$=-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{21\sqrt{1+c^2x^2}}-\frac{38bc^3d^2x^7\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{sinh}\right.}{441\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^5\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{21\sqrt{1+c^2x^2}}$$

$$=\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{2bd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{189c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^5\sqrt{d+}}{}$$

$$=\frac{134b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{122b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{11907c^4}$$

$$=-\frac{160b^2d^2\sqrt{d+c^2dx^2}}{3969c^4}+\frac{4abd^2x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{80b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{11907c^4}$$

Mathematica [A] time = 0.446444, size = 277, normalized size = 0.44

$$\frac{d^2\sqrt{c^2dx^2+d}\left(3969a^2\left(7c^2x^2-2\right)\left(c^2x^2+1\right)^4-126abcx\left(49c^8x^8+171c^6x^6+189c^4x^4+21c^2x^2-126\right)\sqrt{c^2x^2+1}-126\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3969*a^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 + 4
9*c^8*x^8) + 2*b^2*(-6140 - 7039*c^2*x^2 + 106*c^4*x^4 + 2152*c^6*x^6 + 149
0*c^8*x^8 + 343*c^10*x^10) - 126*b*(-63*a*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)
+ b*c*x*Sqrt[1 + c^2*x^2]*(-126 + 21*c^2*x^2 + 189*c^4*x^4 + 171*c^6*x^6 +
49*c^8*x^8))*ArcSinh[c*x] + 3969*b^2*(1 + c^2*x^2)^4*(-2 + 7*c^2*x^2)*ArcSi
nh[c*x]^2))/(250047*c^4*(1 + c^2*x^2))
```

Maple [B] time = 0.388, size = 2014, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] a^2*(1/9*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(c^2*d*x^2+d)^(7/2))+b^2*
(1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^2+1)^(1/2
)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^5*(c^2*x^
2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*c*x*(c^2*
x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2-18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3
/175616*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*
c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1
/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x
)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*
(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*
arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*
x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1
)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*
x)^2+2*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)-1/1728*(d*(c^2*x^2+1))^(1/2)*(4*
c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9
*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+1)+3/175616*(d*(c^2*x^2+
```

```
1))^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*
(c^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x
*(c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^
2+1)+1/373248*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10-256*c^9*x^9*(c^2*x^2+1)^
(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6-432*c^5*x^5*(c^
2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(
c^2*x^2+1)^(1/2)+1)*(81*arcsinh(c*x)^2+18*arcsinh(c*x)+2)*d^2/c^4/(c^2*x^2+
1))+2*a*b*(1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c^10*x^10+256*c^9*x^9*(c^2*x^
2+1)^(1/2)+704*c^8*x^8+576*c^7*x^7*(c^2*x^2+1)^(1/2)+688*c^6*x^6+432*c^5*x^
5*(c^2*x^2+1)^(1/2)+280*c^4*x^4+120*c^3*x^3*(c^2*x^2+1)^(1/2)+41*c^2*x^2+9*
c*x*(c^2*x^2+1)^(1/2)+1)*(-1+9*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d
*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+11
2*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2
*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/5
76*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3
*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-3/256*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/
c^4/(c^2*x^2+1)-3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+
1)*(1+arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)-1/576*(d*(c^2*x^2+1))^(1/2)*(4*c^4*
x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*a
rcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-
64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*
c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2*x^2+1)^(1/2)+1)*
(1+7*arcsinh(c*x))*d^2/c^4/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*c
^10*x^10-256*c^9*x^9*(c^2*x^2+1)^(1/2)+704*c^8*x^8-576*c^7*x^7*(c^2*x^2+1)^
(1/2)+688*c^6*x^6-432*c^5*x^5*(c^2*x^2+1)^(1/2)+280*c^4*x^4-120*c^3*x^3*(c^
2*x^2+1)^(1/2)+41*c^2*x^2-9*c*x*(c^2*x^2+1)^(1/2)+1)*(1+9*arcsinh(c*x))*d^2
/c^4/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.20798, size = 1181, normalized size = 1.89

$$\frac{3969\left(7\,b^2c^{10}d^2x^{10}+26\,b^2c^8d^2x^8+34\,b^2c^6d^2x^6+16\,b^2c^4d^2x^4-b^2c^2d^2x^2-2\,b^2d^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 + 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 + 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 - 2*b^2*d^2)*sqrt(c^2*d*x^2 + d)*
log(c*x + sqrt(c^2*x^2 + 1))^2 + 126*(441*a*b*c^10*d^2*x^10 + 1638*a*b*c^8*
d^2*x^8 + 2142*a*b*c^6*d^2*x^6 + 1008*a*b*c^4*d^2*x^4 - 63*a*b*c^2*d^2*x^2
- 126*a*b*d^2 - (49*b^2*c^9*d^2*x^9 + 171*b^2*c^7*d^2*x^7 + 189*b^2*c^5*d^2
*x^5 + 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*
x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + (343*(81*a^2 + 2*b^2)*c^10*d^2*x^10
 + 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152*b^2)*c^6*d^2*
x^6 + 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*b^2)*c^2*d^2*x
^2 - 2*(3969*a^2 + 6140*b^2)*d^2 - 126*(49*a*b*c^9*d^2*x^9 + 171*a*b*c^7*d^
2*x^7 + 189*a*b*c^5*d^2*x^5 + 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(c^
2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^6*x^2 + c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.275 $\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=536

$$-\frac{bc^5d^2x^8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{32\sqrt{c^2x^2+1}}-\frac{17bc^3d^2x^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{144\sqrt{c^2x^2+1}}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}\right.}{384\sqrt{c^2x^2+1}}$$

```
[Out] (-359*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/(36864*c^2) + (1079*b^2*d^2*x^3*Sqrt[d
 + c^2*d*x^2])/55296 + (209*b^2*c^2*d^2*x^5*Sqrt[d + c^2*d*x^2])/13824 + (b
^2*c^4*d^2*x^7*Sqrt[d + c^2*d*x^2])/256 + (359*b^2*d^2*Sqrt[d + c^2*d*x^2]*
ArcSinh[c*x])/(36864*c^3*Sqrt[1 + c^2*x^2]) - (5*b*d^2*x^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(128*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(384*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*x^6*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(144*Sqrt[1 + c^2*x^2]) - (
b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(32*Sqrt[1 + c^2*x^
2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(128*c^2) + (5*d
^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/64 + (5*d*x^3*(d + c^2*d
*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/48 + (x^3*(d + c^2*d*x^2)^(5/2)*(a + b*
ArcSinh[c*x])^2)/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(38
4*b*c^3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 1.04717, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5744, 5742, 5758, 5675, 5661, 321, 215, 14, 5730, 12, 459, 266, 43, 1267}

$$-\frac{bc^5d^2x^8\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{32\sqrt{c^2x^2+1}}-\frac{17bc^3d^2x^6\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{144\sqrt{c^2x^2+1}}-\frac{59bcd^2x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}\right.}{384\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (-359*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/(36864*c^2) + (1079*b^2*d^2*x^3*Sqrt[d
 + c^2*d*x^2])/55296 + (209*b^2*c^2*d^2*x^5*Sqrt[d + c^2*d*x^2])/13824 + (b
^2*c^4*d^2*x^7*Sqrt[d + c^2*d*x^2])/256 + (359*b^2*d^2*Sqrt[d + c^2*d*x^2]*
ArcSinh[c*x])/(36864*c^3*Sqrt[1 + c^2*x^2]) - (5*b*d^2*x^2*Sqrt[d + c^2*d*x
^2]*(a + b*ArcSinh[c*x]))/(128*c*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*x^4*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(384*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*x^6*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(144*Sqrt[1 + c^2*x^2]) - (
b*c^5*d^2*x^8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(32*Sqrt[1 + c^2*x^
```

```
2]) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(128*c^2) + (5*d
^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/64 + (5*d*x^3*(d + c^2*d
*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/48 + (x^3*(d + c^2*d*x^2)^(5/2)*(a + b*
ArcSinh[c*x])^2)/8 - (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(38
4*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rubi steps

$$\int x^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{1}{8}x^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{8}(5d)\int x^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\right.$$

$$= -\frac{bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{16\sqrt{1+c^2x^2}} - \frac{bc^3d^2x^6\sqrt{d+c^2dx^2}\left(a+b\sinh^{-}\right.}{12\sqrt{1+c^2x^2}}$$

$$= -\frac{11bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{96\sqrt{1+c^2x^2}} - \frac{17bc^3d^2x^6\sqrt{d+c^2dx^2}\left(a+b\,\mathrm{s}\right.}{144\sqrt{1+c^2x^2}}$$

$$= \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2dx^2} - \frac{59bcd^2x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{384\sqrt{1+c^2x^2}} - \frac{17bc}{}$$

$$= \frac{5}{512}b^2d^2x^3\sqrt{d+c^2dx^2} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824} + \frac{1}{256}b^2c^4d^2x^7\sqrt{d+c^2d}$$

$$= \frac{5b^2d^2x\sqrt{d+c^2dx^2}}{1024c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2dx^2}}{13824}$$

$$= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2d}}{13824}$$

$$= -\frac{359b^2d^2x\sqrt{d+c^2dx^2}}{36864c^2} + \frac{1079b^2d^2x^3\sqrt{d+c^2dx^2}}{55296} + \frac{209b^2c^2d^2x^5\sqrt{d+c^2d}}{13824}$$

Mathematica [A] time = 2.04668, size = 619, normalized size = 1.15

$$\frac{d^2\left(110592a^2c^7x^7\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 313344a^2c^5x^5\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 271872a^2c^3x^3\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(34560*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 271872*a^2*c^3*
x^3*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 313344*a^2*c^5*x^5*Sqrt[1 + c^2
*x^2]*Sqrt[d + c^2*d*x^2] + 110592*a^2*c^7*x^7*Sqrt[1 + c^2*x^2]*Sqrt[d + c
^2*d*x^2] - 11520*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 + 13824*a*b*Sqrt[d
 + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 3456*a*b*Sqrt[d + c^2*d*x^2]*Cosh[4*Ar
cSinh[c*x]] - 1536*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 216*a*b*S
```

```
qrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] - 34560*a^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6912*b^2*Sqrt[d + c^2*d*x^2]*
Sinh[2*ArcSinh[c*x]] + 864*b^2*Sqrt[d + c^2*d*x^2]*Sinh[4*ArcSinh[c*x]] + 2
56*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] + 27*b^2*Sqrt[d + c^2*d*x^2
]*Sinh[8*ArcSinh[c*x]] + 24*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(576*b*Cosh[
2*ArcSinh[c*x]] - 144*b*Cosh[4*ArcSinh[c*x]] - 64*b*Cosh[6*ArcSinh[c*x]] -
9*b*Cosh[8*ArcSinh[c*x]] - 1152*a*Sinh[2*ArcSinh[c*x]] + 576*a*Sinh[4*ArcSi
nh[c*x]] + 384*a*Sinh[6*ArcSinh[c*x]] + 72*a*Sinh[8*ArcSinh[c*x]]) + 288*b*
Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(-120*a - 48*b*Sinh[2*ArcSinh[c*x]] + 24
*b*Sinh[4*ArcSinh[c*x]] + 16*b*Sinh[6*ArcSinh[c*x]] + 3*b*Sinh[8*ArcSinh[c*
x]])))/(884736*c^3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.426, size = 1204, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/8*a^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-5/192*a^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5
/128*a^2/c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+1081/110592*b^2*(d*(c^2*x^2+1))^(1/2
)*d^2/(c^2*x^2+1)*x^3+5/64*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/c^2/(c^2*x^2+1)*ar
csinh(c*x)*x+127/96*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c
*x)*x^5+1/4*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^9+
23/24*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^7-5/128*
a^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+359
/36864*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/2
56*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*x^9+263/13824*b^2*(d*(c^2*
x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*x^7+1915/55296*b^2*(d*(c^2*x^2+1))^(1/2)*
d^2*c^2/(c^2*x^2+1)*x^5-359/36864*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c^2/(c^2*x^
2+1)*x-5/384*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^3
*d^2+133/384*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+3
59/36864*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/c^3/(c^2*x^2+1)^(1/2)-5/128*a*b*(d*(
c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)*x^2-59/384*a*b*(d*(c^2*x^2+1))^(1
/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^4-17/144*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c
^2*x^2+1)^(1/2)*x^6+1/8*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsi
nh(c*x)^2*x^9+23/48*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c
*x)^2*x^7+127/192*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x
)^2*x^5+5/128*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c^2/(c^2*x^2+1)*arcsinh(c*x)^2*
x-1/32*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*x^8-5/128*b^2*(d
*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-1/32*b^2*(d*(c
```

```
^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^8-17/144*b^2*(d*(
c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^6-59/384*b^2*(d*
(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^4-5/128*a*b*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3*arcsinh(c*x)^2*d^2+133/192*a*b*(d*(c
^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/48*a^2/c^2*x*(c^2*d*x^2
+d)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^4d^2x^6 + 2\,a^2c^2d^2x^4 + a^2d^2x^2 + \left(b^2c^4d^2x^6 + 2\,b^2c^2d^2x^4 + b^2d^2x^2\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abc^4d^2x^6 + 2\,abc^2d^2x^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^6 + 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 + 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
+ 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2, x)
```

3.276 $\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=366

$$-\frac{2bc^5d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{6bc^3d^2x^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{c^2x^2+1}}-\frac{2bcd^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}\right.}{7\sqrt{c^2x^2+1}}$$

```
[Out] (32*b^2*d^2*Sqrt[d + c^2*d*x^2])/(245*c^2) + (16*b^2*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/(735*c^2) + (12*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2
])/(1225*c^2) + (2*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^2) -
 (2*b*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c*Sqrt[1 + c^2*x^2
]) - (2*b*c*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*Sqrt[1 + c
^2*x^2]) - (6*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(35*S
qrt[1 + c^2*x^2]) - (2*b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
]))/(49*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x])^2)
/(7*c^2*d)
```

Rubi [A] time = 0.293957, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5717, 194, 5679, 12, 1799, 1850}

$$-\frac{2bc^5d^2x^7\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{49\sqrt{c^2x^2+1}}-\frac{6bc^3d^2x^5\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{35\sqrt{c^2x^2+1}}-\frac{2bcd^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}\right.}{7\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (32*b^2*d^2*Sqrt[d + c^2*d*x^2])/(245*c^2) + (16*b^2*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/(735*c^2) + (12*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2
])/(1225*c^2) + (2*b^2*d^2*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2])/(343*c^2) -
 (2*b*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c*Sqrt[1 + c^2*x^2
]) - (2*b*c*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*Sqrt[1 + c
^2*x^2]) - (6*b*c^3*d^2*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(35*S
qrt[1 + c^2*x^2]) - (2*b*c^5*d^2*x^7*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
]))/(49*Sqrt[1 + c^2*x^2]) + ((d + c^2*d*x^2)^(7/2)*(a + b*ArcSinh[c*x])^2)
/(7*c^2*d)
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left(d+c^2dx^2\right)^{7/2}\left(a+b\sinh^{-1}(cx)\right)^2}{7c^2d}-\frac{\left(2bd^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^3\left(a+\right.}{7c\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{7\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{7\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{7\sqrt{1+c^2x^2}}$$

$$=-\frac{2bd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{7c\sqrt{1+c^2x^2}}-\frac{2bcd^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{7\sqrt{1+c^2x^2}}$$

$$=\frac{32b^2d^2\sqrt{d+c^2dx^2}}{245c^2}+\frac{16b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{735c^2}+\frac{12b^2d^2\left(1+c^2x^2\right)^2\sqrt{}}{1225c^2}$$

Mathematica [A] time = 0.410314, size = 224, normalized size = 0.61

$$\frac{d^2\sqrt{c^2dx^2+d}\left(3675a^2\left(c^2x^2+1\right)^4-210abcx\left(5c^6x^6+21c^4x^4+35c^2x^2+35\right)\sqrt{c^2x^2+1}+210b\sinh^{-1}(cx)\left(35a\left(c^2x^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d + c^2*d*x^2]*(3675*a^2*(1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[1 + c
^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(2161 + 2918*c^2
*x^2 + 1108*c^4*x^4 + 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*(1 + c^2*x^2)
^4 - b*c*x*Sqrt[1 + c^2*x^2]*(35 + 35*c^2*x^2 + 21*c^4*x^4 + 5*c^6*x^6))*Ar
cSinh[c*x] + 3675*b^2*(1 + c^2*x^2)^4*ArcSinh[c*x]^2))/(25725*c^2*(1 + c^2*
x^2))
```

Maple [B] time = 0.34, size = 1773, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/7*a^2/c^2/d*(c^2*d*x^2+d)^(7/2)+b^2*(1/43904*(d*(c^2*x^2+1))^(1/2)*(64*c^
8*x^8+64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2
)+104*c^4*x^4+56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/
2)+1)*(49*arcsinh(c*x)^2-14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3200*(d*(
c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^
3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c
*x)^2-10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2)*(4
*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(
9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1)
)^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)
*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(
1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/384*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*
x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/3
200*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x
^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*a
rcsinh(c*x)^2+10*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+1)+1/43904*(d*(c^2*x^2+1)
)^(1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c
^2*x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(
c^2*x^2+1)^(1/2)+1)*(49*arcsinh(c*x)^2+14*arcsinh(c*x)+2)*d^2/c^2/(c^2*x^2+
1))+2*a*b*(1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*c^7*x^7*(c^2*x^2+1)^
(1/2)+144*c^6*x^6+112*c^5*x^5*(c^2*x^2+1)^(1/2)+104*c^4*x^4+56*c^3*x^3*(c^2
*x^2+1)^(1/2)+25*c^2*x^2+7*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+7*arcsinh(c*x))*d^2
/c^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x^5*(c^2*x^
2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c*x*(c^2*x^
2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(
1/2)+1)*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)
*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+5/
128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x)
)*d^2/c^2/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))*d^2/c
^2/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+
1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+
1)^(1/2)+1)*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(
1/2)*(64*c^8*x^8-64*c^7*x^7*(c^2*x^2+1)^(1/2)+144*c^6*x^6-112*c^5*x^5*(c^2*
x^2+1)^(1/2)+104*c^4*x^4-56*c^3*x^3*(c^2*x^2+1)^(1/2)+25*c^2*x^2-7*c*x*(c^2
*x^2+1)^(1/2)+1)*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1))
```

Maxima [A] time = 1.33913, size = 370, normalized size = 1.01

$$\frac{\left(c^2dx^2+d\right)^{\frac{7}{2}}b^2\operatorname{arsinh}(cx)^2}{7\,c^2d}+\frac{2\left(c^2dx^2+d\right)^{\frac{7}{2}}ab\operatorname{arsinh}(cx)}{7\,c^2d}+\frac{2}{25725}b^2\left(\frac{75\,\sqrt{c^2x^2+1}c^4d^{\frac{7}{2}}x^6+351\,\sqrt{c^2x^2+1}c^2d^{\frac{7}{2}}x^4+}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*(c^2*d*x^2 + d)^(7/2)*b^2*arcsinh(c*x)^2/(c^2*d) + 2/7*(c^2*d*x^2 + d)^
(7/2)*a*b*arcsinh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 + 1)*c^4*d^(
7/2)*x^6 + 351*sqrt(c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(c^2*x^2 + 1)*d^
(7/2)*x^2 + 2161*sqrt(c^2*x^2 + 1)*d^(7/2)/c^2)/d - 105*(5*c^6*d^(7/2)*x^7
+ 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*arcsinh(c*x)/(c*d
)) + 1/7*(c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*d^(7/2)*x^7 + 21*
c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 + 35*d^(7/2)*x)*a*b/(c*d)
```

Fricas [A] time = 3.06258, size = 980, normalized size = 2.68

$$\frac{3675\left(b^2c^8d^2x^8+4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4+4\,b^2c^2d^2x^2+b^2d^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+210\left(35\,abc^8d^2x^8+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 + 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 + 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))
^2 + 210*(35*a*b*c^8*d^2*x^8 + 140*a*b*c^6*d^2*x^6 + 210*a*b*c^4*d^2*x^4 +
140*a*b*c^2*d^2*x^2 + 35*a*b*d^2 - (5*b^2*c^7*d^2*x^7 + 21*b^2*c^5*d^2*x^5
+ 35*b^2*c^3*d^2*x^3 + 35*b^2*c*d^2*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 + 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 + 12*(12
25*a^2 + 71*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 + 4*(36
75*a^2 + 1459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2 - 210*(5*a*b*c^7
*d^2*x^7 + 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 + 35*a*b*c*d^2*x)*sqrt(c
^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^4*x^2 + c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.277 $\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=420

$$\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-}\right.}{18c}$$

```
[Out] (245*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/1728 + (b^2*d^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/108
 - (115*b^2*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[1 + c^2*x^2]
) - (5*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*Sqrt[1 + c
^2*x^2]) - (5*b*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(48*c) - (b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(18*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/16
+ (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/24 + (x*(d + c^2*d*x
^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.403579, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{5d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\sqrt{c^2x^2+1}}+\frac{5}{16}d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{bd^2\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-}\right.}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d + c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])/1728 + (b^2*d^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/108
 - (115*b^2*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(1152*c*Sqrt[1 + c^2*x^2]
) - (5*b*c*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*Sqrt[1 + c
^2*x^2]) - (5*b*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(48*c) - (b*d^2*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(18*c) + (5*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/16
+ (5*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/24 + (x*(d + c^2*d*x
^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x])^3)/(48*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{1}{6}x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{1}{6}(5d)\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

$$=-\frac{bd^2\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{5}{24}dx\left(d+c^2dx^2\right)^{3/2}\left(a+\right.$$

$$=\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{5bd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{48c}$$

$$=\frac{65b^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}-\frac{5bcd^2x}{}$$

$$=\frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152}+\frac{65b^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x\right.$$

$$=\frac{245b^2d^2x\sqrt{d+c^2dx^2}}{1152}+\frac{65b^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{1728}+\frac{1}{108}b^2d^2x\left(1+c^2x\right.$$

Mathematica [A] time = 1.53273, size = 499, normalized size = 1.19

$$d^2\left(2304a^2c^5x^5\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+7488a^2c^3x^3\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+9504a^2cx\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}+4320a^2\sqrt{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 7488*a^2*c^3*x^3
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 + c^2*x^2]
*Sqrt[d + c^2*d*x^2] + 1440*b^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 3240*a
*b*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 324*a*b*Sqrt[d + c^2*d*x^2]*C
osh[4*ArcSinh[c*x]] - 24*a*b*Sqrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] + 432
0*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] +
1620*b^2*Sqrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + 81*b^2*Sqrt[d + c^2*d*x
^2]*Sinh[4*ArcSinh[c*x]] + 4*b^2*Sqrt[d + c^2*d*x^2]*Sinh[6*ArcSinh[c*x]] -
 12*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*C
osh[4*ArcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]]
 - 108*a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*Sqrt[d +
c^2*d*x^2]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]] + 9*b*Sinh[4*Ar
cSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]])))/(13824*c*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.286, size = 966, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 5/16*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*arcsinh(c*x)^2*d^2+11/8*
a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x-11/16*a*b*(d*(c^2*
x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*x^2-1/18*a*b*(d*(c^2*x^2+1))^(1/2)*d^
2*c^5/(c^2*x^2+1)^(1/2)*x^6-13/48*a*b*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^
2+1)^(1/2)*x^4-1/18*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^5/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*x^6-13/48*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^3/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*x^4-11/16*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c/(c^2*x^2+1)^(1/2)*arcsi
nh(c*x)*x^2+1/6*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)^
2*x^7+17/24*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)^2*x^
5+59/48*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+5/
24*a^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a^2*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a^2*
d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+59/24*a*b*(
d*(c^2*x^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+1/3*a*b*(d*(c^2*x
^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*arcsinh(c*x)*x^7+17/12*a*b*(d*(c^2*x^2+1))
^(1/2)*d^2*c^4/(c^2*x^2+1)*arcsinh(c*x)*x^5+1/6*x*(c^2*d*x^2+d)^(5/2)*a^2+2
99/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*x+1091/3456*b^2*(d*(c^2*x
```

```
^2+1))^(1/2)*d^2*c^2/(c^2*x^2+1)*x^3+5/48*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/c*arcsinh(c*x)^3*d^2+11/16*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^
2+1)*arcsinh(c*x)^2*x-299/1152*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)-299/1152*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/c/(c^2*x^2+1)^(1/
2)+1/108*b^2*(d*(c^2*x^2+1))^(1/2)*d^2*c^6/(c^2*x^2+1)*x^7+113/1728*b^2*(d*
(c^2*x^2+1))^(1/2)*d^2*c^4/(c^2*x^2+1)*x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2, x)
```

3.278 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=635

$$-\frac{2bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(c\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] (598*b^2*d^2*Sqrt[d + c^2*d*x^2])/225 - (2*a*b*c*d^2*x*Sqrt[d + c^2*d*x^2])
/Sqrt[1 + c^2*x^2] + (74*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/675 + (
2*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/125 - (2*b^2*c*d^2*x*Sqrt[d
+ c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (16*b*c*d^2*x*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 + c^2*x^2]) - (22*b*c^3*d^2*x^3*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*Sqrt[1 + c^2*x^2]) - (2*b*c^5*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) + d^2
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + (d*(d + c^2*d*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/3 + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/5 -
(2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] - (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyL
og[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b^2*d
^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*
b^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.90794, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5744, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 5679, 444, 43, 194, 12, 1247, 698}

$$-\frac{2bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{2bd^2\sqrt{c^2dx^2+d}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(c\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```


```
[Out] (598*b^2*d^2*Sqrt[d + c^2*d*x^2])/225 - (2*a*b*c*d^2*x*Sqrt[d + c^2*d*x^2])
/Sqrt[1 + c^2*x^2] + (74*b^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/675 + (
2*b^2*d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2])/125 - (2*b^2*c*d^2*x*Sqrt[d
+ c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - (16*b*c*d^2*x*Sqrt[d + c^2*d
```

```
*x^2]*(a + b*ArcSinh[c*x]))/(15*Sqrt[1 + c^2*x^2]) - (22*b*c^3*d^2*x^3*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(45*Sqrt[1 + c^2*x^2]) - (2*b*c^5*d^2
*x^5*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(25*Sqrt[1 + c^2*x^2]) + d^2
*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2 + (d*(d + c^2*d*x^2)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/3 + ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/5 -
(2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] - (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyL
og[2, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b*d^2*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (2*b^2*d
^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (2*
b^2*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x}\,dx = \frac{1}{5}\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2 + d\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{x}$$

$$= -\frac{2bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{5\sqrt{1+c^2x^2}} - \frac{4bc^3d^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.}{15\sqrt{1+c^2x^2}}$$

$$= -\frac{16bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}} - \frac{22bc^3d^2x^3\sqrt{d+c^2dx^2}\left(a+b\sinh\right.}{45\sqrt{1+c^2x^2}}$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{15\sqrt{1+c^2x^2}} - \frac{22bc^3d^2x^3}{}$$

$$= -\frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{2b^2cd^2x\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - \frac{16bcd^2x\sqrt{d+c^2dx}}{15\sqrt{1}}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}$$

$$= \frac{598}{225}b^2d^2\sqrt{d+c^2dx^2} - \frac{2abcd^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{74}{675}b^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}$$

Mathematica [A] time = 4.386, size = 710, normalized size = 1.12

$$\frac{d^2\left(-108000ab\sqrt{c^2dx^2+d}\left(-\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\sqrt{c^2x^2+1}\sinh^{-1}(cx)+cx-\sinh\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x,x]
```

```
[Out] (d^2*(3600*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(23 + 11*c^2*x^2 + 3*c
^4*x^4) - 24000*a*b*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^
(3/2)*ArcSinh[c*x]) - 480*a*b*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 + 9
*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) -
 b^2*Sqrt[d + c^2*d*x^2]*(480*c*x*(-30 + 5*c^2*x^2 + 9*c^4*x^4)*ArcSinh[c*x
```

```
] + 6750*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 125*(2 + 9*ArcSinh[c*x]^2
)*Cosh[3*ArcSinh[c*x]] - 27*(2 + 25*ArcSinh[c*x]^2)*Cosh[5*ArcSinh[c*x]]) +
 54000*a^2*Sqrt[d]*Sqrt[1 + c^2*x^2]*Log[c*x] - 54000*a^2*Sqrt[d]*Sqrt[1 +
c^2*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 108000*a*b*Sqrt[d + c^2*d*x
^2]*(c*x - Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - ArcSinh[c*x]*Log[1 - E^(-ArcSin
h[c*x])] + ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - PolyLog[2, -E^(-ArcSin
h[c*x])] + PolyLog[2, E^(-ArcSinh[c*x])]) + 54000*b^2*Sqrt[d + c^2*d*x^2]*(
2*Sqrt[1 + c^2*x^2] - 2*c*x*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2
 + ArcSinh[c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])])
 + 2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-ArcSinh[
c*x])]) + 2*(PolyLog[3, -E^(-ArcSinh[c*x])] - PolyLog[3, E^(-ArcSinh[c*x])]
)) + 1000*b^2*Sqrt[d + c^2*d*x^2]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2
) + (2 + 9*ArcSinh[c*x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + S
inh[3*ArcSinh[c*x]]))))/(54000*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.361, size = 1321, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x)
```

```
[Out] 28/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4*c^4+68/15*
a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2*c^2+2/5*a*b*(d*(
c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^6*c^6+1/3*a^2*d*(c^2*d*x^2
+d)^(3/2)-a^2*d^(5/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+a^2*(c^2*d*
x^2+d)^(1/2)*d^2+1/5*(c^2*d*x^2+d)^(5/2)*a^2+9394/3375*b^2*(d*(c^2*x^2+1))^
(1/2)*d^2/(c^2*x^2+1)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog
(3,-c*x-(c^2*x^2+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*d^2-2/25*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^5*c^5-22/45*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3*c^3-46/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x*c-22/45*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*
x^2+1)^(1/2)*c^3*x^3-46/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*
c*x-2/25*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)*c^5*x^5-2*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2)
)*d^2+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(
c^2*x^2+1)^(1/2))*d^2+34/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsi
nh(c*x)^2*x^2*c^2+1/5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x
)^2*x^6*c^6+14/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*
x^4*c^4+23/15*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*arcsinh(c*x)^2+2*b^
```

```
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x
^2+1)^(1/2))*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)
*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*d^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*d^2-b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))*d^2+2/12
5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)*c^6*x^6+532/3375*b^2*(d*(c^2*x^
2+1))^(1/2)*d^2/(c^2*x^2+1)*c^4*x^4+9872/3375*b^2*(d*(c^2*x^2+1))^(1/2)*d^2
/(c^2*x^2+1)*c^2*x^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))*d^2+46/15*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+
1)*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x
+(c^2*x^2+1)^(1/2))*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arsinh}(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2/x, x)
```

3.279 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=530

$$-\frac{b^2cd^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{15bc^3d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\left(a+b\,\mathrm{si}\right.$$

```
[Out] (31*b^2*c^2*d^2*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*c^2*d^2*x*(1 + c^2*x^2)*Sq
rt[d + c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64
*Sqrt[1 + c^2*x^2]) - (15*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(8*Sqrt[1 + c^2*x^2]) + b*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]) - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/8 + (15*c^2*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/8 + (c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((d +
c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x + (5*c*d^2*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^3)/(8*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d^2*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] -
 (b^2*c*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[1 + c
^2*x^2]
```

Rubi [A] time = 0.612697, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5739, 5684, 5682, 5675, 5661, 321, 215, 5717, 195, 5726, 5659, 3716, 2190, 2279, 2391}

$$\frac{b^2cd^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2x^2+1}}-\frac{15bc^3d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}+\frac{15}{8}c^2d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```


```
[Out] (31*b^2*c^2*d^2*x*Sqrt[d + c^2*d*x^2])/64 + (b^2*c^2*d^2*x*(1 + c^2*x^2)*Sq
rt[d + c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(64
*Sqrt[1 + c^2*x^2]) - (15*b*c^3*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[
c*x]))/(8*Sqrt[1 + c^2*x^2]) + b*c*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
]*(a + b*ArcSinh[c*x]) - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x]))/8 + (15*c^2*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
```

```
*x])^2)/8 - (c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[1 + c^2
*x^2] + (5*c^2*d*x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 - ((d +
c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x + (5*c*d^2*Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^3)/(8*b*Sqrt[1 + c^2*x^2]) + (2*b*c*d^2*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2*x^2] +
(b^2*c*d^2*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[1 + c^2
*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x} + \left(5c^2d\right)\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.$$

$$= \frac{1}{2}bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{5}{4}c^2dx\left(d+c^2dx^2\right)^{3/2}\left(a\right.$$

$$= -\frac{1}{8}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} + bcd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}\right.$$

$$= -\frac{11}{16}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{15bc^3d^2x^2\sqrt{d}}{}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{11b^2cd^2\sqrt{d+c^2}}{16\sqrt{1}}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2}}{64\sqrt{1}}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2}}{64\sqrt{1}}$$

$$= \frac{31}{64}b^2c^2d^2x\sqrt{d+c^2dx^2} + \frac{1}{32}b^2c^2d^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2} - \frac{89b^2cd^2\sqrt{d+c^2}}{64\sqrt{1}}$$

Mathematica [A] time = 2.25686, size = 550, normalized size = 1.04

$$\frac{d^2\left(-256b^2cx\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right) + 64a^2c^4x^4\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d} + 288a^2c^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-256*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 288*a^2*c^2*x^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 160*b^2*c*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 - 128*a*b*c*
x*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 4*a*b*c*x*Sqrt[d + c^2*d*x^2]*
Cosh[4*ArcSinh[c*x]] + 512*a*b*c*x*Sqrt[d + c^2*d*x^2]*Log[c*x] + 480*a^2*c
*Sqrt[d]*x*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 256
```

```
*b^2*c*x*Sqrt[d + c^2*d*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])] + 64*b^2*c*x*S
qrt[d + c^2*d*x^2]*Sinh[2*ArcSinh[c*x]] + b^2*c*x*Sqrt[d + c^2*d*x^2]*Sinh[
4*ArcSinh[c*x]] - 4*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(128*a*Sqrt[1 + c^2*
x^2] + 32*b*c*x*Cosh[2*ArcSinh[c*x]] + b*c*x*Cosh[4*ArcSinh[c*x]] - 128*b*c
*x*Log[1 - E^(-2*ArcSinh[c*x])] - 64*a*c*x*Sinh[2*ArcSinh[c*x]] - 4*a*c*x*S
inh[4*ArcSinh[c*x]]) + 8*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(60*a*c*x + 3
2*b*c*x - 32*b*Sqrt[1 + c^2*x^2] + 16*b*c*x*Sinh[2*ArcSinh[c*x]] + b*c*x*Si
nh[4*ArcSinh[c*x]])))/(256*x*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.338, size = 1223, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x)
```

```
[Out] 11/4*a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^3+1/4*a*b
*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*x)*x+1/2*a*b*(d*(c^2*x
^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^5+5/4*a^2*c^2*d*x*(c^2*d*x^
2+d)^(3/2)-a^2/d/x*(c^2*d*x^2+d)^(7/2)+a^2*c^2*x*(c^2*d*x^2+d)^(5/2)-33/64*
a*b*(d*(c^2*x^2+1))^(1/2)*c*d^2/(c^2*x^2+1)^(1/2)-33/64*b^2*(d*(c^2*x^2+1))
^(1/2)*c*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+5/8*b^2*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c*d^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)^2*c*d^2+1/32*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2
+1)*x^5+35/64*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*x^3+33/64*b^2*(
d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2
*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c*d^2+2*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c*d^2-b^2*(d*(c^2
*x^2+1))^(1/2)*arcsinh(c*x)^2*d^2/x/(c^2*x^2+1)-1/8*a*b*(d*(c^2*x^2+1))^(1/
2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x^4-9/8*a*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2
*x^2+1)^(1/2)*x^2-9/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*a
rcsinh(c*x)*x^2-1/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*x^4+15/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)
^2*c*d^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)*d^2/x/(c^2*x^2+1)-2*a*b*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c*d^2+2*a*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c*d^2+2*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/
2))*c*d^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))*c*d^2+1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+
1)*arcsinh(c*x)^2*x^5+11/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*ar
csinh(c*x)^2*x^3+1/8*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(
```

```
c*x)^2*x+15/8*a^2*c^2*d^2*x*(c^2*d*x^2+d)^(1/2)+15/8*a^2*c^2*d^3*ln(x*c^2*d
/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2/x^2, x)
```

3.280 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=687

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{\sqrt{c^2x^2+1}}$$

```
[Out] (40*b^2*c^2*d^2*Sqrt[d + c^2*d*x^2])/9 - (5*a*b*c^3*d^2*x*Sqrt[d + c^2*d*x^
2])/Sqrt[1 + c^2*x^2] + (2*b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/2
7 - (5*b^2*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
(b*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2]) +
(b*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 +
c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (5*c
^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/6 - ((d + c^2*d*x^2)^(5/
2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*d^2*S
qrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (5*b*c^2
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*P
olyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (5*b^2*c^2*d^2*Sqrt[d + c^2*
d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b^2*c^2*d^2*Sqrt
[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rubi [A] time = 0.988336, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 20, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5739, 5744, 5742, 5760, 4182, 2531, 2282, 6589, 5653, 261, 5679, 444, 43, 270, 5730, 12, 1251, 897, 1153, 208}

$$-\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2x^2+1}}+\frac{5bc^2d^2\sqrt{c^2dx^2+d}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh\right.}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] (40*b^2*c^2*d^2*Sqrt[d + c^2*d*x^2])/9 - (5*a*b*c^3*d^2*x*Sqrt[d + c^2*d*x^
2])/Sqrt[1 + c^2*x^2] + (2*b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])/2
7 - (5*b^2*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] -
```

```
(b*c*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[1 + c^2*x^2]) +
(b*c^3*d^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*Sqrt[1 + c^2*x^2]
) - (2*b*c^5*d^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*Sqrt[1 +
c^2*x^2]) + (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/2 + (5*c
^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/6 - ((d + c^2*d*x^2)^(5/
2)*(a + b*ArcSinh[c*x])^2)/(2*x^2) - (5*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (b^2*c^2*d^2*S
qrt[d + c^2*d*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] - (5*b*c^2
*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/
Sqrt[1 + c^2*x^2] + (5*b*c^2*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*P
olyLog[2, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] + (5*b^2*c^2*d^2*Sqrt[d + c^2*
d*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2] - (5*b^2*c^2*d^2*Sqrt
[d + c^2*d*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[1 + c^2*x^2]
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5744

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^IntP
art[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 + c^2*x^2)^FracPart[p])
, Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x
], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5730

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2
*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && I
GtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2x^2} + \frac{1}{2}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{x}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{2bc^3d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{1+c^2x^2}} + \frac{bc^3d^2x\sqrt{d+c^2}}{3}$$

$$= -\frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} - \frac{5b^2c^3d^2x\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}} - \frac{bcd^2\sqrt{d+c^2dx^2}\left(\right.}{x\sqrt{1+}}$$

$$= \frac{55}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{5}{27}b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx}$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx}$$

$$= \frac{40}{9}b^2c^2d^2\sqrt{d+c^2dx^2} - \frac{5abc^3d^2x\sqrt{d+c^2dx^2}}{\sqrt{1+c^2x^2}} + \frac{2}{27}b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx}$$

Mathematica [A] time = 8.00153, size = 990, normalized size = 1.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*((7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^4*d^2
*x^2)/3) + 2*a*b*c^2*d^2*(-(c*x*Sqrt[d*(1 + c^2*x^2)]*(3 + c^2*x^2))/(9*Sqr
t[1 + c^2*x^2]) + ((1 + c^2*x^2)*Sqrt[d*(1 + c^2*x^2)]*ArcSinh[c*x])/3) + (
5*a^2*c^2*d^(5/2)*Log[x])/2 - (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d*(1
```

```
+ c^2*x^2)]])/2 + (4*a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-(c*x) + Sqrt[1 + c
^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*
x]*Log[1 + E^(-ArcSinh[c*x])] + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2,
 E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + 2*b^2*c^2*d^2*Sqrt[d*(1 + c^2*x^2
)]*(2 - (2*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ArcSinh[c*x]^2 + (ArcSinh[
c*x]^2*(Log[1 - E^(-ArcSinh[c*x])] - Log[1 + E^(-ArcSinh[c*x])]))/Sqrt[1 +
c^2*x^2] + (2*ArcSinh[c*x]*(PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(
-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2] + (2*(PolyLog[3, -E^(-ArcSinh[c*x])] -
PolyLog[3, E^(-ArcSinh[c*x])]))/Sqrt[1 + c^2*x^2]) + (b^2*c^2*d^2*Sqrt[d*(1
 + c^2*x^2)]*(27*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + (2 + 9*ArcSinh[c*
x]^2)*Cosh[3*ArcSinh[c*x]] - 6*ArcSinh[c*x]*(9*c*x + Sinh[3*ArcSinh[c*x]]))
)/(108*Sqrt[1 + c^2*x^2]) + (a*b*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-2*Coth[Arc
Sinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]*Log[1 -
 E^(-ArcSinh[c*x])] - 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] + 4*PolyLog
[2, -E^(-ArcSinh[c*x])] - 4*PolyLog[2, E^(-ArcSinh[c*x])] - ArcSinh[c*x]*Se
ch[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(4*Sqrt[1 + c^2*x^2]) + (b^
2*c^2*d^2*Sqrt[d*(1 + c^2*x^2)]*(-4*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Arc
Sinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c
*x])] - 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[Tanh[ArcSinh[c*
x]/2]] + 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 8*ArcSinh[c*x]*Pol
yLog[2, E^(-ArcSinh[c*x])] + 8*PolyLog[3, -E^(-ArcSinh[c*x])] - 8*PolyLog[3
, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 4*ArcSinh[c*
x]*Tanh[ArcSinh[c*x]/2]))/(8*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.415, size = 1404, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x)
```

```
[Out] 1/2*a^2*c^2*(c^2*d*x^2+d)^(5/2)-5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2*d^2+2/3*a*b*(d*(c^2*x^2+1))
^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)*
c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+5*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2-1/2*a^2/d/x^2*(c^2
*d*x^2+d)^(7/2)+5/6*a^2*c^2*d*(c^2*d*x^2+d)^(3/2)-5/2*a^2*c^2*d^(5/2)*ln((2
*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)+5/2*a^2*c^2*(c^2*d*x^2+d)^(1/2)*d^2+12
2/27*b^2*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)-2/9*b^2*(d*(c^2*x^2+1))^
(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^3-14/3*b^2*(d*(c^2*x^2+1))^(
1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x-b^2*arcsinh(c*x)*(d*(c^2*x^2+
```

```
1))^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c-5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2-5/2*b^2*(d*(c
^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2
))*c^2*d^2+5/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*l
n(1-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2+5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2+1/3*b^2*(d*(c^2
*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^4+8/3*b^2*(d*(c^2*x^2+1
))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2-2/9*a*b*(d*(c^2*x^2+1))^(1/
2)*c^5*d^2/(c^2*x^2+1)^(1/2)*x^3-14/3*a*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^
2*x^2+1)^(1/2)*x-a*b*(d*(c^2*x^2+1))^(1/2)*d^2/x/(c^2*x^2+1)^(1/2)*c+11/3*a
*b*(d*(c^2*x^2+1))^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*x)-5*a*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2*d^2+5*a*
b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(2,c*x+(c^2*x^2+1)^(1/2))*
c^2*d^2-a*b*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)*d^2/x^2/(c^2*x^2+1)+5*b^2*(d
*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2
*d^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arctanh(c*x+(c^2*x^2+1)^
(1/2))*c^2*d^2+2/27*b^2*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*x^4+124/2
7*b^2*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*x^2+11/6*b^2*(d*(c^2*x^2+1)
)^(1/2)*c^2*d^2/(c^2*x^2+1)*arcsinh(c*x)^2-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x
^2+1))^(1/2)*d^2/x^2/(c^2*x^2+1)-5*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*polylog(3,c*x+(c^2*x^2+1)^(1/2))*c^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\mathrm{arsinh}\left(cx\right)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + abd\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2/x^3, x)
```

3.281 $\int \frac{(d+c^2dx^2)^{5/2}(a+b\sinh^{-1}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=561

$$-\frac{7b^2c^3d^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}-\frac{5bc^5d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\left(a+b\sin\right.$$

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d + c^2*d*x^2])/12 - (b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])/(3*x) - (23*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(
12*Sqrt[1 + c^2*x^2]) - (5*b*c^5*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (7*b*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2]*(a
 + b*ArcSinh[c*x])^2)/2 + (7*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/(3*Sqrt[1 + c^2*x^2]) - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
(5*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*Sqrt[1 + c^2*x^
2]) + (14*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(-2*
ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) - (7*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*
PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.850528, antiderivative size = 561, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5739, 5682, 5675, 5661, 321, 215, 5726, 5659, 3716, 2190, 2279, 2391, 195, 5728, 277}

$$\frac{7b^2c^3d^2\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2x^2+1}}-\frac{5bc^5d^2x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{5}{2}c^4d^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d + c^2*d*x^2])/12 - (b^2*c^2*d^2*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2])/(3*x) - (23*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(
12*Sqrt[1 + c^2*x^2]) - (5*b*c^5*d^2*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (7*b*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/3 - (b*c*d^2*(1 + c^2*x^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/(3*x^2) + (5*c^4*d^2*x*Sqrt[d + c^2*d*x^2]*(a
```

```
 + b*ArcSinh[c*x])^2)/2 - (7*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^2)/(3*Sqrt[1 + c^2*x^2]) - (5*c^2*d*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh
[c*x])^2)/(3*x) - ((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*x^3) +
(5*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*Sqrt[1 + c^2*x^
2]) + (14*b*c^3*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])*Log[1 - E^(2*A
rcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2]) + (7*b^2*c^3*d^2*Sqrt[d + c^2*d*x^2]*P
olyLog[2, E^(2*ArcSinh[c*x])])/(3*Sqrt[1 + c^2*x^2])
```

Rule 5739

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m
+ 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[(b*c*n*d^Int
Part[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int
[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5726

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcSinh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]))/x, x], x] - Dist[(b*c*d^p)
/(2*p), Int[(1 + c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 5728

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcSinh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*d^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*(1
+ c^2*x^2)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x]), x], x]) /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(5c^2d\right)\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{x^2}$$

$$= -\frac{bcd^2\left(1+c^2x^2\right)^{3/2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2}-\frac{5c^2d\left(d+c^2dx^2\right)^{3/2}\left(a+b\right.}{3x}$$

$$= -\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}+\frac{7}{3}bc^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.$$

$$= -\frac{2}{3}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{5bc^5d^2x^2\sqrt{d+c^2dx^2}}{2\sqrt{1+}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{2b^2c^3d^2\sqrt{d+c^2dx^2}\,\mathrm{si}}{3\sqrt{1+c^2x^2}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c^3d^2\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c^3d^2\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x}}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d+c^2dx^2}-\frac{b^2c^2d^2\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}}{3x}-\frac{23b^2c^3d^2\sqrt{d+c^2dx^2}}{12\sqrt{1+c^2x}}$$

<u>Mathematica</u> [A] time = 2.37959, size = 616, normalized size = 1.1

$$\frac{d^2\left(-56b^2c^3x^3\sqrt{c^2dx^2+d}\,\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+12a^2c^4x^4\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}-56a^2c^2x^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}-8\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/x^4,x]
```

```
[Out] (d^2*(-8*a*b*c*x*Sqrt[d + c^2*d*x^2] - 8*a^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2] - 56*a^2*c^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] - 8*b^2*c^2*
x^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 12*a^2*c^4*x^4*Sqrt[1 + c^2*x^2
]*Sqrt[d + c^2*d*x^2] + 20*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^3 -
 6*a*b*c^3*x^3*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 112*a*b*c^3*x^3*S
qrt[d + c^2*d*x^2]*Log[c*x] + 60*a^2*c^3*Sqrt[d]*x^3*Sqrt[1 + c^2*x^2]*Log[
```

```
c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 56*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*P
olyLog[2, E^(-2*ArcSinh[c*x])] + 3*b^2*c^3*x^3*Sqrt[d + c^2*d*x^2]*Sinh[2*A
rcSinh[c*x]] - 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(4*b*c*x + 8*a*Sqrt[1 +
 c^2*x^2] + 56*a*c^2*x^2*Sqrt[1 + c^2*x^2] + 3*b*c^3*x^3*Cosh[2*ArcSinh[c*x
]] - 56*b*c^3*x^3*Log[1 - E^(-2*ArcSinh[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcSinh
[c*x]]) + 2*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2*(30*a*c^3*x^3 - 4*b*(-7*c^
3*x^3 + Sqrt[1 + c^2*x^2] + 7*c^2*x^2*Sqrt[1 + c^2*x^2]) + 3*b*c^3*x^3*Sinh
[2*ArcSinh[c*x]])))/(24*x^3*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.375, size = 3311, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x)
```

```
[Out] -1/2*b^2*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2+4
9/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*arcsinh(c*x
)*c^6+7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x*arcsinh
(c*x)*c^4+21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^4/(c
^2*x^2+1)^(1/2)*c^7+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsin
h(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3*d^2+5*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/
(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5+7/3*b^2*(d*(c^2*x^2+1))
^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3-5
*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*
arcsinh(c*x)*c^3-5*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/
(c^2*x^2+1)^(1/2)*c^3-1/2*a*b*(d*(c^2*x^2+1))^(1/2)*c^5*d^2/(c^2*x^2+1)^(1/
2)*x^2+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*ln(1+c
*x+(c^2*x^2+1)^(1/2))*c^3*d^2-56/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^
4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*c^8-71/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*
c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-16/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^
2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4-1/3*b^2*(d*(c^2*x^2+1))^(1/2)
*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)*c^2-1/3*b^2*(d*(c^2*x^2+1))^(1
/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)^2+1/2*b^2*(d
*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/2*b^2*(d*(c^2*
x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x+5/2*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d^2-28/3*a*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^3*d^2+14/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3*d^2+5/3*a^2*c^4*d*x*(c^
2*d*x^2+d)^(3/2)-294*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1
)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-406*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^
```

```
4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)*c^6-380/3*a*b*(d*(c^2*x^2+
1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-46/3
*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x^2+1)*arcs
inh(c*x)*c^2+70*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2
/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^5+294*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c
^4*x^4+15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*c^7+5/2*a^2*c^4*d^2
*x*(c^2*d*x^2+d)^(1/2)+5/2*a^2*c^4*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+
d)^(1/2))/(c^2*d)^(1/2)-56/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c
^2*x^2+1)*x^3/(c^2*x^2+1)*c^6-7/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4
+15*c^2*x^2+1)*x/(c^2*x^2+1)*c^4+a*b*(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2
+1)*arcsinh(c*x)*x^3+a*b*(d*(c^2*x^2+1))^(1/2)*c^4*d^2/(c^2*x^2+1)*arcsinh(
c*x)*x-1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x^2/(c^2
*x^2+1)^(1/2)*arcsinh(c*x)*c+147*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+
15*c^2*x^2+1)*x^4/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^7+35*b^2*(d*(c^2*x^2+1
))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2
*c^5-21*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^2/(c^2*x^
2+1)^(1/2)*arcsinh(c*x)*c^5-2/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+1
5*c^2*x^2+1)/x^3/(c^2*x^2+1)*arcsinh(c*x)-49/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^
2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)*c^8-203*b^2*(d*(c^
2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)^
2*c^6-23/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/x/(c^2*x
^2+1)*arcsinh(c*x)^2*c^2-190/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15
*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)^2*c^4-7/3*b^2*(d*(c^2*x^2+1))^(1/2)*
d^2/(63*c^4*x^4+15*c^2*x^2+1)*x/(c^2*x^2+1)*arcsinh(c*x)*c^4-56/3*b^2*(d*(c
^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3/(c^2*x^2+1)*arcsinh(c*x)
*c^6-49/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*
x^2+1)*c^8+14/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)/(c^
2*x^2+1)^(1/2)*arcsinh(c*x)*c^3-21*a*b*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^
4+15*c^2*x^2+1)*x^2/(c^2*x^2+1)^(1/2)*c^5-1/3*a*b*(d*(c^2*x^2+1))^(1/2)*d^2
/(63*c^4*x^4+15*c^2*x^2+1)/x^2/(c^2*x^2+1)^(1/2)*c-147*b^2*(d*(c^2*x^2+1))^
(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^5/(c^2*x^2+1)*arcsinh(c*x)^2*c^8+5/6*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^3*c^3*d^2+1/4*b^2*
(d*(c^2*x^2+1))^(1/2)*c^6*d^2/(c^2*x^2+1)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)
*c^4*d^2/(c^2*x^2+1)*x-1/4*b^2*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1
/2)*arcsinh(c*x)-1/4*a*b*(d*(c^2*x^2+1))^(1/2)*c^3*d^2/(c^2*x^2+1)^(1/2)-1/
3*a^2/d/x^3*(c^2*d*x^2+d)^(7/2)+4/3*a^2*c^4*x*(c^2*d*x^2+d)^(5/2)-4/3*a^2*c
^2/d/x*(c^2*d*x^2+d)^(7/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4*x^4+15
*c^2*x^2+1)/(c^2*x^2+1)^(1/2)*c^3-7/3*b^2*(d*(c^2*x^2+1))^(1/2)*d^2/(63*c^4
*x^4+15*c^2*x^2+1)*x^3*c^6+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3*d^2+14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3*d^2-14/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c^3*d^2+49/3*a*b*(d*(c^2*x^2
+1))^(1/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x^3*c^6+7/3*a*b*(d*(c^2*x^2+1))^(1
/2)*d^2/(63*c^4*x^4+15*c^2*x^2+1)*x*c^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\mathrm{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2/x^4, x)
```

3.282 $\int \frac{x^4 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=153

$$\frac{x^3\sqrt{a^2x^2+1}}{32a^2}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{4a^2}+\frac{3x^2\sinh^{-1}(ax)}{8a^3}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{8a^4}+\frac{\sinh^{-1}(ax)^3}{8a^5}$$

```
[Out] (-15*x*Sqrt[1 + a^2*x^2])/(64*a^4) + (x^3*Sqrt[1 + a^2*x^2])/(32*a^2) + (15
*ArcSinh[a*x])/(64*a^5) + (3*x^2*ArcSinh[a*x])/(8*a^3) - (x^4*ArcSinh[a*x])
/(8*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(8*a^4) + (x^3*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^2)/(4*a^2) + ArcSinh[a*x]^3/(8*a^5)
```

Rubi [A] time = 0.28968, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5758, 5675, 5661, 321, 215}

$$\frac{x^3\sqrt{a^2x^2+1}}{32a^2}-\frac{15x\sqrt{a^2x^2+1}}{64a^4}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{4a^2}+\frac{3x^2\sinh^{-1}(ax)}{8a^3}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{8a^4}+\frac{\sinh^{-1}(ax)^3}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-15*x*Sqrt[1 + a^2*x^2])/(64*a^4) + (x^3*Sqrt[1 + a^2*x^2])/(32*a^2) + (15
*ArcSinh[a*x])/(64*a^5) + (3*x^2*ArcSinh[a*x])/(8*a^3) - (x^4*ArcSinh[a*x])
/(8*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(8*a^4) + (x^3*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^2)/(4*a^2) + ArcSinh[a*x]^3/(8*a^5)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{\int x^3\sinh^{-1}(ax)\,dx}{2a}$$

$$= -\frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{4a^2} + \frac{1}{8}\int\frac{x^4}{\sqrt{1+a^2x^2}}\,dx + \cdots$$

$$= \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\sinh\cdots}{4a^2}$$

$$= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{8a^4} + \cdots$$

$$= -\frac{15x\sqrt{1+a^2x^2}}{64a^4} + \frac{x^3\sqrt{1+a^2x^2}}{32a^2} + \frac{15\sinh^{-1}(ax)}{64a^5} + \frac{3x^2\sinh^{-1}(ax)}{8a^3} - \frac{x^4\sinh^{-1}(ax)}{8a} - \frac{3x\sqrt{1+a^2\cdots}}{\cdots}$$

Mathematica [A] time = 0.0694461, size = 98, normalized size = 0.64

$$\frac{ax\sqrt{a^2x^2+1}\left(2a^2x^2-15\right)+8ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^2+\left(-8a^4x^4+24a^2x^2+15\right)\sinh^{-1}(ax)+8\sinh^{-1}(ax)^3\cdots}{64a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2) + (15 + 24*a^2*x^2 - 8*a^4*x^4)*Ar
cSinh[a*x] + 8*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 + 8*Ar
cSinh[a*x]^3)/(64*a^5)
```

Maple [A] time = 0.059, size = 125, normalized size = 0.8

$$\frac{1}{64\,a^5}\left(16\,a^3x^3\left(\mathrm{Arcsinh}\left(ax\right)\right)^2\sqrt{a^2x^2+1}-8\,a^4x^4\mathrm{Arcsinh}\left(ax\right)+2\,a^3x^3\sqrt{a^2x^2+1}-24\,\left(\mathrm{Arcsinh}\left(ax\right)\right)^2ax\sqrt{a^2x^2+1}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/64*(16*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-8*a^4*x^4*arcsinh(a*x)+2*
a^3*x^3*(a^2*x^2+1)^(1/2)-24*arcsinh(a*x)^2*a*x*(a^2*x^2+1)^(1/2)+24*a^2*x^
2*arcsinh(a*x)+8*arcsinh(a*x)^3-15*a*x*(a^2*x^2+1)^(1/2)+15*arcsinh(a*x))/a
^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,\mathrm{arsinh}\left(ax\right)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 3.12243, size = 297, normalized size = 1.94

$$\frac{8\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2+8\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3-\left(8\,a^4x^4-24\,a^2x^2-15\right)\log\left(ax+\sqrt{a^2x^2\right.}{64\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(8*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 8*log(a*x + sqrt(a^2*x^2 + 1))^3 - (8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*
x + sqrt(a^2*x^2 + 1)) + (2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] time = 5.75894, size = 146, normalized size = 0.95

$$\begin{cases} -\frac{x^4 \operatorname{asinh}(ax)}{8a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a^2} + \frac{x^3\sqrt{a^2x^2+1}}{32a^2} + \frac{3x^2\operatorname{asinh}(ax)}{8a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{8a^4} - \frac{15x\sqrt{a^2x^2+1}}{64a^4} + \frac{\operatorname{asinh}^3(ax)}{8a^5} + \frac{15\operatorname{asinh}(ax)}{64a^5} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**4*asinh(a*x)/(8*a) + x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/
(4*a**2) + x**3*sqrt(a**2*x**2 + 1)/(32*a**2) + 3*x**2*asinh(a*x)/(8*a**3)
- 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(8*a**4) - 15*x*sqrt(a**2*x**2 + 1)
/(64*a**4) + asinh(a*x)**3/(8*a**5) + 15*asinh(a*x)/(64*a**5), Ne(a, 0)), (
0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

3.283 $\int \frac{x^3\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=122

$$\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^4}-\frac{14\sqrt{a^2x^2+1}}{9a^4}+\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^4}+\frac{4x\sinh^{-1}(ax)}{3a^3}-\frac{2x^3\sinh^{-1}(ax}{9a}$$

```
[Out] (-14*Sqrt[1 + a^2*x^2])/(9*a^4) + (2*(1 + a^2*x^2)^(3/2))/(27*a^4) + (4*x*A
rcSinh[a*x])/(3*a^3) - (2*x^3*ArcSinh[a*x])/(9*a) - (2*Sqrt[1 + a^2*x^2]*Ar
cSinh[a*x]^2)/(3*a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2)
```

Rubi [A] time = 0.215011, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5758, 5717, 5653, 261, 5661, 266, 43}

$$\frac{2\left(a^2x^2+1\right)^{3/2}}{27a^4}-\frac{14\sqrt{a^2x^2+1}}{9a^4}+\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{3a^4}+\frac{4x\sinh^{-1}(ax)}{3a^3}-\frac{2x^3\sinh^{-1}(ax}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-14*Sqrt[1 + a^2*x^2])/(9*a^4) + (2*(1 + a^2*x^2)^(3/2))/(27*a^4) + (4*x*A
rcSinh[a*x])/(3*a^3) - (2*x^3*ArcSinh[a*x])/(9*a) - (2*Sqrt[1 + a^2*x^2]*Ar
cSinh[a*x]^2)/(3*a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a^2)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{2\int x^2\sinh^{-1}(ax)\,dx}{3a}$$

$$= -\frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^2} + \frac{2}{9}\int \frac{x^3}{\sqrt{1+a^2x^2}}\,dx +$$

$$= \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^2} + \frac{1}{9}\mathrm{Su}$$

$$= -\frac{4\sqrt{1+a^2x^2}}{3a^4} + \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh}{3a^2}$$

$$= -\frac{14\sqrt{1+a^2x^2}}{9a^4} + \frac{2\left(1+a^2x^2\right)^{3/2}}{27a^4} + \frac{4x\sinh^{-1}(ax)}{3a^3} - \frac{2x^3\sinh^{-1}(ax)}{9a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{3a^4}$$

Mathematica [A] time = 0.0611081, size = 79, normalized size = 0.65

$$\frac{2\left(a^2x^2-20\right)\sqrt{a^2x^2+1}+9\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2-6ax\left(a^2x^2-6\right)\sinh^{-1}(ax)}{27a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] - 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] +
9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(27*a^4)
```

Maple [A] time = 0.058, size = 113, normalized size = 0.9

$$\frac{1}{27\,a^4}\Big(9\,(\mathrm{Arcsinh}\,(ax))^2\,x^4a^4 - 9\,(\mathrm{Arcsinh}\,(ax))^2\,a^2x^2 - 6\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}a^3x^3 + 2\,x^4a^4 - 38\,a^2x^2 - 18\,(\mathrm{Arcs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/27/a^4/(a^2*x^2+1)^(1/2)*(9*arcsinh(a*x)^2*x^4*a^4-9*arcsinh(a*x)^2*a^2*x
^2-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+2*x^4*a^4-38*a^2*x^2-18*arcsinh
(a*x)^2+36*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-40)
```

Maxima [A] time = 1.22056, size = 136, normalized size = 1.11

$$\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^2 + \frac{2\left(\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}\right)}{27a^2} - \frac{2\left(a^2x^3-6x\right)\operatorname{arsinh}(ax)}{9a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^2 +
2/27*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)/a^2 - 2/9*(a^2*x^3
- 6*x)*arcsinh(a*x)/a^3
```

Fricas [A] time = 3.17186, size = 223, normalized size = 1.83

$$\frac{9\sqrt{a^2x^2+1}\left(a^2x^2-2\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\left(a^3x^3-6ax\right)\log\left(ax+\sqrt{a^2x^2+1}\right) + 2\sqrt{a^2x^2+1}\left(a^2x^2-20\right)}{27a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*
(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1)*(a^2*x
^2 - 20))/a^4
```

Sympy [A] time = 3.07278, size = 121, normalized size = 0.99

$$\begin{cases} -\frac{2x^3\operatorname{asinh}(ax)}{9a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}}{27a^2} + \frac{4x\operatorname{asinh}(ax)}{3a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^4} - \frac{40\sqrt{a^2x^2+1}}{27a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-2*x**3*asinh(a*x)/(9*a) + x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**
2/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)/(27*a**2) + 4*x*asinh(a*x)/(3*a**3)
 - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**4) - 40*sqrt(a**2*x**2 + 1)/(2
7*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.39926, size = 154, normalized size = 1.26

$$\frac{\left(\left(a^2x^2+1\right)^{\frac{3}{2}}-3\,\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{3\,a^4}-\frac{2\left(3\left(a^2x^3-6\,x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}-21\,\sqrt{a^2x^2+1}}{a}\right)}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((a^2*x^2 + 1)^(3/2) - 3*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)
)^2/a^4 - 2/27*(3*(a^2*x^3 - 6*x)*log(a*x + sqrt(a^2*x^2 + 1)) - ((a^2*x^2
+ 1)^(3/2) - 21*sqrt(a^2*x^2 + 1))/a)/a^3
```

3.284 $\int \frac{x^2 \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt{a^2x^2+1}}{4a^2}+\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\sinh^{-1}(ax)}{4a^3}-\frac{x^2\sinh^{-1}(ax)}{2a}$$

```
[Out] (x*Sqrt[1 + a^2*x^2])/(4*a^2) - ArcSinh[a*x]/(4*a^3) - (x^2*ArcSinh[a*x])/(
2*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3
)
```

Rubi [A] time = 0.153914, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5758, 5675, 5661, 321, 215}

$$\frac{x\sqrt{a^2x^2+1}}{4a^2}+\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\sinh^{-1}(ax)}{4a^3}-\frac{x^2\sinh^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (x*Sqrt[1 + a^2*x^2])/(4*a^2) - ArcSinh[a*x]/(4*a^3) - (x^2*ArcSinh[a*x])/(
2*a) + (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*a^2) - ArcSinh[a*x]^3/(6*a^3
)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx &= \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{2a^2}-\frac{\int x\sinh^{-1}(ax)\,dx}{a}\\
&= -\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}+\frac{1}{2}\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx\\
&= \frac{x\sqrt{1+a^2x^2}}{4a^2}-\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}-\frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{4a^2}\\
&= \frac{x\sqrt{1+a^2x^2}}{4a^2}-\frac{\sinh^{-1}(ax)}{4a^3}-\frac{x^2\sinh^{-1}(ax)}{2a}+\frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2a^2}-\frac{\sinh^{-1}(ax)^3}{6a^3}
\end{aligned}
$$

Mathematica [A] time = 0.046353, size = 72, normalized size = 0.83

$$
\frac{3ax\sqrt{a^2x^2+1}+6ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2-3\left(2a^2x^2+1\right)\sinh^{-1}(ax)-2\sinh^{-1}(ax)^3}{12a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3*a*x*Sqrt[1 + a^2*x^2] - 3*(1 + 2*a^2*x^2)*ArcSinh[a*x] + 6*a*x*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3)/(12*a^3)
```

Maple [A] time = 0.046, size = 69, normalized size = 0.8

$$-\frac{1}{12\,a^{3}}\left(-6\,\left(\mathrm{Arcsinh}\,(ax)\right)^{2}ax\sqrt{a^{2}x^{2}+1}+6\,a^{2}x^{2}\mathrm{Arcsinh}\,(ax)+2\,\left(\mathrm{Arcsinh}\,(ax)\right)^{3}-3\,ax\sqrt{a^{2}x^{2}+1}+3\,\mathrm{Arcsinh}\,(ax)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/12*(-6*arcsinh(a*x)^2*a*x*(a^2*x^2+1)^(1/2)+6*a^2*x^2*arcsinh(a*x)+2*arc
sinh(a*x)^3-3*a*x*(a^2*x^2+1)^(1/2)+3*arcsinh(a*x))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}\,\mathrm{arsinh}\,(ax)^{2}}{\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 2.82618, size = 239, normalized size = 2.75

$$\frac{6\,\sqrt{a^{2}x^{2}+1}ax\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}-2\,\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}+3\,\sqrt{a^{2}x^{2}+1}ax-3\left(2\,a^{2}x^{2}+1\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}{12\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*log(a*x +
sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 + 1)))/a^3
```

Sympy [A] time = 1.713, size = 78, normalized size = 0.9

$$\begin{cases} -\frac{x^2 \operatorname{asinh}(ax)}{2a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{2a^2} + \frac{x\sqrt{a^2x^2+1}}{4a^2} - \frac{\operatorname{asinh}^3(ax)}{6a^3} - \frac{\operatorname{asinh}(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**2*asinh(a*x)/(2*a) + x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(2*
a**2) + x*sqrt(a**2*x**2 + 1)/(4*a**2) - asinh(a*x)**3/(6*a**3) - asinh(a*x
)/(4*a**3), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

3.285 $\int \frac{x \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}$$

```
[Out] (2*Sqrt[1 + a^2*x^2])/a^2 - (2*x*ArcSinh[a*x])/a + (Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^2)/a^2
```

Rubi [A] time = 0.0779984, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5717, 5653, 261}

$$\frac{2\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{a^2}-\frac{2x\sinh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2])/a^2 - (2*x*ArcSinh[a*x])/a + (Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^2)/a^2
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx &= \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{a^2} - \frac{2\int \sinh^{-1}(ax)\,dx}{a} \\
&= -\frac{2x\sinh^{-1}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{a^2} + 2\int \frac{x}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{2\sqrt{1+a^2x^2}}{a^2} - \frac{2x\sinh^{-1}(ax)}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0328073, size = 48, normalized size = 0.92

$$\frac{2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2-2ax\sinh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2] - 2*a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
2)/a^2
```

Maple [A] time = 0.043, size = 64, normalized size = 1.2

$$\frac{1}{a^2}\left((\mathrm{Arcsinh}\,(ax))^2 a^2x^2 + (\mathrm{Arcsinh}\,(ax))^2 - 2\,\mathrm{Arcsinh}\,(ax)\sqrt{a^2x^2+1}ax + 2\,a^2x^2 + 2\right)\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^2*a^2*x^2+arcsinh(a*x)^2-2*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2+2)
```

Maxima [A] time = 1.21249, size = 65, normalized size = 1.25

$$\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)^2}{a^2}-\frac{2\left(ax\,\mathrm{arsinh}\,(ax)-\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a^2 - 2*(a*x*arcsinh(a*x) - sqrt(a^2*x^2 +
 1))/a^2
```

Fricas [A] time = 3.11211, size = 157, normalized size = 3.02

$$-\frac{2\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\,\sqrt{a^2x^2+1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2
*x^2 + 1))^2 - 2*sqrt(a^2*x^2 + 1))/a^2
```

Sympy [A] time = 0.986788, size = 49, normalized size = 0.94

$$\begin{cases}-\frac{2x\,\mathrm{asinh}\,(ax)}{a}+\frac{\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{a^2}+\frac{2\sqrt{a^2x^2+1}}{a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-2*x*asinh(a*x)/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**2/a**2 + 2*s
qrt(a**2*x**2 + 1)/a**2, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.3837, size = 100, normalized size = 1.92

$$\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2}-\frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{\sqrt{a^2x^2+1}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(
a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a)/a
```

3.286 $\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

```
[Out] ArcSinh[a*x]^3/(3*a)
```

Rubi [A] time = 0.0338568, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]

[Out] ArcSinh[a*x]^3/(3*a)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^3}{3a}$$

Mathematica [A] time = 0.0072849, size = 13, normalized size = 1.

$$\frac{\sinh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^2/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^3/(3*a)
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$\frac{(\mathrm{Arcsinh}\,(ax))^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/3*arcsinh(a*x)^3/a
```

Maxima [A] time = 1.13512, size = 15, normalized size = 1.15

$$\frac{\mathrm{arsinh}\,(ax)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsinh(a*x)^3/a
```

Fricas [B] time = 2.93021, size = 51, normalized size = 3.92

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/a
```

Sympy [A] time = 0.569962, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^{3}(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

Giac [B] time = 1.43879, size = 31, normalized size = 2.38

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/a
```

3.287 $\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=68

$$-2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)-2\text{PolyLog}$$

```
[Out] -2*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 2*ArcSinh[a*x]*PolyLog[2, -E^Ar
cSinh[a*x]] + 2*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] + 2*PolyLog[3, -E^A
rcSinh[a*x]] - 2*PolyLog[3, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.148574, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5760, 4182, 2531, 2282, 6589}

$$-2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)-2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -2*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 2*ArcSinh[a*x]*PolyLog[2, -E^Ar
cSinh[a*x]] + 2*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] + 2*PolyLog[3, -E^A
rcSinh[a*x]] - 2*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + 2\,\text{Subst}\left(\int x\,\text{lo}\right.$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) +$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) +$$
$$= -2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 2\sinh^{-1}(ax)\text{Li}_2\left(e^{\sinh^{-1}(ax)}\right) +$$

Mathematica [A] time = 0.11136, size = 100, normalized size = 1.47

$$2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) - 2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right) - 2\text{PolyLo}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] - ArcSinh[a*x]^2*Log[1 + E^(-ArcS
inh[a*x])] + 2*ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*ArcSinh[a*x]
*PolyLog[2, E^(-ArcSinh[a*x])] + 2*PolyLog[3, -E^(-ArcSinh[a*x])] - 2*PolyL
og[3, E^(-ArcSinh[a*x])]
```

Maple [A] time = 0.052, size = 144, normalized size = 2.1

$$-(\text{Arcsinh}(ax))^2 \ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 2\,\text{Arcsinh}(ax)\,\text{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right) + 2\,\text{polylog}\left(3,-ax-\sqrt{a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)*polylog(2,-a*x-(
a^2*x^2+1)^(1/2))+2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^2*ln(1-a
*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*pol
ylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\text{arsinh}(ax)^2}{a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x), x)
```

3.288 $\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=66

$$a\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{x}-a\sinh^{-1}(ax)^2+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

```
[Out] -(a*ArcSinh[a*x]^2) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x + 2*a*ArcSinh[a*
x]*Log[1 - E^(2*ArcSinh[a*x])] + a*PolyLog[2, E^(2*ArcSinh[a*x])]
```

Rubi [A] time = 0.162083, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5723, 5659, 3716, 2190, 2279, 2391}

$$a\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2}{x}-a\sinh^{-1}(ax)^2+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```


```
[Out] -(a*ArcSinh[a*x]^2) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/x + 2*a*ArcSinh[a*
x]*Log[1 - E^(2*ArcSinh[a*x])] + a*PolyLog[2, E^(2*ArcSinh[a*x])]
```


Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx &= -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{x}+(2a)\int \frac{\sinh^{-1}(ax)}{x}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{x}+(2a)\,\text{Subst}\left(\int x\coth(x)\,dx,x,\sinh^{-1}(ax)\right)\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{x}-(4a)\,\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx,x,\sinh^{-1}(ax)\right)\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-(2a)\,\text{Subst}\left(\int \log\right.\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)-a\,\text{Subst}\left(\int \frac{\log(1}{x}\right.\\
&= -a\sinh^{-1}(ax)^2-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{x}+2a\sinh^{-1}(ax)\log\left(1-e^{2\sinh^{-1}(ax)}\right)+a\,\text{Li}_2\left(e^{2\sinh^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A] time = 0.320649, size = 65, normalized size = 0.98

$$a\left(\sinh^{-1}(ax)\left(-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{ax}+\sinh^{-1}(ax)+2\log\left(1-e^{-2\sinh^{-1}(ax)}\right)\right)-\text{PolyLog}\left(2,e^{-2\sinh^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] a*(ArcSinh[a*x]*(ArcSinh[a*x] - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(a*x) + 2*
Log[1 - E^(-2*ArcSinh[a*x])]) - PolyLog[2, E^(-2*ArcSinh[a*x])])
```

Maple [A] time = 0.078, size = 132, normalized size = 2.

$$\frac{(\text{Arcsinh}\,(ax))^2}{x}\left(ax-\sqrt{a^2x^2+1}\right)-2\,a\,(\text{Arcsinh}\,(ax))^2+2\,a\text{Arcsinh}\,(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+2\,a\text{polylog}\left(2,-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^2-2*a*arcsinh(a*x)^2+2*a*arcsinh(a*x
)*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*a*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*arc
sinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{x}+\int\frac{2\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2x^2+1}ax^2+\left(a^2x^2+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(sqrt(a^2*x^2 +
 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)^2}{a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^4 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^2(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^2), x)
```

3.289 $\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=135

$$a^2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-a^2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-a^2\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)+a^2\text{PolyL}$$

```
[Out] -((a*ArcSinh[a*x])/x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) + a^2*Ar
cSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^2*ArcTanh[Sqrt[1 + a^2*x^2]] + a^2
*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^2*ArcSinh[a*x]*PolyLog[2, E^A
rcSinh[a*x]] - a^2*PolyLog[3, -E^ArcSinh[a*x]] + a^2*PolyLog[3, E^ArcSinh[a
*x]]
```

Rubi [A] time = 0.262416, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5747, 5760, 4182, 2531, 2282, 6589, 5661, 266, 63, 208}

$$a^2\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-a^2\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-a^2\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)+a^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```


```
[Out] -((a*ArcSinh[a*x])/x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) + a^2*Ar
cSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^2*ArcTanh[Sqrt[1 + a^2*x^2]] + a^2
*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^2*ArcSinh[a*x]*PolyLog[2, E^A
rcSinh[a*x]] - a^2*PolyLog[3, -E^ArcSinh[a*x]] + a^2*PolyLog[3, E^ArcSinh[a
*x]]
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} + a\int \frac{\sinh^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^2\,\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \sinh^{-1}(ax)\right) + a^2\int \frac{}{x\sqrt{}}$$

$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{}{x}\right.$$

$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) + a^2\sinh^{-1}(ax)\text{Li}_2$$

$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - a^2\tanh^{-1}\left(\sqrt{1+\right.$$

$$= -\frac{a\sinh^{-1}(ax)}{x} - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2}{2x^2} + a^2\sinh^{-1}(ax)^2\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - a^2\tanh^{-1}\left(\sqrt{1+\right.$$

Mathematica [A] time = 1.20026, size = 188, normalized size = 1.39

$$\frac{1}{8}a^2\left(-8\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) + 8\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{-\sinh^{-1}(ax)}\right) - 8\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a^2*(-4*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] - ArcSinh[a*x]^2*Csch[ArcSinh[a*
x]/2]^2 - 4*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 4*ArcSinh[a*x]^2*Lo
g[1 + E^(-ArcSinh[a*x])] + 8*Log[Tanh[ArcSinh[a*x]/2]] - 8*ArcSinh[a*x]*Pol
yLog[2, -E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])]
- 8*PolyLog[3, -E^(-ArcSinh[a*x])] + 8*PolyLog[3, E^(-ArcSinh[a*x])] - ArcS
inh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 + 4*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2]))/8
```

Maple [A] time = 0.116, size = 233, normalized size = 1.7

$$-\frac{\text{Arcsinh}\,(ax)}{2\,x^2}\left(a^2x^2\text{Arcsinh}\,(ax)+2\,ax\sqrt{a^2x^2+1}+\text{Arcsinh}\,(ax)\right)\frac{1}{\sqrt{a^2x^2+1}}+\frac{a^2\,(\text{Arcsinh}\,(ax))^2}{2}\ln\left(1+ax+\sqrt{a^2x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)*(a^2*x^2*arcsinh(a*x)+2*a*x*(a^2*x^
2+1)^(1/2)+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))
+a^2*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-(a^2
*x^2+1)^(1/2))-1/2*a^2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-a^2*arcsi
nh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))+a^2*polylog(3,a*x+(a^2*x^2+1)^(1/2
))-2*a^2*arctanh(a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\,(ax)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)^2}{a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/(a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{asinh}^2\,(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**2/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{arsinh}\,(ax)^2}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(sqrt(a^2*x^2 + 1)*x^3), x)
```

3.290 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=383

$$-\frac{16abx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}}-\frac{2bx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}+\frac{8bx^3\sqrt{c^2x^2+1}\left(a+b\sinh\right.}{45c^3\sqrt{c^2dx^2+d}}$$

```
[Out] (-16*a*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (298*b^2*(1 +
c^2*x^2))/(225*c^6*Sqrt[d + c^2*d*x^2]) - (76*b^2*(1 + c^2*x^2)^2)/(675*c^6
*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^3)/(125*c^6*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(15*c^5*Sqrt[d + c^2*d*x^2])
 + (8*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3*Sqrt[d + c^2*d*
x^2]) - (2*b*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c*Sqrt[d + c^2
*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^6*d) - (4*x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4*d) + (x^4*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(5*c^2*d)
```

Rubi [A] time = 0.554273, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5758, 5717, 5653, 261, 5661, 266, 43}

$$-\frac{16abx\sqrt{c^2x^2+1}}{15c^5\sqrt{c^2dx^2+d}}-\frac{2bx^5\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{c^2dx^2+d}}+\frac{x^4\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d}+\frac{8bx^3\sqrt{c^2x^2+1}\left(a+b\sinh\right.}{45c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```


```
[Out] (-16*a*b*x*Sqrt[1 + c^2*x^2])/(15*c^5*Sqrt[d + c^2*d*x^2]) + (298*b^2*(1 +
c^2*x^2))/(225*c^6*Sqrt[d + c^2*d*x^2]) - (76*b^2*(1 + c^2*x^2)^2)/(675*c^6
*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^3)/(125*c^6*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(15*c^5*Sqrt[d + c^2*d*x^2])
 + (8*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(45*c^3*Sqrt[d + c^2*d*
x^2]) - (2*b*x^5*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c*Sqrt[d + c^2
*d*x^2]) + (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^6*d) - (4*x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(15*c^4*d) + (x^4*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(5*c^2*d)
```


Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^4\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{5c^2d} - \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{5c^2} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int x^4\left(a+b\,\mathrm{s}\right.}{5c\sqrt{d+c^2dx^2}}$$

$$= -\frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{d+c^2dx^2}} - \frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{15c^4d} + \frac{x^4\sqrt{d+c^2dx^2}}{}$$

$$= \frac{8bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{25c\sqrt{d+c^2dx^2}} + \frac{8\sqrt{d+c^2dx^2}\left(a\right.}{15c}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{8bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{45c^3\sqrt{d+c^2dx^2}} - \frac{2bx^5\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{25c\sqrt{d+c^2dx^2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{25c^6\sqrt{d+c^2dx^2}} - \frac{4b^2\left(1+c^2x^2\right)^2}{75c^6\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^3}{125c^6\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1}}{15c^5}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{15c^5\sqrt{d+c^2dx^2}} + \frac{298b^2\left(1+c^2x^2\right)}{225c^6\sqrt{d+c^2dx^2}} - \frac{76b^2\left(1+c^2x^2\right)^2}{675c^6\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^3}{125c^6\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{}}{15}$$

Mathematica [A] time = 0.365514, size = 230, normalized size = 0.6

$$\frac{225a^2\left(3c^6x^6-c^4x^4+4c^2x^2+8\right)-30abcx\sqrt{c^2x^2+1}\left(9c^4x^4-20c^2x^2+120\right)+30b\sinh^{-1}(cx)\left(15a\left(3c^6x^6-c^4x^4+4c^2x^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-30*a*b*c*x*Sqrt[1 + c^2*x^2]*(120 - 20*c^2*x^2 + 9*c^4*x^4) + 225*a^2*(8
+ 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6) + 2*b^2*(2072 + 1936*c^2*x^2 - 109*c^4*x
^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[1 + c^2*x^2]*(-120 + 20*c^2*x^2 - 9*c^4
*x^4) + 15*a*(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6))*ArcSinh[c*x] + 225*b^2*
```

```
(8 + 4*c^2*x^2 - c^4*x^4 + 3*c^6*x^6)*ArcSinh[c*x]^2)/(3375*c^6*Sqrt[d + c^
2*d*x^2])
```

Maple [B] time = 0.355, size = 1227, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(1/5*x^4/c^2/d*(c^2*d*x^2+d)^(1/2)-4/5/c^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)
^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2)))+b^2*(1/4000*(d*(c^2*x^2+1))^(1/2)*(1
6*c^6*x^6+16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1
/2)+13*c^2*x^2+5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2-10*arcsinh(c*x
)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^
2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arc
sinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2
*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+5/16*(
d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*ar
csinh(c*x)+2)/c^6/d/(c^2*x^2+1)-5/864*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^
3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x
)^2+6*arcsinh(c*x)+2)/c^6/d/(c^2*x^2+1)+1/4000*(d*(c^2*x^2+1))^(1/2)*(16*c^
6*x^6-16*c^5*x^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+
13*c^2*x^2-5*c*x*(c^2*x^2+1)^(1/2)+1)*(25*arcsinh(c*x)^2+10*arcsinh(c*x)+2)
/c^6/d/(c^2*x^2+1))+2*a*b*(1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6+16*c^5*x
^5*(c^2*x^2+1)^(1/2)+28*c^4*x^4+20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2+5*c
*x*(c^2*x^2+1)^(1/2)+1)*(-1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*
x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^6/d/(c^2*x^2+1)+5/16*(d*(c^2*x^2+1))^
(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^6/d/(c^2*x^2+1)
+5/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*
x))/c^6/d/(c^2*x^2+1)-5/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(1+3*arcsinh(c*x))/c^6/d
/(c^2*x^2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*c^6*x^6-16*c^5*x^5*(c^2*x^2+1)
^(1/2)+28*c^4*x^4-20*c^3*x^3*(c^2*x^2+1)^(1/2)+13*c^2*x^2-5*c*x*(c^2*x^2+1)
^(1/2)+1)*(1+5*arcsinh(c*x))/c^6/d/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.31596, size = 711, normalized size = 1.86

$$\frac{225 \left(3 b^{2} c^{6} x^{6} - b^{2} c^{4} x^{4} + 4 b^{2} c^{2} x^{2} + 8 b^{2}\right) \sqrt{c^{2} d x^{2} + d} \log \left(c x + \sqrt{c^{2} x^{2} + 1}\right)^{2} + 30 \left(45 a b c^{6} x^{6} - 15 a b c^{4} x^{4} + 60 a b c^{2} x^{2} + 12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - b^2*c^4*x^4 + 4*b^2*c^2*x^2 + 8*b^2)*sqrt(c^2*
d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 30*(45*a*b*c^6*x^6 - 15*a*b*c^4
*x^4 + 60*a*b*c^2*x^2 + 120*a*b - (9*b^2*c^5*x^5 - 20*b^2*c^3*x^3 + 120*b^2
*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
 (27*(25*a^2 + 2*b^2)*c^6*x^6 - (225*a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 +
968*b^2)*c^2*x^2 + 1800*a^2 + 4144*b^2 - 30*(9*a*b*c^5*x^5 - 20*a*b*c^3*x^3
 + 120*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))/(c^8*d*x^2 + c^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^2 x^5}{\sqrt{c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^5/sqrt(c^2*d*x^2 + d), x)
```

3.291 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=323

$$-\frac{bx^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d}+\frac{3bx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{c^2dx^2+d}}-\frac{3x\sqrt{c^2dx^2}}{}$$

```
[Out] (-15*b^2*x*(1 + c^2*x^2))/(64*c^4*Sqrt[d + c^2*d*x^2]) + (b^2*x^3*(1 + c^2*
x^2))/(32*c^2*Sqrt[d + c^2*d*x^2]) + (15*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(64*c^5*Sqrt[d + c^2*d*x^2]) + (3*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(8*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(8*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
c^2*d) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c^5*Sqrt[d + c^2*d
*x^2])
```

Rubi [A] time = 0.478891, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5758, 5677, 5675, 5661, 321, 215}

$$-\frac{bx^4\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{c^2dx^2+d}}+\frac{x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d}+\frac{3bx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{c^2dx^2+d}}-\frac{3x\sqrt{c^2dx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 + c^2*x^2))/(64*c^4*Sqrt[d + c^2*d*x^2]) + (b^2*x^3*(1 + c^2*
x^2))/(32*c^2*Sqrt[d + c^2*d*x^2]) + (15*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(64*c^5*Sqrt[d + c^2*d*x^2]) + (3*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x]))/(8*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(8*c*Sqrt[d + c^2*d*x^2]) - (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(8*c^4*d) + (x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
c^2*d) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(8*b*c^5*Sqrt[d + c^2*d
*x^2])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
```

```
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2d} - \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{4c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x^3\left(a+b\,\mathrm{si}\right.}{2c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{d+c^2dx^2}} - \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{8c^4d} + \frac{x^3\sqrt{d+c^2dx^2}\left(\right.}{4}$$

$$= \frac{b^2x^3\left(1+c^2x^2\right)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c\sqrt{d+c^2dx^2}}$$

$$= -\frac{15b^2x\left(1+c^2x^2\right)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3\left(1+c^2x^2\right)}{32c^2\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c^3\sqrt{d+c^2dx^2}} - \frac{bx^4\sqrt{1+c^2x}}{8c}$$

$$= -\frac{15b^2x\left(1+c^2x^2\right)}{64c^4\sqrt{d+c^2dx^2}} + \frac{b^2x^3\left(1+c^2x^2\right)}{32c^2\sqrt{d+c^2dx^2}} + \frac{15b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{64c^5\sqrt{d+c^2dx^2}} + \frac{3bx^2\sqrt{1+c^2x^2}\left(a+\right.}{8c^3\sqrt{d+c^2}}$$

Mathematica [A] time = 0.834756, size = 268, normalized size = 0.83

$$\frac{32a^2c\sqrt{d}x\left(c^2x^2+1\right)\left(2c^2x^2-3\right)+96a^2\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right)+4ab\sqrt{d}\sqrt{c^2x^2+1}\left(4\sinh^{-1}(cx)\left(6\sinh^{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(1 + c^2*x^2)*(-3 + 2*c^2*x^2) + 96*a^2*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*Sqrt[1 + c^2*x^
2]*(32*ArcSinh[c*x]^3 - 4*ArcSinh[c*x]*(-16*Cosh[2*ArcSinh[c*x]] + Cosh[4*A
rcSinh[c*x]]) - 32*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]] + 8*ArcSinh[
c*x]^2*(-8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + 4*a*b*Sqrt[d]*Sq
rt[1 + c^2*x^2]*(16*Cosh[2*ArcSinh[c*x]] - Cosh[4*ArcSinh[c*x]] + 4*ArcSinh
[c*x]*(6*ArcSinh[c*x] - 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(2
56*c^5*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.38, size = 760, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/4*a^2*x^3/c^2/d*(c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(c^2*d*x^2+d)^(1/2)+3
/8*a^2/c^4*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/8*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d*arcsinh(c*x)^3+3/8*b^2*(d
*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-3/8*b^2*(d*(c^
2*x^2+1))^(1/2)/c^4/d/(c^2*x^2+1)*arcsinh(c*x)^2*x+15/64*b^2*(d*(c^2*x^2+1)
)^(1/2)/c^5/d/(c^2*x^2+1)^(1/2)*arcsinh(c*x)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/
d/(c^2*x^2+1)*arcsinh(c*x)^2*x^5-1/8*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x
^2+1)*arcsinh(c*x)^2*x^3-1/8*b^2*(d*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2
)*arcsinh(c*x)*x^4+1/32*b^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*x^5-13/64*b
^2*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2+1)*x^3-15/64*b^2*(d*(c^2*x^2+1))^(1
/2)/c^4/d/(c^2*x^2+1)*x+3/8*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5
/d*arcsinh(c*x)^2+3/8*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+1)^(1/2)*x^2
-3/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d/(c^2*x^2+1)*arcsinh(c*x)*x+1/2*a*b*(d*
(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*x^5-1/8*a*b*(d*(c^2*x^2+1))^(
1/2)/c/d/(c^2*x^2+1)^(1/2)*x^4-1/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2
+1)*arcsinh(c*x)*x^3+15/64*a*b*(d*(c^2*x^2+1))^(1/2)/c^5/d/(c^2*x^2+1)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4\,\text{arsinh}\,(cx)^2 + 2\,abx^4\,\text{arsinh}\,(cx) + a^2x^4}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)/sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{asinh}(cx))^2}{\sqrt{d\left(c^2x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^4}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/sqrt(c^2*d*x^2 + d), x)
```

3.292 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=265

$$\frac{4abx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}-\frac{2bx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(c\right.}{3c^4d}$$

```
[Out] (4*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (14*b^2*(1 + c^2*
x^2))/(9*c^4*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^4*Sqrt[d
+ c^2*d*x^2]) + (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^3*Sqrt[d + c^
2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c*Sqrt[d +
c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d) + (x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```

Rubi [A] time = 0.329653, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5758, 5717, 5653, 261, 5661, 266, 43}

$$\frac{4abx\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}-\frac{2bx^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{c^2dx^2+d}}+\frac{x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d}-\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(c\right.}{3c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```


```
[Out] (4*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (14*b^2*(1 + c^2*
x^2))/(9*c^4*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^4*Sqrt[d
+ c^2*d*x^2]) + (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^3*Sqrt[d + c^
2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c*Sqrt[d +
c^2*d*x^2]) - (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d) + (x
^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^2*d)
```


Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3 \left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d} - \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{3c^2} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int x^2\left(a+b\right.}{3c\sqrt{d+c^2dx}}$$

$$= -\frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d} + \frac{x^2\sqrt{d+c^2dx^2}}{}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c\sqrt{d+c^2dx^2}} - \frac{2\sqrt{a}}{}$$

$$= \frac{4abx\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{14b^2\left(1+c^2x^2\right)}{9c^4\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)^2}{27c^4\sqrt{d+c^2dx^2}} + \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3\sqrt{d+c^2dx^2}} - \frac{2bx}{}$$

Mathematica [A] time = 0.280226, size = 176, normalized size = 0.66

$$\frac{9a^2\left(c^4x^4-c^2x^2-2\right)-6abcx\left(c^2x^2-6\right)\sqrt{c^2x^2+1}-6b\sinh^{-1}(cx)\left(a\left(-3c^4x^4+3c^2x^2+6\right)+bcx\sqrt{c^2x^2+1}\left(c^2x^2-6\right)\right)}{27c^4\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-6*a*b*c*x*(-6 + c^2*x^2)*Sqrt[1 + c^2*x^2] + 2*b^2*(-20 - 19*c^2*x^2 + c^
4*x^4) + 9*a^2*(-2 - c^2*x^2 + c^4*x^4) - 6*b*(b*c*x*(-6 + c^2*x^2)*Sqrt[1
+ c^2*x^2] + a*(6 + 3*c^2*x^2 - 3*c^4*x^4))*ArcSinh[c*x] + 9*b^2*(-2 - c^2*
x^2 + c^4*x^4)*ArcSinh[c*x]^2)/(27*c^4*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.267, size = 706, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(1/3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(c^2*d*x^2+d)^(1/2))+b^2*(
1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^
2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2-6*arcsinh(c*x)+2)/c^4/d/(c^2
*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(arcsin
h(c*x)^2-2*arcsinh(c*x)+2)/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2
*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*arcsinh(c*x)+2)/c^4/d/(c^2*
x^2+1)+1/216*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5
*c^2*x^2-3*c*x*(c^2*x^2+1)^(1/2)+1)*(9*arcsinh(c*x)^2+6*arcsinh(c*x)+2)/c^4
/d/(c^2*x^2+1))+2*a*b*(1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*c^3*x^3*(c^2
*x^2+1)^(1/2)+5*c^2*x^2+3*c*x*(c^2*x^2+1)^(1/2)+1)*(-1+3*arcsinh(c*x))/c^4/
d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(c^2*x^2+1)^(1/2)+1)*(
-1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)-3/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(
c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^4/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))
^(1/2)*(4*c^4*x^4-4*c^3*x^3*(c^2*x^2+1)^(1/2)+5*c^2*x^2-3*c*x*(c^2*x^2+1)^(
1/2)+1)*(1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.12222, size = 536, normalized size = 2.02

$$\frac{9\left(b^{2}c^{4}x^{4}-b^{2}c^{2}x^{2}-2\,b^{2}\right)\sqrt{c^{2}dx^{2}+d}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+6\left(3\,abc^{4}x^{4}-3\,abc^{2}x^{2}-6\,ab-\left(b^{2}c^{3}x^{3}-6\,b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - b^2*c^2*x^2 - 2*b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + s
qrt(c^2*x^2 + 1))^2 + 6*(3*a*b*c^4*x^4 - 3*a*b*c^2*x^2 - 6*a*b - (b^2*c^3*x
^3 - 6*b^2*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x
^2 + 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - (9*a^2 + 38*b^2)*c^2*x^2 - 18*a^2 - 4
0*b^2 - 6*(a*b*c^3*x^3 - 6*a*b*c*x)*sqrt(c^2*x^2 + 1))*sqrt(c^2*d*x^2 + d))
/(c^6*d*x^2 + c^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^3}{\sqrt{c^2dx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^3/sqrt(c^2*d*x^2 + d), x)
```

3.293 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc^3\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}+\frac{b^2x\left(c^2x^2+1\right)}{4c^2\sqrt{c^2dx^2+d}}$$

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x])/(4*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b
*c^3*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.275487, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5758, 5677, 5675, 5661, 321, 215}

$$-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc^3\sqrt{c^2dx^2+d}}+\frac{x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{c^2dx^2+d}}+\frac{b^2x\left(c^2x^2+1\right)}{4c^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x])/(4*c^3*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d*x^2]) + (x*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(2*c^2*d) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b
*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{2c^2} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int x\left(a+b\sinh^{-1}(\right.}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} + \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int}{2\sqrt{d+c^2}}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^2\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2d} - \cdots$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^3\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}} + \frac{x\sqrt{d+c^2}}{}$$

Mathematica [A] time = 0.880818, size = 198, normalized size = 0.97

$$\frac{12a^2cx\left(c^2dx^2+d\right) - 12a^2\sqrt{d}\sqrt{c^2dx^2+d}\log\left(\sqrt{d}\sqrt{c^2dx^2+d}+cdx\right) - 6abd\sqrt{c^2x^2+1}\left(2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)-\sinh\left(2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (12*a^2*c*x*(d + c^2*d*x^2) - 12*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x
+ Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 6*a*b*d*Sqrt[1 + c^2*x^2]*(Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*(ArcSinh[c*x] - Sinh[2*ArcSinh[c*x]])) - b^2*d*Sqrt[
1 + c^2*x^2]*(4*ArcSinh[c*x]^3 + 6*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 3*(1
 + 2*ArcSinh[c*x]^2)*Sinh[2*ArcSinh[c*x]]))/(24*c^3*d*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.262, size = 530, normalized size = 2.6

$$\frac{a^2x}{2\,c^2d}\sqrt{c^2dx^2+d} - \frac{a^2}{2\,c^2}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}} + \frac{b^2x^3}{4\,d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)} + \frac{b^2x}{4\,c^2d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a^2/c^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c
^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1
)*x^3+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d/(c^2*x^2+1)*x-1/6*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d*arcsinh(c*x)^3+1/2*b^2*(d*(c^2*x^2+1))^(
1/2)/d/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d/(
c^2*x^2+1)*arcsinh(c*x)^2*x-1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+1)
^(1/2)*arcsinh(c*x)-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*arc
sinh(c*x)*x^2-1/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d*arcsinh
(c*x)^2+a*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)*x^3-1/2*a*b*(d
*(c^2*x^2+1))^(1/2)/c/d/(c^2*x^2+1)^(1/2)*x^2+a*b*(d*(c^2*x^2+1))^(1/2)/c^2
/d/(c^2*x^2+1)*arcsinh(c*x)*x-1/4*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d/(c^2*x^2+
1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\operatorname{arsinh}(cx)^2 + 2\,abx^2\operatorname{arsinh}(cx) + a^2x^2}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)/sqrt(c
^2*d*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{asinh}(cx))^2}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^2}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/sqrt(c^2*d*x^2 + d), x)
```

3.294 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2abx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}+\frac{2b^2\left(c^2x^2+1\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{2b^2x\sqrt{c^2x^2+1}\sinh^{-1}(cx)}{c\sqrt{c^2dx^2+d}}$$

```
[Out] (-2*a*b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)
)/(c^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*S
qrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d)
```

Rubi [A] time = 0.124207, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {5717, 5653, 261}

$$-\frac{2abx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d}+\frac{2b^2\left(c^2x^2+1\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{2b^2x\sqrt{c^2x^2+1}\sinh^{-1}(cx)}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)
)/(c^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(c*S
qrt[d + c^2*d*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^2*d)
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(a+b\sinh^{-1}(cx)\right)\,dx}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} - \frac{\left(2b^2\sqrt{1+c^2x^2}\right)\int\sinh^{-1}(cx)\,dx}{c\sqrt{d+c^2dx^2}}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d} + \frac{\left(2b^2\sqrt{1}\right.}{}$$

$$= -\frac{2abx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^2\sqrt{d+c^2dx^2}} - \frac{2b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c\sqrt{d+c^2dx^2}} + \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-}\right.}{c^2d}$$

Mathematica [A] time = 0.28431, size = 127, normalized size = 0.92

$$\frac{\sqrt{c^2dx^2+d}\left(a^2\sqrt{c^2x^2+1}-2b\sinh^{-1}(cx)\left(bcx-a\sqrt{c^2x^2+1}\right)-2abcx+2b^2\sqrt{c^2x^2+1}+b^2\sqrt{c^2x^2+1}\sinh^{-1}(cx)^2\right)}{c^2d\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x^2] + 2*b^2*Sqrt[1 + c
^2*x^2] - 2*b*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + b^2*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]^2))/(c^2*d*Sqrt[1 + c^2*x^2])
```

Maple [B] time = 0.155, size = 296, normalized size = 2.1

$$\frac{a^2}{c^2 d}\sqrt{c^2 d x^2 + d} + b^2\left(\frac{(\mathrm{Arcsinh}\,(cx))^2 - 2\,\mathrm{Arcsinh}\,(cx) + 2}{2\,c^2 d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}\left(c^2x^2 + cx\sqrt{c^2x^2+1}+1\right) + \frac{(\mathrm{Arcsinh}\,(cx))^2 -}{2\,c^2 d\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2/c^2/d*(c^2*d*x^2+d)^(1/2)+b^2*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+c*x*(
c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2-2*arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1)+1/2
*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(arcsinh(c*x)^2+2*
arcsinh(c*x)+2)/c^2/d/(c^2*x^2+1))+2*a*b*(1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^
2+c*x*(c^2*x^2+1)^(1/2)+1)*(-1+arcsinh(c*x))/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2-c*x*(c^2*x^2+1)^(1/2)+1)*(1+arcsinh(c*x))/c^2/d/(c^2
*x^2+1))
```

Maxima [A] time = 1.27355, size = 169, normalized size = 1.22

$$-2\,b^2\left(\frac{x\,\mathrm{arsinh}\,(cx)}{c\sqrt{d}} - \frac{\sqrt{c^2x^2+1}}{c^2\sqrt{d}}\right) - \frac{2\,abx}{c\sqrt{d}} + \frac{\sqrt{c^2dx^2+d}\,b^2\,\mathrm{arsinh}\,(cx)^2}{c^2 d} + \frac{2\,\sqrt{c^2dx^2+d}\,ab\,\mathrm{arsinh}\,(cx)}{c^2 d} + \frac{\sqrt{c^2dx^2+d}\,a^2}{c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*b^2*(x*arcsinh(c*x)/(c*sqrt(d)) - sqrt(c^2*x^2 + 1)/(c^2*sqrt(d))) - 2*a
*b*x/(c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b^2*arcsinh(c*x)^2/(c^2*d) + 2*sqrt(
c^2*d*x^2 + d)*a*b*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2 + d)*a^2/(c^2*d)
```

Fricas [A] time = 3.20107, size = 385, normalized size = 2.79

$$\frac{\left(b^2c^2x^2+b^2\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(abc^2x^2-\sqrt{c^2x^2+1}b^2cx+ab\right)\sqrt{c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(\right.}{c^4dx^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*x^2 + b^2)*sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2
*(a*b*c^2*x^2 - sqrt(c^2*x^2 + 1)*b^2*c*x + a*b)*sqrt(c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 + 1)) + ((a^2 + 2*b^2)*c^2*x^2 - 2*sqrt(c^2*x^2 + 1)*a*b*c
*x + a^2 + 2*b^2)*sqrt(c^2*d*x^2 + d))/(c^4*d*x^2 + c^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{asinh}(cx)\right)^2}{\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arsinh}(cx)+a\right)^2x}{\sqrt{c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/sqrt(c^2*d*x^2 + d), x)
```

3.295 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{c^2dx^2+d}}$$

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.0947017, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {5677, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} = \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.130499, size = 62, normalized size = 1.32

$$\frac{\sqrt{c^2x^2+1}\sinh^{-1}(cx)\left(3a^2+3ab\sinh^{-1}(cx)+b^2\sinh^{-1}(cx)^2\right)}{3c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(3*a^2 + 3*a*b*ArcSinh[c*x] + b^2*ArcSinh[c
*x]^2))/(3*c*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.052, size = 120, normalized size = 2.6

$$a^2\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b^2\left(\mathrm{Arcsinh}\left(cx\right)\right)^3}{3\,cd}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}+\frac{ab\left(\mathrm{Arcsinh}\left(cx\right)\right)^2}{cd}\sqrt{d\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^3+a*b*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{\sqrt{c^2dx^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(c^2*d*x^2 + d
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{\sqrt{d\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(c^2*d*x^2 + d), x)
```

3.296 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=223

$$-\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+$$

```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[
d + c^2*d*x^2] - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^
ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b^2*Sqrt[1 + c^2
*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rubi [A] time = 0.34232, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5764, 5760, 4182, 2531, 2282, 6589}

$$-\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```


```
[Out] (-2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[
d + c^2*d*x^2] - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^
ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (2*b^2*Sqrt[1 + c^2
*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (2*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```


Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
```

```
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2\mathbf{csch}(x)\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\,\text{lo}\right.}{\sqrt{d+c^2d}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\mathbf{Li}_2}{\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.880241, size = 266, normalized size = 1.19

$$\frac{2ab\sqrt{c^2x^2+1}\left(\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)\left(\log\left(1-e^{-\sinh^{-1}(cx)}\right)-\log\left(e^{-\sinh^{-1}}\right.\right.\right.}{\sqrt{c^2dx^2+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/Sqrt[d]
 + (2*a*b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(Log[1 - E^(-ArcSinh[c*x])] - Log
[1 + E^(-ArcSinh[c*x])]) + PolyLog[2, -E^(-ArcSinh[c*x])] - PolyLog[2, E^(-
ArcSinh[c*x])]))/Sqrt[d + c^2*d*x^2] + (b^2*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]
^2*Log[1 - E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] +
 2*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] - 2*ArcSinh[c*x]*PolyLog[2,
E^(-ArcSinh[c*x])] + 2*PolyLog[3, -E^(-ArcSinh[c*x])] - 2*PolyLog[3, E^(-Ar
cSinh[c*x])]))/Sqrt[d + c^2*d*x^2]
```

Maple [B] time = 0.234, size = 564, normalized size = 2.5

$$-a^2\ln\left(\frac{1}{x}\left(2d+2\sqrt{d}\sqrt{c^2dx^2+d}\right)\right)\frac{1}{\sqrt{d}}-\frac{b^2\left(\operatorname{Arcsinh}\left(cx\right)\right)^2}{d}\sqrt{d\left(c^2x^2+1\right)}\ln\left(1+cx+\sqrt{c^2x^2+1}\right)\frac{1}{\sqrt{c^2x^2+1}}-2\frac{b^2\sqrt{d\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c^2*d*x^2+d)^(1/2))/x)-b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/2))-2*b^2*
(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,-c*x-(c^2*
x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,-c*
x-(c^2*x^2+1)^(1/2))+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(
c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2))-2*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2+1)^(1/2))-2*a*b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))-2
*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(
1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x
-(c^2*x^2+1)^(1/2))+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog
(2,c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2\right)}{c^2dx^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^3 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x\sqrt{d(c^2x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + d}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x), x)
```

3.297 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}+\frac{c\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}}+\frac{2bc\sqrt{c^2x}}{}$$

```
[Out] (c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2] - (Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x) + (2*b*c*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])*Log[1 - E^(-2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2] - (b^2*c*
Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2]
```

Rubi [A] time = 0.226659, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5723, 5659, 3716, 2190, 2279, 2391}

$$\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{dx}-\frac{c\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}}+\frac{2bc\sqrt{c^2x^2+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2]) - (Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(d*x) + (2*b*c*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x])*Log[1 - E^(2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2] + (b^2*
c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/Sqrt[d + c^2*d*x^2]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \sinh^{-1}(\right.}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} - \frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}}{\sqrt{}}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}{\sqrt{}}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}{\sqrt{}}$$

$$= -\frac{c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{dx} + \frac{2bc\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}{\sqrt{}}$$

Mathematica [A] time = 0.449452, size = 168, normalized size = 1.01

$$\frac{-b^2cx\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right) - a\left(ac^2x^2+a-2bcx\sqrt{c^2x^2+1}\log(cx)\right) - 2b\sinh^{-1}(cx)\left(ac^2x^2+a-bcx\sqrt{c^2x\right.}{x\sqrt{c^2dx^2+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (b^2*(-1 - c^2*x^2 + c*x*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - 2*b*ArcSinh[c*
x]*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c*x])]) -
 a*(a + a*c^2*x^2 - 2*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x]) - b^2*c*x*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(x*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.219, size = 526, normalized size = 3.2

$$-\frac{a^2}{dx}\sqrt{c^2dx^2+d} - \frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2xc^2}{d\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)} - \frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2c}{d}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}} - \frac{b^2\left(\text{Arcsinh}\,(c\right.}{xd\left(c^2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/d/x*(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*
x^2+1)/d*x*c^2-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/d
*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)/d/x+2*b^2*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c
+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)
^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1
-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*p
olylog(2,c*x+(c^2*x^2+1)^(1/2))*c-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(
1/2)/d*arcsinh(c*x)*c-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/
d*x*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/d/x+2*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^2dx^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^4 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x^2\sqrt{d(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\sqrt{c^2dx^2 + d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^2), x)
```

3.298 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=360

$$\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-$$

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[d + c^2*d*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (
b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[d + c^2*d*x^2] +
 (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[d + c^2*d*x^2] - (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (b^2*c^2*Sqrt[1 + c^2*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b^2*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rubi [A] time = 0.570366, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5747, 5764, 5760, 4182, 2531, 2282, 6589, 5661, 266, 63, 208}

$$\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-\frac{bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```


```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(x*Sqrt[d + c^2*d*x^2])) - (
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*d*x^2) + (c^2*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (
b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/Sqrt[d + c^2*d*x^2] +
 (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])
/Sqrt[d + c^2*d*x^2] - (b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLo
g[2, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] - (b^2*c^2*Sqrt[1 + c^2*x^2]*Poly
Log[3, -E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2] + (b^2*c^2*Sqrt[1 + c^2*x^2]*P
olyLog[3, E^ArcSinh[c*x]])/Sqrt[d + c^2*d*x^2]
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{1}{2}c^2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh}{x^2}}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2x^2}\right)\int \frac{(a+b\,\mathrm{si}}{x\sqrt{}}}{2\sqrt{d+c^2dx}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} - \frac{\left(c^2\sqrt{1+c^2x^2}\right)\mathrm{Subst}\,(}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{x\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2} + \frac{c^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{\sqrt{}}$$

Mathematica [A] time = 5.4416, size = 455, normalized size = 1.26

$$\frac{\frac{2abc^2d^2\left(c^2x^2+1\right)^{3/2}\left(-4\mathrm{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)+4\mathrm{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-4\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)+4\sinh^{-1}(cx)\log\left(e^{-\sinh^{-1}(cx)}+1\right)+2\tanh\left(\frac{1}{2}\right.\right.}{\left(c^2dx^2+d\right)^{3/2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((-4*a^2*Sqrt[d + c^2*d*x^2])/x^2 - 4*a^2*c^2*Sqrt[d]*Log[x] + 4*a^2*c^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (2*a*b*c^2*d^2*(1 + c^2*x^2)^(
3/2)*(-2*Coth[ArcSinh[c*x]/2] - ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 4*Arc
Sinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c
*x])] - 4*PolyLog[2, -E^(-ArcSinh[c*x])] + 4*PolyLog[2, E^(-ArcSinh[c*x])]
- ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d
*x^2)^(3/2) + (b^2*c^2*d^2*(1 + c^2*x^2)^(3/2)*(-4*ArcSinh[c*x]*Coth[ArcSin
h[c*x]/2] - ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 4*ArcSinh[c*x]^2*Log[1
```

```
- E^(-ArcSinh[c*x])] + 4*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Log[
Tanh[ArcSinh[c*x]/2]] - 8*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + 8*A
rcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 8*PolyLog[3, -E^(-ArcSinh[c*x])
] + 8*PolyLog[3, E^(-ArcSinh[c*x])] - ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2
 + 4*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(d + c^2*d*x^2)^(3/2))/(8*d)
```

Maple [B] time = 0.342, size = 901, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a^2/d/x^2*(c^2*d*x^2+d)^(1/2)+1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(c
^2*d*x^2+d)^(1/2))/x)-1/2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x
^2+1)*c^2-b^2*arcsinh(c*x)*(d*(c^2*x^2+1))^(1/2)/x/d/(c^2*x^2+1)^(1/2)*c-1/
2*b^2*arcsinh(c*x)^2*(d*(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)+1/2*b^2*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1+c*x+(c^2*x^2+1)^(1/
2))*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
*polylog(3,-c*x-(c^2*x^2+1)^(1/2))*c^2-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/d*arcsinh(c*x)^2*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^2-b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*polylog(2,c*x+(c^2*x^2+1)^(1/2
))*c^2+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(3,c*x+(c^2*x^2
+1)^(1/2))*c^2-2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arctanh(c*x+
(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/d/(c^2*x^2+1)*arcsinh(c*x)
*c^2-a*b*(d*(c^2*x^2+1))^(1/2)/x/d/(c^2*x^2+1)^(1/2)*c-a*b*arcsinh(c*x)*(d*
(c^2*x^2+1))^(1/2)/x^2/d/(c^2*x^2+1)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^2+a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^2-a*b*(d*(c^2*x
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c
^2-a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^
(1/2))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2\right)}{c^2dx^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^5 + d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\text{asinh}\,(cx))^2}{x^3\sqrt{d\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)^2}{\sqrt{c^2dx^2+d}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^3), x)
```

3.299 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=299

$$\frac{2b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2dx^2+d}}-\frac{2c^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2dx^2+d}}+\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx}-\frac{bc}{}$$

```
[Out] -(b^2*c^2*(1 + c^2*x^2))/(3*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/(3*x^2*Sqrt[d + c^2*d*x^2]) - (2*c^3*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1
- E^(-2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2]) + (2*b^2*c^3*Sqrt[1 + c^2*x
^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.427373, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5747, 5723, 5659, 3716, 2190, 2279, 2391, 5661, 264}

$$-\frac{2b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3\sqrt{c^2dx^2+d}}+\frac{2c^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{c^2dx^2+d}}+\frac{2c^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx}-\frac{bc}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -(b^2*c^2*(1 + c^2*x^2))/(3*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x]))/(3*x^2*Sqrt[d + c^2*d*x^2]) + (2*c^3*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x])^2)/(3*Sqrt[d + c^2*d*x^2]) - (Sqrt[d + c^2*d*x^2]*(
a + b*ArcSinh[c*x])^2)/(3*d*x^3) + (2*c^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*d*x) - (4*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1
- E^(2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^3*Sqrt[1 + c^2*x^
2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\sqrt{d+c^2dx^2}}\,dx = -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} - \frac{1}{3}\left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\sqrt{d+c^2dx^2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+\cdots}{\cdots}}{3\sqrt{d+c^2\cdots}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{2c^2\sqrt{d+c^2dx^2}\left(a+b\cdots\right.}{3dx}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} - \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3} + \frac{2c^2\cdots}{\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{\cdots}}{\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{\cdots}}{\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{\cdots}}{\cdots}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3x\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3x^2\sqrt{d+c^2dx^2}} + \frac{2c^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+c^2dx^2}} - \frac{\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 0.629257, size = 278, normalized size = 0.93

$$2b^2c^3x^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+2a^2c^4x^4+a^2c^2x^2-a^2-abcx\sqrt{c^2x^2+1}-4abc^3x^3\sqrt{c^2x^2+1}\log(cx)-b\,\text{s}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-a^2 + a^2*c^2*x^2 - b^2*c^2*x^2 + 2*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x*S
qrt[1 + c^2*x^2] + b^2*(-1 + c^2*x^2 + 2*c^4*x^4 - 2*c^3*x^3*Sqrt[1 + c^2*x
^2])*ArcSinh[c*x]^2 - b*ArcSinh[c*x]*(b*c*x*Sqrt[1 + c^2*x^2] - 2*a*(-1 + c
^2*x^2 + 2*c^4*x^4) + 4*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 - E^(-2*ArcSinh[c
*x])]) - 4*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[c*x] + 2*b^2*c^3*x^3*Sqrt[1 +
c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*x^3*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.309, size = 2147, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*ln(1-c*x-(c
^2*x^2+1)^(1/2))*c^3+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/
d*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4
+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3+8/3*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)*c^3+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*
c^2*x^2-1)/d*x^3*c^6-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/
d*x*c^4-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)
^(1/2)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^3*arcsinh(
c*x)-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*ln((c*x+(c^2*x^2+1)^
(1/2))^2-1)*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*c
^4+2/3*a^2*c^2/d/x*(c^2*d*x^2+d)^(1/2)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4
*x^4+2*c^2*x^2-1)/d*x*(c^2*x^2+1)*c^4+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c^4+4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^4*
x^4+2*c^2*x^2-1)/d*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-8/3*a*b*(d*(c^2*x^2+1
))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*x^2+1
))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^2*c*(c^2*x^2+1)^(1/2)-4/3*a*b*(d*(c^2*
```

```
x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*c^6+4*a*b*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*c^6-4/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*(c^2*x^2+1)*c^6-2
*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*arcsinh(c*x)^2*(c^
2*x^2+1)^(1/2)*c^5+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*
x*arcsinh(c*x)*(c^2*x^2+1)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c
^2*x^2-1)/d/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-4*a*b*(d*(c^2*x^2+1))^(1/2
)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5-1/3*a^2/
d/x^3*(c^2*d*x^2+d)^(1/2)-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d
*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^3+4/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*arcsinh(c*x)*c^8+2*b^2*(d*(c^2*x^2+1))^(1/2)
/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)^2*c^6-2/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*(c^2*x^2+1)*c^6+2/3*b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*arcsinh(c*x)*c^6+b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^2*(c^2*x^2+1)^(1/2)*c^5+1/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)^2*c^4-2/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x*arcsinh(c*x)*c^4-4/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*arcsinh(c*x)^2*c^2+1/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^4*x^4+2*c^2*x^2-1)/d/x^3*arcsinh(c*x)^2-4/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/d*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3+4/3*b^2*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*arcsinh(c*x)^2*c^3-1/3*b^2*(d*(c^2*x^2+1)
)^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*c^3*(c^2*x^2+1)^(1/2)-4/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3+2/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^5*c^8-1/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^4*x^4+2*c^2*x^2-1)/d*x^3*c^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^2dx^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^2*d*x^6 + d*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{asinh}(cx))^2}{x^4\sqrt{d(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**4*sqrt(d*(c**2*x**2 + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{c^2dx^2+d}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(sqrt(c^2*d*x^2 + d)*x^4), x)
```

3.300 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2}$$

```
[Out] (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (32*b^2*(1 + c
^2*x^2))/(9*c^6*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^6*d*
Sqrt[d + c^2*d*x^2]) + (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d*S
qrt[d + c^2*d*x^2]) - (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9
*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d +
c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^2) +
(4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^6*d*Sqrt[d + c
^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(c^6*d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.792561, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5751, 5758, 5717, 5653, 261, 5661, 266, 43, 5767, 5693, 4180, 2279, 2391}

$$-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^6d\sqrt{c^2dx^2+d}}+\frac{4x^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```


```
[Out] (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d*Sqrt[d + c^2*d*x^2]) - (32*b^2*(1 + c
^2*x^2))/(9*c^6*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2)^2)/(27*c^6*d*
Sqrt[d + c^2*d*x^2]) + (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d*S
qrt[d + c^2*d*x^2]) - (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9
*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d +
```

```
c^2*d*x^2]) - (8*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^2) +
(4*x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2) + (4*b*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^6*d*Sqrt[d + c
^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(c^6*d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^A
rcSinh[c*x]])/(c^6*d*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4\int\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}\cdots$$

$$=\frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^3d\sqrt{d+c^2dx^2}}-\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{4x^2\sqrt{d+c^2dx^2}\left(a+b\sinh\cdots\right.}{3c^4d^2}\cdots$$

$$=-\frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}}-\frac{2bx^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3d\sqrt{d+c^2dx^2}}-\frac{x^4\left(a+b\sinh\cdots\right.}{c^2d\sqrt{d+c\cdots}}\cdots$$

$$=\frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}}+\frac{2b^2\left(1+c^2x^2\right)}{c^6d\sqrt{d+c^2dx^2}}-\frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^5d\sqrt{d+c^2dx^2}}-\frac{2bx^3\sqrt{1+c^2x^2}\cdots}{9c^3d\sqrt{\cdots}}\cdots$$

$$=\frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}}+\frac{8b^2\left(1+c^2x^2\right)}{3c^6d\sqrt{d+c^2dx^2}}-\frac{2b^2\left(1+c^2x^2\right)^2}{9c^6d\sqrt{d+c^2dx^2}}+\frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d\sqrt{d+c^2dx^2}}-\cdots$$

$$=\frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}}-\frac{32b^2\left(1+c^2x^2\right)}{9c^6d\sqrt{d+c^2dx^2}}+\frac{2b^2\left(1+c^2x^2\right)^2}{27c^6d\sqrt{d+c^2dx^2}}+\frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d\sqrt{d+c^2dx^2}}-\cdots$$

$$=\frac{16abx\sqrt{1+c^2x^2}}{3c^5d\sqrt{d+c^2dx^2}}-\frac{32b^2\left(1+c^2x^2\right)}{9c^6d\sqrt{d+c^2dx^2}}+\frac{2b^2\left(1+c^2x^2\right)^2}{27c^6d\sqrt{d+c^2dx^2}}+\frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d\sqrt{d+c^2dx^2}}-\cdots$$

Mathematica [A] time = 0.605713, size = 427, normalized size = 0.83

$$\frac{-54ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+54ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+9a^2c^4x^4-36a^2c^2x^2-72a^2-6}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-72*a^2 - 94*b^2 - 36*a^2*c^2*x^2 - 92*b^2*c^2*x^2 + 9*a^2*c^4*x^4 + 2*b^2
*c^4*x^4 + 90*a*b*c*x*Sqrt[1 + c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 + c^2*x^2] -
 144*a*b*ArcSinh[c*x] - 72*a*b*c^2*x^2*ArcSinh[c*x] + 18*a*b*c^4*x^4*ArcSin
h[c*x] + 90*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - 6*b^2*c^3*x^3*Sqrt[1 +
 c^2*x^2]*ArcSinh[c*x] - 72*b^2*ArcSinh[c*x]^2 - 36*b^2*c^2*x^2*ArcSinh[c*x
]^2 + 9*b^2*c^4*x^4*ArcSinh[c*x]^2 + 108*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[
ArcSinh[c*x]/2]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^Ar
cSinh[c*x]] + (54*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh
[c*x]] - (54*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (54
*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(27*c^6*d*Sqrt[d +
c^2*d*x^2])
```

Maple [A] time = 0.369, size = 933, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/3*a^2*x^4/c^2/d/(c^2*d*x^2+d)^(1/2)-4/3*a^2/c^4*x^2/d/(c^2*d*x^2+d)^(1/2)
-8/3*a^2/c^6/d/(c^2*d*x^2+d)^(1/2)-94/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/
(c^2*x^2+1)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)^
2*x^4-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2+
2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*arcsinh(c*x)*ln(1-I
*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c
^6/d^2*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^
2/d^2/(c^2*x^2+1)*x^4-92/27*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*x
^2+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*ln(c*x+(c^2*x^2+
1)^(1/2)+I)-2/9*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*arcsinh
(c*x)*x^3+10/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(
c*x)*x+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^2*dilog(1-I*(c
```

```
*x+(c^2*x^2+1)^(1/2)))-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^2/(c^2*x^2+1)*ar
csinh(c*x)^2+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)
*x^4-2/9*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x^3-8/3*a*b*(d
*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2+10/3*a*b*(d*(c^2*x
^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*x-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/c^6
/d^2/(c^2*x^2+1)*arcsinh(c*x)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/c^6/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^6/d^2*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^5\,\mathrm{arsinh}\,(cx)^2+2\,abx^5\,\mathrm{arsinh}\,(cx)+a^2x^5\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^5}{(c^2 dx^2 + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^5/(c^2*d*x^2 + d)^(3/2), x)
```

3.301 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=400

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c^5d\sqrt{c^2dx^2+d}}+\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{2c^3d\sqrt{}}$$

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^4*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/(4*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(2*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x
])^2)/(c^2*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^2)/(c^5*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/(2*c^4*d^2) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 +
 E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.66494, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5751, 5758, 5677, 5675, 5661, 321, 215, 5767, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c^5d\sqrt{c^2dx^2+d}}+\frac{3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{2c^4d^2}-\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{bx^2\sqrt{c^2x^2+1}}{2c^3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 + c^2*x^2))/(4*c^4*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2
]*ArcSinh[c*x])/(4*c^5*d*Sqrt[d + c^2*d*x^2]) - (b*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(2*c^3*d*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x
])^2)/(c^2*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^2)/(c^5*d*Sqrt[d + c^2*d*x^2]) + (3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x])^2)/(2*c^4*d^2) - (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c^5*d
*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 +
 E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*
```

```
PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^5*d*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{2c^4d^2}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)}{2c^4d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{3x\sqrt{d+}}{}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{2c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{c^2d\sqrt{}}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{c^2d\sqrt{}}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{c^2d\sqrt{}}$$

$$= \frac{b^2x\left(1+c^2x^2\right)}{4c^4d\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c^5d\sqrt{d+c^2dx^2}} - \frac{bx^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c^3d\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{c^2d\sqrt{}}$$

Mathematica [A] time = 1.9296, size = 288, normalized size = 0.72

$$\frac{b^2\sqrt{d}\left(8\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\left(-4\sinh^{-1}(cx)^3+2\left(\sinh\left(2\sinh^{-1}(cx)\right)-4\right)\sinh^{-1}(cx)^2+\text{si}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a^2*c*Sqrt[d]*x*(3 + c^2*x^2) - 12*a^2*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*Sqrt[d]*(8*c*x*ArcSinh[c*x]^2 + 8*Sqrt[1
+ c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*(-4*ArcSinh
[c*x]^3 - 2*ArcSinh[c*x]*(Cosh[2*ArcSinh[c*x]] + 8*Log[1 + E^(-2*ArcSinh[c*
x])]) + 2*ArcSinh[c*x]^2*(-4 + Sinh[2*ArcSinh[c*x]]) + Sinh[2*ArcSinh[c*x]]
)) + 2*a*b*Sqrt[d]*(8*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(6*ArcSinh[c*x]^
2 + Cosh[2*ArcSinh[c*x]] + 4*Log[1 + c^2*x^2] - 2*ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.386, size = 816, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/2*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(c^2*d*x^2+d)^(1/2)-3
/2*a^2/c^4/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-2*
b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)*ln(1+(c*x+
(c^2*x^2+1)^(1/2))^2)+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*x^3
+1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^2-1/2*b^2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^2*arcsinh(c*x)^3+1/2*b^2*(d*(c^2*x^2
+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^3+3/2*b^2*(d*(c^2*x^2+1))^(
1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x-1/4*b^2*(d*(c^2*x^2+1))^(1/2)/c^5
/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/2*b^2*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(
c^2*x^2+1)^(1/2)*arcsinh(c*x)*x^2-3/2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/c^5/d^2*arcsinh(c*x)^2+a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)
*arcsinh(c*x)*x^3-1/2*a*b*(d*(c^2*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x
^2+3*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*arcsinh(c*x)*x+2*a*b*(d*
(c^2*x^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)-1/4*a*b*(d*(c^2*x
^2+1))^(1/2)/c^5/d^2/(c^2*x^2+1)^(1/2)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^5/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4\,\text{arsinh}\,(cx)^2+2\,abx^4\,\text{arsinh}\,(cx)+a^2x^4\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4\,(a+b\,\text{asinh}\,(cx))^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{2}x^{4}}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^(3/2), x)
```

3.302 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=383

$$\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2}-$$

```
[Out] (-4*a*b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*
x^2))/(c^4*d*Sqrt[d + c^2*d*x^2]) - (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[
d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^4*d^2)
- (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^4*
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*Poly
Log[2, I*E^ArcSinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.455493, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5751, 5717, 5653, 261, 5767, 5693, 4180, 2279, 2391}

$$\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^4d\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```



```
[Out] (-4*a*b*x*Sqrt[1 + c^2*x^2])/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*
x^2))/(c^4*d*Sqrt[d + c^2*d*x^2]) - (4*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
]))/(c^3*d*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[
d + c^2*d*x^2]) + (2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(c^4*d^2)
- (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^4*
d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Arc
Sinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*Poly
Log[2, I*E^ArcSinh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(\right.}{c^4d^2}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh\right.}{c^2d\sqrt{d+c^2}}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} - \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}\left(a+b\right.}{c^3d\sqrt{d+c^2d}}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}\left(a+b\right.}{c^3d\sqrt{d+c^2d}}$$

$$= -\frac{4abx\sqrt{1+c^2x^2}}{c^3d\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^4d\sqrt{d+c^2dx^2}} - \frac{4b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{c^3d\sqrt{d+c^2dx^2}} + \frac{2bx\sqrt{1+c^2x^2}\left(a+b\right.}{c^3d\sqrt{d+c^2d}}$$

Mathematica [A] time = 0.460229, size = 318, normalized size = 0.83

$$\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right) - 2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right) + a^2c^2x^2 + 2a^2 - 2abcx\sqrt{c^2x^2+1} +}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a^2 + 2*b^2 + a^2*c^2*x^2 + 2*b^2*c^2*x^2 - 2*a*b*c*x*Sqrt[1 + c^2*x^2]
+ 4*a*b*ArcSinh[c*x] + 2*a*b*c^2*x^2*ArcSinh[c*x] - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + 2*b^2*ArcSinh[c*x]^2 + b^2*c^2*x^2*ArcSinh[c*x]^2 - 4*a
*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*A
rcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog
[2, (-I)/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSi
nh[c*x]])/(c^4*d*Sqrt[d + c^2*d*x^2])
```

Maple [A] time = 0.319, size = 703, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2*x^2/c^2/d/(c^2*d*x^2+d)^(1/2)+2*a^2/d/c^4/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c
^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)^2*x^2-2*b^2*(d*(c^2*x^2+1
))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+2*b^2*(d*(c^2*x^2+1))^(1/
2)/c^2/d^2/(c^2*x^2+1)*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)*
arcsinh(c*x)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^2/(c^2*x^2+1)+2*I*b^2*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x
^2+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*arcsi
nh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^4/d^2*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^2*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*a
*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*arcsinh(c*x)*x^2-2*a*b*(d*(c^2
*x^2+1))^(1/2)/c^3/d^2/(c^2*x^2+1)^(1/2)*x+4*a*b*(d*(c^2*x^2+1))^(1/2)/c^4/
d^2/(c^2*x^2+1)*arcsinh(c*x)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)/c^4/d^2*ln(c*x+(c^2*x^2+1)^(1/2)+I)+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^
2+1)^(1/2)/c^4/d^2*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^3\operatorname{arsinh}(cx)^2 + 2\,abx^3\operatorname{arsinh}(cx) + a^2x^3\right)\sqrt{c^2dx^2+d}}{c^4d^2x^4 + 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{asinh}(cx))^2}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^3}{\left(c^2 dx^2 + d\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^3/(c^2*d*x^2 + d)^(3/2), x)
```

3.303 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c^3d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh\right.}{c^3d\sqrt{c^2dx^2+d}}$$

```
[Out] -((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) - (Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^3*d*Sqrt[d + c
^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^3*d
*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.383471, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5751, 5677, 5675, 5714, 3718, 2190, 2279, 2391}

$$\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c^3d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh\right.}{c^3d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```


```
[Out] -((x*(a + b*ArcSinh[c*x])^2)/(c^2*d*Sqrt[d + c^2*d*x^2])) - (Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])^2)/(c^3*d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^3)/(3*b*c^3*d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c^3*d*Sqrt[d + c
^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c^3*d
*Sqrt[d + c^2*d*x^2])
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
```

```
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx}{c^2d}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^3d\sqrt{d+c^2dx^2}}+$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d+c^2dx^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d+c^2dx^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d+c^2dx^2}}$$

$$=-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{c^3d\sqrt{d+c^2dx^2}}+\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 1.03183, size = 215, normalized size = 0.92

$$b^2d\left(\sinh^{-1}(cx)\left(\sqrt{c^2x^2+1}\left(\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+3\right)+6\log\left(e^{-2\sinh^{-1}(cx)}+1\right)\right)-3cx\sinh^{-1}(cx)\right)-3\sqrt{c^2x^2+1}\text{Poly}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*c*d*x - 3*a*b*d*(2*c*x*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(ArcSinh[c*
x]^2 + Log[1 + c^2*x^2])) + 3*a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*d*x + S
qrt[d]*Sqrt[d + c^2*d*x^2]] + b^2*d*(ArcSinh[c*x]*(-3*c*x*ArcSinh[c*x] + Sq
rt[1 + c^2*x^2]*(ArcSinh[c*x]*(3 + ArcSinh[c*x]) + 6*Log[1 + E^(-2*ArcSinh[
c*x])])) - 3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^3*d^
```

```
2*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.257, size = 478, normalized size = 2.1

$$-\frac{a^2x}{c^2d}\frac{1}{\sqrt{c^2dx^2+d}}+\frac{a^2}{c^2d}\ln\left(c^2dx\frac{1}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)\frac{1}{\sqrt{c^2d}}+\frac{b^2\left(\mathrm{Arcsinh}\left(cx\right)\right)^3}{3\,d^2c^3}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}-\frac{b^2\left(\mathrm{Arcsin}\right.}{c^2d^2\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2*x/c^2/d/(c^2*d*x^2+d)^(1/2)+a^2/c^2/d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*
x^2+d)^(1/2))/(c^2*d)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)
/c^3/d^2*arcsinh(c*x)^3-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^2/d^2/(c
^2*x^2+1)*x-b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c^3/d^2/(c^2*x^2+1)^(1
/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3
/d^2*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)/c^3/d^2*arcsinh(c*x)^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^3/d^2*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/c^2/d^2
/(c^2*x^2+1)*x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^3/d^2*ln(1+(
c*x+(c^2*x^2+1)^(1/2))^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2} x^{2} \operatorname{arsinh}\left(c x\right)^{2} + 2 \, a b x^{2} \operatorname{arsinh}\left(c x\right) + a^{2} x^{2}\right)\sqrt{c^{2} d x^{2} + d}}{c^{4} d^{2} x^{4} + 2 \, c^{2} d^{2} x^{2} + d^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{2}}{\left(d \left(c^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arsinh}\left(c x\right) + a\right)^{2} x^{2}}{\left(c^{2} d x^{2} + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(3/2), x)
```

3.304 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{4b\sqrt{c^2x^2+}}{}$$

```
[Out] -((a + b*ArcSinh[c*x])^2/(c^2*d*Sqrt[d + c^2*d*x^2])) + (4*b*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^2*d*Sqrt[d + c^2*d*x^2]
) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d*Sq
rt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.186987, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5717, 5693, 4180, 2279, 2391}

$$-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}+\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{c^2dx^2+d}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{c^2dx^2+d}}+\frac{4b\sqrt{c^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(c^2*d*Sqrt[d + c^2*d*x^2])) + (4*b*Sqrt[1 + c^2*x
^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c^2*d*Sqrt[d + c^2*d*x^2]
) - ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d*Sq
rt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*
x]])/(c^2*d*Sqrt[d + c^2*d*x^2])
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{1+c^2x^2}\,dx}{cd\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\mathbf{sech}(x)\,dx, x, \sinh^{-1}(cx)\right)}{c^2d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c^2\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{\left(2ib^2\sqrt{1+c\right.}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{c^2d\sqrt{d+c^2dx^2}} + \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{c^2d\sqrt{d+c^2dx^2}} - \frac{2ib^2\sqrt{1+c^2}}{c^2d}$$

Mathematica [A] time = 0.454932, size = 217, normalized size = 1.15

$$-\frac{2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-2ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+a^2-4ab\sqrt{c^2x^2+1}\tan^{-1}\left(\tanh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] -((a^2 + 2*a*b*ArcSinh[c*x] + b^2*ArcSinh[c*x]^2 - 4*a*b*Sqrt[1 + c^2*x^2]*
ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log
[1 - I/E^ArcSinh[c*x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I
/E^ArcSinh[c*x]] + (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*
x]] - (2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c^2*d*Sqrt
[d + c^2*d*x^2]))
```

Maple [B] time = 0.151, size = 446, normalized size = 2.4

$$-\frac{a^2}{c^2d}\frac{1}{\sqrt{c^2dx^2+d}}-\frac{b^2\left(\text{Arcsinh}\left(cx\right)\right)^2}{d^2c^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}-\frac{2\,ib^2\text{Arcsinh}\left(cx\right)}{d^2c^2}\sqrt{d\left(c^2x^2+1\right)}\ln\left(1+i\left(cx+\sqrt{c^2x^2+1}\right)\right)\frac{}{\sqrt{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2/c^2/d/(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1
)*arcsinh(c*x)^2-2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ar
csinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c^2/d^2*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))-2*I*b^
2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*dilog(1+I*(c*x+(c^2*x^2+1
)^(1/2)))+2*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*dilog(1-I
*(c*x+(c^2*x^2+1)^(1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^2/d^2/(c^2*x^2+1)*a
rcsinh(c*x)+2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^2*ln(c*x+
(c^2*x^2+1)^(1/2)+I)-2*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^
2*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{\sqrt{c^2dx^2+d}c^2d}+\int\frac{b^2x\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^2/(sqrt(c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2*x^2 +
 1))^2/(c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(c^2*d*
x^2 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2x\,\text{arsinh}\,(cx)^2+2\,abx\,\text{arsinh}\,(cx)+a^2x\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}}{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}x}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(3/2), x)
```

3.305 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\sinh^{-1}(cx)}\ldots\right.}{c\ldots}$$

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])^2)/(c*d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2]) -
(b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d
*x^2])
```

Rubi [A] time = 0.183529, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{cd\sqrt{c^2dx^2+d}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log\left(e^{2\ldots}\right.}{c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])^2)/(c*d*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2]) -
(b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*d*Sqrt[d + c^2*d
*x^2])
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \sinh^{-1}(cx)\right)}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,d\right.}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{cd\sqrt{d+c^2dx^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{cd\sqrt{d+c^2dx^2}} - \frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{cd\sqrt{d+c^2dx^2}}$$

Mathematica [A] time = 0.455164, size = 152, normalized size = 0.85

$$\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+a\left(acx-b\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)\right)+2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\log\left(e^{-2}\right.\right.}{cd\sqrt{c^2dx^2+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] (-(b^2*(-(c*x) + Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2) + 2*b*ArcSinh[c*x]*(a*c
*x - b*Sqrt[1 + c^2*x^2]*Log[1 + E^(-2*ArcSinh[c*x])]) + a*(a*c*x - b*Sqrt[
1 + c^2*x^2]*Log[1 + c^2*x^2]) + b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*Ar
cSinh[c*x])])/(c*d*Sqrt[d + c^2*d*x^2])
```

Maple [A] time = 0.17, size = 343, normalized size = 1.9

$$\frac{a^2x}{d}\frac{1}{\sqrt{c^2dx^2+d}}+\frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2x}{d^2\left(c^2x^2+1\right)}\sqrt{d\left(c^2x^2+1\right)}+\frac{b^2\left(\text{Arcsinh}\,(cx)\right)^2}{d^2c}\sqrt{d\left(c^2x^2+1\right)}\frac{1}{\sqrt{c^2x^2+1}}-2\,\frac{b^2\sqrt{d\left(c^2x^2+1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2*x/d/(c^2*d*x^2+d)^(1/2)+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/d^2/(c
^2*x^2+1)*x+b^2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2/c/d^2/(c^2*x^2+1)^(1/2
)-2*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)-b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1)
)^(1/2)/c/d^2*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d^2/(c^
2*x^2+1)*x-2*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^2*ln(1+(c*x+(c
^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc\sqrt{\frac{1}{c^4d}}\log\left(x^2+\frac{1}{c^2}\right)}{d}+b^2\int\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx+\frac{2\,abx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dx^2+d}d}+\frac{a^2x}{\sqrt{c^2dx^2+d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*b*c*sqrt(1/(c^4*d))*log(x^2 + 1/c^2)/d + b^2*integrate(log(c*x + sqrt(c^
2*x^2 + 1))^2/(c^2*d*x^2 + d)^(3/2), x) + 2*a*b*x*arcsinh(c*x)/(sqrt(c^2*d*
x^2 + d)*d) + a^2*x/(sqrt(c^2*d*x^2 + d)*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2\right)}{c^4d^2x^4+2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{asinh}\,(cx))^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(3/2), x)
```

3.306 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=412

$$-\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+$$

```
[Out] (a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcS
inh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*
x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d*Sqrt[d
 + c^2*d*x^2])
```

Rubi [A] time = 0.59453, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391}

$$-\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcSinh[c*x])^2/(d*Sqrt[d + c^2*d*x^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - (2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcS
inh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2
, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*
```

```
x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(d*Sqrt[d + c
^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d*Sqrt[d
 + c^2*d*x^2])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}\,dx}{d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{1+c^2x^2}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{1+c^2x^2}}\,dx}{d\sqrt{d+c^2dx^2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{sec}\right.}{d\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} + \frac{\sqrt{1+c^2x^2}\,\text{Subst}}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,(a+b}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,(a+b}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,(a+b}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\tan^{-1}\left(e^{\sinh^{-1}(cx)}\right)}{d\sqrt{d+c^2dx^2}} - \frac{2\sqrt{1+c^2x^2}\,(a+b}{}$$

Mathematica [A] time = 1.56519, size = 568, normalized size = 1.38

$$\frac{2abd\left(\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)+\sqrt{c^2x^2+1}\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*d + a^2*Sqrt[d]*Sqrt[d + c^2*d*x^2]*Log[c*x] - a^2*Sqrt[d]*Sqrt[d + c^
2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2*a*b*d*(ArcSinh[c*x] - 2*S
qrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + Sqrt[1 + c^2*x^2]*ArcSinh[c
*x]*Log[1 - E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(
-ArcSinh[c*x])] + Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*x])] - Sqrt[1
 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])]) + b^2*d*(ArcSinh[c*x]^2 + Sqrt[1
 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] + (2*I)*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (2*I)*Sqrt[1 + c^2*x^2]*ArcSi
nh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1
```

```
+ E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-Arc
Sinh[c*x])] + (2*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (2*
I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] - 2*Sqrt[1 + c^2*x^2]*Arc
Sinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] + 2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E
^(-ArcSinh[c*x])] - 2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])]))/(d^
2*Sqrt[d + c^2*d*x^2])
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{Arcsinh}\left(cx\right)\right)^{2}}{x}\left(c^{2}d\,x^{2}+d\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^{2}dx^{2}+d}\left(b^{2}\,\mathrm{arsinh}\left(cx\right)^{2}+2\,ab\,\mathrm{arsinh}\left(cx\right)+a^{2}\right)}{c^{4}d^{2}x^{5}+2\,c^{2}d^{2}x^{3}+d^{2}x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{x\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x), x)
```

3.307 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{2c\sqrt{c^2x^2}}{}$$

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSi
nh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (2*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (4*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d*x^2]) + (4*b*c*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c
^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*S
qrt[d + c^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x]
)])/(d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.4642, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5747, 5687, 5714, 3718, 2190, 2279, 2391, 5720, 5461, 4182}

$$\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{c^2dx^2+d}}-\frac{2c\sqrt{c^2x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(d*x*Sqrt[d + c^2*d*x^2])) - (2*c^2*x*(a + b*ArcSi
nh[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (2*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])^2)/(d*Sqrt[d + c^2*d*x^2]) - (4*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d*x^2]) + (4*b*c*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c
^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*S
qrt[d + c^2*d*x^2]) + (b^2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x]
)])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)}\,dx}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} + \frac{\left(4bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int(a+bx)\text{csch}(2\right.}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{}}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\sqrt{d+c^2dx^2}} - \frac{2c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{2c\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{d\sqrt{d+c^2dx^2}} - \frac{4bc\sqrt{}}{}$$

Mathematica [A] time = 0.897617, size = 296, normalized size = 0.97

$$-\frac{b^2cx\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+b^2cx\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+2a^2c^2x^2+a^2-2abcx\sqrt{c^2x^2+1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a^2 + 2*a^2*c^2*x^2 + 2*a*b*ArcSinh[c*x] + 4*a*b*c^2*x^2*ArcSinh[c*x] +
b^2*ArcSinh[c*x]^2 + 2*b^2*c^2*x^2*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x]^2 - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-
2*ArcSinh[c*x])] - 2*b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*A
rcSinh[c*x])] - 2*a*b*c*x*Sqrt[1 + c^2*x^2]*Log[c*x] - a*b*c*x*Sqrt[1 + c^2
*x^2]*Log[1 + c^2*x^2] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSin
h[c*x])] + b^2*c*x*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(d*x*
Sqrt[d + c^2*d*x^2]))
```

Maple [B] time = 0.227, size = 660, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2/d/x/(c^2*d*x^2+d)^(1/2)-2*a^2*c^2/d*x/(c^2*d*x^2+d)^(1/2)-2*b^2*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)/d^2*x*c^2-2*b^2*(d*(c^2*x^2+1))^
(1/2)*arcsinh(c*x)^2/(c^2*x^2+1)^(1/2)/d^2*c-b^2*(d*(c^2*x^2+1))^(1/2)*arcs
inh(c*x)^2/(c^2*x^2+1)/d^2/x+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
d^2*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/
(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^
2)*c+b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-(c*x+(c^2*x
^2+1)^(1/2))^2)*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh
(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c-4*a*b*(d*(c^2*x^2+1))^(1/2)/(
c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c-4*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x
)/(c^2*x^2+1)/d^2*x*c^2-2*a*b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1
)/d^2/x+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+
1)^(1/2))^4-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^4d^2x^6+2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{x^2\left(d\left(c^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**2*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\left(c^2dx^2 + d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^2), x)
```

3.308 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=573

$$\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}$$

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x*Sqrt[d + c^2*d*x^2])) -
 (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[
c*x])^2/(2*d*x^2*Sqrt[d + c^2*d*x^2]) + (4*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*c^2*Sqrt[
1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^
2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d*Sqrt[
d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*c
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
- (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]
])/(d*Sqrt[d + c^2*d*x^2]) - (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^Arc
Sinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[
3, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.925848, antiderivative size = 573, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5747, 5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391, 266, 63, 208}

$$\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}-\frac{3bc^2\sqrt{c^2x^2+1}\,\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```


```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(d*x*Sqrt[d + c^2*d*x^2])) -
 (3*c^2*(a + b*ArcSinh[c*x])^2)/(2*d*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[
c*x])^2/(2*d*x^2*Sqrt[d + c^2*d*x^2]) + (4*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*A
rcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*c^2*Sqrt[
```

```
1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSinh[c*x]])/(d*Sqrt[d + c^
2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d*Sqrt[
d + c^2*d*x^2]) + (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2
, -E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) - ((2*I)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + ((2*I)*b^2*c
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
- (3*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]
])/(d*Sqrt[d + c^2*d*x^2]) - (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^Arc
Sinh[c*x]])/(d*Sqrt[d + c^2*d*x^2]) + (3*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[
3, E^ArcSinh[c*x]])/(d*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
```

```
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} - \frac{1}{2}\left(3c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,d}{d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} - \frac{\left(3c^2\right)\int}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} - \frac{\left(3c^2\sqrt{\right.}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{}}{}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{dx\sqrt{d+c^2dx^2}} - \frac{3c^2\left(a+b\sinh^{-1}(cx)\right)^2}{2d\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\sqrt{d+c^2dx^2}} + \frac{4bc^2\sqrt{}}{}$$

Mathematica [A] time = 7.49549, size = 884, normalized size = 1.54

$$-\frac{3a^2\log(x)c^2}{2d^{3/2}} + \frac{3a^2\log\left(d+\sqrt{d\left(c^2x^2+1\right)}\sqrt{d}\right)c^2}{2d^{3/2}} + \frac{ab\left(-\sqrt{c^2x^2+1}\sinh^{-1}(cx)\mathrm{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - \sqrt{c^2x^2+1}\sinh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-a^2/(2*d^2*x^2) - (a^2*c^2)/(d^2*(1 + c^2*x^2))) -
(3*a^2*c^2*Log[x])/(2*d^(3/2)) + (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2
*x^2)]])/(2*d^(3/2)) + (a*b*c^2*(-8*ArcSinh[c*x] + 16*Sqrt[1 + c^2*x^2]*Arc
Tan[Tanh[ArcSinh[c*x]/2]] - 2*Sqrt[1 + c^2*x^2]*Coth[ArcSinh[c*x]/2] - Sqrt
```

```
[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*Ar
cSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*L
og[1 + E^(-ArcSinh[c*x])] - 12*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-ArcSinh[c*
x])] + 12*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 2*Sqrt[1 + c^2*x^2]*Tanh[ArcSinh[c
*x]/2]))/(4*d*Sqrt[d*(1 + c^2*x^2)]) + (b^2*c^2*(-8*ArcSinh[c*x]^2 - 4*Sqrt
[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSinh[c*x]/2] - Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1
- E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^Ar
cSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x
]] + 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 8*Sqr
t[1 + c^2*x^2]*Log[Tanh[ArcSinh[c*x]/2]] - 24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
]*PolyLog[2, -E^(-ArcSinh[c*x])] - (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)
/E^ArcSinh[c*x]] + (16*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] +
24*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 24*Sqrt[1
 + c^2*x^2]*PolyLog[3, -E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*PolyLog[3
, E^(-ArcSinh[c*x])] - Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2
]^2 + 4*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(8*d*Sqrt[d*(
1 + c^2*x^2)])
```

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \frac{(a + b\mathrm{Arcsinh}\,(cx))^2}{x^3} \left(c^2dx^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^4d^2x^7+2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{asinh}(cx))^2}{x^3\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(x**3*(d*(c**2*x**2 + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^3), x)
```

3.309 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=452

$$-\frac{b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{5b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3d\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\sqrt{c^2dx^2+d}}+\frac{8c^3\sqrt{}}{}$$

```
[Out] -(b^2*c^2*(1 + c^2*x^2))/(3*d*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/(3*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x
])^2/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x])^2)/(3*d*x*
Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2*d
*x^2]) + (8*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2
*d*x^2]) + (20*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*Ar
cSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (16*b*c^3*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (b
^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d
*x^2]) - (5*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d*
Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.840918, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5747, 5687, 5714, 3718, 2190, 2279, 2391, 5720, 5461, 4182, 264}

$$-\frac{b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{d\sqrt{c^2dx^2+d}}-\frac{5b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3d\sqrt{c^2dx^2+d}}+\frac{8c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\sqrt{c^2dx^2+d}}+\frac{8c^3\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```


```
[Out] -(b^2*c^2*(1 + c^2*x^2))/(3*d*x*Sqrt[d + c^2*d*x^2]) - (b*c*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/(3*d*x^2*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x
])^2/(3*d*x^3*Sqrt[d + c^2*d*x^2]) + (4*c^2*(a + b*ArcSinh[c*x])^2)/(3*d*x*
Sqrt[d + c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2*d
*x^2]) + (8*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d*Sqrt[d + c^2
*d*x^2]) + (20*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*Ar
cSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (16*b*c^3*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*d*Sqrt[d + c^2*d*x^2]) - (b
```

```
^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(d*Sqrt[d + c^2*d
*x^2]) - (5*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d*
Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} - \frac{1}{3}\left(4c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{3/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(1+c^2x^2\right)}}{3d\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh^{-1}(cx)\right)^2}{3dx\sqrt{d+c^2dx^2}} + \frac{1}{3}\left(8c^4\right.$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh\right.}{3dx\sqrt{d+c}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh\right.}{3dx\sqrt{d+c}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh\right.}{3dx\sqrt{d+c}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh\right.}{3dx\sqrt{d+c}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh\right.}{3dx\sqrt{d+c}}$$

$$= -\frac{b^2c^2\left(1+c^2x^2\right)}{3dx\sqrt{d+c^2dx^2}} - \frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3dx^2\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\sqrt{d+c^2dx^2}} + \frac{4c^2\left(a+b\sinh\right.}{3dx\sqrt{d+c}}$$

Mathematica [A] time = 0.848985, size = 438, normalized size = 0.97

$$\frac{3b^2c^3x^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+5b^2c^3x^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+8a^2c^4x^4+4a^2c^2x^2-a^2-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-a^2 + 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 - a*b*c*x
*Sqrt[1 + c^2*x^2] - 2*a*b*ArcSinh[c*x] + 8*a*b*c^2*x^2*ArcSinh[c*x] + 16*a
*b*c^4*x^4*ArcSinh[c*x] - b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b^2*ArcS
inh[c*x]^2 + 4*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSinh[c*x]^2 -
8*b^2*c^3*x^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - 10*b^2*c^3*x^3*Sqrt[1 + c^
```

```
2*x^2]*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 6*b^2*c^3*x^3*Sqrt[1 + c
^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 10*a*b*c^3*x^3*Sqrt[1 +
 c^2*x^2]*Log[c*x] - 3*a*b*c^3*x^3*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + 3*b
^2*c^3*x^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 5*b^2*c^3*x
^3*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*d*x^3*Sqrt[d + c^2
*d*x^2])
```

Maple [B] time = 0.314, size = 2609, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)^
2+4/3*a^2*c^2/d/x/(c^2*d*x^2+d)^(1/2)+16*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x
^4+7*c^2*x^2-1)/d^2*x*arcsinh(c*x)*c^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^
4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-8*a*b*(d*(c^2*x^2
+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*x
^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x^2*c*(c^2*x^2+1)^(1/2)-64/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*arcsinh(c*x)^2*(c^2*x^
2+1)^(1/2)*c^5+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*
arcsinh(c*x)*(c^2*x^2+1)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2
*x^2-1)/d^2/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-64/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*arcsinh(c*x)*(c^2*x^2+1)*c^8-32/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)*(c^2*x^2
+1)*c^6-64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2
*x^2+1)*c^8-32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*
(c^2*x^2+1)*c^6+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2
*x^3*arcsinh(c*x)*c^6+8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)
/d^2*x*(c^2*x^2+1)*c^4-128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2
-1)/d^2*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5-32/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*(c^2*x^2+1)*c^8+32*b^2*(d*(c^2*x^2+1))^(
1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*arcsinh(c*x)*c^8+64/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)^2*c^6-8/3*b^2*(d*(c^
2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*(c^2*x^2+1)*c^6+8*b^2*(d*(c
^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*arcsinh(c*x)*c^6+8/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^2*c^5*(c^2*x^2+1)^(1/2)+
32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*c^3+64/3*
a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10+32*a*b*(d*(c
^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8+8*a*b*(d*(c^2*x^2+1))^
```

```
(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^3*c^6-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*
c^4*x^4+7*c^2*x^2-1)/d^2*x*c^4-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c
^2*x^2-1)/d^2*c^3*(c^2*x^2+1)^(1/2)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^
4+7*c^2*x^2-1)/d^2/x^3*arcsinh(c*x)-2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)
^(1/2)/d^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-10/3*a*b*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/d^2*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c^3-1/3*a^2/d/x^3/
(c^2*d*x^2+d)^(1/2)+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*ar
csinh(c*x)^2*c^3-b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/d^2*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c^3-10/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/d^2*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c^3-1/3*b^2*(d*(
c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*c^3*(c^2*x^2+1)^(1/2)+32/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^7*c^10+40/3*b^2*(d*(c
^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x^5*c^8-7/3*b^2*(d*(c^2*x^2+1)
)^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*
c^4*x^4+7*c^2*x^2-1)/d^2/x*c^2+8/3*a^2*c^4/d*x/(c^2*d*x^2+d)^(1/2)+8*b^2*(d
*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh(c*x)^2*c^4-8/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2*x*arcsinh(c*x)*c^4-4*b^
2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/d^2/x*arcsinh(c*x)^2*c^2-10
/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1-c*x-(c
^2*x^2+1)^(1/2))*c^3+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*c^2*x^2-1)/
d^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^4*x^4+7*c^2*x^2-1)/d^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-2*b^2*(d*(c^2*x^
2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))
^2)*c^3-10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^2*arcsinh(c*x)*l
n(1+c*x+(c^2*x^2+1)^(1/2))*c^3+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^4*x^4+7*
c^2*x^2-1)/d^2*x^7*arcsinh(c*x)*c^10
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2ab\operatorname{arsinh}(cx)+a^2\right)}{c^4d^2x^8+2c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^4*d^2*x^8 + 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(3/2)*x^4), x)
```

3.310 $\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=512

$$\frac{11ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{11ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{16abx\sqrt{c^2x^2+1}}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{bx^3\left(a+b\right.}{3c^3d^2\sqrt{c^2x^2\cdots}}$$

```
[Out] b^2/(3*c^6*d^2*Sqrt[d + c^2*d*x^2]) - (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2))/(c^6*d^2*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^
2]) - (b*x^3*(a + b*ArcSinh[c*x]))/(3*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2]) + (11*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^5*d^2*Sqrt
[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(3*c^2*d*(d + c^2*d*x^2)^(3
/2)) - (4*x^2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (8*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*c^6*d^2*Sqrt[d + c^2
*d*x^2]) + (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]
])/(c^6*d^2*Sqrt[d + c^2*d*x^2]) - (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.881223, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5751, 5717, 5653, 261, 5767, 5693, 4180, 2279, 2391, 266, 43}

$$\frac{11ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{11ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^6d^2\sqrt{c^2dx^2+d}}-\frac{16abx\sqrt{c^2x^2+1}}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{bx^3\left(a+b\right.}{3c^3d^2\sqrt{c^2x^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```


```
[Out] b^2/(3*c^6*d^2*Sqrt[d + c^2*d*x^2]) - (16*a*b*x*Sqrt[1 + c^2*x^2])/(3*c^5*d
^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*(1 + c^2*x^2))/(c^6*d^2*Sqrt[d + c^2*d*x^2
]) - (16*b^2*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^
2]) - (b*x^3*(a + b*ArcSinh[c*x]))/(3*c^3*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2]) + (11*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^5*d^2*Sqrt
[d + c^2*d*x^2]) - (x^4*(a + b*ArcSinh[c*x])^2)/(3*c^2*d*(d + c^2*d*x^2)^(3
```

```
/2)) - (4*x^2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (8*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^6*d^3) - (22*b*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*c^6*d^2*Sqrt[d + c^2
*d*x^2]) + (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]
])/(c^6*d^2*Sqrt[d + c^2*d*x^2]) - (((11*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c^6*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5767

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(f^2*(m - 1))/(c^2*(m +
 2*p + 1)), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
```

```
- Dist[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(c*(m + 2*p + 1)*(1 + c
^2*x^2)^FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcS
inh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2
*d] && GtQ[n, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[m]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{4\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{4x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{8\int \frac{x(\ldots}{}}{}$$

$$= -\frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{11bx\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{x^4\left(a+b\sinh^{-1}(cx)\right)}{3c^2d\left(d+c^2dx^2\right)^{3/2}}$$

$$= -\frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{11b^2\left(1+c^2x^2\right)}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{bx^3\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{11bx\sqrt{1+c^2x^2}\,(a}{3c^5d^2\sqrt{d}}$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{10b^2\left(1+c^2x^2\right)}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} -$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} -$$

$$= \frac{b^2}{3c^6d^2\sqrt{d+c^2dx^2}} - \frac{16abx\sqrt{1+c^2x^2}}{3c^5d^2\sqrt{d+c^2dx^2}} + \frac{2b^2\left(1+c^2x^2\right)}{c^6d^2\sqrt{d+c^2dx^2}} - \frac{16b^2x\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} -$$

Mathematica [A] time = 1.73931, size = 333, normalized size = 0.65

$$\frac{\sqrt{c^2dx^2+d}\left(b^2\left(11i\left(c^2x^2+1\right)^{3/2}\left(\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)\right)+3\left(c^2x^2+1\right)^2\left(\sinh^{-1}(cx)^2+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d + c^2*d*x^2]*(a^2*(8 + 12*c^2*x^2 + 3*c^4*x^4) + a*b*(2*(8 + 12*c^2
*x^2 + 3*c^4*x^4)*ArcSinh[c*x] - Sqrt[1 + c^2*x^2]*(c*x*(5 + 6*c^2*x^2) + 2
```

```
2*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) + b^2*(c*x*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x] - 6*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x] - ArcSinh[c*x]^2 + 3
*(1 + c^2*x^2)^2*(2 + ArcSinh[c*x]^2) + (1 + c^2*x^2)*(1 + 6*ArcSinh[c*x]^2
) + (11*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(Log[1 - I/E^ArcSinh[c*x]] - Lo
g[1 + I/E^ArcSinh[c*x]]) + (11*I)*(1 + c^2*x^2)^(3/2)*(PolyLog[2, (-I)/E^Ar
cSinh[c*x]] - PolyLog[2, I/E^ArcSinh[c*x]]))))/(3*c^6*d^3*(1 + c^2*x^2)^2)
```

Maple [B] time = 0.342, size = 1040, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 4*a^2/c^4*x^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*
x^2+1)^2/c^6+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+1)-11/3*I*b^2*(d*
(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*
x^2+1)^(1/2)))+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*a
rcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)))+11/3*I*a*b*(d*(c^2*x^2+1))^(1/2
)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)-11/3*I*a*b*(d*(c^2*
x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)+2*a*b*(
d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*arcsinh(c*x)*x^2+4*a*b*(d*(c^2*x^2
+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)*x^2+8/3*a^2/c^6/d/(c^2*d*x^2+
d)^(3/2)+a^2*x^4/c^2/d/(c^2*d*x^2+d)^(3/2)-2*b^2*(d*(c^2*x^2+1))^(1/2)/c^5/
d^3/(c^2*x^2+1)^(1/2)*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2
*x^2+1)^(3/2)/c^5*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2
+1)^2/c^4*x^2+5/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsinh(c
*x)^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*x^2+b^2*(d*(c^2*x^2+1
))^(1/2)/c^6/d^3/(c^2*x^2+1)*arcsinh(c*x)^2+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/d
^3/(c^2*x^2+1)^(3/2)/c^5*x-2*a*b*(d*(c^2*x^2+1))^(1/2)/c^5/d^3/(c^2*x^2+1)^
(1/2)*x+11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog(1+
I*(c*x+(c^2*x^2+1)^(1/2)))+b^2*(d*(c^2*x^2+1))^(1/2)/c^4/d^3/(c^2*x^2+1)*ar
csinh(c*x)^2*x^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(
c*x)^2*x^2-11/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^6/d^3*dilog
(1-I*(c*x+(c^2*x^2+1)^(1/2)))+2*a*b*(d*(c^2*x^2+1))^(1/2)/c^6/d^3/(c^2*x^2+
1)*arcsinh(c*x)+10/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^6*arcsin
h(c*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^5\,\text{arsinh}\,(cx)^2 + 2\,abx^5\,\text{arsinh}\,(cx) + a^2x^5\right)\sqrt{c^2dx^2 + d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^5*arcsinh(c*x)^2 + 2*a*b*x^5*arcsinh(c*x) + a^2*x^5)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2x^5}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^5/(c^2*d*x^2 + d)^(5/2), x)
```

3.311 $\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{4b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}\right.}{3bc^5d^2\sqrt{c^2dx^2+d}}$$

```
[Out] -(b^2*x)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c
*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (b*x^2*(a + b*ArcSinh[c*x]))/(3*c^3*
d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(
3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d^2*Sqrt[d
 + c^2*d*x^2]) - (4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^5*d^2*Sq
rt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^5*d^
2*Sqrt[d + c^2*d*x^2]) + (8*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1
+ E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) + (4*b^2*Sqrt[1 + c^
2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.761133, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5751, 5677, 5675, 5714, 3718, 2190, 2279, 2391, 288, 215}

$$\frac{4b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c^5d^2\sqrt{c^2dx^2+d}}-\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}\right.}{3bc^5d^2\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c
*x])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) - (b*x^2*(a + b*ArcSinh[c*x]))/(3*c^3*
d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^3*(a + b*ArcSinh[c*x])^2)/(
3*c^2*d*(d + c^2*d*x^2)^(3/2)) - (x*(a + b*ArcSinh[c*x])^2)/(c^4*d^2*Sqrt[d
 + c^2*d*x^2]) - (4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^5*d^2*Sq
rt[d + c^2*d*x^2]) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c^5*d^
2*Sqrt[d + c^2*d*x^2]) + (8*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1
+ E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2]) + (4*b^2*Sqrt[1 + c^
```

```
2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^5*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}dx}{c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{(a+b}{\sqrt{}}}{}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{x\left(a+b\sin\right.}{c^4d^2\sqrt{d}}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{3c^2d\left(d\right.}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{3c^2d\left(d\right.}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{3c^2d\left(d\right.}$$

$$= -\frac{b^2x}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^5d^2\sqrt{d+c^2dx^2}} - \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^3\left(a+b\right.}{3c^2d\left(d\right.}$$

Mathematica [A] time = 1.24282, size = 359, normalized size = 0.9

$$\frac{-b^2\sqrt{d}\left(4\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+c^3x^3+4c^3x^3\sinh^{-1}(cx)^2-\left(c^2x^2+1\right)^{3/2}\sinh^{-1}(cx)^3-4\left(c^2x^2+1\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*c*Sqrt[d]*x*(3 + 4*c^2*x^2)) + a*b*Sqrt[d]*(Sqrt[1 + c^2*x^2] + 2*c*
x*ArcSinh[c*x] - 8*c*x*(1 + c^2*x^2)*ArcSinh[c*x] + (1 + c^2*x^2)^(3/2)*(3*
ArcSinh[c*x]^2 + 4*Log[1 + c^2*x^2])) + 3*a^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*
x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - b^2*Sqrt[d]*(c*x + c^3*x^3
- Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 3*c*x*ArcSinh[c*x]^2 + 4*c^3*x^3*ArcSinh
```

```
[c*x]^2 - 4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - (1 + c^2*x^2)^(3/2)*ArcSin
h[c*x]^3 - 8*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])]
+ 4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])]))/(3*c^5*d^(5/2)*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.378, size = 3705, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a^2/c^4/d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)-1/3
*a^2*x^3/c^2/d/(c^2*d*x^2+d)^(3/2)-17*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8
+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*x^5+84*b^2*(d*(c^2*x^2+1))^(1/2)
/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)^2*(c^
2*x^2+1)^(1/2)*x^4+8*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c
^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4+4*b^2*(d*(c^
2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*a
rcsinh(c*x)*(c^2*x^2+1)*x+28/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6
*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^3+220/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)
/c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2+13*b^2*(d*(c^2*x^2+1))^(1/2)/
(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2
*x^2+1)^(1/2)*x^2+32*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c
^4*x^4+71*c^2*x^2+16)*c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^6-64*a*b*(d*
(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^
3*arcsinh(c*x)*x^7+8*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c
^4*x^4+71*c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x^4+28/3*a*b*(d*(c^2*x^2+1))^
(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*(c^2*x^2+1)
*x^3-362/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*
c^2*x^2+16)/c^2/d^3*arcsinh(c*x)*x^3+13*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x
^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*x^2*(c^2*x^2+1)^(1/2)+4*a*
b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c
^4/d^3*(c^2*x^2+1)*x-32*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+11
8*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)*x+128/3*a*b*(d*(c^2*x^2+1))^(
1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*arcsinh(c*x)
*(c^2*x^2+1)^(1/2)+a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcs
inh(c*x)^2+64*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+
71*c^2*x^2+16)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6+168*a*b*(d*(c^2*x^2
+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*arcsinh(
```

c*x)*(c^2*x^2+1)^(1/2)*x^4+440/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c
^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^
2-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)-16/
3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+1
6)*c^2/d^3*x^7+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^
4*x^4+71*c^2*x^2+16)/d^3*(c^2*x^2+1)*x^5-152*a*b*(d*(c^2*x^2+1))^(1/2)/(24*
c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)*x^5-40/3*a*b
(d(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^
2/d^3*x^3-4*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71
*c^2*x^2+16)/c^4/d^3*x+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^
6+118*c^4*x^4+71*c^2*x^2+16)/c^5/d^3*(c^2*x^2+1)^(1/2)+8/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-8/3*b^2
(d(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^
2/d^3*(c^2*x^2+1)*x^3-40/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6
+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*arcsinh(c*x)*x^3+55/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^3/d^3*x^2*(c^2
*x^2+1)^(1/2)-16*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x
^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)^2*x-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(2
4*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*(c^2*x^2+1)*x+64/3*
b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)
/c^5/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)-4*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c
^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^4/d^3*arcsinh(c*x)*x+16/3*b^
2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c
^5/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^
8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^
5+8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)*ln(1
+(c*x+(c^2*x^2+1)^(1/2))^2)-32*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6
*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*arcsinh(c*x)^2*x^7-16/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*ar
csinh(c*x)*x^7+8*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x
^4+71*c^2*x^2+16)*c/d^3*(c^2*x^2+1)^(1/2)*x^6+21*b^2*(d*(c^2*x^2+1))^(1/2)/
(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c/d^3*(c^2*x^2+1)^(1/2)*x
^4-181/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^
2*x^2+16)/c^2/d^3*arcsinh(c*x)^2*x^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2
+1)^(1/2)/c^5/d^3*arcsinh(c*x)^3-76*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+8
7*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*arcsinh(c*x)^2*x^5-4/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*(c^2*x
^2+1)*x^5-44/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4
+71*c^2*x^2+16)/d^3*arcsinh(c*x)*x^5-20/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8
*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)*c^2/d^3*x^7-43/3*b^2*(d*(c^2*x^2
+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/c^2/d^3*x^3-4*
b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)
/c^4/d^3*x+16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^
4+71*c^2*x^2+16)/c^5/d^3*(c^2*x^2+1)^(1/2)-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c
^2*x^2+1)^(1/2)/c^5/d^3*arcsinh(c*x)^2+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x

```
^2+1)^(1/2)/c^5/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)-44/3*a*b*(d*(c^2*
x^2+1))^(1/2)/(24*c^8*x^8+87*c^6*x^6+118*c^4*x^4+71*c^2*x^2+16)/d^3*x^5-a^2
/c^4/d^2*x/(c^2*d*x^2+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2}x^{4}\operatorname{arsinh}(cx)^{2}+2\,abx^{4}\operatorname{arsinh}(cx)+a^{2}x^{4}\right)\sqrt{c^{2}dx^{2}+d}}{c^{6}d^{3}x^{6}+3\,c^{4}d^{3}x^{4}+3\,c^{2}d^{3}x^{2}+d^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^4}{(c^2 d x^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^4/(c^2*d*x^2 + d)^(5/2), x)
```

3.312 $\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{5ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{2\left(a\right.}{3c}$$

```
[Out] -b^2/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (b*x*(a + b*ArcSinh[c*x]))/(3*c^3*d^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(3*
c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d
 + c^2*d*x^2]) + (10*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (((5*I)/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (((5*I)
/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c
^2*d*x^2])
```

Rubi [A] time = 0.504904, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5751, 5717, 5693, 4180, 2279, 2391, 261}

$$-\frac{5ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}+\frac{5ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^4d^2\sqrt{c^2dx^2+d}}-\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{2\left(a\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```


```
[Out] -b^2/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (b*x*(a + b*ArcSinh[c*x]))/(3*c^3*d^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (x^2*(a + b*ArcSinh[c*x])^2)/(3*
c^2*d*(d + c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSinh[c*x])^2)/(3*c^4*d^2*Sqrt[d
 + c^2*d*x^2]) + (10*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(3*c^4*d^2*Sqrt[d + c^2*d*x^2]) - (((5*I)/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c^2*d*x^2]) + (((5*I)
/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^4*d^2*Sqrt[d + c
^2*d*x^2])
```


Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3c^2d} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d+c^2dx^2}} + \frac{\left(b\sqrt{1+\right.}{3}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh\right.}{3c^4d^2\sqrt{d+}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh\right.}{3c^4d^2\sqrt{d+}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh\right.}{3c^4d^2\sqrt{d+}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d+c^2dx^2}} - \frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3c^3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}} - \frac{2\left(a+b\sinh\right.}{3c^4d^2\sqrt{d+}}$$

Mathematica [A] time = 1.09377, size = 301, normalized size = 0.98

$$\frac{-b^2\left(5i\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-5i\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+c^2x^2+3c^2x^2\sinh^{-1}(cx)^2+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*(2 + 3*c^2*x^2)) + a*b*(-2*(2 + 3*c^2*x^2)*ArcSinh[c*x] + Sqrt[1 + c
^2*x^2]*(-(c*x) + 10*(1 + c^2*x^2)*ArcTan[Tanh[ArcSinh[c*x]/2]])) - b^2*(1
```

```
+ c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 2*ArcSinh[c*x]^2 + 3*c^2*x
^2*ArcSinh[c*x]^2 + (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^ArcS
inh[c*x]] - (5*I)*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] + (5*I)*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (5*I)*(1 +
c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^4*d^2*(1 + c^2*x^2)*Sqrt
[d + c^2*d*x^2])
```

Maple [B] time = 0.286, size = 705, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a^2*x^2/c^2/d/(c^2*d*x^2+d)^(3/2)-2/3*a^2/d/c^4/(c^2*d*x^2+d)^(3/2)-b^2*(d
*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)^2*x^2-1/3*b^2*(d*(c^
2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*arcsinh(c*x)*x-1/3*b^2*(d*(c^2*x^
2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*x^2-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^
2*x^2+1)^2/c^4*arcsinh(c*x)^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)
^2/c^4-5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1+I*
(c*x+(c^2*x^2+1)^(1/2)))+5/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/
c^4/d^3*arcsinh(c*x)*ln(1-I*(c*x+(c^2*x^2+1)^(1/2)))+5/3*I*a*b*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)+5/3*I*b^2*(
d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(
1/2)))-2*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)*x^2-1
/3*a*b*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c^3*x-4/3*a*b*(d*(c^2*x^
2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^4*arcsinh(c*x)-5/3*I*a*b*(d*(c^2*x^2+1))^(1
/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)-5/3*I*b^2*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^4/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+
1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\,abc\left(\frac{x}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}-\frac{5\arctan\left(cx\right)}{c^5d^{\frac{5}{2}}}\right)-\frac{2}{3}\,ab\left(\frac{3\,x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\frac{2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)\mathrm{arsinh}\left(cx\right)-\frac{1}{3}\,a^2\left(\frac{3\,x^2}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(x/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) - 5*arctan(c*x)/(c^5*d^(5/2))
) - 2/3*a*b*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((c^2*d*x^2 + d)^(3/2)
*c^4*d))*arcsinh(c*x) - 1/3*a^2*(3*x^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + 2/((
c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2*integrate(x^3*log(c*x + sqrt(c^2*x^2 + 1
))^2/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^3\,\text{arsinh}\,(cx)^2 + 2\,abx^3\,\text{arsinh}\,(cx) + a^2x^3\right)\sqrt{c^2dx^2 + d}}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,(a + b\,\text{asinh}\,(cx))^2}{\left(d\left(c^2x^2 + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^3}{(c^2 d x^2 + d)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^3/(c^2*d*x^2 + d)^(5/2), x)
```

3.313 $\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=312

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c^3d^2\sqrt{c^2dx^2+d}}+\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log}{}$$

```
[Out] (b^2*x)/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) + (b*x^2*(a + b*ArcSinh[c*x]))/(3*c*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[c*x])^2)/(3*d
*(d + c^2*d*x^2)^(3/2)) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^3
*d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c
^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.364395, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {5723, 5751, 5714, 3718, 2190, 2279, 2391, 288, 215}

$$-\frac{b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c^3d^2\sqrt{c^2dx^2+d}}+\frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{3c^3d^2\sqrt{c^2dx^2+d}}-\frac{2b\sqrt{c^2x^2+1}\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```


```
[Out] (b^2*x)/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*
x])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) + (b*x^2*(a + b*ArcSinh[c*x]))/(3*c*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x^3*(a + b*ArcSinh[c*x])^2)/(3*d
*(d + c^2*d*x^2)^(3/2)) + (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c^3
*d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2]) - (b^2*Sqrt[1 + c
^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c^3*d^2*Sqrt[d + c^2*d*x^2])
```


Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}dx = \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(b^2\sqrt{1+c^2x^2}\right)\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}}dx}{3d^2\sqrt{d+c^2dx^2}} - \cdots$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(2b\sqrt{1+c^2x^2}\right.}{}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sin\right.}{3d\left(d+c\right.}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sin\right.}{3d\left(d+c\right.}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sin\right.}{3d\left(d+c\right.}$$

$$= \frac{b^2x}{3c^2d^2\sqrt{d+c^2dx^2}} - \frac{b^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{3c^3d^2\sqrt{d+c^2dx^2}} + \frac{bx^2\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x^3\left(a+b\sin\right.}{3d\left(d+c\right.}$$

Mathematica [A] time = 0.96228, size = 280, normalized size = 0.9

$$\frac{b^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+a^2c^3x^3-ab\sqrt{c^2x^2+1}-abc^2x^2\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)-ab\sqrt{c^2x^2+1}\log\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (b^2*c*x + a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 + c^2*x^2] - b^2*(-(c^3*x
^3) + Sqrt[1 + c^2*x^2] + c^2*x^2*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 - b*Arc
Sinh[c*x]*(-2*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b*(1 + c^2*x^2)^(3/2)*Log
[1 + E^(-2*ArcSinh[c*x])]) - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - a*b*c
^2*x^2*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + b^2*(1 + c^2*x^2)^(3/2)*PolyLog
```

```
[2, -E^(-2*ArcSinh[c*x])])/(3*c^3*d^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.281, size = 3112, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a^2/c^2/d^2*x/(c^2*d*x^2+d)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+
9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*
x^6+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1
)*c^4/d^3*arcsinh(c*x)*x^7-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x
^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5+2*a*b*(d*(c^2*x^2+1))^(1
/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)*x^5-a
*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3
*(c^2*x^2+1)^(1/2)*x^4-a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^
4*x^4+5*c^2*x^2+1)/c/d^3*x^2*(c^2*x^2+1)^(1/2)-b^2*(d*(c^2*x^2+1))^(1/2)/(3
*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(
1/2)*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^
2*x^2+1)*c^2/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(
3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^3*arcsinh(c*x)^2*(c^2*x^2+1
)^(1/2)*x^4-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2
*x^2+1)*c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-4/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*arcsinh(c*x)^2*(c^2*
x^2+1)^(1/2)*x^2-2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*
x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2/3*a*b*(d*(c^2*x^2
+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5-1/3*a*b
*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^
2*x^2+1)*x^3+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+
5*c^2*x^2+1)/d^3*arcsinh(c*x)*x^3-1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+
9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)-2/3*a*b/(c^2*x^
2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c^3/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-4*
a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c/d^
3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-8/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x
^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x
^2-2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
*c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^6-1/3*a^2/c^2*x/d/(c^2*d*x^2+d)^(
3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^
2+1)/d^3*x^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+
5*c^2*x^2+1)*c^2/d^3*(c^2*x^2+1)*x^5-2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+
```

```
1))^(1/2)/c^3/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)-1/3*b^2*(d*(
c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(
c*x)*(c^2*x^2+1)*x^3-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*
c^4*x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)-4/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c/d^3*x^2*(c^2*
x^2+1)^(1/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+
5*c^2*x^2+1)/c^2/d^3*(c^2*x^2+1)*x-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8
+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)
+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)
*c^2/d^3*arcsinh(c*x)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+
10*c^4*x^4+5*c^2*x^2+1)*c/d^3*(c^2*x^2+1)^(1/2)*x^4+b^2*(d*(c^2*x^2+1))^(1/
2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*arcsinh(c*x)^2*x^7+
1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*
c^4/d^3*arcsinh(c*x)*x^7-b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*
c^4*x^4+5*c^2*x^2+1)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^6+b^2*(d*(c^2*x^2+1))^(1/2
)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*arcsinh(c*x)^2*x^5+4
/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c^3/d^3*arcsinh(c*x)+1/3*a*b
*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3
*x^7-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^
2+1)/c^3/d^3*(c^2*x^2+1)^(1/2)+1/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c
^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*x^3+2/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^
2+1))^(1/2)/c^3/d^3*arcsinh(c*x)^2+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8
+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*arcsinh(c*x)^2*x^3+2/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)/d^3*(c^2*x^2+1)*x
^3+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+
1)/d^3*arcsinh(c*x)*x^3-1/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c^3
/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3
*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^4/d^3*x^7+b^2*(d*(c^2*x^2+1))^
(1/2)/(3*c^8*x^8+9*c^6*x^6+10*c^4*x^4+5*c^2*x^2+1)*c^2/d^3*x^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\,abc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2+c^4d^{\frac{5}{2}}}+\frac{\log\left(c^2x^2+1\right)}{c^4d^{\frac{5}{2}}}\right)+\frac{2}{3}\,ab\left(\frac{x}{\sqrt{c^2dx^2+d}c^2d^2}-\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\operatorname{arsinh}(cx)+\frac{1}{3}\,a^2\left(\frac{x}{\sqrt{c^2dx^2+dc^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 + c^4*d^(5/2)) + log(c^2*x^2 + 1)/(c^4*d^(5/
```

```
2))) + 2/3*a*b*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d*x^2 + d)^(3/2)*
c^2*d))*arcsinh(c*x) + 1/3*a^2*(x/(sqrt(c^2*d*x^2 + d)*c^2*d^2) - x/((c^2*d
*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2*log(c*x + sqrt(c^2*x^2 + 1))^2/
(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^2\,\text{arsinh}\,(cx)^2+2\,abx^2\,\text{arsinh}\,(cx)+a^2x^2\right)\sqrt{c^2dx^2+d}}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2)*sqrt(c
^2*d*x^2 + d)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(a+b\,\text{asinh}\,(cx))^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\text{arsinh}\,(cx)+a)^2x^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2*x^2/(c^2*d*x^2 + d)^(5/2), x)
```

3.314 $\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x}}{}$$

```
[Out] b^2/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^2*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*c^2*d*(d
+ c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^
ArcSinh[c*x]])/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - ((I/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d*x^2]) + ((I/3)*
b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d
*x^2])
```

Rubi [A] time = 0.21746, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5717, 5690, 5693, 4180, 2279, 2391, 261}

$$-\frac{ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{ib^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c^2d^2\sqrt{c^2dx^2+d}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```


```
[Out] b^2/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) + (b*x*(a + b*ArcSinh[c*x]))/(3*c*d^2*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*c^2*d*(d
+ c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^
ArcSinh[c*x]])/(3*c^2*d^2*Sqrt[d + c^2*d*x^2]) - ((I/3)*b^2*Sqrt[1 + c^2*x^
2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d*x^2]) + ((I/3)*
b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(c^2*d^2*Sqrt[d + c^2*d
*x^2])
```


Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\left(2b\sqrt{1+c^2x^2}\right)\int\frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d+c^2dx^2}}$$

$$=\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}-\frac{\left(b^2\sqrt{1+c^2x^2}\right)\int\frac{x}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3d^2\sqrt{d+c^2dx^2}}+\frac{\left(b\sqrt{1\cdots\right.}{\cdots}$$

$$=\frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{\left(b\sqrt{1+c^2x^2}\right)\mathrm{Sub}\cdots}{\cdots}$$

$$=\frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{1+c^2x^2}\left(a+b\cdots\right.}{3c\cdots}$$

$$=\frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{1+c^2x^2}\left(a+b\cdots\right.}{3c\cdots}$$

$$=\frac{b^2}{3c^2d^2\sqrt{d+c^2dx^2}}+\frac{bx\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3c^2d\left(d+c^2dx^2\right)^{3/2}}+\frac{2b\sqrt{1+c^2x^2}\left(a+b\cdots\right.}{3c\cdots}$$

Mathematica [A] time = 1.02615, size = 254, normalized size = 0.94

$$\frac{b^2\left(-i\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+i\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+c^2x^2+cx\sqrt{c^2x^2+1}\sinh^{-1}(c\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (-a^2 + a*b*(-2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*(c*x + 2*(1 + c^2*x^2)*Arc
Tan[Tanh[ArcSinh[c*x]/2]])) + b^2*(1 + c^2*x^2 + c*x*Sqrt[1 + c^2*x^2]*ArcS
inh[c*x] - ArcSinh[c*x]^2 - I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 - I/E^
ArcSinh[c*x]] + I*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]
] - I*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]] + I*(1 + c^2*x^2)
```

```
^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]]))/(3*c^2*d*(d + c^2*d*x^2)^(3/2))
```

Maple [B] time = 0.184, size = 591, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/3*a^2/c^2/d/(c^2*d*x^2+d)^(3/2)+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x
^2+1)^(3/2)/c*arcsinh(c*x)*x+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^
2*x^2-1/3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)^2+1/
3*b^2*(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^2/c^2-1/3*I*b^2*(d*(c^2*x^2+1))
^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*ln(1+I*(c*x+(c^2*x^2+1)^(1/2)
))+1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*arcsinh(c*x)*l
n(1-I*(c*x+(c^2*x^2+1)^(1/2)))-1/3*I*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^
(1/2)/c^2/d^3*dilog(1+I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*I*b^2*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*dilog(1-I*(c*x+(c^2*x^2+1)^(1/2)))+1/3*a*b*
(d*(c^2*x^2+1))^(1/2)/d^3/(c^2*x^2+1)^(3/2)/c*x-2/3*a*b*(d*(c^2*x^2+1))^(1/
2)/d^3/(c^2*x^2+1)^2/c^2*arcsinh(c*x)+1/3*I*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*
x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)+I)-1/3*I*a*b*(d*(c^2*x^2+1))^
(1/2)/(c^2*x^2+1)^(1/2)/c^2/d^3*ln(c*x+(c^2*x^2+1)^(1/2)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{3\left(c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\int\frac{b^2x\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a^2/((c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c^2
*x^2 + 1))^2/(c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c^2*x^2 + 1))/(
c^2*d*x^2 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2x\,\text{arsinh}\,(cx)^2+2\,abx\,\text{arsinh}\,(cx)+a^2x\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(a+b\,\text{asinh}\,(cx))^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx)+a)^2x}{\left(c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x/(c^2*d*x^2 + d)^(5/2), x)
```

3.315 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{2b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2x^2+1}\left(a+b\,\text{si}\right.}{3cd^2\sqrt{c^2dx^2}}$$

```
[Out] -(b^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*d^2*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*d*(d +
 c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2
]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*d^2*Sqrt[d + c^2*d*x
^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x
])])/(3*c*d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E
^(2*ArcSinh[c*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.294435, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191}

$$-\frac{2b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3cd^2\sqrt{c^2dx^2+d}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\frac{2\sqrt{c^2x^2+1}\left(a+b\,\text{si}\right.}{3cd^2\sqrt{c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```



```
[Out] -(b^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b*(a + b*ArcSinh[c*x]))/(3*c*d^2*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (x*(a + b*ArcSinh[c*x])^2)/(3*d*(d +
 c^2*d*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2
]) + (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*c*d^2*Sqrt[d + c^2*d*x
^2]) - (4*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x
])])/(3*c*d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, -E
^(2*ArcSinh[c*x])])/(3*c*d^2*Sqrt[d + c^2*d*x^2])
```


Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
```

```
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx}{3d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{\left(b^2\sqrt{1+c^2 \cdots\right.}{3d^2 \cdots}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(\cdots\right.}{3d^2\sqrt{d+c^2dx \cdots}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(\cdots\right.}{3d^2\sqrt{d+c^2dx \cdots}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(\cdots\right.}{3d^2\sqrt{d+c^2dx \cdots}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(\cdots\right.}{3d^2\sqrt{d+c^2dx \cdots}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d+c^2dx^2}} + \frac{b\left(a+b\sinh^{-1}(cx)\right)}{3cd^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{2x\left(a+b\sinh^{-1}(\cdots\right.}{3d^2\sqrt{d+c^2dx \cdots}}$$

Mathematica [A] time = 1.3074, size = 236, normalized size = 0.81

$$-b^2\left(-2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+c^3x^3-2cx\left(c^2x^2+1\right)\sinh^{-1}(cx)^2-\sqrt{c^2x^2+1}\sinh^{-1}(cx)+2\left(c^2x^2+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] (a^2*c*x*(3 + 2*c^2*x^2) + a*b*((6*c*x + 4*c^3*x^3)*ArcSinh[c*x] + Sqrt[1 +
 c^2*x^2]*(1 - 2*(1 + c^2*x^2)*Log[1 + c^2*x^2])) - b^2*(c*x + c^3*x^3 - Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] - c*x*ArcSinh[c*x]^2 - 2*c*x*(1 + c^2*x^2)*Arc
Sinh[c*x]^2 + 2*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(ArcSinh[c*x] + 2*Log[1 +
E^(-2*ArcSinh[c*x])]) - 2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x
])]))/(3*d^2*(c + c^3*x^2)*Sqrt[d + c^2*d*x^2])
```

Maple [B] time = 0.194, size = 2729, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a^2*x/d/(c^2*d*x^2+d)^(3/2)+2/3*a^2/d^2*x/(c^2*d*x^2+d)^(1/2)+4/3*b^2*(
d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*
x)*(c^2*x^2+1)*x^5-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2
*x^2+4)*c^3/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^4+10/3*a*b*(d*(c^2*x^2+1
))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*(c^2*x^2+1)*x^3+34/3*a
*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsin
h(c*x)*x^3+a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/
d^3*x^2*(c^2*x^2+1)^(1/2)-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*
x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4*a*b*(d*(c^2*x^2+1)
)^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+4/3*a*
b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^
2+1)*x^5-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)
*c/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*x^2+10/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)*(c^2*x^2+1)*x^3+b^
2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c
*x)*(c^2*x^2+1)^(1/2)*x^2+8/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c
/d^3*arcsinh(c*x)-4/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^
```

```
2*x^2+4)*c^6/d^3*x^7-14/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+1
1*c^2*x^2+4)*c^4/d^3*x^5-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x
^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)*x^5+b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*
x^6+10*c^4*x^4+11*c^2*x^2+4)*c^3/d^3*(c^2*x^2+1)^(1/2)*x^4+17/3*b^2*(d*(c^2
*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*arcsinh(c*x)^2*x
^3+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^
3*(c^2*x^2+1)*x^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c
^2*x^2+4)*c^2/d^3*arcsinh(c*x)*x^3+7/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*x^2*(c^2*x^2+1)^(1/2)-8/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*arcsinh(c*x)^2*(c^2*x^2+
1)^(1/2)+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/
c/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6
+10*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*(c^2*x^2+1)*x-4/3*b^2/(c^2*x^2+1
)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2
))^2)-16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^
2/d^3*x^3+2*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d
^3*(c^2*x^2+1)*x+8*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x
^2+4)/d^3*arcsinh(c*x)*x-4*a*b*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+
11*c^2*x^2+4)*c^3/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*x^4-28/3*a*b*(d*(c^2*x
^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c/d^3*arcsinh(c*x)*(c^2*x^
2+1)^(1/2)*x^2+4*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2
+4)/d^3*arcsinh(c*x)^2*x+2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^
4+11*c^2*x^2+4)/d^3*(c^2*x^2+1)*x-2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10
*c^4*x^4+11*c^2*x^2+4)/d^3*arcsinh(c*x)*x-2/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^6/d^3*x^7-2*a*b*(d*(c^2*x^2+1))^(1/2)/(3
*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x-3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*
x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*x^5-13/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*
c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^2/d^3*x^3+4/3*b^2*(d*(c^2*x^2+1))^(1/2)/
(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^(1/2)-2/3*b^2/(c^2*x^
2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2
)+4/3*b^2/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*arcsinh(c*x)^2-2*b^
2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/d^3*x+4/3*a*b*(
d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)/c/d^3*(c^2*x^2+1)^
(1/2)-4/3*a*b/(c^2*x^2+1)^(1/2)*(d*(c^2*x^2+1))^(1/2)/c/d^3*ln(1+(c*x+(c^2*
x^2+1)^(1/2))^2)-4/3*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2
*x^2+4)*c^6/d^3*arcsinh(c*x)*x^7+2*b^2*(d*(c^2*x^2+1))^(1/2)/(3*c^6*x^6+10*
c^4*x^4+11*c^2*x^2+4)*c^4/d^3*arcsinh(c*x)^2*x^5+2/3*b^2*(d*(c^2*x^2+1))^(1
/2)/(3*c^6*x^6+10*c^4*x^4+11*c^2*x^2+4)*c^4/d^3*(c^2*x^2+1)*x^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,abc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}}\right)+\frac{2}{3}\,ab\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\frac{x}{\left(c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\mathrm{arsinh}\left(cx\right)+\frac{1}{3}\,a^2\left(\frac{2\,x}{\sqrt{c^2dx^2+d}d^2}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 + c^2*d^(5/2)) - 2*log(c^2*x^2 + 1)/(c^2*d^(5
/2))) + 2/3*a*b*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 + d)^(3/2)*d
))*arcsinh(c*x) + 1/3*a^2*(2*x/(sqrt(c^2*d*x^2 + d)*d^2) + x/((c^2*d*x^2 +
d)^(3/2)*d)) + b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/(c^2*d*x^2 + d)
^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\,\mathrm{arsinh}\left(cx\right)^2+2\,ab\,\mathrm{arsinh}\left(cx\right)+a^2\right)}{c^6d^3x^6+3\,c^4d^3x^4+3\,c^2d^3x^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{asinh}\left(cx\right)\right)^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2}{(c^2dx^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(c^2*d*x^2 + d)^(5/2), x)
```

3.316 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=518

$$-\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{7i}{}$$

```
[Out] -b^2/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x])^2/(3*d*(d + c^2*d
*x^2)^(3/2)) + (a + b*ArcSinh[c*x])^2/(d^2*Sqrt[d + c^2*d*x^2]) - (14*b*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*d^2*Sqrt[d +
 c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((7*I)/3)*b
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x
^2]) - (((7*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[
2, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*Po
lyLog[3, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 0.863378, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391, 5690, 261}

$$-\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{2b\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}+\frac{7i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -b^2/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) + (a + b*ArcSinh[c*x])^2/(3*d*(d + c^2*d
*x^2)^(3/2)) + (a + b*ArcSinh[c*x])^2/(d^2*Sqrt[d + c^2*d*x^2]) - (14*b*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3*d^2*Sqrt[d +
```

```
 c^2*d*x^2]) - (2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2*ArcTanh[E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((7*I)/3)*b
^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x
^2]) - (((7*I)/3)*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*
Sqrt[d + c^2*d*x^2]) + (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[
2, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (2*b^2*Sqrt[1 + c^2*x^2]*Po
lyLog[3, -E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (2*b^2*Sqrt[1 + c^2*
x^2]*PolyLog[3, E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{3/2}}\,dx}{d} - \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} + \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\sqrt{d+c^2dx^2}}}{d^2}$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} +$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} -$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} -$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} -$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} -$$

$$= -\frac{b^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bcx\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d^2\sqrt{d+c^2dx^2}} -$$

Mathematica [A] time = 4.12413, size = 547, normalized size = 1.06

$$\frac{abd^2\left(c^2x^2+1\right)^{3/2}\left(6\text{PolyLog}\left(2,-e^{-\sinh^{-1}(cx)}\right)-6\text{PolyLog}\left(2,e^{-\sinh^{-1}(cx)}\right)-\frac{cx}{c^2x^2+1}+\frac{6\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}}+\frac{2\sinh^{-1}(cx)}{\left(c^2x^2+1\right)^{3/2}}+6\sinh^{-1}(cx)\log\left(1-e^{-\sinh^{-1}(cx)}\right)-6\sinh^{-1}(cx)\,\text{lo}\right.}{\left(c^2dx^2+d\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(4 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2])/(1 + c^2*x^2)^2 + 3*a^2*Sqrt[d]*
Log[c*x] - 3*a^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + (a*b*d^2*(1
 + c^2*x^2)^(3/2)*(-((c*x)/(1 + c^2*x^2)) + (2*ArcSinh[c*x])/(1 + c^2*x^2)^
(3/2) + (6*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 14*ArcTan[Tanh[ArcSinh[c*x]/2]
] + 6*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] - 6*ArcSinh[c*x]*Log[1 + E^(-
ArcSinh[c*x])] + 6*PolyLog[2, -E^(-ArcSinh[c*x])] - 6*PolyLog[2, E^(-ArcSin
h[c*x])]))/(d + c^2*d*x^2)^(3/2) + (b^2*d^2*(1 + c^2*x^2)^(3/2)*(-(1/Sqrt[1
 + c^2*x^2]) - (c*x*ArcSinh[c*x])/(1 + c^2*x^2) + ArcSinh[c*x]^2/(1 + c^2*x
^2)^(3/2) + (3*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + 3*ArcSinh[c*x]^2*Log[1 -
 E^(-ArcSinh[c*x])] + (7*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - (7*I)*
ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] - 3*ArcSinh[c*x]^2*Log[1 + E^(-ArcSi
nh[c*x])] + 6*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])] + (7*I)*PolyLog[2
, (-I)/E^ArcSinh[c*x]] - (7*I)*PolyLog[2, I/E^ArcSinh[c*x]] - 6*ArcSinh[c*x
]*PolyLog[2, E^(-ArcSinh[c*x])] + 6*PolyLog[3, -E^(-ArcSinh[c*x])] - 6*Poly
Log[3, E^(-ArcSinh[c*x])]))/(d + c^2*d*x^2)^(3/2))/(3*d^3)
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{x} \left(c^2dx^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^7+3\,c^4d^3x^5+3\,c^2d^3x^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x), x)
```

3.317 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=421

$$\frac{5b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{8c^2x\left(\right.}{3}$$

```
[Out] (b^2*c^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(d*x*(d +
c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*c
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4
*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d
^2*Sqrt[d + c^2*d*x^2]) + (16*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b^
2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(d^2*Sqrt[d + c^2*d*x
^2])
```

Rubi [A] time = 0.655093, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5747, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5755, 5720, 5461, 4182}

$$\frac{5b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{b^2c\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{c^2x^2+1}\sqrt{c^2dx^2+d}}-\frac{8c^2x\left(\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```


```
[Out] (b^2*c^2*x)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(d*x*(d +
c^2*d*x^2)^(3/2)) - (4*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(
3/2)) - (8*c^2*x*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*c
*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (4
*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(d
```

```
^2*Sqrt[d + c^2*d*x^2]) + (16*b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Lo
g[1 + E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c*Sqrt[1 +
c^2*x^2]*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) + (b^
2*c*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(d^2*Sqrt[d + c^2*d*x
^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
```

```
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \left(4c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x\left(1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d+c^2dx^2}}$$

$$= \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx)\right)^2}{3d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(8c^2\right)\int \frac{\left(a+b\sin\right.}{\left(d+c\right.}}{3d}$$

$$= -\frac{b^2c^2x}{d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx\right.}{3d\left(d+c^2dx^2\right)^{3/2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx\right)}{3d\left(d+c^2dx^2\right)^{3/2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx\right)}{3d\left(d+c^2dx^2\right)^{3/2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx\right)}{3d\left(d+c^2dx^2\right)^{3/2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx\right)}{3d\left(d+c^2dx^2\right)^{3/2}}$$

$$= \frac{b^2c^2x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} - \frac{4c^2x\left(a+b\sinh^{-1}(cx\right)}{3d\left(d+c^2dx^2\right)^{3/2}}$$

Mathematica [A] time = 1.85731, size = 408, normalized size = 0.97

$$-\frac{5b^2cx\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+3b^2cx\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)+8a^2c^4x^4+12a^2c^2x^2+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^2*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(3*a^2 + 12*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 - b^2*c^4*x^4 + a*b*
c*x*Sqrt[1 + c^2*x^2] + 6*a*b*ArcSinh[c*x] + 24*a*b*c^2*x^2*ArcSinh[c*x] +
```

```
16*a*b*c^4*x^4*ArcSinh[c*x] + b^2*c*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 3*b^
2*ArcSinh[c*x]^2 + 12*b^2*c^2*x^2*ArcSinh[c*x]^2 + 8*b^2*c^4*x^4*ArcSinh[c*
x]^2 - 8*b^2*c*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2 - 6*b^2*c*x*(1 + c^2*x^
2)^(3/2)*ArcSinh[c*x]*Log[1 - E^(-2*ArcSinh[c*x])] - 10*b^2*c*x*(1 + c^2*x^
2)^(3/2)*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] - 6*a*b*c*x*(1 + c^2*x^2
)^(3/2)*Log[c*x] - 5*a*b*c*x*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2] + 5*b^2*c
*x*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(-2*ArcSinh[c*x])] + 3*b^2*c*x*(1 + c^
2*x^2)^(3/2)*PolyLog[2, E^(-2*ArcSinh[c*x])])/(3*d*x*(d + c^2*d*x^2)^(3/2))
```

Maple [B] time = 0.276, size = 3517, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)^2*c-4/3*
a^2*c^2*x/d/(c^2*d*x^2+d)^(3/2)+128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+
25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5+272/3*a
*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)*c^3-a^2/d/x/(c^2*d*x^2+d)^(3/2)+64/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*arcsinh(c*x)^2*(c^
2*x^2+1)^(1/2)*c^5+40*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^
2*x^2+9)/d^3*x^3*arcsinh(c*x)*(c^2*x^2+1)*c^4+136/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*arcsinh(c*x)^2*(c^2*x^2+1)^(
1/2)*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/
d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*(c^2*x^2+1)*c^2+64/3*b^2
*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*arcsinh(
c*x)*(c^2*x^2+1)*c^8+160/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+
26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*(c^2*x^2+1)*c^6+64/3*a*b*(d*(c^2*x^2+1))
^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8+160/3*a*
b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^
2+1)*c^6-128/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9
)/d^3*x^5*arcsinh(c*x)*c^6+40*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x
^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-112*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c
^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4-8/3*a*b*(d*(c^2*x^
2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*c^3*(c^2*x^2+1)^(1/
2)+8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c
^2*x^2+1)*c^2-88*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2
+9)/d^3*x*arcsinh(c*x)*c^2+48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x
```

^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*c-64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*c^10-224/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*c^8-280/3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*c^6-48*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*c^4-8*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-3*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)-18*a*b*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)+2*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(1/2))^2-1)*c+10/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*arcsinh(c*x)*c^10+32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*(c^2*x^2+1)*c^8-224/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7*arcsinh(c*x)*c^8-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)^2*c^6+88/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*(c^2*x^2+1)*c^6-280/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*arcsinh(c*x)*c^6-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^4*c^5*(c^2*x^2+1)^(1/2)-56*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)^2*c^4+80/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*(c^2*x^2+1)*c^4-48*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*arcsinh(c*x)*c^4-17/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^2*c^3*(c^2*x^2+1)^(1/2)-44*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)^2*c^2+8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*(c^2*x^2+1)*c^2-8*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*arcsinh(c*x)*c^2+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c+10/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c+24*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c-3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-160/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^5*c^6-3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*c*(c^2*x^2+1)^(1/2)+5/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+1)^(1/2))*c+2*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,-c*x-(c^2*x^2+1)^(1/2))*c-29*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^3*c^4-5*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x*c^2-9*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3/x*arcsinh(c*x)^2-32/3*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^9*c^

```
10-40*b^2*(d*(c^2*x^2+1))^(1/2)/(8*c^6*x^6+25*c^4*x^4+26*c^2*x^2+9)/d^3*x^7
*c^8-8/3*a^2*c^2/d^2*x/(c^2*d*x^2+d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^8+3\,c^4d^3x^6+3\,c^2d^3x^4+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^8 + 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 + d^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^2), x)
```

3.318 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=687

$$\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-$$

```
[Out] (b^2*c^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(d^2*x*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (2*b*c^3*x*(a + b*ArcSinh[c*x]))/(3
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x])^2
)/(6*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])^2/(2*d*x^2*(d + c^2*d*
x^2)^(3/2)) - (5*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^2*Sqrt[d + c^2*d*x^2]) +
(26*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3
*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2
*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2
*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d
 + c^2*d*x^2]) - (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d
+ c^2*d*x^2]) - (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(
d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 1.259, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5747, 5755, 5764, 5760, 4182, 2531, 2282, 6589, 5693, 4180, 2279, 2391, 5690, 261, 266, 51, 63, 208}

$$\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-\frac{5bc^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{\sinh^{-1}(cx)}\right)\left(a+b\sinh^{-1}(cx)\right)}{d^2\sqrt{c^2dx^2+d}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(d^2*x*S
qrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (2*b*c^3*x*(a + b*ArcSinh[c*x]))/(3
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]) - (5*c^2*(a + b*ArcSinh[c*x])^2
)/(6*d*(d + c^2*d*x^2)^(3/2)) - (a + b*ArcSinh[c*x])^2/(2*d*x^2*(d + c^2*d*
x^2)^(3/2)) - (5*c^2*(a + b*ArcSinh[c*x])^2)/(2*d^2*Sqrt[d + c^2*d*x^2]) +
(26*b*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(3
*d^2*Sqrt[d + c^2*d*x^2]) + (5*c^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2
*ArcTanh[E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (b^2*c^2*Sqrt[1 + c^2
*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(d^2*Sqrt[d + c^2*d*x^2]) + (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, -E^ArcSinh[c*x]])/(d^2*Sqrt[d
 + c^2*d*x^2]) - (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)*E^Ar
cSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) + (((13*I)/3)*b^2*c^2*Sqrt[1 + c^2*x
^2]*PolyLog[2, I*E^ArcSinh[c*x]])/(d^2*Sqrt[d + c^2*d*x^2]) - (5*b*c^2*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*PolyLog[2, E^ArcSinh[c*x]])/(d^2*Sqrt[d
+ c^2*d*x^2]) - (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^ArcSinh[c*x]])/(
d^2*Sqrt[d + c^2*d*x^2]) + (5*b^2*c^2*Sqrt[1 + c^2*x^2]*PolyLog[3, E^ArcSin
h[c*x]])/(d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5764

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[
```

```
((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&  !GtQ[d, 0] && (Integer
Q[m] || EqQ[n, 1])
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
```

```
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^3\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{1}{2}\left(5c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^2\left(1+c^2x^2\right)^2}\,d}{d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2dx^2\left(d+c^2dx^2\right)^{3/2}} - \frac{\left(5c^2\right)\int \frac{(a+b}{x(d}}{2}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-1}(cx)\right)^2}{6d\left(d+c^2dx^2\right)^{3/2}} - \frac{(a+b}{2dx^2(}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh\right.}{6d\left(d+c^2dx^2\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-}\right.}{6d\left(d+c^2dx^2\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-}\right.}{6d\left(d+c^2dx^2\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-}\right.}{6d\left(d+c^2dx^2\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-}\right.}{6d\left(d+c^2dx^2\right.}$$

$$= \frac{b^2c^2}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{d^2x\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{2bc^3x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{5c^2\left(a+b\sinh^{-}\right.}{6d\left(d+c^2dx^2\right.}$$

Mathematica [A] time = 7.97946, size = 983, normalized size = 1.43

$$-\frac{5a^2\log(x)c^2}{2d^{5/2}} + \frac{5a^2\log\left(d+\sqrt{d\left(c^2x^2+1\right)}\sqrt{d}\right)c^2}{2d^{5/2}} + \frac{ab\left(-3\sqrt{c^2x^2+1}\sinh^{-1}(cx)\operatorname{csch}^2\left(\frac{1}{2}\sinh^{-1}(cx)\right) - 3\sqrt{c^2x^2+1}\sin\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^3*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[d*(1 + c^2*x^2)]*(-a^2/(2*d^3*x^2) - (a^2*c^2)/(3*d^3*(1 + c^2*x^2)^2)
 - (2*a^2*c^2)/(d^3*(1 + c^2*x^2))) - (5*a^2*c^2*Log[x])/(2*d^(5/2)) + (5*a
^2*c^2*Log[d + Sqrt[d]*Sqrt[d*(1 + c^2*x^2)]])/(2*d^(5/2)) + (a*b*c^2*((4*c
*x)/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x] - (8*ArcSinh[c*x])/(1 + c^2*x^2) +
104*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - 6*Sqrt[1 + c^2*x^2]*Co
th[ArcSinh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Csch[ArcSinh[c*x]/2]^
2 - 60*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - E^(-ArcSinh[c*x])] + 60*Sqrt[
1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + E^(-ArcSinh[c*x])] - 60*Sqrt[1 + c^2*x^2]
*PolyLog[2, -E^(-ArcSinh[c*x])] + 60*Sqrt[1 + c^2*x^2]*PolyLog[2, E^(-ArcSi
nh[c*x])] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Sech[ArcSinh[c*x]/2]^2 + 6*Sqr
t[1 + c^2*x^2]*Tanh[ArcSinh[c*x]/2]))/(12*d^2*Sqrt[d*(1 + c^2*x^2)]) + (b^2
*c^2*(8 + (8*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] - 48*ArcSinh[c*x]^2 - (8*A
rcSinh[c*x]^2)/(1 + c^2*x^2) - 12*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Coth[ArcSi
nh[c*x]/2] - 3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Csch[ArcSinh[c*x]/2]^2 - 60
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Log[1 - E^(-ArcSinh[c*x])] - (104*I)*Sqrt
[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + (104*I)*Sqrt[1 + c^2
*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSinh[c*x]] + 60*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]^2*Log[1 + E^(-ArcSinh[c*x])] + 24*Sqrt[1 + c^2*x^2]*Log[Tanh[ArcSinh[
c*x]/2]] - 120*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*PolyLog[2, -E^(-ArcSinh[c*x])
] - (104*I)*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + (104*I)*Sqr
t[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]] + 120*Sqrt[1 + c^2*x^2]*ArcSinh
[c*x]*PolyLog[2, E^(-ArcSinh[c*x])] - 120*Sqrt[1 + c^2*x^2]*PolyLog[3, -E^(
-ArcSinh[c*x])] + 120*Sqrt[1 + c^2*x^2]*PolyLog[3, E^(-ArcSinh[c*x])] - 3*S
qrt[1 + c^2*x^2]*ArcSinh[c*x]^2*Sech[ArcSinh[c*x]/2]^2 + 12*Sqrt[1 + c^2*x^
2]*ArcSinh[c*x]*Tanh[ArcSinh[c*x]/2]))/(24*d^2*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int \frac{(a + b\mathrm{Arcsinh}\,(cx))^2}{x^3}\left(c^2dx^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^9+3\,c^4d^3x^7+3\,c^2d^3x^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^9 + 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 + d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**3/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^3/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^3), x)
```

3.319 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=506

$$-\frac{8b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{8b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\ldots$$

```
[Out] -(b^2*c^2)/(3*d^2*x*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d + c^
2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d
+ c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*d*x^3*(d + c^2*d*x^2)^(3/2)) + (2
*c^2*(a + b*ArcSinh[c*x])^2)/(d*x*(d + c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*
ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x
])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (16*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (
32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])]
)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^
2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rubi [A] time = 1.09419, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5747, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5755, 5720, 5461, 4182, 271}

$$-\frac{8b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}-\frac{8b^2c^3\sqrt{c^2x^2+1}\text{PolyLog}\left(2,e^{2\sinh^{-1}(cx)}\right)}{3d^2\sqrt{c^2dx^2+d}}+\frac{16c^4x\left(a+b\sinh^{-1}(cx)\right)^2}{3d^2\sqrt{c^2dx^2+d}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b^2*c^2)/(3*d^2*x*Sqrt[d + c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d + c^
2*d*x^2]) - (b*c*(a + b*ArcSinh[c*x]))/(3*d^2*x^2*Sqrt[1 + c^2*x^2]*Sqrt[d
+ c^2*d*x^2]) - (a + b*ArcSinh[c*x])^2/(3*d*x^3*(d + c^2*d*x^2)^(3/2)) + (2
*c^2*(a + b*ArcSinh[c*x])^2)/(d*x*(d + c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*
ArcSinh[c*x])^2)/(3*d*(d + c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSinh[c*x
```

```
])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (16*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x])^2)/(3*d^2*Sqrt[d + c^2*d*x^2]) + (32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b
*ArcSinh[c*x])*ArcTanh[E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (
32*b*c^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])]
)/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^2]*PolyLog[2, -E^
(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2]) - (8*b^2*c^3*Sqrt[1 + c^2*x^
2]*PolyLog[2, E^(2*ArcSinh[c*x])])/(3*d^2*Sqrt[d + c^2*d*x^2])
```

Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5755

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcSinh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 + c^2*x^2)^FracPa
```

```
rt[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n -
 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 5720

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Dist[1/d, Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x, Arc
Sinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^4\left(d+c^2dx^2\right)^{5/2}}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} - \left(2c^2\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{x^2\left(d+c^2dx^2\right)^{5/2}}\,dx + \frac{\left(2bc\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{x^3\left(1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} + \frac{2c^2\left(a+b\sinh^{-1}(cx)\right)^2}{dx\left(d+c^2dx^2\right)^{3/2}} + \left(8c^4\right)\int$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{8bc^3\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh\right.}{3dx^3\left(d+c\right.}$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} + \frac{2b^2c^4x}{d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} +$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} +$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} +$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} +$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} +$$

$$= -\frac{b^2c^2}{3d^2x\sqrt{d+c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d+c^2dx^2}} - \frac{bc\left(a+b\sinh^{-1}(cx)\right)}{3d^2x^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{3dx^3\left(d+c^2dx^2\right)^{3/2}} +$$

Mathematica [A] time = 3.02378, size = 417, normalized size = 0.82

$$\frac{b^2c^3\left(c^2x^2+1\right)^{3/2}\left(8\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(cx)}\right)+8\text{PolyLog}\left(2,e^{-2\sinh^{-1}(cx)}\right)-\frac{\sqrt{c^2x^2+1}}{cx}-\frac{cx}{\sqrt{c^2x^2+1}}+\frac{8\sqrt{c^2x^2+1}\sinh^{-1}(cx)^2}{cx}-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(x^4*(d + c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(-1 + 6*c^2*x^2 + 24*c^4*x^4 + 16*c^6*x^6))/x^3 - (a*b*(-2*(-1 + 6*c^
2*x^2 + 24*c^4*x^4 + 16*c^6*x^6)*ArcSinh[c*x] + c*x*Sqrt[1 + c^2*x^2]*(1 +
16*(c^2*x^2 + c^4*x^4)*Log[c*x] + 8*(c^2*x^2 + c^4*x^4)*Log[1 + c^2*x^2])))
/x^3 + b^2*c^3*(1 + c^2*x^2)^(3/2)*(-((c*x)/Sqrt[1 + c^2*x^2]) - Sqrt[1 + c
^2*x^2]/(c*x) - ArcSinh[c*x]/(c^2*x^2) + ArcSinh[c*x]/(1 + c^2*x^2) - 16*Ar
cSinh[c*x]^2 + (c*x*ArcSinh[c*x]^2)/(1 + c^2*x^2)^(3/2) + (8*c*x*ArcSinh[c*
x]^2)/Sqrt[1 + c^2*x^2] - (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c^3*x^3) + (8
*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*x) - 16*ArcSinh[c*x]*Log[1 - E^(-2*Ar
cSinh[c*x])] - 16*ArcSinh[c*x]*Log[1 + E^(-2*ArcSinh[c*x])] + 8*PolyLog[2,
-E^(-2*ArcSinh[c*x])] + 8*PolyLog[2, E^(-2*ArcSinh[c*x])]))/(3*d*(d + c^2*d
*x^2)^(3/2))
```

Maple [B] time = 0.33, size = 4955, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 2*a^2*c^2/d/x/(c^2*d*x^2+d)^(3/2)+256/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x
^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^11*c^14+896/3*a*b*(d*(c^2*x^2+
1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12+1120
/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1
)/d^3*x^7*c^10+560/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^
4*x^4+10*c^2*x^2-1)/d^3*x^5*c^8+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+
36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*c^6-16/3*a*b*(d*(c^2*x^2+1))^(1
/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4-16/3*a*b*(d*(
c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*ln((c*x+(c^2*x^2+1)^(1/2))^4-1)*c^3
+64/3*a*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*c^3-4*a*
b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3
*c^3*(c^2*x^2+1)^(1/2)+2/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6
+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c*x)-16/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1-c*x-(c^2*x^2+1)^(1/2))*c^3+256/3
*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3*x^11*arcsinh(c*x)*c^14-64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^
6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2*x^2+1)*c^12+896/3*b^2*(d*(c^2*x
^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*arcsin
h(c*x)*c^12+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^
4+10*c^2*x^2-1)/d^3*x^11*c^14-128*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*
```

c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^9
-256*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2
-1)/d^3*x^4*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^7-352/3*a*b*(d*(c^2*x^2+1))^(1
/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*arcsinh(c*x)*(c
^2*x^2+1)^(1/2)*c^5+64*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*
c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)^2*c^10-160/3*b^2*(d*(c^2*x^2+1))
^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*
c^10+1120/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*
c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10+8*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*(c^2*x^2+1)^(1/2)*c^9+160*b^2
(d(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*
x^5*arcsinh(c*x)^2*c^8-128/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x
^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*(c^2*x^2+1)*c^8+560/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*
x)*c^8+16*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^
2*x^2-1)/d^3*x^4*(c^2*x^2+1)^(1/2)*c^7+344/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*
c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)^2*c^6-32/3
b^2(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3*x^3*(c^2*x^2+1)*c^6+64/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x
^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*arcsinh(c*x)*c^6+22/3*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*c^5*(c^2*x
^2+1)^(1/2)+12*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+
10*c^2*x^2-1)/d^3*x*arcsinh(c*x)^2*c^4-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c
^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*c^4+16/3*b^2*
(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*a
rcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^3-4*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8
+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3
-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+(c*
x+(c^2*x^2+1)^(1/2))^2)*c^3-6*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*
x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*arcsinh(c*x)^2*c^2-16/3*b^2*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)*ln(1+c*x+(c^2*x^2+1)^(1/2))*c^
3-4*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-
1)/d^3*x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^5+16/3*b^2*(d*(c^2*x^2+1))^(1/2
)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c*x)*(c^2*x
^2+1)*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+1
0*c^2*x^2-1)/d^3/x^2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c-160*b^2*(d*(c^2*x^2+1
))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*
x)*(c^2*x^2+1)*c^8-128*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*
c^4*x^4+10*c^2*x^2-1)/d^3*x^4*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^7-80/3*b^2
(d(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*
x^3*arcsinh(c*x)*(c^2*x^2+1)*c^6-256/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*arcsinh(c*x)*(c^2*x^2+1)*c^12
-640/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x
^2-1)/d^3*x^7*arcsinh(c*x)*(c^2*x^2+1)*c^10-80/3*a*b*(d*(c^2*x^2+1))^(1/2)/
(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^3*(c^2*x^2+1)*c^6+688

```
/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1
)/d^3*x^3*arcsinh(c*x)*c^6-4*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x
^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*c^5*(c^2*x^2+1)^(1/2)+24*a*b*(d*(c^2*x^
2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x*arcsinh(c
*x)*c^4+16/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10
*c^2*x^2-1)/d^3*x*(c^2*x^2+1)*c^4-12*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+
36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*arcsinh(c*x)*c^2+1/3*a*b*(d*(c^2*
x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^2*c*(c^
2*x^2+1)^(1/2)+32/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4
*x^4+10*c^2*x^2-1)/d^3*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^3-256/3*a*b*(d*(c^2
*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*(c^2
*x^2+1)*c^12-640/3*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*
x^4+10*c^2*x^2-1)/d^3*x^7*(c^2*x^2+1)*c^10+128*a*b*(d*(c^2*x^2+1))^(1/2)/(1
2*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*arcsinh(c*x)*c^10-160
*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/
d^3*x^5*(c^2*x^2+1)*c^8+320*a*b*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^
6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^5*arcsinh(c*x)*c^8-64*b^2*(d*(c^2*x^2+1))^
(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^6*arcsinh(c*x)^
2*(c^2*x^2+1)^(1/2)*c^9-176/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*
x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^2*arcsinh(c*x)^2*(c^2*x^2+1)^(1/2)*c^5-1
/3*a^2/d/x^3/(c^2*d*x^2+d)^(3/2)+16/3*a^2*c^4/d^2*x/(c^2*d*x^2+d)^(1/2)+8/3
*a^2*c^4*x/d/(c^2*d*x^2+d)^(3/2)+224/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^9*c^12+88*b^2*(d*(c^2*x^2+1))^(
1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*x^7*c^10+100/3*b^2
*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*
x^5*c^8-14/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10
*c^2*x^2-1)/d^3*x^3*c^6-3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^8+36*c^6*x^6+
35*c^4*x^4+10*c^2*x^2-1)/d^3*x*c^4+1/3*b^2*(d*(c^2*x^2+1))^(1/2)/(12*c^8*x^
8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x*c^2+1/3*b^2*(d*(c^2*x^2+1))^(1/
2)/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3/x^3*arcsinh(c*x)^2-1
6/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(2,c*x+(c^2*x^2+
1)^(1/2))*c^3-16/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/d^3*polylog(
2,-c*x-(c^2*x^2+1)^(1/2))*c^3-8/3*b^2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/d^3*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))^2)*c^3+32/3*b^2*(d*(c^2*x^2+1))^(
1/2)/(c^2*x^2+1)^(1/2)/d^3*arcsinh(c*x)^2*c^3-2/3*b^2*(d*(c^2*x^2+1))^(1/2)
/(12*c^8*x^8+36*c^6*x^6+35*c^4*x^4+10*c^2*x^2-1)/d^3*c^3*(c^2*x^2+1)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2+d}\left(b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2\right)}{c^6d^3x^{10}+3\,c^4d^3x^8+3\,c^2d^3x^6+d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)/(c^6*d^3*x^10 + 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 + d^3*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/x**4/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{arsinh}(cx)+a)^2}{\left(c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/x^4/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsinh(c*x) + a)^2/((c^2*d*x^2 + d)^(5/2)*x^4), x)
```

3.320 $\int \frac{\sinh^{-1}(ax)^2}{(c+a^2cx^2)^{7/2}} \, dx$

Optimal. Leaf size=366

$$-\frac{8\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{15ac^3\sqrt{a^2cx^2+c}}-\frac{x}{3c^3\sqrt{a^2cx^2+c}}-\frac{x}{30c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)^2}{15c^3\sqrt{a^2cx^2+c}}+\frac{8\sqrt{a^2x^2+\ldots}}{15ac^3\ldots}$$

```
[Out] -x/(3*c^3*Sqrt[c + a^2*c*x^2]) - x/(30*c^3*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2
]) + ArcSinh[a*x]/(10*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (4*A
rcSinh[a*x])/(15*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[
a*x]^2)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^2)/(15*c^2*(c + a^2
*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^2)/(15*c^3*Sqrt[c + a^2*c*x^2]) + (8*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^2)/(15*a*c^3*Sqrt[c + a^2*c*x^2]) - (16*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]*Log[1 + E^(2*ArcSinh[a*x])])/(15*a*c^3*Sqrt[c + a^2
*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(15*a*c^3*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.352804, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 192}

$$-\frac{8\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{15ac^3\sqrt{a^2cx^2+c}}-\frac{x}{3c^3\sqrt{a^2cx^2+c}}-\frac{x}{30c^3\left(a^2x^2+1\right)\sqrt{a^2cx^2+c}}+\frac{8x\sinh^{-1}(ax)^2}{15c^3\sqrt{a^2cx^2+c}}+\frac{8\sqrt{a^2x^2+\ldots}}{15ac^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```


```
[Out] -x/(3*c^3*Sqrt[c + a^2*c*x^2]) - x/(30*c^3*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2
]) + ArcSinh[a*x]/(10*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]) + (4*A
rcSinh[a*x])/(15*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[
a*x]^2)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^2)/(15*c^2*(c + a^2
*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^2)/(15*c^3*Sqrt[c + a^2*c*x^2]) + (8*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^2)/(15*a*c^3*Sqrt[c + a^2*c*x^2]) - (16*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]*Log[1 + E^(2*ArcSinh[a*x])])/(15*a*c^3*Sqrt[c + a^2
*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(15*a*c^3*
Sqrt[c + a^2*c*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(2a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^2}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)^2}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)^2}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{15c^2} - \cdots$$

$$= -\frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}(ax)}{15ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \cdots$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}\cdots}{15ac^3\sqrt{1+a^2x\cdots}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}\cdots}{15ac^3\sqrt{1+a^2x\cdots}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}\cdots}{15ac^3\sqrt{1+a^2x\cdots}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}\cdots}{15ac^3\sqrt{1+a^2x\cdots}}$$

$$= -\frac{x}{3c^3\sqrt{c+a^2cx^2}} - \frac{x}{30c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)}{10ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{4\sinh^{-1}\cdots}{15ac^3\sqrt{1+a^2x\cdots}}$$

Mathematica [A] time = 0.761946, size = 178, normalized size = 0.49

$$\frac{16\sqrt{a^2x^2+1}\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right) + ax\left(-\frac{1}{a^2x^2+1}-10\right) + \left(\frac{2ax\left(8a^4x^4+20a^2x^2+15\right)}{\left(a^2x^2+1\right)^2} - 16\sqrt{a^2x^2+1}\right)\sinh^{-1}(ax)^2 + \frac{\sinh^{-1}(a\cdots}{\cdots}}{30ac^3\sqrt{a^2cx^2+c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^2/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (a*x*(-10 - (1 + a^2*x^2)^(-1)) + (-16*Sqrt[1 + a^2*x^2] + (2*a*x*(15 + 20*
a^2*x^2 + 8*a^4*x^4))/(1 + a^2*x^2)^2)*ArcSinh[a*x]^2 + (ArcSinh[a*x]*(11 +
```

```
 8*a^2*x^2 - 32*(1 + a^2*x^2)^2*Log[1 + E^(-2*ArcSinh[a*x])]))/(1 + a^2*x^2
)^(3/2) + 16*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(-2*ArcSinh[a*x])])/(30*a*c^3*
Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.178, size = 570, normalized size = 1.6

$$\frac{1}{\left(1200\,a^{10}x^{10}+6450\,x^{8}a^{8}+14070\,x^{6}a^{6}+15510\,x^{4}a^{4}+8610\,a^{2}x^{2}+1920\right)ac^{4}}\sqrt{c\left(a^{2}x^{2}+1\right)}\left(8\,x^{5}a^{5}-8\,a^{4}x^{4}\sqrt{a^{2}x^{2}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/30*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*x^3*a^
3-16*a^2*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(-64*arcsinh(a*x
)*x^8*a^8-64*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*x^7*a^7-32*x^8*a^8-32*(a^2*x^2+
1)^(1/2)*x^7*a^7-280*arcsinh(a*x)*x^6*a^6-248*(a^2*x^2+1)^(1/2)*arcsinh(a*x
)*x^5*a^5-142*x^6*a^6-126*(a^2*x^2+1)^(1/2)*x^5*a^5+80*arcsinh(a*x)^2*x^4*a
^4-456*a^4*x^4*arcsinh(a*x)-340*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-265*
x^4*a^4-156*a^3*x^3*(a^2*x^2+1)^(1/2)+190*arcsinh(a*x)^2*a^2*x^2-328*a^2*x^
2*arcsinh(a*x)-165*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-235*a^2*x^2-62*a*x*(a
^2*x^2+1)^(1/2)+128*arcsinh(a*x)^2-88*arcsinh(a*x)-80)/(40*a^10*x^10+215*a^
8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4+16/15/(a^2*x^2+1)^(1/2)
*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2-16/15/(a^2*x^2+1)^(1/2)*(c*(a^2
*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-8/15/(a^2
*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*polylog(2,-(a*x+(a^2*x^2+1)^(1/2)
)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{2}}{\left(a^{2}cx^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^2}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^2/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**2/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^2}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^2/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/(a^2*c*x^2 + c)^(7/2), x)
```

3.321 $\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=935

$$\frac{\left(c^2dx^2+d\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2x^{m+1}}{m+6}+\frac{5d\left(c^2dx^2+d\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2x^{m+1}}{(m+4)(m+6)}+\frac{15d^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(c\right.}{(m+6)\left(m^2+6m+8\right)}$$

```
[Out] (10*b^2*c^2*d^2*x^(3 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^3*(6 + m)) + (2*b^2
*c^2*d^2*(52 + 15*m + m^2)*x^(3 + m)*Sqrt[d + c^2*d*x^2])/((4 + m)^2*(6 + m
)^3) + (2*b^2*c^4*d^2*x^(5 + m)*Sqrt[d + c^2*d*x^2])/(6 + m)^3 - (30*b*c*d^
2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*(4 + m)*(6
 + m)*Sqrt[1 + c^2*x^2]) - (10*b*c*d^2*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x]))/((6 + m)*(8 + 6*m + m^2)*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*x^
(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((12 + 8*m + m^2)*Sqrt[1
+ c^2*x^2]) - (10*b*c^3*d^2*x^(4 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/((4 + m)^2*(6 + m)*Sqrt[1 + c^2*x^2]) - (4*b*c^3*d^2*x^(4 + m)*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) - (2
*b*c^5*d^2*x^(6 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((6 + m)^2*S
qrt[1 + c^2*x^2]) + (15*d^2*x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/((6 + m)*(8 + 6*m + m^2)) + (5*d*x^(1 + m)*(d + c^2*d*x^2)^(3/2)*(a
+ b*ArcSinh[c*x])^2)/((4 + m)*(6 + m)) + (x^(1 + m)*(d + c^2*d*x^2)^(5/2)*(
a + b*ArcSinh[c*x])^2)/(6 + m) + (30*b^2*c^2*d^2*x^(3 + m)*Sqrt[d + c^2*d*x
^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)^2*(3
 + m)*(4 + m)*(6 + m)*Sqrt[1 + c^2*x^2]) + (10*b^2*c^2*d^2*(10 + 3*m)*x^(3
+ m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2
*x^2)])/((2 + m)*(3 + m)*(4 + m)^3*(6 + m)*Sqrt[1 + c^2*x^2]) + (2*b^2*c^2*
d^2*(264 + 130*m + 15*m^2)*x^(3 + m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)*(3 + m)*(4 + m)^2*(6 + m)^
3*Sqrt[1 + c^2*x^2]) + (15*d^3*Unintegrable[(x^m*(a + b*ArcSinh[c*x])^2)/Sq
rt[d + c^2*d*x^2], x])/((6 + m)*(8 + 6*m + m^2))
```

Rubi [A] time = 0.152192, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx=\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 2.72439, size = 0, normalized size = 0.

$$\int x^m \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[x^m*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 1.248, size = 0, normalized size = 0.

$$\int x^m \left(c^2dx^2+d\right)^{\frac{5}{2}} (a+b\mathrm{Arcsinh}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}} (b\,\mathrm{arsinh}\,(cx)+a)^2 x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2c^4d^2x^4 + 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 + 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arsinh}\,(cx)^2 + 2\left(abc^4d^2x^4 + 2\,abc^2d^2x^2 + ab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
+ 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 + 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.322 $\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=487

$$\frac{3d^2\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}},x\right)}{m^2+6m+8}+\frac{2b^2c^2d(3m+10)x^{m+3}\sqrt{c^2dx^2+d}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)(m+3)(m+4)^3\sqrt{c^2x^2+1}}$$

```
[Out] (2*b^2*c^2*d*x^(3 + m)*Sqrt[d + c^2*d*x^2])/(4 + m)^3 - (6*b*c*d*x^(2 + m)*
Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*(4 + m)*Sqrt[1 + c^2*x
^2]) - (2*b*c*d*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((8 + 6
*m + m^2)*Sqrt[1 + c^2*x^2]) - (2*b*c^3*d*x^(4 + m)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x]))/((4 + m)^2*Sqrt[1 + c^2*x^2]) + (3*d*x^(1 + m)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(8 + 6*m + m^2) + (x^(1 + m)*(d + c^2*d*
x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(4 + m) + (6*b^2*c^2*d*x^(3 + m)*Sqrt[d
+ c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/((2
+ m)^2*(3 + m)*(4 + m)*Sqrt[1 + c^2*x^2]) + (2*b^2*c^2*d*(10 + 3*m)*x^(3 +
m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x
^2)])/((2 + m)*(3 + m)*(4 + m)^3*Sqrt[1 + c^2*x^2]) + (3*d^2*Unintegrable[(
x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x])/(8 + 6*m + m^2)
```

Rubi [A] time = 0.151321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx=\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 0.420097, size = 0, normalized size = 0.

$$\int x^m \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[x^m*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.997, size = 0, normalized size = 0.

$$\int x^m \left(c^2dx^2+d\right)^{\frac{3}{2}} (a+b\mathrm{Arcsinh}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}} (b \,\mathrm{arsinh}\,(cx)+a)^2 x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2c^2dx^2+a^2d+\left(b^2c^2dx^2+b^2d\right)\mathrm{arsinh}\,(cx)^2+2\left(abc^2dx^2+abd\right)\mathrm{arsinh}\,(cx)\right)\sqrt{c^2dx^2+d}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral((a^2*c^2*d*x^2 + a^2*d + (b^2*c^2*d*x^2 + b^2*d)*arcsinh(c*x)^2 +
2*(a*b*c^2*d*x^2 + a*b*d)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.323 $\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=198

$$\frac{d\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}},x\right)}{m+2}+\frac{2b^2c^2x^{m+3}\sqrt{c^2dx^2+d}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-c^2x^2\right)}{(m+2)^2(m+3)\sqrt{c^2x^2+1}}+\frac{x^{m+1}\sqrt{c^2d}}{}$$

```
[Out] (-2*b*c*x^(2 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/((2 + m)^2*Sqrt
[1 + c^2*x^2]) + (x^(1 + m)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2
+ m) + (2*b^2*c^2*x^(3 + m)*Sqrt[d + c^2*d*x^2]*Hypergeometric2F1[1/2, (3 +
 m)/2, (5 + m)/2, -(c^2*x^2)])/((2 + m)^2*(3 + m)*Sqrt[1 + c^2*x^2]) + (d*U
nintegrable[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x])/(2 + m)
```

Rubi [A] time = 0.136242, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Mathematica [A] time = 0.29456, size = 0, normalized size = 0.

$$\int x^m\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[x^m*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.852, size = 0, normalized size = 0.

$$\int x^{m}\sqrt{c^{2}d x^{2}+d}\,(a+b\mathrm{Arcsinh}\,(cx))^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^{2}d x^{2}+d}(b\,\mathrm{arsinh}\,(cx)+a)^{2}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^2*x^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c^{2}d x^{2}+d}\left(b^{2}\,\mathrm{arsinh}\,(cx)^{2}+2\,ab\,\mathrm{arsinh}\,(cx)+a^{2}\right)x^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*d*x**2+d)**(1/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*d*x^2+d)^(1/2)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.324 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{c^2dx^2+d}},x\right)$$

```
[Out] Unintegrable[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Rubi [A] time = 0.142021, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]

[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Mathematica [A] time = 2.78973, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/Sqrt[d + c^2*d*x^2], x]
```

Maple [A] time = 0.285, size = 0, normalized size = 0.

$$\int x^{m} \left(a + b\text{Arcsinh}\left(cx\right)\right)^{2} \frac{1}{\sqrt{c^{2}dx^{2} + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^{2} x^{m}}{\sqrt{c^{2}dx^{2} + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2} \operatorname{arsinh}\left(cx\right)^{2} + 2\, ab \operatorname{arsinh}\left(cx\right) + a^{2}\right)x^{m}}{\sqrt{c^{2}dx^{2} + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*x^m/sqrt(c^2*d*x^2
 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (a + b \operatorname{asinh}(cx))^2}{\sqrt{d (c^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^m}{\sqrt{c^2dx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/sqrt(c^2*d*x^2 + d), x)
```

3.325 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(c^2dx^2+d\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Rubi [A] time = 0.159924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.2265, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(3/2), x]
```

Maple [A] time = 0.444, size = 0, normalized size = 0.

$$\int x^{m} (a+b\mathrm{Arcsinh}\,(cx))^{2} \left(c^{2}dx^{2}+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{arsinh}\,(cx)+a)^{2}x^{m}}{\left(c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^{2}dx^{2}+d}\left(b^{2}\,\mathrm{arsinh}\,(cx)^{2}+2\,ab\,\mathrm{arsinh}\,(cx)+a^{2}\right)x^{m}}{c^{4}d^{2}x^{4}+2\,c^{2}d^{2}x^{2}+d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m (a + b\,\text{asinh}\,(cx))^2}{\left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**m*(a + b*asinh(c*x))**2/(d*(c**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x^m}{\left(c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(3/2), x)
```

3.326 $\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(c^2dx^2+d\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Rubi [A] time = 0.159274, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]

[Out] Defer[Int][(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx=\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 4.31537, size = 0, normalized size = 0.

$$\int \frac{x^m\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^m*(a + b*ArcSinh[c*x])^2)/(d + c^2*d*x^2)^(5/2), x]
```

Maple [A] time = 0.433, size = 0, normalized size = 0.

$$\int x^m (a + b\text{Arcsinh}\,(cx))^2 \left(c^2dx^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{arsinh}\,(cx) + a)^2 x^m}{\left(c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2 + d}\left(b^2\,\text{arsinh}\,(cx)^2 + 2\,ab\,\text{arsinh}\,(cx) + a^2\right)x^m}{c^6d^3x^6 + 3\,c^4d^3x^4 + 3\,c^2d^3x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
)*x^m/(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(c*x))**2/(c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2 x^m}{\left(c^2 dx^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(c*x))^2/(c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*x^m/(c^2*d*x^2 + d)^(5/2), x)
```

3.327 $\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^m \sinh^{-1}(ax)^2}{\sqrt{a^2x^2+1}}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Rubi [A] time = 0.0917486, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]

[Out] Defer[Int][(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx = \int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$$

Mathematica [A] time = 0.485458, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh^{-1}(ax)^2}{\sqrt{1+a^2x^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Integrate[(x^m*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2], x]
```

Maple [A] time = 0.249, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{Arcsinh}\left(ax\right)\right)^{2} \frac{1}{\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \operatorname{arsinh}\left(ax\right)^{2}}{\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^{m} \operatorname{arsinh}\left(ax\right)^{2}}{\sqrt{a^{2}x^{2}+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{asinh}^2(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**2/sqrt(a**2*x**2 + 1), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{arsinh}(ax)^2}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^2/sqrt(a^2*x^2 + 1), x)
```

3.328 $\int \left(c+a^2cx^2\right)^3 \sinh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=359

$$-\frac{6c^3\left(a^2x^2+1\right)^{7/2}}{2401a}-\frac{2664c^3\left(a^2x^2+1\right)^{5/2}}{214375a}-\frac{30256c^3\left(a^2x^2+1\right)^{3/2}}{385875a}-\frac{413312c^3\sqrt{a^2x^2+1}}{128625a}+\frac{6}{343}a^6c^3x^7\sinh^{-1}(ax)+\frac{702}{}$$

```
[Out] (-413312*c^3*Sqrt[1 + a^2*x^2])/(128625*a) - (30256*c^3*(1 + a^2*x^2)^(3/2)
)/(385875*a) - (2664*c^3*(1 + a^2*x^2)^(5/2))/(214375*a) - (6*c^3*(1 + a^2*
x^2)^(7/2))/(2401*a) + (4322*c^3*x*ArcSinh[a*x])/1225 + (1514*a^2*c^3*x^3*A
rcSinh[a*x])/3675 + (702*a^4*c^3*x^5*ArcSinh[a*x])/6125 + (6*a^6*c^3*x^7*Ar
cSinh[a*x])/343 - (48*c^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(35*a) - (8*c^3
*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(35*a) - (18*c^3*(1 + a^2*x^2)^(5/2)*A
rcSinh[a*x]^2)/(175*a) - (3*c^3*(1 + a^2*x^2)^(7/2)*ArcSinh[a*x]^2)/(49*a)
+ (16*c^3*x*ArcSinh[a*x]^3)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/35
+ (6*c^3*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcS
inh[a*x]^3)/7
```

Rubi [A] time = 0.727717, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {5684, 5653, 5717, 261, 5679, 444, 43, 194, 12, 1247, 698, 1799, 1850}

$$-\frac{6c^3\left(a^2x^2+1\right)^{7/2}}{2401a}-\frac{2664c^3\left(a^2x^2+1\right)^{5/2}}{214375a}-\frac{30256c^3\left(a^2x^2+1\right)^{3/2}}{385875a}-\frac{413312c^3\sqrt{a^2x^2+1}}{128625a}+\frac{6}{343}a^6c^3x^7\sinh^{-1}(ax)+\frac{702}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (-413312*c^3*Sqrt[1 + a^2*x^2])/(128625*a) - (30256*c^3*(1 + a^2*x^2)^(3/2)
)/(385875*a) - (2664*c^3*(1 + a^2*x^2)^(5/2))/(214375*a) - (6*c^3*(1 + a^2*
x^2)^(7/2))/(2401*a) + (4322*c^3*x*ArcSinh[a*x])/1225 + (1514*a^2*c^3*x^3*A
rcSinh[a*x])/3675 + (702*a^4*c^3*x^5*ArcSinh[a*x])/6125 + (6*a^6*c^3*x^7*Ar
cSinh[a*x])/343 - (48*c^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(35*a) - (8*c^3
*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(35*a) - (18*c^3*(1 + a^2*x^2)^(5/2)*A
rcSinh[a*x]^2)/(175*a) - (3*c^3*(1 + a^2*x^2)^(7/2)*ArcSinh[a*x]^2)/(49*a)
+ (16*c^3*x*ArcSinh[a*x]^3)/35 + (8*c^3*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/35
+ (6*c^3*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/35 + (c^3*x*(1 + a^2*x^2)^3*ArcS
inh[a*x]^3)/7
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^3 \sinh^{-1}(ax)^3\,dx = \frac{1}{7}c^3x\left(1+a^2x^2\right)^3\sinh^{-1}(ax)^3+\frac{1}{7}(6c)\int \left(c+a^2cx^2\right)^2\sinh^{-1}(ax)^3\,dx-\frac{1}{7}\left(3ac^3\right)\int x\,($$

$$= -\frac{3c^3\left(1+a^2x^2\right)^{7/2}\sinh^{-1}(ax)^2}{49a}+\frac{6}{35}c^3x\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^3+\frac{1}{7}c^3x\left(1+a^2x^2\right)^3\text{si}$$

$$= \frac{6}{49}c^3x\sinh^{-1}(ax)+\frac{6}{49}a^2c^3x^3\sinh^{-1}(ax)+\frac{18}{245}a^4c^3x^5\sinh^{-1}(ax)+\frac{6}{343}a^6c^3x^7\sinh^{-}$$

$$= \frac{402c^3x\sinh^{-1}(ax)}{1225}+\frac{318a^2c^3x^3\sinh^{-1}(ax)}{1225}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{343}a^6c^3x^7\text{sin}$$

$$= \frac{962c^3x\sinh^{-1}(ax)}{1225}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{343}a^6c^3x^7\text{sin}$$

$$= \frac{4322c^3x\sinh^{-1}(ax)}{1225}+\frac{1514a^2c^3x^3\sinh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\sinh^{-1}(ax)}{6125}+\frac{6}{343}a^6c^3x^7\text{s}$$

$$= -\frac{960c^3\sqrt{1+a^2x^2}}{343a}-\frac{16c^3\left(1+a^2x^2\right)^{3/2}}{1715a}-\frac{36c^3\left(1+a^2x^2\right)^{5/2}}{8575a}-\frac{6c^3\left(1+a^2x^2\right)^{7/2}}{2401a}+\frac{432}{}$$

$$= -\frac{413312c^3\sqrt{1+a^2x^2}}{128625a}-\frac{30256c^3\left(1+a^2x^2\right)^{3/2}}{385875a}-\frac{2664c^3\left(1+a^2x^2\right)^{5/2}}{214375a}-\frac{6c^3\left(1+a^2x^2\right.}{2401a}$$

Mathematica [A] time = 0.258246, size = 169, normalized size = 0.47

$$\frac{c^3\left(-2\sqrt{a^2x^2+1}\left(16875a^6x^6+134541a^4x^4+747937a^2x^2+22329151\right)+385875ax\left(5a^6x^6+21a^4x^4+35a^2x^2+35\right)\text{sin}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3*ArcSinh[a*x]^3,x]
```

```
[Out] (c^3*(-2*Sqrt[1 + a^2*x^2]*(22329151 + 747937*a^2*x^2 + 134541*a^4*x^4 + 16
875*a^6*x^6) + 210*a*x*(226905 + 26495*a^2*x^2 + 7371*a^4*x^4 + 1125*a^6*x^
6)*ArcSinh[a*x] - 11025*Sqrt[1 + a^2*x^2]*(2161 + 757*a^2*x^2 + 351*a^4*x^4
 + 75*a^6*x^6)*ArcSinh[a*x]^2 + 385875*a*x*(35 + 35*a^2*x^2 + 21*a^4*x^4 +
5*a^6*x^6)*ArcSinh[a*x]^3))/(13505625*a)
```

Maple [A] time = 0.055, size = 270, normalized size = 0.8

$$\frac{c^3}{13505625\,a}\left(1929375\,(\mathrm{Arcsinh}\,(ax))^3\,a^7x^7-826875\,(\mathrm{Arcsinh}\,(ax))^2\,\sqrt{a^2x^2+1}a^6x^6+8103375\,(\mathrm{Arcsinh}\,(ax))^3\,a^5x^5+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x)
```

```
[Out] 1/13505625/a*c^3*(1929375*arcsinh(a*x)^3*a^7*x^7-826875*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)*a^6*x^6+8103375*arcsinh(a*x)^3*a^5*x^5+236250*arcsinh(a*x)*a^
7*x^7-3869775*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a^4*x^4-33750*a^6*x^6*(a^2*x
^2+1)^(1/2)+13505625*arcsinh(a*x)^3*a^3*x^3+1547910*arcsinh(a*x)*a^5*x^5-83
45925*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-269082*a^4*x^4*(a^2*x^2+1)^(
1/2)+13505625*arcsinh(a*x)^3*a*x+5563950*arcsinh(a*x)*a^3*x^3-23825025*arcs
inh(a*x)^2*(a^2*x^2+1)^(1/2)-1495874*a^2*x^2*(a^2*x^2+1)^(1/2)+47650050*a*x
*arcsinh(a*x)-44658302*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.22839, size = 373, normalized size = 1.04

$$-\frac{1}{1225}\left(75\,\sqrt{a^2x^2+1}a^4c^3x^6+351\,\sqrt{a^2x^2+1}a^2c^3x^4+757\,\sqrt{a^2x^2+1}c^3x^2+\frac{2161\,\sqrt{a^2x^2+1}c^3}{a^2}\right)a\,\mathrm{arsinh}\,(ax)^2+\frac{1}{35}\left(5\,a^6c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/1225*(75*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 351*sqrt(a^2*x^2 + 1)*a^2*c^3*x
^4 + 757*sqrt(a^2*x^2 + 1)*c^3*x^2 + 2161*sqrt(a^2*x^2 + 1)*c^3/a^2)*a*arcs
inh(a*x)^2 + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3
*x)*arcsinh(a*x)^3 - 2/13505625*(16875*sqrt(a^2*x^2 + 1)*a^4*c^3*x^6 + 1345
41*sqrt(a^2*x^2 + 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 + 1)*c^3*x^2 + 22329
151*sqrt(a^2*x^2 + 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 + 7371*a^4*c^3*x^5 +
26495*a^2*c^3*x^3 + 226905*c^3*x)*arcsinh(a*x)/a)*a
```

Fricas [A] time = 2.24818, size = 603, normalized size = 1.68

$$\frac{385875\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-11025\left(75\,a^6c^3x^6+351\,a^4c^3x^4+757\,a^2c^3x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/13505625*(385875*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*
c^3*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 11025*(75*a^6*c^3*x^6 + 351*a^4*c^3
*x^4 + 757*a^2*c^3*x^2 + 2161*c^3)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1))^2 + 210*(1125*a^7*c^3*x^7 + 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 + 2
26905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(16875*a^6*c^3*x^6 + 134541
*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 + 22329151*c^3)*sqrt(a^2*x^2 + 1))/a
```

Sympy [A] time = 25.4759, size = 355, normalized size = 0.99

$$\begin{cases} \frac{a^6c^3x^7 \operatorname{asinh}^3(ax)}{7} + \frac{6a^6c^3x^7 \operatorname{asinh}(ax)}{343} - \frac{3a^5c^3x^6\sqrt{a^2x^2+1} \operatorname{asinh}^2(ax)}{49} - \frac{6a^5c^3x^6\sqrt{a^2x^2+1}}{2401} + \frac{3a^4c^3x^5 \operatorname{asinh}^3(ax)}{5} + \frac{702a^4c^3x^5 \operatorname{asinh}(ax)}{6125} - \frac{351a^3c \cdots}{} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**6*c**3*x**7*asinh(a*x)**3/7 + 6*a**6*c**3*x**7*asinh(a*x)/343
 - 3*a**5*c**3*x**6*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/49 - 6*a**5*c**3*x**6
*sqrt(a**2*x**2 + 1)/2401 + 3*a**4*c**3*x**5*asinh(a*x)**3/5 + 702*a**4*c**
3*x**5*asinh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)*
*2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 + 1)/1500625 + a**2*c**3*x**3
*asinh(a*x)**3 + 1514*a**2*c**3*x**3*asinh(a*x)/3675 - 757*a*c**3*x**2*sqrt
(a**2*x**2 + 1)*asinh(a*x)**2/1225 - 1495874*a*c**3*x**2*sqrt(a**2*x**2 + 1
)/13505625 + c**3*x*asinh(a*x)**3 + 4322*c**3*x*asinh(a*x)/1225 - 2161*c**3
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
+ 1)/(13505625*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.67381, size = 333, normalized size = 0.93

$$\frac{1}{13505625}\left(210\left(1125\,a^6x^7 + 7371\,a^4x^5 + 26495\,a^2x^3 + 226905\,x\right)\log\left(ax + \sqrt{a^2x^2+1}\right) - \frac{11025\left(75\left(a^2x^2+1\right)^{\frac{7}{2}} + 126 \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/13505625*(210*(1125*a^6*x^7 + 7371*a^4*x^5 + 26495*a^2*x^3 + 226905*x)*lo
g(a*x + sqrt(a^2*x^2 + 1)) - 11025*(75*(a^2*x^2 + 1)^(7/2) + 126*(a^2*x^2 +
 1)^(5/2) + 280*(a^2*x^2 + 1)^(3/2) + 1680*sqrt(a^2*x^2 + 1))*log(a*x + sqr
t(a^2*x^2 + 1))^2/a - 2*(16875*(a^2*x^2 + 1)^(7/2) + 83916*(a^2*x^2 + 1)^(5
/2) + 529480*(a^2*x^2 + 1)^(3/2) + 21698880*sqrt(a^2*x^2 + 1))/a)*c^3 + 1/3
5*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*log(a*x + sq
rt(a^2*x^2 + 1))^3
```

3.329 $\int \left(c + a^2cx^2\right)^2 \sinh^{-1}(ax)^3\, dx$

Optimal. Leaf size=265

$$-\frac{6c^2\left(a^2x^2+1\right)^{5/2}}{625a} - \frac{272c^2\left(a^2x^2+1\right)^{3/2}}{3375a} - \frac{4144c^2\sqrt{a^2x^2+1}}{1125a} + \frac{6}{125}a^4c^2x^5\sinh^{-1}(ax) + \frac{76}{225}a^2c^2x^3\sinh^{-1}(ax) + \frac{1}{5}c^2x\,($$

```
[Out] (-4144*c^2*Sqrt[1 + a^2*x^2])/(1125*a) - (272*c^2*(1 + a^2*x^2)^(3/2))/(337
5*a) - (6*c^2*(1 + a^2*x^2)^(5/2))/(625*a) + (298*c^2*x*ArcSinh[a*x])/75 +
(76*a^2*c^2*x^3*ArcSinh[a*x])/225 + (6*a^4*c^2*x^5*ArcSinh[a*x])/125 - (8*c
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(5*a) - (4*c^2*(1 + a^2*x^2)^(3/2)*Arc
Sinh[a*x]^2)/(15*a) - (3*c^2*(1 + a^2*x^2)^(5/2)*ArcSinh[a*x]^2)/(25*a) + (
8*c^2*x*ArcSinh[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/15 + (c
^2*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5
```

Rubi [A] time = 0.421442, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5684, 5653, 5717, 261, 5679, 444, 43, 194, 12, 1247, 698}

$$-\frac{6c^2\left(a^2x^2+1\right)^{5/2}}{625a} - \frac{272c^2\left(a^2x^2+1\right)^{3/2}}{3375a} - \frac{4144c^2\sqrt{a^2x^2+1}}{1125a} + \frac{6}{125}a^4c^2x^5\sinh^{-1}(ax) + \frac{76}{225}a^2c^2x^3\sinh^{-1}(ax) + \frac{1}{5}c^2x\,($$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```


```
[Out] (-4144*c^2*Sqrt[1 + a^2*x^2])/(1125*a) - (272*c^2*(1 + a^2*x^2)^(3/2))/(337
5*a) - (6*c^2*(1 + a^2*x^2)^(5/2))/(625*a) + (298*c^2*x*ArcSinh[a*x])/75 +
(76*a^2*c^2*x^3*ArcSinh[a*x])/225 + (6*a^4*c^2*x^5*ArcSinh[a*x])/125 - (8*c
^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(5*a) - (4*c^2*(1 + a^2*x^2)^(3/2)*Arc
Sinh[a*x]^2)/(15*a) - (3*c^2*(1 + a^2*x^2)^(5/2)*ArcSinh[a*x]^2)/(25*a) + (
8*c^2*x*ArcSinh[a*x]^3)/15 + (4*c^2*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/15 + (c
^2*x*(1 + a^2*x^2)^2*ArcSinh[a*x]^3)/5
```


Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^3\,dx = \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^3+\frac{1}{5}(4c)\int \left(c+a^2cx^2\right)\sinh^{-1}(ax)^3\,dx-\frac{1}{5}\left(3ac^2\right)\int x\left(1+\right.$$

$$= -\frac{3c^2\left(1+a^2x^2\right)^{5/2}\sinh^{-1}(ax)^2}{25a}+\frac{4}{15}c^2x\left(1+a^2x^2\right)\sinh^{-1}(ax)^3+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\sinh$$

$$= \frac{6}{25}c^2x\sinh^{-1}(ax)+\frac{4}{25}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)-\frac{4c^2\left(1+a^2x^2\right)^{3/2}\text{si}}{15a}$$

$$= \frac{58}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)-\frac{8c^2\sqrt{1+a^2x^2}\,\text{sin}}{5a}$$

$$= \frac{298}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax)-\frac{8c^2\sqrt{1+a^2x^2}\,\text{si}}{5a}$$

$$= -\frac{16c^2\sqrt{1+a^2x^2}}{5a}+\frac{298}{75}c^2x\sinh^{-1}(ax)+\frac{76}{225}a^2c^2x^3\sinh^{-1}(ax)+\frac{6}{125}a^4c^2x^5\sinh^{-1}(ax$$

$$= -\frac{4144c^2\sqrt{1+a^2x^2}}{1125a}-\frac{272c^2\left(1+a^2x^2\right)^{3/2}}{3375a}-\frac{6c^2\left(1+a^2x^2\right)^{5/2}}{625a}+\frac{298}{75}c^2x\sinh^{-1}(ax)+\frac{7}{22}$$

Mathematica [A] time = 0.144417, size = 137, normalized size = 0.52

$$\frac{c^2\left(-2\sqrt{a^2x^2+1}\left(81a^4x^4+842a^2x^2+31841\right)+1125ax\left(3a^4x^4+10a^2x^2+15\right)\sinh^{-1}(ax)^3-225\sqrt{a^2x^2+1}\left(9a^4x^4+38a\right.\right.}{16875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*(-2*Sqrt[1 + a^2*x^2]*(31841 + 842*a^2*x^2 + 81*a^4*x^4) + 30*a*x*(223
5 + 190*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(149 + 3
8*a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a*x*(15 + 10*a^2*x^2 + 3*a^4*x
^4)*ArcSinh[a*x]^3))/(16875*a)
```

Maple [A] time = 0.046, size = 200, normalized size = 0.8

$$\frac{c^2}{16875\,a}\left(3375\,(\text{Arcsinh}\,(ax))^3\,a^5x^5-2025\,(\text{Arcsinh}\,(ax))^2\sqrt{a^2x^2+1}a^4x^4+11250\,(\text{Arcsinh}\,(ax))^3\,a^3x^3+810\,\text{Arcsinh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x)
```

```
[Out] 1/16875/a*c^2*(3375*arcsinh(a*x)^3*a^5*x^5-2025*arcsinh(a*x)^2*(a^2*x^2+1)^
(1/2)*a^4*x^4+11250*arcsinh(a*x)^3*a^3*x^3+810*arcsinh(a*x)*a^5*x^5-8550*a^
2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-162*a^4*x^4*(a^2*x^2+1)^(1/2)+16875*
arcsinh(a*x)^3*a*x+5700*arcsinh(a*x)*a^3*x^3-33525*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)-1684*a^2*x^2*(a^2*x^2+1)^(1/2)+67050*a*x*arcsinh(a*x)-63682*(a^2*x
^2+1)^(1/2))
```

Maxima [A] time = 1.21228, size = 284, normalized size = 1.07

$$-\frac{1}{75}\left(9\sqrt{a^2x^2+1}a^2c^2x^4+38\sqrt{a^2x^2+1}c^2x^2+\frac{149\sqrt{a^2x^2+1}c^2}{a^2}\right)a\operatorname{arsinh}(ax)^2+\frac{1}{15}\left(3a^4c^2x^5+10a^2c^2x^3+15c^2x\right)\operatorname{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/75*(9*sqrt(a^2*x^2 + 1)*a^2*c^2*x^4 + 38*sqrt(a^2*x^2 + 1)*c^2*x^2 + 149
*sqrt(a^2*x^2 + 1)*c^2/a^2)*a*arcsinh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 + 10*a^2
*c^2*x^3 + 15*c^2*x)*arcsinh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 + 1)*a^2*c^2
*x^4 + 842*sqrt(a^2*x^2 + 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 + 190*a^2*c^2*x^3
 + 2235*c^2*x)*arcsinh(a*x)/a + 31841*sqrt(a^2*x^2 + 1)*c^2/a^2)*a
```

Fricas [A] time = 2.17195, size = 467, normalized size = 1.76

$$1125\left(3a^5c^2x^5+10a^3c^2x^3+15ac^2x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\left(9a^4c^2x^4+38a^2c^2x^2+149c^2\right)\sqrt{a^2x^2+1}\log\left(ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 + 1))^3 - 225*(9*a^4*c^2*x^4 + 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(27*a^5*c^2*x^5 + 190*a^3*c^2*x
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(81*a^4*c^2*x^4 + 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 + 1))/a
```

Sympy [A] time = 9.06585, size = 262, normalized size = 0.99

$$\begin{cases} \frac{a^4c^2x^5\operatorname{asinh}^3(ax)}{5} + \frac{6a^4c^2x^5\operatorname{asinh}(ax)}{125} - \frac{3a^3c^2x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25} - \frac{6a^3c^2x^4\sqrt{a^2x^2+1}}{625} + \frac{2a^2c^2x^3\operatorname{asinh}^3(ax)}{3} + \frac{76a^2c^2x^3\operatorname{asinh}(ax)}{225} - \frac{38ac^2x^2\sqrt{}}{} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*asinh(a*x)**3/5 + 6*a**4*c**2*x**5*asinh(a*x)/125
 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/25 - 6*a**3*c**2*x**4
*sqrt(a**2*x**2 + 1)/625 + 2*a**2*c**2*x**3*asinh(a*x)**3/3 + 76*a**2*c**2*
x**3*asinh(a*x)/225 - 38*a*c**2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/75 -
 1684*a*c**2*x**2*sqrt(a**2*x**2 + 1)/16875 + c**2*x*asinh(a*x)**3 + 298*c*
*2*x*asinh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(75*a) - 63
682*c**2*sqrt(a**2*x**2 + 1)/(16875*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.72186, size = 273, normalized size = 1.03

$$\frac{1}{15}\left(3\,a^4c^2x^5 + 10\,a^2c^2x^3 + 15\,c^2x\right)\log\left(ax + \sqrt{a^2x^2+1}\right)^3 + \frac{1}{16875}\left(30\left(27\,a^4x^5 + 190\,a^2x^3 + 2235\,x\right)\log\left(ax + \sqrt{a^2x^2+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*log(a*x + sqrt(a^2*x^2 + 1
))^3 + 1/16875*(30*(27*a^4*x^5 + 190*a^2*x^3 + 2235*x)*log(a*x + sqrt(a^2*x
^2 + 1)) - 225*(9*(a^2*x^2 + 1)^(5/2) + 20*(a^2*x^2 + 1)^(3/2) + 120*sqrt(a
^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2/a - 2*(81*(a^2*x^2 + 1)^(5/2) +
 680*(a^2*x^2 + 1)^(3/2) + 31080*sqrt(a^2*x^2 + 1))/a)*c^2
```

3.330 $\int \left(c+a^2cx^2\right)\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=153

$$-\frac{2c\left(a^2x^2+1\right)^{3/2}}{27a}-\frac{40c\sqrt{a^2x^2+1}}{9a}+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)+\frac{1}{3}cx\left(a^2x^2+1\right)\sinh^{-1}(ax)^3-\frac{c\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)^2}{3a}-2$$

```
[Out] (-40*c*Sqrt[1 + a^2*x^2])/(9*a) - (2*c*(1 + a^2*x^2)^(3/2))/(27*a) + (14*c*
x*ArcSinh[a*x])/3 + (2*a^2*c*x^3*ArcSinh[a*x])/9 - (2*c*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^2)/a - (c*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(3*a) + (2*c*x*Ar
cSinh[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3
```

Rubi [A] time = 0.217183, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5684, 5653, 5717, 261, 5679, 444, 43}

$$-\frac{2c\left(a^2x^2+1\right)^{3/2}}{27a}-\frac{40c\sqrt{a^2x^2+1}}{9a}+\frac{2}{9}a^2cx^3\sinh^{-1}(ax)+\frac{1}{3}cx\left(a^2x^2+1\right)\sinh^{-1}(ax)^3-\frac{c\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)^2}{3a}-2$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```


```
[Out] (-40*c*Sqrt[1 + a^2*x^2])/(9*a) - (2*c*(1 + a^2*x^2)^(3/2))/(27*a) + (14*c*
x*ArcSinh[a*x])/3 + (2*a^2*c*x^3*ArcSinh[a*x])/9 - (2*c*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^2)/a - (c*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x]^2)/(3*a) + (2*c*x*Ar
cSinh[a*x]^3)/3 + (c*x*(1 + a^2*x^2)*ArcSinh[a*x]^3)/3
```


Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
```

```
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(c + a^2cx^2\right)\sinh^{-1}(ax)^3\,dx = \frac{1}{3}cx\left(1 + a^2x^2\right)\sinh^{-1}(ax)^3 + \frac{1}{3}(2c)\int \sinh^{-1}(ax)^3\,dx - (ac)\int x\sqrt{1 + a^2x^2}\sinh^{-1}(ax)^2$$

$$= -\frac{c\left(1 + a^2x^2\right)^{3/2}\sinh^{-1}(ax)^2}{3a} + \frac{2}{3}cx\sinh^{-1}(ax)^3 + \frac{1}{3}cx\left(1 + a^2x^2\right)\sinh^{-1}(ax)^3 + \frac{1}{3}(2c)$$

$$= \frac{2}{3}cx\sinh^{-1}(ax) + \frac{2}{9}a^2cx^3\sinh^{-1}(ax) - \frac{2c\sqrt{1 + a^2x^2}\sinh^{-1}(ax)^2}{a} - \frac{c\left(1 + a^2x^2\right)^{3/2}\sinh}{3a}$$

$$= \frac{14}{3}cx\sinh^{-1}(ax) + \frac{2}{9}a^2cx^3\sinh^{-1}(ax) - \frac{2c\sqrt{1 + a^2x^2}\sinh^{-1}(ax)^2}{a} - \frac{c\left(1 + a^2x^2\right)^{3/2}\sin}{3a}$$

$$= -\frac{4c\sqrt{1 + a^2x^2}}{a} + \frac{14}{3}cx\sinh^{-1}(ax) + \frac{2}{9}a^2cx^3\sinh^{-1}(ax) - \frac{2c\sqrt{1 + a^2x^2}\sinh^{-1}(ax)^2}{a} - \frac{c}{}$$

$$= -\frac{40c\sqrt{1 + a^2x^2}}{9a} - \frac{2c\left(1 + a^2x^2\right)^{3/2}}{27a} + \frac{14}{3}cx\sinh^{-1}(ax) + \frac{2}{9}a^2cx^3\sinh^{-1}(ax) - \frac{2c\sqrt{1 + a}}{}$$

Mathematica [A] time = 0.0819733, size = 99, normalized size = 0.65

$$\frac{c\left(-2\sqrt{a^2x^2 + 1}\left(a^2x^2 + 61\right) + 9ax\left(a^2x^2 + 3\right)\sinh^{-1}(ax)^3 - 9\sqrt{a^2x^2 + 1}\left(a^2x^2 + 7\right)\sinh^{-1}(ax)^2 + 6ax\left(a^2x^2 + 21\right)\sinh^{-}\right.}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*(-2*Sqrt[1 + a^2*x^2]*(61 + a^2*x^2) + 6*a*x*(21 + a^2*x^2)*ArcSinh[a*x]
 - 9*Sqrt[1 + a^2*x^2]*(7 + a^2*x^2)*ArcSinh[a*x]^2 + 9*a*x*(3 + a^2*x^2)*A
rcSinh[a*x]^3))/(27*a)
```

Maple [A] time = 0.036, size = 128, normalized size = 0.8

$$\frac{c}{27\,a}\left(9\,\left(\mathrm{Arcsinh}\,(ax)\right)^3 a^3x^3 - 9\,a^2x^2\left(\mathrm{Arcsinh}\,(ax)\right)^2\sqrt{a^2x^2+1} + 27\,\left(\mathrm{Arcsinh}\,(ax)\right)^3 ax + 6\,\mathrm{Arcsinh}\,(ax)\,a^3x^3 - 63\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)*arcsinh(a*x)^3,x)
```

```
[Out] 1/27/a*c*(9*arcsinh(a*x)^3*a^3*x^3-9*a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/
2)+27*arcsinh(a*x)^3*a*x+6*arcsinh(a*x)*a^3*x^3-63*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)-2*a^2*x^2*(a^2*x^2+1)^(1/2)+126*a*x*arcsinh(a*x)-122*(a^2*x^2+1)^(
1/2))
```

Maxima [A] time = 1.20982, size = 167, normalized size = 1.09

$$-\frac{1}{3}\left(\sqrt{a^2x^2+1}cx^2+\frac{7\sqrt{a^2x^2+1}c}{a^2}\right)a\,\mathrm{arsinh}\,(ax)^2+\frac{1}{3}\left(a^2cx^3+3\,cx\right)\mathrm{arsinh}\,(ax)^3-\frac{2}{27}\left(\sqrt{a^2x^2+1}cx^2-\frac{3\left(a^2cx^3+21\,cx\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(a^2*x^2 + 1)*c*x^2 + 7*sqrt(a^2*x^2 + 1)*c/a^2)*a*arcsinh(a*x)^2
 + 1/3*(a^2*c*x^3 + 3*c*x)*arcsinh(a*x)^3 - 2/27*(sqrt(a^2*x^2 + 1)*c*x^2 -
 3*(a^2*c*x^3 + 21*c*x)*arcsinh(a*x)/a + 61*sqrt(a^2*x^2 + 1)*c/a^2)*a
```

Fricas [A] time = 2.09969, size = 315, normalized size = 2.06

$$\frac{9\left(a^3cx^3+3\,acx\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-9\left(a^2cx^2+7\,c\right)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+6\left(a^3cx^3+21\,acx\right)\log\left(ax+\right.}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(a^3*c*x^3 + 3*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*(a^2*c*x^2
 + 7*c)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*c*x^3 + 2
1*a*c*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(a^2*c*x^2 + 61*c)*sqrt(a^2*x^2 +
 1))/a
```

Sympy [A] time = 2.46731, size = 150, normalized size = 0.98

$$\begin{cases}\frac{a^2cx^3\,\mathrm{asinh}^3\,(ax)}{3}+\frac{2a^2cx^3\,\mathrm{asinh}\,(ax)}{9}-\frac{acx^2\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{3}-\frac{2acx^2\sqrt{a^2x^2+1}}{27}+cx\,\mathrm{asinh}^3\,(ax)+\frac{14cx\,\mathrm{asinh}\,(ax)}{3}-\frac{7c\sqrt{a^2x^2+1}\,\mathrm{asinh}^2\,(ax)}{3a} \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)*asinh(a*x)**3,x)
```

```
[Out] Piecewise((a**2*c*x**3*asinh(a*x)**3/3 + 2*a**2*c*x**3*asinh(a*x)/9 - a*c*x
**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/3 - 2*a*c*x**2*sqrt(a**2*x**2 + 1)/27
 + c*x*asinh(a*x)**3 + 14*c*x*asinh(a*x)/3 - 7*c*sqrt(a**2*x**2 + 1)*asinh(
a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 + 1)/(27*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.67821, size = 196, normalized size = 1.28

$$\frac{1}{3}\left(a^2cx^3+3\,cx\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+\frac{1}{27}\left(6\left(a^2x^3+21\,x\right)\log\left(ax+\sqrt{a^2x^2+1}\right)-\frac{9\left(\left(a^2x^2+1\right)^{\frac{3}{2}}+6\sqrt{a^2x^2+1}\right)\log}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*c*x^3 + 3*c*x)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 1/27*(6*(a^2*x^3 +
 21*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 9*((a^2*x^2 + 1)^(3/2) + 6*sqrt(a^2*x
^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2/a - 2*((a^2*x^2 + 1)^(3/2) + 60*sqr
t(a^2*x^2 + 1))/a)*c
```

3.331 $\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx$

Optimal. Leaf size=174

$$-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{6i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\sinh^{-}}\right.}{ac}$$

```
[Out] (2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c) - ((3*I)*ArcSinh[a*x]^2*Pol
yLog[2, (-I)*E^ArcSinh[a*x]])/(a*c) + ((3*I)*ArcSinh[a*x]^2*PolyLog[2, I*E^
ArcSinh[a*x]])/(a*c) + ((6*I)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])
/(a*c) - ((6*I)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c) - ((6*I)*P
olyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c) + ((6*I)*PolyLog[4, I*E^ArcSinh[a*x]]
)/(a*c)
```

Rubi [A] time = 0.130053, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5693, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{ac}+\frac{6i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\sinh^{-}}\right.}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```


```
[Out] (2*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c) - ((3*I)*ArcSinh[a*x]^2*Pol
yLog[2, (-I)*E^ArcSinh[a*x]])/(a*c) + ((3*I)*ArcSinh[a*x]^2*PolyLog[2, I*E^
ArcSinh[a*x]])/(a*c) + ((6*I)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])
/(a*c) - ((6*I)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c) - ((6*I)*P
olyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c) + ((6*I)*PolyLog[4, I*E^ArcSinh[a*x]]
)/(a*c)
```


Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx = \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \sinh^{-1}(ax)\right)}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{(3i)\,\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \sinh^{-1}(ax)\right)}{ac} + \frac{(3i)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right.}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right.}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right.}{ac}$$

$$= \frac{2\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac} - \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(-ie^{\sinh^{-1}(ax)}\right)}{ac} + \frac{3i\sinh^{-1}(ax)^2\text{Li}_2\left(ie^{\sinh^{-1}(ax)}\right.}{ac}$$

Mathematica [B] time = 0.219007, size = 454, normalized size = 2.61

$$-\frac{i\left(192\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)+192i\pi\sinh^{-1}(ax)\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)+384\sinh^{-1}(ax)\text{PolyLog}\left(3,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2),x]
```

```
[Out] ((-I/64)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 - (32*I
)*Pi*ArcSinh[a*x]^3 - 16*ArcSinh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[a*
x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] - (96*I)*Pi*ArcSinh[a*
x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]
] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSinh[a*x]] + (96*I)*Pi*ArcSinh[a*x]
^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 64*Ar
cSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*Arc
Sinh[a*x])/4]] - 48*(Pi - (2*I)*ArcSinh[a*x])^2*PolyLog[2, (-I)/E^ArcSinh[a
*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 48*Pi^2*PolyLog
[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*x]
] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcSinh[a*x]*PolyLog[3
, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]] -
 (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)/E^ArcSinh[a*
x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{Arcsinh}(ax))^3}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{a^2cx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^2x^2+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c),x)
```

```
[Out] Integral(asinh(a*x)**3/(a**2*x**2 + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{a^2cx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c), x)
```

3.332 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=294

$$-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\sinh^{-1}(ax)}\right)}{ac^2}$$

```
[Out] (3*ArcSinh[a*x]^2)/(2*a*c^2*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(2*c^2*
(1 + a^2*x^2)) - (6*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + (ArcSinh
[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*PolyLog[2, (-I)*E^ArcSinh[
a*x]])/(a*c^2) - (((3*I)/2)*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]])
/(a*c^2) - ((3*I)*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + (((3*I)/2)*ArcSin
h[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*ArcSinh[a*x]*PolyLo
g[3, (-I)*E^ArcSinh[a*x]])/(a*c^2) - ((3*I)*ArcSinh[a*x]*PolyLog[3, I*E^Arc
Sinh[a*x]])/(a*c^2) - ((3*I)*PolyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^2) + ((3
*I)*PolyLog[4, I*E^ArcSinh[a*x]])/(a*c^2)
```

Rubi [A] time = 0.306182, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5690, 5693, 4180, 2531, 6609, 2282, 6589, 5717, 2279, 2391}

$$-\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{2ac^2}+\frac{3i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\sinh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```


```
[Out] (3*ArcSinh[a*x]^2)/(2*a*c^2*Sqrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(2*c^2*
(1 + a^2*x^2)) - (6*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + (ArcSinh
[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*PolyLog[2, (-I)*E^ArcSinh[
a*x]])/(a*c^2) - (((3*I)/2)*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]])
/(a*c^2) - ((3*I)*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + (((3*I)/2)*ArcSin
h[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^2) + ((3*I)*ArcSinh[a*x]*PolyLo
g[3, (-I)*E^ArcSinh[a*x]])/(a*c^2) - ((3*I)*ArcSinh[a*x]*PolyLog[3, I*E^Arc
Sinh[a*x]])/(a*c^2) - ((3*I)*PolyLog[4, (-I)*E^ArcSinh[a*x]])/(a*c^2) + ((3
*I)*PolyLog[4, I*E^ArcSinh[a*x]])/(a*c^2)
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx = \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{(3a)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\sinh^{-1}(ax)^3}{c+a^2cx^2}\,dx}{2c}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{3\int \frac{\sinh^{-1}(ax)}{1+a^2x^2}\,dx}{c^2} + \frac{\text{Subst}\left(\int x^3\,\textbf{sech}(x)\,dx, x, \sinh^{-1}(ax)\right)}{2ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} - \frac{(3i)\,\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,d\right.}{2ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}$$

$$= \frac{3\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{2c^2\left(1+a^2x^2\right)} - \frac{6\sinh^{-1}(ax)\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2} + \frac{\sinh^{-1}(ax)^3\tan^{-1}\left(e^{\sinh^{-1}(ax)}\right)}{ac^2}$$

Mathematica [A] time = 2.53661, size = 568, normalized size = 1.93

$$-\frac{i\left(192\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)+192i\pi\sinh^{-1}(ax)\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)+384\sinh^{-1}(ax)\text{PolyLog}\left(3,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^2,x]
```

```
[Out] ((-I/128)*(7*Pi^4 + (8*I)*Pi^3*ArcSinh[a*x] + 24*Pi^2*ArcSinh[a*x]^2 + ((19
2*I)*ArcSinh[a*x]^2)/Sqrt[1 + a^2*x^2] - (32*I)*Pi*ArcSinh[a*x]^3 + ((64*I)
*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 16*ArcSinh[a*x]^4 - 384*ArcSinh[a*x]*L
og[1 - I/E^ArcSinh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcSinh[a*x]] + 384*ArcSi
nh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 48*Pi^2*ArcSinh[a*x]*Log[1 + I/E^ArcSin
h[a*x]] - (96*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^ArcSinh[a*x]] - 64*ArcSinh[a
*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 48*Pi^2*ArcSinh[a*x]*Log[1 - I*E^ArcSinh[
a*x]] + (96*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^ArcSinh[a*x]] - (8*I)*Pi^3*Log
```

```
[1 + I*E^ArcSinh[a*x]] + 64*ArcSinh[a*x]^3*Log[1 + I*E^ArcSinh[a*x]] + (8*I
)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*ArcS
inh[a*x] - 4*ArcSinh[a*x]^2)*PolyLog[2, (-I)/E^ArcSinh[a*x]] + 384*PolyLog[
2, I/E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^2*PolyLog[2, (-I)*E^ArcSinh[a*x]] -
 48*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]] + (192*I)*Pi*ArcSinh[a*x]*PolyLog[2,
I*E^ArcSinh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcSinh[a*x]] + 384*ArcSin
h[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x]] - 384*ArcSinh[a*x]*PolyLog[3, (-I)*E
^ArcSinh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcSinh[a*x]] + 384*PolyLog[4, (
-I)/E^ArcSinh[a*x]] + 384*PolyLog[4, (-I)*E^ArcSinh[a*x]]))/(a*c^2)
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \frac{(\text{Arcsinh}\,(ax))^3}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsinh}\,(ax)^3}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{arsinh}\,(ax)^3}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(asinh(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.333 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=409

$$-\frac{9i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\sinh}\right.}{4ac^3}$$

```
[Out] -1/(4*a*c^3*Sqrt[1 + a^2*x^2]) - (x*ArcSinh[a*x])/(4*c^3*(1 + a^2*x^2)) + A
rcSinh[a*x]^2/(4*a*c^3*(1 + a^2*x^2)^(3/2)) + (9*ArcSinh[a*x]^2)/(8*a*c^3*S
qrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcSi
nh[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (5*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/
(a*c^3) + (3*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(4*a*c^3) + (((5*I)/2)*
PolyLog[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/8)*ArcSinh[a*x]^2*PolyLo
g[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((5*I)/2)*PolyLog[2, I*E^ArcSinh[a*x]
])/(a*c^3) + (((9*I)/8)*ArcSinh[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^3
) + (((9*I)/4)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9
*I)/4)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/4)*Poly
Log[4, (-I)*E^ArcSinh[a*x]])/(a*c^3) + (((9*I)/4)*PolyLog[4, I*E^ArcSinh[a*
x]])/(a*c^3)
```

Rubi [A] time = 0.516848, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5690, 5693, 4180, 2531, 6609, 2282, 6589, 5717, 2279, 2391, 261}

$$-\frac{9i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right)}{8ac^3}+\frac{9i\sinh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\sinh}\right.}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```


```
[Out] -1/(4*a*c^3*Sqrt[1 + a^2*x^2]) - (x*ArcSinh[a*x])/(4*c^3*(1 + a^2*x^2)) + A
rcSinh[a*x]^2/(4*a*c^3*(1 + a^2*x^2)^(3/2)) + (9*ArcSinh[a*x]^2)/(8*a*c^3*S
qrt[1 + a^2*x^2]) + (x*ArcSinh[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) + (3*x*ArcSi
nh[a*x]^3)/(8*c^3*(1 + a^2*x^2)) - (5*ArcSinh[a*x]*ArcTan[E^ArcSinh[a*x]])/
(a*c^3) + (3*ArcSinh[a*x]^3*ArcTan[E^ArcSinh[a*x]])/(4*a*c^3) + (((5*I)/2)*
PolyLog[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/8)*ArcSinh[a*x]^2*PolyLo
g[2, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((5*I)/2)*PolyLog[2, I*E^ArcSinh[a*x]
```

```
])/(a*c^3) + (((9*I)/8)*ArcSinh[a*x]^2*PolyLog[2, I*E^ArcSinh[a*x]])/(a*c^3
) + (((9*I)/4)*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]])/(a*c^3) - (((9
*I)/4)*ArcSinh[a*x]*PolyLog[3, I*E^ArcSinh[a*x]])/(a*c^3) - (((9*I)/4)*Poly
Log[4, (-I)*E^ArcSinh[a*x]])/(a*c^3) + (((9*I)/4)*PolyLog[4, I*E^ArcSinh[a*
x]])/(a*c^3)
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx = \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} - \frac{(3a)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\sinh^{-1}(ax)^3}{8c^3\left(1+a^2x^2\right)} - \frac{\int \frac{\sinh^{-1}(ax)}{\left(1+a^2x^2\right)^2}\,dx}{2c^3} - \frac{(9a)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^{3/2}}\,dx}{8c^3} + \frac{3}{}$$

$$= -\frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\sinh^{-1}(ax)^3}{8c^3\left(1+a^2x^2\right)} - \int \frac{\sin}{1}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\,\mathrm{si}}{8c^3(}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\,\mathrm{si}}{8c^3(}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\,\mathrm{si}}{8c^3(}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\,\mathrm{si}}{8c^3(}$$

$$= -\frac{1}{4ac^3\sqrt{1+a^2x^2}} - \frac{x\sinh^{-1}(ax)}{4c^3\left(1+a^2x^2\right)} + \frac{\sinh^{-1}(ax)^2}{4ac^3\left(1+a^2x^2\right)^{3/2}} + \frac{9\sinh^{-1}(ax)^2}{8ac^3\sqrt{1+a^2x^2}} + \frac{x\sinh^{-1}(ax)^3}{4c^3\left(1+a^2x^2\right)^2} + \frac{3x\,\mathrm{si}}{8c^3(}$$

<u>Mathematica</u> [A] time = 5.36398, size = 654, normalized size = 1.6

$$-\frac{i\left(576\sinh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(ax)}\right) + 576i\pi\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(ax)}\right) + 1152\sinh^{-1}(ax)\mathrm{PolyLog}\left(3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^3,x]
```

```
[Out] ((-I/512)*(21*Pi^4 - (128*I)/Sqrt[1 + a^2*x^2] + (24*I)*Pi^3*ArcSinh[a*x] -
 ((128*I)*a*x*ArcSinh[a*x])/(1 + a^2*x^2) + 72*Pi^2*ArcSinh[a*x]^2 + ((128*
```

```
I)*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + ((576*I)*ArcSinh[a*x]^2)/Sqrt[1 +
a^2*x^2] - (96*I)*Pi*ArcSinh[a*x]^3 + ((128*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2
*x^2)^2 + ((192*I)*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 48*ArcSinh[a*x]^4 -
1280*ArcSinh[a*x]*Log[1 - I/E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[1 + I/E^ArcSi
nh[a*x]] + 1280*ArcSinh[a*x]*Log[1 + I/E^ArcSinh[a*x]] + 144*Pi^2*ArcSinh[a
*x]*Log[1 + I/E^ArcSinh[a*x]] - (288*I)*Pi*ArcSinh[a*x]^2*Log[1 + I/E^ArcSi
nh[a*x]] - 192*ArcSinh[a*x]^3*Log[1 + I/E^ArcSinh[a*x]] - 144*Pi^2*ArcSinh[
a*x]*Log[1 - I*E^ArcSinh[a*x]] + (288*I)*Pi*ArcSinh[a*x]^2*Log[1 - I*E^ArcS
inh[a*x]] - (24*I)*Pi^3*Log[1 + I*E^ArcSinh[a*x]] + 192*ArcSinh[a*x]^3*Log[
1 + I*E^ArcSinh[a*x]] + (24*I)*Pi^3*Log[Tan[(Pi + (2*I)*ArcSinh[a*x])/4]] -
 16*(80 + 9*Pi^2 - (36*I)*Pi*ArcSinh[a*x] - 36*ArcSinh[a*x]^2)*PolyLog[2, (
-I)/E^ArcSinh[a*x]] + 1280*PolyLog[2, I/E^ArcSinh[a*x]] + 576*ArcSinh[a*x]^
2*PolyLog[2, (-I)*E^ArcSinh[a*x]] - 144*Pi^2*PolyLog[2, I*E^ArcSinh[a*x]] +
 (576*I)*Pi*ArcSinh[a*x]*PolyLog[2, I*E^ArcSinh[a*x]] + (576*I)*Pi*PolyLog[
3, (-I)/E^ArcSinh[a*x]] + 1152*ArcSinh[a*x]*PolyLog[3, (-I)/E^ArcSinh[a*x]]
 - 1152*ArcSinh[a*x]*PolyLog[3, (-I)*E^ArcSinh[a*x]] - (576*I)*Pi*PolyLog[3
, I*E^ArcSinh[a*x]] + 1152*PolyLog[4, (-I)/E^ArcSinh[a*x]] + 1152*PolyLog[4
, (-I)*E^ArcSinh[a*x]]))/(a*c^3)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{Arcsinh}(ax))^3}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

```
[Out] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{(a^2cx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{asinh}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**3,x)
```

```
[Out] Integral(asinh(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

3.334 $\int \left(c+a^2cx^2\right)^{5/2} \sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=509

$$-\frac{65a^3c^2x^4\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{865ac^2x^2\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{c^2\left(a^2x^2+1\right)^{5/2}\sqrt{a^2cx^2+c}}{216a}-\frac{15ac^2x^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{32\sqrt{a^2x^2+1}}+\frac{5}{16}c^2x$$

```
[Out] (-865*a*c^2*x^2*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (65*a^3*c^2
*x^4*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (c^2*(1 + a^2*x^2)^(5/
2)*Sqrt[c + a^2*c*x^2])/(216*a) + (245*c^2*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x])/384 + (65*c^2*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/576 + (
c^2*x*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/36 - (115*c^2*Sqrt[
c + a^2*c*x^2]*ArcSinh[a*x]^2)/(768*a*Sqrt[1 + a^2*x^2]) - (15*a*c^2*x^2*Sq
rt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*Sqrt[1 + a^2*x^2]) - (5*c^2*(1 + a^2*
x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*a) - (c^2*(1 + a^2*x^2)^
(5/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x]^3)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/24 +
 (x*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3)/6 + (5*c^2*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x]^4)/(64*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.594754, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5684, 5682, 5675, 5661, 5758, 30, 5717, 14, 261}

$$-\frac{65a^3c^2x^4\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{865ac^2x^2\sqrt{a^2cx^2+c}}{2304\sqrt{a^2x^2+1}}-\frac{c^2\left(a^2x^2+1\right)^{5/2}\sqrt{a^2cx^2+c}}{216a}-\frac{15ac^2x^2\sqrt{a^2cx^2+c}\,\sinh^{-1}(ax)^2}{32\sqrt{a^2x^2+1}}+\frac{5}{16}c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (-865*a*c^2*x^2*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (65*a^3*c^2
*x^4*Sqrt[c + a^2*c*x^2])/(2304*Sqrt[1 + a^2*x^2]) - (c^2*(1 + a^2*x^2)^(5/
2)*Sqrt[c + a^2*c*x^2])/(216*a) + (245*c^2*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*
x])/384 + (65*c^2*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/576 + (
c^2*x*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/36 - (115*c^2*Sqrt[
c + a^2*c*x^2]*ArcSinh[a*x]^2)/(768*a*Sqrt[1 + a^2*x^2]) - (15*a*c^2*x^2*Sq
rt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*Sqrt[1 + a^2*x^2]) - (5*c^2*(1 + a^2*
x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(32*a) - (c^2*(1 + a^2*x^2)^
(5/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(12*a) + (5*c^2*x*Sqrt[c + a^2*c*
```

```
x^2]*ArcSinh[a*x]^3)/16 + (5*c*x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3)/24 +
 (x*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3)/6 + (5*c^2*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x]^4)/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

```
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^3\,dx = \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^3+\frac{1}{6}(5c)\int\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3\,dx-\frac{\left(ac^2\sqrt{c+a^2c\right.}{}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{12a}+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3+\frac{1}{6}x\left(c+\right.$$

$$= \frac{1}{36}c^2x\left(1+a^2x^2\right)^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)-\frac{5c^2\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{32a}-$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}}{216a}+\frac{65}{576}c^2x\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sinh^{-1}(ax)+\frac{1}{36}c^2x\left(1+\right.$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}}{216a}+\frac{245}{384}c^2x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)+\frac{65}{576}c^2x\left(1+a^2x^2\right)\sqrt{c\right.}$$

$$= -\frac{865ac^2x^2\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}-\frac{65a^3c^2x^4\sqrt{c+a^2cx^2}}{2304\sqrt{1+a^2x^2}}-\frac{c^2\left(1+a^2x^2\right)^{5/2}\sqrt{c+a^2cx^2}}{216a}+\frac{245}{384}c^2x$$

Mathematica [A] time = 0.685039, size = 177, normalized size = 0.35

$$\frac{c^2\sqrt{a^2cx^2+c}\left(4320\sinh^{-1}(ax)^4+288\left(45\sinh\left(2\sinh^{-1}(ax)\right)+9\sinh\left(4\sinh^{-1}(ax)\right)+\sinh\left(6\sinh^{-1}(ax)\right)\right)\sinh^{-1}(ax\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(4320*ArcSinh[a*x]^4 - 9720*Cosh[2*ArcSinh[a*x]] -
 243*Cosh[4*ArcSinh[a*x]] - 8*Cosh[6*ArcSinh[a*x]] - 72*ArcSinh[a*x]^2*(270
*Cosh[2*ArcSinh[a*x]] + 27*Cosh[4*ArcSinh[a*x]] + 2*Cosh[6*ArcSinh[a*x]]) +
 288*ArcSinh[a*x]^3*(45*Sinh[2*ArcSinh[a*x]] + 9*Sinh[4*ArcSinh[a*x]] + Sin
h[6*ArcSinh[a*x]]) + 12*ArcSinh[a*x]*(1620*Sinh[2*ArcSinh[a*x]] + 81*Sinh[4
*ArcSinh[a*x]] + 4*Sinh[6*ArcSinh[a*x]])))/(55296*a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.176, size = 802, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x)
```

```
[Out] 5/64*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c^2+1/13824*(
c*(a^2*x^2+1))^(1/2)*(32*x^7*a^7+32*a^6*x^6*(a^2*x^2+1)^(1/2)+64*x^5*a^5+48
*a^4*x^4*(a^2*x^2+1)^(1/2)+38*x^3*a^3+18*a^2*x^2*(a^2*x^2+1)^(1/2)+6*a*x+(a
^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3-18*arcsinh(a*x)^2+6*arcsinh(a*x)-1)*c^2
/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5+8*a^4*x^4*(a^2*x^2+1
)^(1/2)+12*x^3*a^3+8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x+(a^2*x^2+1)^(1/2))*(32
*arcsinh(a*x)^3-24*arcsinh(a*x)^2+12*arcsinh(a*x)-3)*c^2/a/(a^2*x^2+1)+15/5
12*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*
x^2+1)^(1/2))*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c^2/a/(a
^2*x^2+1)+15/512*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*a^2*x^2*(a^2*x^2+1)^(1/
2)+2*a*x-(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*
x)+3)*c^2/a/(a^2*x^2+1)+3/4096*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*a^4*x^4*(
a^2*x^2+1)^(1/2)+12*x^3*a^3-8*a^2*x^2*(a^2*x^2+1)^(1/2)+4*a*x-(a^2*x^2+1)^(
1/2))*(32*arcsinh(a*x)^3+24*arcsinh(a*x)^2+12*arcsinh(a*x)+3)*c^2/a/(a^2*x^
2+1)+1/13824*(c*(a^2*x^2+1))^(1/2)*(32*x^7*a^7-32*a^6*x^6*(a^2*x^2+1)^(1/2)
+64*x^5*a^5-48*a^4*x^4*(a^2*x^2+1)^(1/2)+38*x^3*a^3-18*a^2*x^2*(a^2*x^2+1)^
(1/2)+6*a*x-(a^2*x^2+1)^(1/2))*(36*arcsinh(a*x)^3+18*arcsinh(a*x)^2+6*arcsi
nh(a*x)+1)*c^2/a/(a^2*x^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4c^2x^4 + 2\,a^2c^2x^2 + c^2\right)\sqrt{a^2cx^2 + c}\,\text{arsinh}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arcsinh(a*
x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)*asinh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.335 $\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=348

$$-\frac{3a^3cx^4\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{51acx^2\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{9acx^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{16\sqrt{a^2x^2+1}}+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\sinh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{a^2cx^2}$$

```
[Out] (-51*a*c*x^2*Sqrt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) - (3*a^3*c*x^4*Sq
rt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) + (45*c*x*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x])/64 + (3*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/32
- (27*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(128*a*Sqrt[1 + a^2*x^2]) - (9*
a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*Sqrt[1 + a^2*x^2]) - (3*c*(
1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*a) + (3*c*x*Sqrt
[c + a^2*c*x^2]*ArcSinh[a*x]^3)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3
)/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(32*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.347992, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5684, 5682, 5675, 5661, 5758, 30, 5717, 14}

$$-\frac{3a^3cx^4\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{51acx^2\sqrt{a^2cx^2+c}}{128\sqrt{a^2x^2+1}}-\frac{9acx^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{16\sqrt{a^2x^2+1}}+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\sinh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{a^2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```


```
[Out] (-51*a*c*x^2*Sqrt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) - (3*a^3*c*x^4*Sq
rt[c + a^2*c*x^2])/(128*Sqrt[1 + a^2*x^2]) + (45*c*x*Sqrt[c + a^2*c*x^2]*Ar
cSinh[a*x])/64 + (3*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x])/32
- (27*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(128*a*Sqrt[1 + a^2*x^2]) - (9*
a*c*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*Sqrt[1 + a^2*x^2]) - (3*c*(
1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(16*a) + (3*c*x*Sqrt
[c + a^2*c*x^2]*ArcSinh[a*x]^3)/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3
)/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^4)/(32*a*Sqrt[1 + a^2*x^2])
```


Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
```

```
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3\,dx=\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^3+\frac{1}{4}(3c)\int\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3\,dx-\frac{\left(3ac\sqrt{c+a^2cx\right.}{}$$

$$=-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{16a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3+\frac{1}{4}x\left(c+a^2\right.$$

$$=\frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sinh^{-1}(ax)-\frac{9acx^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{16\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2\right.}{}$$

$$=\frac{45}{64}cx\sqrt{c+a^2cx^2}\sinh^{-1}(ax)+\frac{3}{32}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sinh^{-1}(ax)-\frac{9acx^2\sqrt{c+a}}{16\sqrt{}}$$

$$=-\frac{51acx^2\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}-\frac{3a^3cx^4\sqrt{c+a^2cx^2}}{128\sqrt{1+a^2x^2}}+\frac{45}{64}cx\sqrt{c+a^2cx^2}\sinh^{-1}(ax)+\frac{3}{32}cx\left(1+\right.$$

Mathematica [A] time = 0.281254, size = 136, normalized size = 0.39

$$\frac{c\sqrt{a^2cx^2+c}\left(96\sinh^{-1}(ax)^4+32\left(8\sinh\left(2\sinh^{-1}(ax)\right)+\sinh\left(4\sinh^{-1}(ax)\right)\right)\sinh^{-1}(ax)^3+12\left(32\sinh\left(2\sinh^{-1}(ax\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^3,x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(96*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*(16*Cosh[2*Ar
cSinh[a*x]] + Cosh[4*ArcSinh[a*x]]) - 3*(64*Cosh[2*ArcSinh[a*x]] + Cosh[4*A
```

```
rcSinh[a*x]]) + 32*ArcSinh[a*x]^3*(8*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[
a*x]]) + 12*ArcSinh[a*x]*(32*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])))
/(1024*a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.135, size = 484, normalized size = 1.4

$$\frac{3\,(\text{Arcsinh}\,(ax))^4\,c}{32\,a}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{\left(32\,(\text{Arcsinh}\,(ax))^3-24\,(\text{Arcsinh}\,(ax))^2+12\,\text{Arcsinh}\,(ax)-3\right)c}{\left(2048\,a^2x^2+2048\right)a}\sqrt{c\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x)
```

```
[Out] 3/32*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4*c+1/2048*(c*(
a^2*x^2+1))^(1/2)*(8*x^5*a^5+8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*x^3*a^3+8*a^2*x
^2*(a^2*x^2+1)^(1/2)+4*a*x+(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3-24*arcsinh
(a*x)^2+12*arcsinh(a*x)-3)*c/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2+1))^(1/2)*(2*x^
3*a^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*x^2+1)^(1/2))*(4*arcsinh(a*x)^
3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)*c/(a^2*x^2+1)/a+1/32*(c*(a^2*x^2+1))^(
1/2)*(2*x^3*a^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x-(a^2*x^2+1)^(1/2))*(4*arc
sinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)*c/(a^2*x^2+1)/a+1/2048*(c*(a
^2*x^2+1))^(1/2)*(8*x^5*a^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+12*x^3*a^3-8*a^2*x^
2*(a^2*x^2+1)^(1/2)+4*a*x-(a^2*x^2+1)^(1/2))*(32*arcsinh(a*x)^3+24*arcsinh(
a*x)^2+12*arcsinh(a*x)+3)*c/(a^2*x^2+1)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2cx^2+c\right)^{\frac{3}{2}}\,\text{arsinh}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.336 $\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3\,dx$

Optimal. Leaf size=205

$$-\frac{3ax^2\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^4}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^3-\frac{3ax^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{4\sqrt{a^2x^2+1}}-\frac{3\sqrt{a^2cx^2+c}}{8a\sqrt{a^2}}$$

```
[Out] (-3*a*x^2*Sqrt[c + a^2*c*x^2])/(8*Sqrt[1 + a^2*x^2]) + (3*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x])/4 - (3*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(8*a*Sqrt[1 +
 a^2*x^2]) - (3*a*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(4*Sqrt[1 + a^2*x
^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcS
inh[a*x]^4)/(8*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.176458, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5682, 5675, 5661, 5758, 30}

$$-\frac{3ax^2\sqrt{a^2cx^2+c}}{8\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^4}{8a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^3-\frac{3ax^2\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^2}{4\sqrt{a^2x^2+1}}-\frac{3\sqrt{a^2cx^2+c}}{8a\sqrt{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x^2*Sqrt[c + a^2*c*x^2])/(8*Sqrt[1 + a^2*x^2]) + (3*x*Sqrt[c + a^2*c*
x^2]*ArcSinh[a*x])/4 - (3*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(8*a*Sqrt[1 +
 a^2*x^2]) - (3*a*x^2*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^2)/(4*Sqrt[1 + a^2*x
^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3)/2 + (Sqrt[c + a^2*c*x^2]*ArcS
inh[a*x]^4)/(8*a*Sqrt[1 + a^2*x^2])
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3 + \frac{\sqrt{c+a^2cx^2}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c+a^2cx^2}\right)\int x\sinh^{-1}(\ldots}{2\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3 + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^4}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{4}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax) - \frac{3ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{4\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^3 \ldots$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}}{8\sqrt{1+a^2x^2}} + \frac{3}{4}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax) - \frac{3\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^2}{8a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+\ldots}}{4\ldots}$$

Mathematica [A] time = 0.140813, size = 86, normalized size = 0.42

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(2\sinh^{-1}(ax)\left(\sinh^{-1}(ax)^3+\left(2\sinh^{-1}(ax)^2+3\right)\sinh\left(2\sinh^{-1}(ax)\right)\right)-3\left(2\sinh^{-1}(ax)^2+1\right)\cosh\left(2\sinh\right.\right.}{16a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^3,x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(-3*(1 + 2*ArcSinh[a*x]^2)*Cosh[2*ArcSinh[a*x]] + 2*
ArcSinh[a*x]*(ArcSinh[a*x]^3 + (3 + 2*ArcSinh[a*x]^2)*Sinh[2*ArcSinh[a*x]])
))/(16*a*Sqrt[1 + a^2*x^2])
```

Maple [A] time = 0.118, size = 231, normalized size = 1.1

$$\frac{(\text{Arcsinh}(ax))^4}{8a}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}+\frac{4(\text{Arcsinh}(ax))^3-6(\text{Arcsinh}(ax))^2+6\,\text{Arcsinh}(ax)-3}{\left(32a^2x^2+32\right)a}\sqrt{c\left(a^2x^2+1\right)}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/8*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a*arcsinh(a*x)^4+1/32*(c*(a^2*x
^2+1))^(1/2)*(2*x^3*a^3+2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x+(a^2*x^2+1)^(1/2)
)*(4*arcsinh(a*x)^3-6*arcsinh(a*x)^2+6*arcsinh(a*x)-3)/(a^2*x^2+1)/a+1/32*(
c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+2*a*x-(a^2*x^2+
1)^(1/2))*(4*arcsinh(a*x)^3+6*arcsinh(a*x)^2+6*arcsinh(a*x)+3)/(a^2*x^2+1)/
a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c\left(a^2x^2+1\right)}\,\text{asinh}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3, x)
```

3.337 $\int \frac{\sinh^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0784065, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5677, 5675}

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^4}{4a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0366589, size = 40, normalized size = 1.

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^4}{4a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^4)/(4*a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.032, size = 39, normalized size = 1.

$$\frac{(\text{Arcsinh}\,(ax))^4}{4\,ca}\sqrt{c\left(a^2x^2+1\right)}\frac{1}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(c*(a^2*x^2+1))^(1/2)/(a^2*x^2+1)^(1/2)/a/c*arcsinh(a*x)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2cx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.338 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{3\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\sinh^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2 \cdots}}{ac\cdots}$$

```
[Out] (x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x
]^3)/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1
+ E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) + (3*S
qrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(2*a*c*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] time = 0.188048, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5687, 5714, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{a^2cx^2+c}}+\frac{3\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\sinh^{-1}(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2 \cdots}}{ac\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*ArcSinh[a*x
]^3)/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1
+ E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c*Sqrt[c + a^2*c*x^2]) + (3*S
qrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(2*a*c*Sqrt[c + a^2*c*x^2
])
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
```

```
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{1+a^2x^2}\,dx}{c\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{\left(3\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \sinh^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{\left(6\sqrt{1+a^2x^2}\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)}{ac\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} + \frac{\left(6\sqrt{1\right.}{}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+}}{}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+}}{}$$

$$= \frac{x\sinh^{-1}(ax)^3}{c\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^2\log\left(1+e^{2\sinh^{-1}(ax)}\right)}{ac\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+}}{}$$

Mathematica [A] time = 0.260056, size = 133, normalized size = 0.61

$$\frac{6\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right)+3\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(ax)}\right)-2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^2\left(\right.}{2ac\sqrt{c\left(a^2x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (2*a*x*ArcSinh[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*(ArcSinh[a*x] +
3*Log[1 + E^(-2*ArcSinh[a*x])]) + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 3*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*ArcSinh[a*
x])])/(2*a*c*Sqrt[c*(1 + a^2*x^2)])
```

Maple [A] time = 0.121, size = 262, normalized size = 1.2

$$\frac{(\mathrm{Arcsinh}\,(ax))^3}{ac^2\,(a^2x^2+1)}\sqrt{c\,(a^2x^2+1)}\left(ax-\sqrt{a^2x^2+1}\right)+2\,\frac{\sqrt{c\,(a^2x^2+1)}\,(\mathrm{Arcsinh}\,(ax))^3}{\sqrt{a^2x^2+1}ac^2}-3\,\frac{\sqrt{c\,(a^2x^2+1)}\,(\mathrm{Arcsinh}\,(ax))^2\ln}{\sqrt{a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] (c*(a^2*x^2+1))^(1/2)*(a*x-(a^2*x^2+1)^(1/2))*arcsinh(a*x)^3/a/c^2/(a^2*x^2
+1)+2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)^3-3/(a^2*x
^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1
)^(1/2))^2)-3/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^2*arcsinh(a*x)*po
lylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(
1/2)/a/c^2*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arsinh}\,(ax)^3}{a^4c^2x^4+2\,a^2c^2x^2+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 +
c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.339 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac^2\sqrt{a^2cx^2+c}}+$$

```
[Out] -((x*ArcSinh[a*x])/(c^2*Sqrt[c + a^2*c*x^2])) + ArcSinh[a*x]^2/(2*a*c^2*Sqr
t[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^
2)^(3/2)) + (2*x*ArcSinh[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^3)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^
2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[c + a^2*c*x^2])
+ (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[
c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(a*c^
2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.328811, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5690, 5687, 5714, 3718, 2190, 2531, 2282, 6589, 5717, 260}

$$-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{ac^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\log\left(a^2x^2+1\right)}{2ac^2\sqrt{a^2cx^2+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```


```
[Out] -((x*ArcSinh[a*x])/(c^2*Sqrt[c + a^2*c*x^2])) + ArcSinh[a*x]^2/(2*a*c^2*Sqr
t[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) + (x*ArcSinh[a*x]^3)/(3*c*(c + a^2*c*x^
2)^(3/2)) + (2*x*ArcSinh[a*x]^3)/(3*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x]^3)/(3*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*Sqrt[1 + a^2*x^
2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[c + a^2*c*x^2])
+ (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^2*Sqrt[c + a^2*c*x^2]) - (2*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSinh[a*x])])/(a*c^2*Sqrt[
c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(2*ArcSinh[a*x])])/(a*c^
2*Sqrt[c + a^2*c*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^2}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)}{\left(1+a^2x^2\right)^{3/2}}\,dx}{c^2\sqrt{c+a^2cx^2}} - \cdots$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} - \frac{\left(2\sqrt{1+a^2x \cdots\right.}{\cdots}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2 \cdots}}{3ac^2\sqrt{c \cdots}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2 \cdots}}{3ac^2\sqrt{c \cdots}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2 \cdots}}{3ac^2\sqrt{c \cdots}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2 \cdots}}{3ac^2\sqrt{c \cdots}}$$

$$= -\frac{x\sinh^{-1}(ax)}{c^2\sqrt{c+a^2cx^2}} + \frac{\sinh^{-1}(ax)^2}{2ac^2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sinh^{-1}(ax)^3}{3c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2 \cdots}}{3ac^2\sqrt{c \cdots}}$$

Mathematica [A] time = 0.535376, size = 195, normalized size = 0.54

$$\frac{\left(a^2x^2+1\right)^{3/2}\left(12\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right) + 6\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(ax)}\right) + 3\log\left(a^2x^2+1\right) + \frac{4ax\sinh^{-1}(ax \cdots}{\sqrt{a^2x^2+1}} \cdots\right.}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a^2*x^2)^(3/2)*((-6*a*x*ArcSinh[a*x])/Sqrt[1 + a^2*x^2] + (3*ArcSinh[
a*x]^2)/(1 + a^2*x^2) - 4*ArcSinh[a*x]^3 + (2*a*x*ArcSinh[a*x]^3)/(1 + a^2*
```

```
x^2)^(3/2) + (4*a*x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2] - 12*ArcSinh[a*x]^2*L
og[1 + E^(-2*ArcSinh[a*x])] + 3*Log[1 + a^2*x^2] + 12*ArcSinh[a*x]*PolyLog[
2, -E^(-2*ArcSinh[a*x])] + 6*PolyLog[3, -E^(-2*ArcSinh[a*x])]))/(6*a*c*(c +
 a^2*c*x^2)^(3/2))
```

Maple [A] time = 0.184, size = 550, normalized size = 1.5

$$\frac{\mathrm{Arcsinh}\,(ax)}{\left(18\,x^6a^6+60\,x^4a^4+66\,a^2x^2+24\right)ac^3}\sqrt{c\left(a^2x^2+1\right)}\left(2\,x^3a^3-2\,a^2x^2\sqrt{a^2x^2+1}+3\,ax-2\,\sqrt{a^2x^2+1}\right)\left(-6\,a^4x^4\mathrm{Arcsinh}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/6*(c*(a^2*x^2+1))^(1/2)*(2*x^3*a^3-2*a^2*x^2*(a^2*x^2+1)^(1/2)+3*a*x-2*(a
^2*x^2+1)^(1/2))*arcsinh(a*x)*(-6*a^4*x^4*arcsinh(a*x)-6*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)*a^3*x^3-6*x^4*a^4-6*a^3*x^3*(a^2*x^2+1)^(1/2)+6*arcsinh(a*x)^2
*a^2*x^2-12*a^2*x^2*arcsinh(a*x)-9*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-18*a^
2*x^2-6*a*x*(a^2*x^2+1)^(1/2)+8*arcsinh(a*x)^2-6*arcsinh(a*x)-12)/(3*a^6*x^
6+10*a^4*x^4+11*a^2*x^2+4)/a/c^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/
a/c^3*ln(a*x+(a^2*x^2+1)^(1/2))+1/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a
/c^3*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+4/3/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^
(1/2)/a/c^3*arcsinh(a*x)^3-2/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^3*
arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)-2/(a^2*x^2+1)^(1/2)*(c*(a^2*
x^2+1))^(1/2)/a/c^3*arcsinh(a*x)*polylog(2,-(a*x+(a^2*x^2+1)^(1/2))^2)+1/(a
^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^3*polylog(3,-(a*x+(a^2*x^2+1)^(1/
2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^3}{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 +
3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{asinh}^3\,(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{arsinh}\,(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.340 $\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}-\frac{1}{20ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+}}$$

```
[Out] -1/(20*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(c^3
*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(10*c^3*(1 + a^2*x^2)*Sqrt[c + a^2
*c*x^2]) + (3*ArcSinh[a*x]^2)/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*
x^2]) + (2*ArcSinh[a*x]^2)/(5*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])
+ (x*ArcSinh[a*x]^3)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^3)/(15
*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^3)/(15*c^3*Sqrt[c + a^2*c*x
^2]) + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(15*a*c^3*Sqrt[c + a^2*c*x^2])
- (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(5*a*c^3
*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^3*Sqrt[
c + a^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSin
h[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(2*ArcSinh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.525902, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5690, 5687, 5714, 3718, 2190, 2531, 2282, 6589, 5717, 260, 261}

$$-\frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{2\sinh^{-1}(ax)}\right)}{5ac^3\sqrt{a^2cx^2+c}}-\frac{1}{20ac^3\sqrt{a^2x^2+1}\sqrt{a^2cx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/(20*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(c^3
*Sqrt[c + a^2*c*x^2]) - (x*ArcSinh[a*x])/(10*c^3*(1 + a^2*x^2)*Sqrt[c + a^2
*c*x^2]) + (3*ArcSinh[a*x]^2)/(20*a*c^3*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*
x^2]) + (2*ArcSinh[a*x]^2)/(5*a*c^3*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])
+ (x*ArcSinh[a*x]^3)/(5*c*(c + a^2*c*x^2)^(5/2)) + (4*x*ArcSinh[a*x]^3)/(15
*c^2*(c + a^2*c*x^2)^(3/2)) + (8*x*ArcSinh[a*x]^3)/(15*c^3*Sqrt[c + a^2*c*x
^2]) + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(15*a*c^3*Sqrt[c + a^2*c*x^2])
```

```
- (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(2*ArcSinh[a*x])])/(5*a*c^3
*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2])/(2*a*c^3*Sqrt[
c + a^2*c*x^2]) - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*PolyLog[2, -E^(2*ArcSin
h[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[1 + a^2*x^2]*PolyLog[3, -
E^(2*ArcSinh[a*x])])/(5*a*c^3*Sqrt[c + a^2*c*x^2])
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{7/2}}\,dx = \frac{x\sinh^{-1}(ax)^3}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx}{5c} - \frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^2}{\left(1+a^2x^2\right)^3}\,dx}{5c^3\sqrt{c+a^2cx^2}}$$

$$= \frac{3\sinh^{-1}(ax)^2}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{x\sinh^{-1}(ax)^3}{5c\left(c+a^2cx^2\right)^{5/2}} + \frac{4x\sinh^{-1}(ax)^3}{15c^2\left(c+a^2cx^2\right)^{3/2}} + \frac{8\int \frac{\sinh^{-1}(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{15c^2} - \cdots$$

$$= -\frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(ax)^2}{20ac^3\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}} + \frac{2\sinh^{-1}(ax)^2}{5ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} + \cdots$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(\cdots}{20ac^3\left(1+a^2x^2\right)^{3/\cdots}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(\cdots}{20ac^3\left(1+a^2x^2\right)^{3/\cdots}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(\cdots}{20ac^3\left(1+a^2x^2\right)^{3/\cdots}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(\cdots}{20ac^3\left(1+a^2x^2\right)^{3/\cdots}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(\cdots}{20ac^3\left(1+a^2x^2\right)^{3/\cdots}}$$

$$= -\frac{1}{20ac^3\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{c^3\sqrt{c+a^2cx^2}} - \frac{x\sinh^{-1}(ax)}{10c^3\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}} + \frac{3\sinh^{-1}(\cdots}{20ac^3\left(1+a^2x^2\right)^{3/\cdots}}$$

Mathematica [A] time = 0.659682, size = 297, normalized size = 0.58

$$96\sqrt{a^2x^2+1}\sinh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\sinh^{-1}(ax)}\right)+48\sqrt{a^2x^2+1}\text{PolyLog}\left(3,-e^{-2\sinh^{-1}(ax)}\right)-\frac{3}{\sqrt{a^2x^2+1}}+30\sqrt{a^2x^2+\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(c + a^2*c*x^2)^(7/2),x]
```

```
[Out] (-3/Sqrt[1 + a^2*x^2] - 60*a*x*ArcSinh[a*x] - (6*a*x*ArcSinh[a*x])/(1 + a^2
*x^2) + (9*ArcSinh[a*x]^2)/(1 + a^2*x^2)^(3/2) + (24*ArcSinh[a*x]^2)/Sqrt[1
 + a^2*x^2] + 32*a*x*ArcSinh[a*x]^3 + (12*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2)
^2 + (16*a*x*ArcSinh[a*x]^3)/(1 + a^2*x^2) - 32*Sqrt[1 + a^2*x^2]*ArcSinh[a
*x]^3 - 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2*Log[1 + E^(-2*ArcSinh[a*x])] +
30*Sqrt[1 + a^2*x^2]*Log[1 + a^2*x^2] + 96*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]*P
olyLog[2, -E^(-2*ArcSinh[a*x])] + 48*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(-2*Ar
cSinh[a*x])])/(60*a*c^3*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.226, size = 888, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x)
```

```
[Out] 1/60*(c*(a^2*x^2+1))^(1/2)*(8*x^5*a^5-8*a^4*x^4*(a^2*x^2+1)^(1/2)+20*x^3*a^
3-16*a^2*x^2*(a^2*x^2+1)^(1/2)+15*a*x-8*(a^2*x^2+1)^(1/2))*(24-1590*a^4*x^4
*arcsinh(a*x)-1410*a^2*x^2*arcsinh(a*x)+105*a^3*x^3*(a^2*x^2+1)^(1/2)+45*a*
x*(a^2*x^2+1)^(1/2)-495*arcsinh(a*x)^2*a*x*(a^2*x^2+1)^(1/2)-372*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)*a*x+96*a^2*x^2+256*arcsinh(a*x)^3-480*arcsinh(a*x)-264
*arcsinh(a*x)^2-1368*arcsinh(a*x)^2*x^4*a^4-984*arcsinh(a*x)^2*a^2*x^2-936*
arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-192*arcsinh(a*x)*x^8*a^8-852*arcsinh
(a*x)*x^6*a^6-1020*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-192*arcsinh(a*x
)^2*(a^2*x^2+1)^(1/2)*x^7*a^7-744*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*x^5*a^5+
24*x^8*a^8+96*x^6*a^6+144*x^4*a^4+24*(a^2*x^2+1)^(1/2)*x^7*a^7+84*(a^2*x^2+
1)^(1/2)*x^5*a^5-840*arcsinh(a*x)^2*x^6*a^6+160*arcsinh(a*x)^3*x^4*a^4+380*
arcsinh(a*x)^3*x^2*a^2-192*arcsinh(a*x)^2*x^8*a^8-192*(a^2*x^2+1)^(1/2)*arc
sinh(a*x)*x^7*a^7-756*(a^2*x^2+1)^(1/2)*arcsinh(a*x)*x^5*a^5)/(40*a^10*x^10
+215*a^8*x^8+469*a^6*x^6+517*a^4*x^4+287*a^2*x^2+64)/a/c^4-2/(a^2*x^2+1)^(1
/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*ln(a*x+(a^2*x^2+1)^(1/2))+1/(a^2*x^2+1)^(1/
2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2)+16/15/(a^2*x
^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^3-8/5/(a^2*x^2+1)^(1/2
)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)^2*ln(1+(a*x+(a^2*x^2+1)^(1/2))^2
)-8/5/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c^4*arcsinh(a*x)*polylog(2,
-(a*x+(a^2*x^2+1)^(1/2))^2)+4/5/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/a/c
^4*polylog(3,-(a*x+(a^2*x^2+1)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2cx^2+c}\,\operatorname{arsinh}(ax)^3}{a^8c^4x^8+4\,a^6c^4x^6+6\,a^4c^4x^4+4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^3/(a^8*c^4*x^8 + 4*a^6*c^4*x^6 +
6*a^4*c^4*x^4 + 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\left(a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(a^2*c*x^2 + c)^(7/2), x)
```

3.341 $\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^m \sinh^{-1}(ax)^3}{\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Rubi [A] time = 0.0884131, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]

[Out] Defer[Int][(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 0.47496, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Integrate[(x^m*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2], x]
```

Maple [A] time = 0.22, size = 0, normalized size = 0.

$$\int x^m (\mathrm{Arcsinh}\,(ax))^3 \frac{1}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{asinh}^3(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**3/sqrt(a**2*x**2 + 1), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

3.342 $\int \frac{x^4 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=187

$$\frac{45x^2}{128a^3}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{4a^2}+\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{32a^2}+\frac{9x^2\sinh^{-1}(ax)^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{8a^4}-\frac{45x\sqrt{a^2}}{}$$

```
[Out] (45*x^2)/(128*a^3) - (3*x^4)/(128*a) - (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(64*a^4) + (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a^2) + (45*ArcSinh[
a*x]^2)/(128*a^5) + (9*x^2*ArcSinh[a*x]^2)/(16*a^3) - (3*x^4*ArcSinh[a*x]^2
)/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^4) + (x^3*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^3)/(4*a^2) + (3*ArcSinh[a*x]^4)/(32*a^5)
```

Rubi [A] time = 0.498473, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5758, 5675, 5661, 30}

$$\frac{45x^2}{128a^3}+\frac{x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{4a^2}+\frac{3x^3\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{32a^2}+\frac{9x^2\sinh^{-1}(ax)^2}{16a^3}-\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{8a^4}-\frac{45x\sqrt{a^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```


```
[Out] (45*x^2)/(128*a^3) - (3*x^4)/(128*a) - (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
)/(64*a^4) + (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a^2) + (45*ArcSinh[
a*x]^2)/(128*a^5) + (9*x^2*ArcSinh[a*x]^2)/(16*a^3) - (3*x^4*ArcSinh[a*x]^2
)/(16*a) - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^4) + (x^3*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^3)/(4*a^2) + (3*ArcSinh[a*x]^4)/(32*a^5)
```


Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^4 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{4a^2} - \frac{3\int \frac{x^2\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{4a^2} - \frac{3\int x^3\sinh^{-1}(ax)^2\,dx}{4a}$$

$$= -\frac{3x^4\sinh^{-1}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{8a^4} + \frac{x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{4a^2} + \frac{3}{8}\int \frac{x^4\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}$$

$$= \frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{32a^2} + \frac{9x^2\sinh^{-1}(ax)^2}{16a^3} - \frac{3x^4\sinh^{-1}(ax)^2}{16a} - \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{8a^4} + $$

$$= -\frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{32a^2} + \frac{9x^2\sinh^{-1}(ax)^2}{16a^3} - \frac{3x^4\sinh^{-}}{16a}$$

$$= \frac{45x^2}{128a^3} - \frac{3x^4}{128a} - \frac{45x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{64a^4} + \frac{3x^3\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{32a^2} + \frac{45\sinh^{-1}(ax)^2}{128a^5} + \frac{9x^2}{}$$

Mathematica [A] time = 0.0768526, size = 121, normalized size = 0.65

$$\frac{-3a^4x^4 + 45a^2x^2 + 16ax\sqrt{a^2x^2+1}\left(2a^2x^2-3\right)\sinh^{-1}(ax)^3 + 6ax\sqrt{a^2x^2+1}\left(2a^2x^2-15\right)\sinh^{-1}(ax) + \left(-24a^4x^4 + 72a\right.}{128a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (45*a^2*x^2 - 3*a^4*x^4 + 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcSinh
[a*x] + (45 + 72*a^2*x^2 - 24*a^4*x^4)*ArcSinh[a*x]^2 + 16*a*x*Sqrt[1 + a^2
*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 12*ArcSinh[a*x]^4)/(128*a^5)
```

Maple [A] time = 0.061, size = 156, normalized size = 0.8

$$\frac{1}{128\,a^5}\Big(32\,(\mathrm{Arcsinh}\,(ax))^3\,\sqrt{a^2x^2+1}a^3x^3-24\,(\mathrm{Arcsinh}\,(ax))^2\,x^4a^4+12\,\mathrm{Arcsinh}\,(ax)\,\sqrt{a^2x^2+1}a^3x^3-3\,x^4a^4-48\,(\mathrm{A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/128*(32*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a^3*x^3-24*arcsinh(a*x)^2*x^4*a^
4+12*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3-3*x^4*a^4-48*arcsinh(a*x)^3*a*x
*(a^2*x^2+1)^(1/2)+72*arcsinh(a*x)^2*a^2*x^2+12*arcsinh(a*x)^4-90*arcsinh(a
*x)*(a^2*x^2+1)^(1/2)*a*x+45*a^2*x^2+45*arcsinh(a*x)^2+48)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\,\mathrm{arsinh}\,(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 2.16705, size = 383, normalized size = 2.05

$$-\frac{3\,a^4x^4-16\left(2\,a^3x^3-3\,ax\right)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2+1}\right)^3-45\,a^2x^2-12\,\log\left(ax+\sqrt{a^2x^2+1}\right)^4+3\left(8\,a^4x^4-24\,a^2x^2\right.}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(3*a^4*x^4 - 16*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 + 1))^3 - 45*a^2*x^2 - 12*log(a*x + sqrt(a^2*x^2 + 1))^4 + 3*(8*a^
4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(2*a^3*x^3 - 15
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^5
```

Sympy [A] time = 9.19402, size = 185, normalized size = 0.99

$$\begin{cases} -\frac{3x^4\operatorname{asinh}^2(ax)}{16a} - \frac{3x^4}{128a} + \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a^2} + \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a^2} + \frac{9x^2\operatorname{asinh}^2(ax)}{16a^3} + \frac{45x^2}{128a^3} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{8a^4} - \frac{45x\sqrt{a^2x^2+}}{64} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x**4*asinh(a*x)**2/(16*a) - 3*x**4/(128*a) + x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**3/(4*a**2) + 3*x**3*sqrt(a**2*x**2 + 1)*asinh(a*x)/(32
*a**2) + 9*x**2*asinh(a*x)**2/(16*a**3) + 45*x**2/(128*a**3) - 3*x*sqrt(a**
2*x**2 + 1)*asinh(a*x)**3/(8*a**4) - 45*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(6
4*a**4) + 3*asinh(a*x)**4/(32*a**5) + 45*asinh(a*x)**2/(128*a**5), Ne(a, 0)
), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

3.343 $\int \frac{x^3 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=153

$$\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^2}-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{3a^4}-\frac{40\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^4}+\frac{40x}{9a^3}+\frac{2x}{}$$

```
[Out] (40*x)/(9*a^3) - (2*x^3)/(27*a) - (40*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^
4) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^2) + (2*x*ArcSinh[a*x]^2)/
a^3 - (x^3*ArcSinh[a*x]^2)/(3*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*
a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a^2)
```

Rubi [A] time = 0.338049, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5758, 5717, 5653, 8, 5661, 30}

$$\frac{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{3a^2}+\frac{2x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^2}-\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{3a^4}-\frac{40\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{9a^4}+\frac{40x}{9a^3}+\frac{2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (40*x)/(9*a^3) - (2*x^3)/(27*a) - (40*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^
4) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^2) + (2*x*ArcSinh[a*x]^2)/
a^3 - (x^3*ArcSinh[a*x]^2)/(3*a) - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*
a^4) + (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*a^2)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx &= \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3a^2} - \frac{2\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{3a^2} - \frac{\int x^2\sinh^{-1}(ax)^2\,dx}{a} \\
&= -\frac{x^3\sinh^{-1}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3a^4} + \frac{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3a^2} + \frac{2}{3}\int \frac{x^3\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,d \\
&= \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} - \frac{x^3\sinh^{-1}(ax)^2}{3a} - \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{3a^4} + \frac{x^2\sqrt{}}{} \\
&= -\frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} - \frac{x^3\sinh^{-1}(ax}{3a} \\
&= \frac{40x}{9a^3} - \frac{2x^3}{27a} - \frac{40\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^4} + \frac{2x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{9a^2} + \frac{2x\sinh^{-1}(ax)^2}{a^3} - \frac{x^3\sinh}{3a}
\end{aligned}$$

Mathematica [A] time = 0.068115, size = 98, normalized size = 0.64

$$\frac{-2ax\left(a^2x^2-60\right)+9\left(a^2x^2-2\right)\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3-9ax\left(a^2x^2-6\right)\sinh^{-1}(ax)^2+6\left(a^2x^2-20\right)\sqrt{a^2x^2+1}\sinh^{-1}(a}{27a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-2*a*x*(-60 + a^2*x^2) + 6*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
- 9*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 + 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*
ArcSinh[a*x]^3)/(27*a^4)
```

Maple [A] time = 0.054, size = 164, normalized size = 1.1

$$\frac{1}{27a^4}\left(9\left(\operatorname{Arcsinh}(ax)\right)^3x^4a^4-9\left(\operatorname{Arcsinh}(ax)\right)^3x^2a^2-9a^3x^3\left(\operatorname{Arcsinh}(ax)\right)^2\sqrt{a^2x^2+1}+6a^4x^4\operatorname{Arcsinh}(ax)-114a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/27/a^4/(a^2*x^2+1)^(1/2)*(9*arcsinh(a*x)^3*x^4*a^4-9*arcsinh(a*x)^3*x^2*a
^2-9*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+6*a^4*x^4*arcsinh(a*x)-114*a^
2*x^2*arcsinh(a*x)-2*a^3*x^3*(a^2*x^2+1)^(1/2)-18*arcsinh(a*x)^3+54*arcsinh
(a*x)^2*a*x*(a^2*x^2+1)^(1/2)-120*arcsinh(a*x)+120*a*x*(a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.16037, size = 171, normalized size = 1.12

$$\frac{1}{3}\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^3+\frac{2}{27}a\left(\frac{3\left(\sqrt{a^2x^2+1}x^2-\frac{20\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsinh}(ax)}{a^3}-\frac{a^2x^3-60x}{a^4}\right)-\frac{\left(a^2x^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arcsinh(a*x)^3 +
2/27*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2 + 1)/a^2)*arcsinh(a*x)/a
```

```
^3 - (a^2*x^3 - 60*x)/a^4) - 1/3*(a^2*x^3 - 6*x)*arcsinh(a*x)^2/a^3
```

Fricas [A] time = 2.14252, size = 296, normalized size = 1.93

$$-\frac{2a^3x^3 - 9\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^3 + 9(a^3x^3-6ax)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 6\sqrt{a^2x^2+1}(a^2x^2-2}{27a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(2*a^3*x^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2
 + 1))^3 + 9*(a^3*x^3 - 6*a*x)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*
x^2 + 1)*(a^2*x^2 - 20)*log(a*x + sqrt(a^2*x^2 + 1)) - 120*a*x)/a^4
```

Sympy [A] time = 4.80225, size = 148, normalized size = 0.97

$$\begin{cases} -\frac{x^3\operatorname{asinh}^2(ax)}{3a} - \frac{2x^3}{27a} + \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^2} + \frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^2} + \frac{2x\operatorname{asinh}^2(ax)}{a^3} + \frac{40x}{9a^3} - \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{3a^4} - \frac{40\sqrt{a^2x^2+1}\operatorname{asinh}}{9a^4} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-x**3*asinh(a*x)**2/(3*a) - 2*x**3/(27*a) + x**2*sqrt(a**2*x**2
+ 1)*asinh(a*x)**3/(3*a**2) + 2*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**2
) + 2*x*asinh(a*x)**2/a**3 + 40*x/(9*a**3) - 2*sqrt(a**2*x**2 + 1)*asinh(a*
x)**3/(3*a**4) - 40*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**4), Ne(a, 0)), (0,
 True))
```

Giac [A] time = 1.43019, size = 193, normalized size = 1.26

$$\frac{\left((a^2x^2+1)^{\frac{3}{2}} - 3\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{3a^4} - \frac{2a^2x^3 + 9(a^2x^3-6x)\log\left(ax+\sqrt{a^2x^2+1}\right)^2 - 120x - \frac{6\left((a^2x^2+1)\right.}{}}{27a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((a^2*x^2 + 1)^(3/2) - 3*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)
)^3/a^4 - 1/27*(2*a^2*x^3 + 9*(a^2*x^3 - 6*x)*log(a*x + sqrt(a^2*x^2 + 1))^
2 - 120*x - 6*((a^2*x^2 + 1)^(3/2) - 21*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a
^2*x^2 + 1))/a)/a^3
```

3.344 $\int \frac{x^2 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=105

$$\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{2a^2}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}-\frac{\sinh^{-1}(ax)^4}{8a^3}-\frac{3\sinh^{-1}(ax)^2}{8a^3}-\frac{3x^2}{8a}-\frac{3x^2\sinh^{-1}(ax)^2}{4a}$$

```
[Out] (-3*x^2)/(8*a) + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*ArcSinh[
a*x]^2)/(8*a^3) - (3*x^2*ArcSinh[a*x]^2)/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3)
```

Rubi [A] time = 0.224513, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5758, 5675, 5661, 30}

$$\frac{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{2a^2}+\frac{3x\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{4a^2}-\frac{\sinh^{-1}(ax)^4}{8a^3}-\frac{3\sinh^{-1}(ax)^2}{8a^3}-\frac{3x^2}{8a}-\frac{3x^2\sinh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-3*x^2)/(8*a) + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(4*a^2) - (3*ArcSinh[
a*x]^2)/(8*a^3) - (3*x^2*ArcSinh[a*x]^2)/(4*a) + (x*Sqrt[1 + a^2*x^2]*ArcSi
nh[a*x]^3)/(2*a^2) - ArcSinh[a*x]^4/(8*a^3)
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
```

```
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2a^2} - \frac{\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{2a^2} - \frac{3\int x\sinh^{-1}(ax)^2\,dx}{2a}$$

$$= -\frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2a^2} - \frac{\sinh^{-1}(ax)^4}{8a^3} + \frac{3}{2}\int \frac{x^2\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$

$$= \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2} - \frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2a^2} - \frac{\sinh^{-1}(ax)^4}{8a^3} - \frac{3\int \frac{\sinh^{-1}}{\sqrt{1+}}}{4a}$$

$$= -\frac{3x^2}{8a} + \frac{3x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{4a^2} - \frac{3\sinh^{-1}(ax)^2}{8a^3} - \frac{3x^2\sinh^{-1}(ax)^2}{4a} + \frac{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2a^2}$$

Mathematica [A] time = 0.0671444, size = 83, normalized size = 0.79

$$-\frac{3a^2x^2 - 4ax\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3 + \left(6a^2x^2+3\right)\sinh^{-1}(ax)^2 - 6ax\sqrt{a^2x^2+1}\sinh^{-1}(ax) + \sinh^{-1}(ax)^4}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -(3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*ArcSin
h[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + ArcSinh[a*x]^4)/(8*a^3)
```

Maple [A] time = 0.052, size = 84, normalized size = 0.8

$$-\frac{1}{8a^3}\left(-4\left(\operatorname{Arcsinh}(ax)\right)^3 ax\sqrt{a^2x^2+1}+6\left(\operatorname{Arcsinh}(ax)\right)^2 a^2x^2+\left(\operatorname{Arcsinh}(ax)\right)^4-6\operatorname{Arcsinh}(ax)\sqrt{a^2x^2+1}ax+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-4*arcsinh(a*x)^3*a*x*(a^2*x^2+1)^(1/2)+6*arcsinh(a*x)^2*a^2*x^2+arcs
inh(a*x)^4-6*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*a^2*x^2+3*arcsinh(a*x)^2+
3)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 2.11824, size = 293, normalized size = 2.79

$$\frac{4\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3-3a^2x^2-\log\left(ax+\sqrt{a^2x^2+1}\right)^4+6\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)-3\left(2a^2\right.}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a^2*x^2 - l
og(a*x + sqrt(a^2*x^2 + 1))^4 + 6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*
x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^3
```

Sympy [A] time = 3.07738, size = 100, normalized size = 0.95

$$\begin{cases} -\frac{3x^2\operatorname{asinh}^2(ax)}{4a} - \frac{3x^2}{8a} + \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{2a^2} + \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{4a^2} - \frac{\operatorname{asinh}^4(ax)}{8a^3} - \frac{3\operatorname{asinh}^2(ax)}{8a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x**2*asinh(a*x)**2/(4*a) - 3*x**2/(8*a) + x*sqrt(a**2*x**2 +
1)*asinh(a*x)**3/(2*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(4*a**2) - a
sinh(a*x)**4/(8*a**3) - 3*asinh(a*x)**2/(8*a**3), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^3/sqrt(a^2*x^2 + 1), x)
```

3.345 $\int \frac{x \sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a^2}+\frac{6\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{6x}{a}-\frac{3x\sinh^{-1}(ax)^2}{a}$$

```
[Out] (-6*x)/a + (6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2 - (3*x*ArcSinh[a*x]^2)/a
+ (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Rubi [A] time = 0.10974, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5717, 5653, 8}

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{a^2}+\frac{6\sqrt{a^2x^2+1}\sinh^{-1}(ax)}{a^2}-\frac{6x}{a}-\frac{3x\sinh^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*x)/a + (6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^2 - (3*x*ArcSinh[a*x]^2)/a
+ (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{x\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a^2} - \frac{3\int \sinh^{-1}(ax)^2\,dx}{a}$$
$$= -\frac{3x\sinh^{-1}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a^2} + 6\int \frac{x\sinh^{-1}(ax)}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{6\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2} - \frac{3x\sinh^{-1}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a^2} - \frac{6\int 1\,dx}{a}$$
$$= -\frac{6x}{a} + \frac{6\sqrt{1+a^2x^2}\sinh^{-1}(ax)}{a^2} - \frac{3x\sinh^{-1}(ax)^2}{a} + \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{a^2}$$

Mathematica [A] time = 0.0327067, size = 58, normalized size = 0.91

$$\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3 + 6\sqrt{a^2x^2+1}\sinh^{-1}(ax) - 6ax - 3ax\sinh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^3)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (-6*a*x + 6*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] - 3*a*x*ArcSinh[a*x]^2 + Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^3)/a^2
```

Maple [A] time = 0.045, size = 90, normalized size = 1.4

$$\frac{1}{a^2}\Big((\text{Arcsinh}\,(ax))^3\,x^2a^2 + (\text{Arcsinh}\,(ax))^3 - 3\,(\text{Arcsinh}\,(ax))^2\,ax\sqrt{a^2x^2+1} + 6\,a^2x^2\text{Arcsinh}\,(ax) + 6\,\text{Arcsinh}\,(ax) - 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/a^2/(a^2*x^2+1)^(1/2)*(arcsinh(a*x)^3*x^2*a^2+arcsinh(a*x)^3-3*arcsinh(a*
x)^2*a*x*(a^2*x^2+1)^(1/2)+6*a^2*x^2*arcsinh(a*x)+6*arcsinh(a*x)-6*a*x*(a^2
*x^2+1)^(1/2))
```

Maxima [A] time = 1.15992, size = 82, normalized size = 1.28

$$-\frac{3\,x\,\mathrm{arsinh}\,(ax)^2}{a}+\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)^3}{a^2}-\frac{6\left(x-\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}(ax)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x*arcsinh(a*x)^2/a + sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a^2 - 6*(x - sqrt(
a^2*x^2 + 1)*arcsinh(a*x)/a)/a
```

Fricas [A] time = 2.10925, size = 209, normalized size = 3.27

$$-\frac{3\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax-6\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a
^2*x^2 + 1))^3 + 6*a*x - 6*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/
a^2
```

Sympy [A] time = 1.47613, size = 61, normalized size = 0.95

$$\begin{cases}-\frac{3x\operatorname{asinh}^2(ax)}{a}-\frac{6x}{a}+\frac{\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a^2}+\frac{6\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-3*x*asinh(a*x)**2/a - 6*x/a + sqrt(a**2*x**2 + 1)*asinh(a*x)**3
/a**2 + 6*sqrt(a**2*x**2 + 1)*asinh(a*x)/a**2, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.48015, size = 136, normalized size = 2.12

$$\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2}-\frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(
a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
/a^2))/a
```

3.346 $\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Rubi [A] time = 0.0310136, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^4}{4a}$$

Mathematica [A] time = 0.006072, size = 13, normalized size = 1.

$$\frac{\sinh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^4/(4*a)
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$\frac{(\text{Arcsinh}\,(ax))^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Maxima [A] time = 1.08582, size = 15, normalized size = 1.15

$$\frac{\text{arsinh}\,(ax)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*arcsinh(a*x)^4/a
```

Fricas [B] time = 2.0812, size = 51, normalized size = 3.92

$$\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(a*x + sqrt(a^2*x^2 + 1))^4/a
```

Sympy [A] time = 0.84327, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{asinh}^{4}(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

Giac [B] time = 1.45238, size = 31, normalized size = 2.38

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(a*x + sqrt(a^2*x^2 + 1))^4/a
```

3.347 $\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=102

$$-3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+6\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)$$

```
[Out] -2*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 3*ArcSinh[a*x]^2*PolyLog[2, -E^
ArcSinh[a*x]] + 3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 6*ArcSinh[a*x
]*PolyLog[3, -E^ArcSinh[a*x]] - 6*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] -
 6*PolyLog[4, -E^ArcSinh[a*x]] + 6*PolyLog[4, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.162071, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5760, 4182, 2531, 6609, 2282, 6589}

$$-3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)+3\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)+6\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```


```
[Out] -2*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 3*ArcSinh[a*x]^2*PolyLog[2, -E^
ArcSinh[a*x]] + 3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 6*ArcSinh[a*x
]*PolyLog[3, -E^ArcSinh[a*x]] - 6*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] -
 6*PolyLog[4, -E^ArcSinh[a*x]] + 6*PolyLog[4, E^ArcSinh[a*x]]
```


Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx = \text{Subst}\left(\int x^3\mathbf{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \sinh^{-1}(ax)\right) + 3\,\text{Subst}\left(\int x^2\right.$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$
$$= -2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right) - 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(-e^{\sinh^{-1}(ax)}\right) + 3\sinh^{-1}(ax)^2\mathbf{Li}_2\left(e^{\sinh^{-1}(ax)}\right)$$

Mathematica [A] time = 0.128932, size = 146, normalized size = 1.43

$$\frac{1}{8}\left(24\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(ax)}\right) + 24\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right) + 48\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\sin}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcSinh[a*x]^4 - 8*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] + 8*
ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*ArcSinh[a*x]^2*PolyLog[2, -E^(-
ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] + 48*ArcSinh[
a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[
a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 48*PolyLog[4, E^ArcSinh[a*x]])/
8
```

Maple [A] time = 0.056, size = 197, normalized size = 1.9

$$-\left(\text{Arcsinh}\left(ax\right)\right)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 3\left(\text{Arcsinh}\left(ax\right)\right)^2\text{polylog}\left(2,-ax-\sqrt{a^2x^2+1}\right) + 6\,\text{Arcsinh}\left(ax\right)\text{polylog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-3*arcsinh(a*x)^2*polylog(2,-a*x
-(a^2*x^2+1)^(1/2))+6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*poly
```

```
log(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*
arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,a*
x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^3}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x), x)
```

3.348 $\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=88

$$3a\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}a\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{x}-a\sinh^{-1}(ax)^3+3a\sin$$

```
[Out] -(a*ArcSinh[a*x]^3) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x + 3*a*ArcSinh[a*
x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 3*a*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh
[a*x])] - (3*a*PolyLog[3, E^(2*ArcSinh[a*x])])/2
```

Rubi [A] time = 0.1888, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5723, 5659, 3716, 2190, 2531, 2282, 6589}

$$3a\sinh^{-1}(ax)\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right)-\frac{3}{2}a\text{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right)-\frac{\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{x}-a\sinh^{-1}(ax)^3+3a\sin$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] -(a*ArcSinh[a*x]^3) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x + 3*a*ArcSinh[a*
x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 3*a*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh
[a*x])] - (3*a*PolyLog[3, E^(2*ArcSinh[a*x])])/2
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
 d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
 NeQ[m, -1]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
```

```
0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} + (3a)\int \frac{\sinh^{-1}(ax)^2}{x}\,dx$$

$$= -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} + (3a)\,\mathrm{Subst}\left(\int x^2\coth(x)\,dx, x, \sinh^{-1}(ax)\right)$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} - (6a)\,\mathrm{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax)\right)$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) - (6a)\,\mathrm{Subst}\left(\int x\right.$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 3a\sinh^{-1}(ax)\mathrm{Li}$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 3a\sinh^{-1}(ax)\mathrm{Li}$$

$$= -a\sinh^{-1}(ax)^3 - \frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{x} + 3a\sinh^{-1}(ax)^2\log\left(1-e^{2\sinh^{-1}(ax)}\right) + 3a\sinh^{-1}(ax)\mathrm{Li}$$

Mathematica [C] time = 0.19918, size = 97, normalized size = 1.1

$$\frac{1}{8}a\left(24\sinh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}(ax)}\right) - 12\mathrm{PolyLog}\left(3,e^{2\sinh^{-1}(ax)}\right) - \frac{8\sqrt{a^2x^2+1}\sinh^{-1}(ax)^3}{ax} - 8\sinh^{-1}(ax)^3 + 2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^3/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(a*x)
+ 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 24*ArcSinh[a*x]*PolyLog[2
, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcSinh[a*x])]))/8
```

Maple [A] time = 0.083, size = 187, normalized size = 2.1

$$\frac{(\mathrm{Arcsinh}\,(ax))^3}{x}\left(ax-\sqrt{a^2x^2+1}\right) - 2\,a\,(\mathrm{Arcsinh}\,(ax))^3 + 3\,a\,(\mathrm{Arcsinh}\,(ax))^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + 6\,a\mathrm{Arcsinh}\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] (a*x-(a^2*x^2+1)^(1/2))/x*arcsinh(a*x)^3-2*a*arcsinh(a*x)^3+3*a*arcsinh(a*x
)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)
^(1/2))-6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a*arcsinh(a*x)^2*ln(1-a*x-(
a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*a*pol
ylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{x}+\int\frac{3\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{\sqrt{a^2x^2+1}ax^2+\left(a^2x^2+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2
+ sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(sqrt(a^2*x^2
 + 1)*a*x^2 + (a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)^3}{a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^4 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{asinh}^3\,(ax)}{x^2\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^2), x)
```

3.349 $\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=210

$$\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-3a^2\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}}\right.$$

```
[Out] (-3*a*ArcSinh[a*x]^2)/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*x^2) -
6*a^2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + a^2*ArcSinh[a*x]^3*ArcTanh[E^A
rcSinh[a*x]] - 3*a^2*PolyLog[2, -E^ArcSinh[a*x]] + (3*a^2*ArcSinh[a*x]^2*Po
lyLog[2, -E^ArcSinh[a*x]])/2 + 3*a^2*PolyLog[2, E^ArcSinh[a*x]] - (3*a^2*Ar
cSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]])/2 - 3*a^2*ArcSinh[a*x]*PolyLog[3,
-E^ArcSinh[a*x]] + 3*a^2*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 3*a^2*Po
lyLog[4, -E^ArcSinh[a*x]] - 3*a^2*PolyLog[4, E^ArcSinh[a*x]]
```

Rubi [A] time = 0.362582, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5747, 5760, 4182, 2531, 6609, 2282, 6589, 5661, 2279, 2391}

$$\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(ax)}\right)-\frac{3}{2}a^2\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-3a^2\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{\sinh^{-1}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```


```
[Out] (-3*a*ArcSinh[a*x]^2)/(2*x) - (Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(2*x^2) -
6*a^2*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + a^2*ArcSinh[a*x]^3*ArcTanh[E^A
rcSinh[a*x]] - 3*a^2*PolyLog[2, -E^ArcSinh[a*x]] + (3*a^2*ArcSinh[a*x]^2*Po
lyLog[2, -E^ArcSinh[a*x]])/2 + 3*a^2*PolyLog[2, E^ArcSinh[a*x]] - (3*a^2*Ar
cSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]])/2 - 3*a^2*ArcSinh[a*x]*PolyLog[3,
-E^ArcSinh[a*x]] + 3*a^2*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 3*a^2*Po
lyLog[4, -E^ArcSinh[a*x]] - 3*a^2*PolyLog[4, E^ArcSinh[a*x]]
```


Rule 5747

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] + (-Dist[(c^2*(m + 2*p + 3))/(f^2*(m +
 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^Frac
```

```
Part[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n
, 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 5760

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Si
nh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^
2*d] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx = -\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\sinh^{-1}(ax)^2}{x^2}\,dx-\frac{1}{2}a^2\int \frac{\sinh^{-1}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}-\frac{1}{2}a^2\,\text{Subst}\left(\int x^3\,\mathbf{csch}(x)\,dx, x, \sinh^{-1}(ax)\right)+\left(3a^2\right)\int$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}+a^2\sinh^{-1}(ax)^3\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+\frac{1}{2}\left(3a^2\right)\text{Subst}\Big($$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+a^2\sinh^{-1}(ax)^3\,\text{ta}$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+a^2\sinh^{-1}(ax)^3\,\text{ta}$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+a^2\sinh^{-1}(ax)^3\,\text{ta}$$

$$= -\frac{3a\sinh^{-1}(ax)^2}{2x}-\frac{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}{2x^2}-6a^2\sinh^{-1}(ax)\tanh^{-1}\left(e^{\sinh^{-1}(ax)}\right)+a^2\sinh^{-1}(ax)^3\,\text{ta}$$

Mathematica [A] time = 4.46347, size = 304, normalized size = 1.45

$$\frac{a\left(-24ax\sinh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\sinh^{-1}(ax)}\right)-48ax\sinh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\sinh^{-1}(ax)}\right)+48ax\sinh^{-1}(ax)\text{PolyLog}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSinh[a*x]^3/(x^3*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] (a*(-(a*Pi^4*x) + 2*a*x*ArcSinh[a*x]^4 - 12*a*x*ArcSinh[a*x]^2*Coth[ArcSinh
[a*x]/2] - 2*a*x*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]/2]^2 + 48*a*x*ArcSinh[a*x
]*Log[1 - E^(-ArcSinh[a*x])] - 48*a*x*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x]
)] + 8*a*x*ArcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 8*a*x*ArcSinh[a*x]^3
*Log[1 - E^ArcSinh[a*x]] - 24*a*x*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-Arc
Sinh[a*x])] - 48*a*x*PolyLog[2, E^(-ArcSinh[a*x])] - 24*a*x*ArcSinh[a*x]^2*
PolyLog[2, E^ArcSinh[a*x]] - 48*a*x*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*
x])] + 48*a*x*ArcSinh[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 48*a*x*PolyLog[4, -
E^(-ArcSinh[a*x])] - 48*a*x*PolyLog[4, E^ArcSinh[a*x]] + 12*a*x*ArcSinh[a*x
]^2*Tanh[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/(16*x)
```

Maple [A] time = 0.133, size = 377, normalized size = 1.8

$$-\frac{(\text{Arcsinh}\,(ax))^2}{2\,x^2}\left(a^2x^2\text{Arcsinh}\,(ax)+3\,ax\sqrt{a^2x^2+1}+\text{Arcsinh}\,(ax)\right)\frac{1}{\sqrt{a^2x^2+1}}+\frac{a^2\,(\text{Arcsinh}\,(ax))^3}{2}\ln\left(1+ax+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/(a^2*x^2+1)^(1/2)/x^2*arcsinh(a*x)^2*(a^2*x^2*arcsinh(a*x)+3*a*x*(a^2*
x^2+1)^(1/2)+arcsinh(a*x))+1/2*a^2*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2
))+3/2*a^2*arcsinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a
*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(4,-a*x-(a^2*x^2+1)^(1/2
))-1/2*a^2*arcsinh(a*x)^3*ln(1-a*x-(a^2*x^2+1)^(1/2))-3/2*a^2*arcsinh(a*x)^
2*polylog(2,a*x+(a^2*x^2+1)^(1/2))+3*a^2*arcsinh(a*x)*polylog(3,a*x+(a^2*x^
2+1)^(1/2))-3*a^2*polylog(4,a*x+(a^2*x^2+1)^(1/2))-3*a^2*arcsinh(a*x)*ln(1+
a*x+(a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+3*a^2*arcsin
h(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*a^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^3}{a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/(a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^3(ax)}{x^3\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**3/x**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**3/(x**3*sqrt(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^3}{\sqrt{a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^3/x^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/(sqrt(a^2*x^2 + 1)*x^3), x)
```

3.350 $\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=67

$$\frac{35c^3\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\mathrm{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\mathrm{Chi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

```
[Out] (35*c^3*CoshIntegral[ArcSinh[a*x]])/(64*a) + (21*c^3*CoshIntegral[3*ArcSinh
[a*x]])/(64*a) + (7*c^3*CoshIntegral[5*ArcSinh[a*x]])/(64*a) + (c^3*CoshInt
egral[7*ArcSinh[a*x]])/(64*a)
```

Rubi [A] time = 0.114914, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5699, 3312, 3301}

$$\frac{35c^3\mathrm{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\mathrm{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\mathrm{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\mathrm{Chi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```


```
[Out] (35*c^3*CoshIntegral[ArcSinh[a*x]])/(64*a) + (21*c^3*CoshIntegral[3*ArcSinh
[a*x]])/(64*a) + (7*c^3*CoshIntegral[5*ArcSinh[a*x]])/(64*a) + (c^3*CoshInt
egral[7*ArcSinh[a*x]])/(64*a)
```


Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+a^2cx^2\right)^3}{\sinh^{-1}(ax)}\,dx &= \frac{c^3\,\text{Subst}\left(\int \frac{\cosh^7(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{c^3\,\text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x}+\frac{21\cosh(3x)}{64x}+\frac{7\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{c^3\,\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a}+\frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{64a}+\frac{\left(21c^3\right)\text{Subs}}{}\\
&= \frac{35c^3\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{21c^3\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\textbf{Chi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{c^3\textbf{Chi}\left(7\sinh^{-1}(ax\right.}{64a}
\end{aligned}$$

Mathematica [A] time = 0.113499, size = 43, normalized size = 0.64

$$\frac{c^3\left(35\textbf{Chi}\left(\sinh^{-1}(ax)\right)+21\textbf{Chi}\left(3\sinh^{-1}(ax)\right)+7\textbf{Chi}\left(5\sinh^{-1}(ax)\right)+\textbf{Chi}\left(7\sinh^{-1}(ax)\right)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x],x]
```

```
[Out] (c^3*(35*CoshIntegral[ArcSinh[a*x]] + 21*CoshIntegral[3*ArcSinh[a*x]] + 7*C
oshIntegral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]]))/(64*a)
```

Maple [A] time = 0.04, size = 42, normalized size = 0.6

$$\frac{c^3\left(35\,\text{Chi}\left(\text{Arcsinh}\left(ax\right)\right)+21\,\text{Chi}\left(3\,\text{Arcsinh}\left(ax\right)\right)+7\,\text{Chi}\left(5\,\text{Arcsinh}\left(ax\right)\right)+\text{Chi}\left(7\,\text{Arcsinh}\left(ax\right)\right)\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x),x)
```

```
[Out] 1/64/a*c^3*(35*Chi(arcsinh(a*x))+21*Chi(3*arcsinh(a*x))+7*Chi(5*arcsinh(a*x
))+Chi(7*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^6c^3x^6+3\,a^4c^3x^4+3\,a^2c^3x^2+c^3}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int \frac{3a^2x^2}{\operatorname{asinh}(ax)}\,dx+\int \frac{3a^4x^4}{\operatorname{asinh}(ax)}\,dx+\int \frac{a^6x^6}{\operatorname{asinh}(ax)}\,dx+\int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x),x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x), x) + Integral(3*a**4*x**4/asinh(a*x)
, x) + Integral(a**6*x**6/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^3}{\operatorname{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x), x)
```

3.351 $\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=50

$$\frac{5c^2\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{5c^2\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

```
[Out] (5*c^2*CoshIntegral[ArcSinh[a*x]])/(8*a) + (5*c^2*CoshIntegral[3*ArcSinh[a*
x]])/(16*a) + (c^2*CoshIntegral[5*ArcSinh[a*x]])/(16*a)
```

Rubi [A] time = 0.100021, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5699, 3312, 3301}

$$\frac{5c^2\text{Chi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{5c^2\text{Chi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (5*c^2*CoshIntegral[ArcSinh[a*x]])/(8*a) + (5*c^2*CoshIntegral[3*ArcSinh[a*
x]])/(16*a) + (c^2*CoshIntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^2}{\sinh^{-1}(ax)}\,dx = \frac{c^2\,\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c^2\,\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x}+\frac{5\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)dx,x,\sinh^{-1}(ax)\right)}{a}$$
$$= \frac{c^2\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{Subs}}{}$$
$$= \frac{5c^2\,\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{5c^2\,\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{c^2\,\textbf{Chi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Mathematica [A] time = 0.0758856, size = 34, normalized size = 0.68

$$\frac{c^2\left(10\textbf{Chi}\left(\sinh^{-1}(ax)\right)+5\textbf{Chi}\left(3\sinh^{-1}(ax)\right)+\textbf{Chi}\left(5\sinh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x],x]
```

```
[Out] (c^2*(10*CoshIntegral[ArcSinh[a*x]] + 5*CoshIntegral[3*ArcSinh[a*x]] + Cosh
Integral[5*ArcSinh[a*x]]))/(16*a)
```

Maple [A] time = 0.033, size = 33, normalized size = 0.7

$$\frac{c^2\,(10\,\text{Chi}\,(\text{Arcsinh}\,(ax))+5\,\text{Chi}\,(3\,\text{Arcsinh}\,(ax))+\text{Chi}\,(5\,\text{Arcsinh}\,(ax)))}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

```
[Out] 1/16/a*c^2*(10*Chi(arcsinh(a*x))+5*Chi(3*arcsinh(a*x))+Chi(5*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2cx^2 + c)^2}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x),x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x), x) + Integral(a**4*x**4/asinh(a*x),
x) + Integral(1/asinh(a*x), x))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^2}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x), x)
```

3.352 $\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{3c\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

```
[Out] (3*c*CoshIntegral[ArcSinh[a*x]])/(4*a) + (c*CoshIntegral[3*ArcSinh[a*x]])/(
4*a)
```

Rubi [A] time = 0.0705044, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5699, 3312, 3301}

$$\frac{3c\text{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\text{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

```
[Out] (3*c*CoshIntegral[ArcSinh[a*x]])/(4*a) + (c*CoshIntegral[3*ArcSinh[a*x]])/(
4*a)
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)}\,dx &= \frac{c\,\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \sinh^{-1}(ax)\right)}{a}\\
&= \frac{c\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a}+\frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a}\\
&= \frac{3c\,\textbf{Chi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{c\,\textbf{Chi}\left(3\sinh^{-1}(ax)\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0126044, size = 23, normalized size = 0.79

$$\frac{c\left(3\textbf{Chi}\left(\sinh^{-1}(ax)\right)+\textbf{Chi}\left(3\sinh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x],x]
```

```
[Out] (c*(3*CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]]))/(4*a)
```

Maple [A] time = 0.028, size = 22, normalized size = 0.8

$$\frac{c\left(3\,\text{Chi}\left(\text{Arcsinh}\left(ax\right)\right)+\text{Chi}\left(3\,\text{Arcsinh}\left(ax\right)\right)\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x),x)
```

```
[Out] 1/4/a*c*(3*Chi(arcsinh(a*x))+Chi(3*arcsinh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{a^2x^2}{\operatorname{asinh}(ax)}\,dx + \int \frac{1}{\operatorname{asinh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x),x)
```

```
[Out] c*(Integral(a**2*x**2/asinh(a*x), x) + Integral(1/asinh(a*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x), x)
```

3.353 $\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{1}{\left(a^2cx^2+c\right)\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Rubi [A] time = 0.0262787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)}\,dx=\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.34705, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]), x]
```

Maple [A] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right) \operatorname{Arcsinh}\left(a x \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

```
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)} \operatorname{arsinh}\left(a x\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{{\left(a^{2} c x^{2} + c\right)} \operatorname{arsinh}\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^2\operatorname{asinh}(ax)+\operatorname{asinh}(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x),x)
```

```
[Out] Integral(1/(a**2*x**2*asinh(a*x) + asinh(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2+c)\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)), x)
```

3.354 $\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{1}{(a^2cx^2+c)^2 \sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Rubi [A] time = 0.0292485, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)}\,dx = \int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.54474, size = 0, normalized size = 0.

$$\int \frac{1}{(c+a^2cx^2)^2 \sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]), x]
```

Maple [A] time = 0.121, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{2} \operatorname{Arcsinh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{2} \operatorname{arsinh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^{4} c^{2} x^{4}+2\,a^{2} c^{2} x^{2}+c^{2}\right) \operatorname{arsinh}\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4 \operatorname{asinh}(ax)+2a^2x^2 \operatorname{asinh}(ax)+\operatorname{asinh}(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*asinh(a*x) + 2*a**2*x**2*asinh(a*x) + asinh(a*x)), x)
/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 \operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)), x)
```

3.355 $\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^5}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\ldots\right.}{32b\ldots}$$

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(32*b*c^5) - (Cos
h[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (Cosh[(6*
a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^5) + Log[a + b*ArcS
inh[c*x]]/(16*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))
/b])/(32*b*c^5) + (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/
(16*b*c^5) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b
*c^5)
```

Rubi [A] time = 0.508286, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^5}-\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^5}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc^5}+\frac{\sinh\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c^5) - (Cosh[
(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^5) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^5) + Log[a + b*ArcSinh[c*
x]]/(16*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32
*b*c^5) + (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^5)
 - (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^5)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} - \frac{\cosh(4x)}{16(a+bx)} + \frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^5} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx, x, \sinh^{-1}(\right.}{32c^5}$$

$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^5} - \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\mathbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^5} - \frac{\cosh\left(\frac{4a}{b}\right)\mathbf{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^5} + \frac{\cosh\left(\frac{6a}{b}\right)\mathbf{Chi}\left(\frac{6a}{b}\right.}{32bc}$$

Mathematica [A] time = 0.352281, size = 152, normalized size = 0.74

$$\frac{-\cosh\left(\frac{2a}{b}\right)\mathbf{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2\cosh\left(\frac{4a}{b}\right)\mathbf{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{6a}{b}\right)\mathbf{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) - 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] + 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^5)
```

Maple [A] time = 0.262, size = 199, normalized size = 1.

$$\frac{\ln(a+b\text{Arcsinh}(cx))}{16c^5b} - \frac{1}{64c^5b}e^{6\frac{a}{b}}\text{Ei}\left(1, 6\,\text{Arcsinh}(cx) + 6\frac{a}{b}\right) + \frac{1}{32c^5b}e^{4\frac{a}{b}}\text{Ei}\left(1, 4\,\text{Arcsinh}(cx) + 4\frac{a}{b}\right) + \frac{1}{64c^5b}e^{2\frac{a}{b}}\text{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/16*ln(a+b*arcsinh(c*x))/b/c^5-1/64/c^5/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6
*a/b)+1/32/c^5/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^5/b*exp(2*a/b
)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^5/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*
a/b)+1/32/c^5/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^5/b*exp(-6*a
/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}\,x^4}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^4}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**4*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}\,x^4}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^4/(b*arcsinh(c*x) + a), x)
```

3.356 $\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=183

$$\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^4}-\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^4}$$

```
[Out] (CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^4) + (CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^4) - (CoshIntegral[(5*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^4) - (Cosh[a/b]*SinhIntegral[
(a + b*ArcSinh[c*x])/b])/(8*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*
ArcSinh[c*x]))/b])/(16*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/(16*b*c^4)
```

Rubi [A] time = 0.53183, antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^4}-\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b*c^4) + (CoshIntegral[(3*a
)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b*c^4) - (CoshIntegral[(5*a)/b + 5
*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b*c^4) - (Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(8*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(16*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(1
6*b*c^4)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(x)}{8(a+bx)} - \frac{\sinh(3x)}{16(a+bx)} + \frac{\sinh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^4} + \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^4} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\right.}{8}$$

$$= -\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sinh^{-1}\right.}{16c^4}$$

$$= \frac{\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^4} + \frac{\mathbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^4} - \frac{\mathbf{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^4}$$

Mathematica [A] time = 0.286365, size = 135, normalized size = 0.74

$$\frac{2\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\sinh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\cosh\left(\frac{a}{b}\right.}{16bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSi
nh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)/b
] - 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - Cosh[(3*a)/b]*SinhIntegr
al[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*
x])])/(16*b*c^4)
```

Maple [A] time = 0.168, size = 178, normalized size = 1.

$$\frac{1}{32\,c^4b}e^{5\,\frac{a}{b}}\operatorname{Ei}\left(1,5\operatorname{Arcsinh}(cx)+5\,\frac{a}{b}\right)-\frac{1}{32\,c^4b}e^{3\,\frac{a}{b}}\operatorname{Ei}\left(1,3\operatorname{Arcsinh}(cx)+3\,\frac{a}{b}\right)-\frac{1}{16\,c^4b}e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{Arcsinh}(cx)+\frac{a}{b}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/32/c^4/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/c^4/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)-1/16/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/32/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}\,x^3}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^3}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}x^3}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a), x)
```

3.357 $\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c^3) - Log[a
+ b*ArcSinh[c*x]]/(8*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh
[c*x]))/b])/(8*b*c^3)
```

Rubi [A] time = 0.273493, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^3) - Log[a +
b*ArcSinh[c*x]]/(8*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh
[c*x]])/(8*b*c^3)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}\\
&= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3}\\
&= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Subst}}{}\\
&= \frac{\cosh\left(\frac{4a}{b}\right)\textbf{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}-\frac{\sinh\left(\frac{4a}{b}\right)\textbf{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^3}
\end{aligned}
$$

Mathematica [A] time = 0.174887, size = 65, normalized size = 0.79

$$\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])])/(8*b*c^3)
```

Maple [A] time = 0.106, size = 79, normalized size = 1.

$$-\frac{\ln\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}{8\,c^3b}-\frac{1}{16\,c^3b}\mathrm{e}^{4\,\frac{a}{b}}\mathrm{Ei}\left(1,4\,\mathrm{Arcsinh}\left(cx\right)+4\,\frac{a}{b}\right)-\frac{1}{16\,c^3b}\mathrm{e}^{-4\,\frac{a}{b}}\mathrm{Ei}\left(1,-4\,\mathrm{Arcsinh}\left(cx\right)-4\,\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] -1/8*ln(a+b*arcsinh(c*x))/b/c^3-1/16/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4
*a/b)-1/16/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}x^2}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}x^2}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a), x)
```

3.358 $\int \frac{x\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^2}$$

```
[Out] -(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b*c^2) - (CoshIntegral
[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^2) + (Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(4*b*c^2) + (Cosh[(3*a)/b]*SinhIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(4*b*c^2)
```

Rubi [A] time = 0.312524, antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^2}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Sh}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b*c^2) - (CoshIntegral[(3*
a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b*c^2) + (Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]])/(4*b*c^2) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*Arc
Sinh[c*x]])/(4*b*c^2)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2} \\
&= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^2}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sinh^{-1}(c
\end{aligned}$$

$$= -\frac{\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^2}-\frac{\textbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh
}{4bc^2}$$

Mathematica [A] time = 0.193129, size = 91, normalized size = 0.75

$$\frac{\sinh\left(\frac{a}{b}\right)\left(-\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\mathrm{S}}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcS
inh[c*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^2)
```

Maple [A] time = 0.092, size = 118, normalized size = 1.

$$\frac{1}{8\,c^2b}\mathrm{e}^{3\,\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\,(cx)+3\,\frac{a}{b}\right)+\frac{1}{8\,c^2b}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right)-\frac{1}{8\,c^2b}\mathrm{e}^{-\frac{a}{b}}\mathrm{Ei}\left(1,-\mathrm{Arcsinh}\,(cx)-\frac{a}{b}\right)-\frac{1}{8\,c^2b}\mathrm{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/8/c^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/8/c^2/b*exp(a/b)*Ei(1,arc
sinh(c*x)+a/b)-1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-1/8/c^2/b*exp(-3
*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}x}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c^2x^2+1}}{a+b\,\text{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}x}{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a), x)
```

3.359 $\int \frac{\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c) + Log[a +
b*ArcSinh[c*x]]/(2*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(2*b*c)
```

Rubi [A] time = 0.182576, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5699, 3312, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c) + Log[a + b*
ArcSinh[c*x]]/(2*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x
]])/(2*b*c)
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c}$$
$$= \frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Subst}\left(\right.}{}$$
$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx\right.}{2bc}$$

Mathematica [A] time = 0.127303, size = 63, normalized size = 0.77

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\log\left(a+b\sinh^{-1}(cx)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c)
```

Maple [A] time = 0.08, size = 79, normalized size = 1.

$$\frac{\ln\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}{2\,bc}-\frac{1}{4\,bc}\mathrm{e}^{2\,\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{Arcsinh}\left(cx\right)+2\,\frac{a}{b}\right)-\frac{1}{4\,bc}\mathrm{e}^{-2\,\frac{a}{b}}\mathrm{Ei}\left(1,-2\,\mathrm{Arcsinh}\left(cx\right)-2\,\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/2*ln(a+b*arcsinh(c*x))/b/c-1/4/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-
1/4/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{b\,\mathrm{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

3.360 $\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)} dx$

Optimal. Leaf size=77

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b}$$

```
[Out] -((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/b) + (Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/b + Unintegrable[1/(x*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.42614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/b) + (Cosh[a/b]*SinhIntegral
[a/b + ArcSinh[c*x]])/b + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int\left(\frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{c^2x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\right)dx$$
$$= c^2\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$
$$= \int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)$$
$$= \cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,\right.$$
$$= -\frac{\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b}+\frac{\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b}+\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\,\text{si}\right.}$$

Mathematica [A] time = 1.88624, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\text{Arcsinh}\left(cx\right)\right)}\sqrt{c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{bx\operatorname{arsinh}(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{x(a+b\operatorname{asinh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x), x)
```

3.361 $\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=44

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)+\frac{c\log\left(a+b\sinh^{-1}(cx)\right)}{b}$$

```
[Out] (c*Log[a + b*ArcSinh[c*x]])/b + Unintegrable[1/(x^2*Sqrt[1 + c^2*x^2]*(a +
b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.299652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Log[a + b*ArcSinh[c*x]])/b + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \int \left(\frac{c^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}+\frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\right)dx\\
&= c^2\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx+\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx\\
&= \frac{c\log\left(a+b\sinh^{-1}(cx)\right)}{b}+\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx
\end{aligned}$$

Mathematica [A] time = 1.10183, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}\sqrt{c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{bx^2\,\mathrm{arsinh}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}\left(cx\right)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^2), x)
```

3.362 $\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.126256, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 4.62366, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(a+b\,\mathrm{Arcsinh}\left(cx\right)\right)}\sqrt{c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^{2}x^{2}+1}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{bx^{3}\,\mathrm{arsinh}\left(cx\right)+ax^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\,\mathrm{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^3), x)
```

3.363 $\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.124778, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.844451, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.399, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a + b\operatorname{Arcsinh}\left(cx\right)\right)} \sqrt{c^{2}x^{2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^{2}x^{2}+1}}{\left(b \operatorname{arsinh}\left(cx\right)+a\right)x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{bx^{4} \operatorname{arsinh}\left(cx\right)+ax^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\operatorname{arsinh}\left(cx\right)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)*x^4), x)
```

3.364 $\int \frac{x^3(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^4}-\frac{\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a}{}\right.}{64bc^4}$$

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(64*b*c^4) + (3*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(64*b*c^4) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(64*b*c^4) - (CoshIntegral[(7*(a
 + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(64*b*c^4) - (3*Cosh[a/b]*SinhIntegra
l[(a + b*ArcSinh[c*x])/b])/(64*b*c^4) - (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(64*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*
ArcSinh[c*x]))/b])/(64*b*c^4) + (Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSi
nh[c*x]))/b])/(64*b*c^4)
```

Rubi [A] time = 0.615652, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64bc^4}+\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64bc^4}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64bc^4}-\frac{\sinh\left(\frac{7a}{b}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(64*b*c^4) + (3*CoshIntegral
[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(64*b*c^4) - (CoshIntegral[(5*a)/
b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(64*b*c^4) - (CoshIntegral[(7*a)/b + 7*A
rcSinh[c*x]]*Sinh[(7*a)/b])/(64*b*c^4) - (3*Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(64*b*c^4) - (3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh
[c*x]])/(64*b*c^4) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])
/(64*b*c^4) + (Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64*b*
c^4)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{3\sinh(x)}{64(a+bx)}-\frac{3\sinh(3x)}{64(a+bx)}+\frac{\sinh(5x)}{64(a+bx)}+\frac{\sinh(7x)}{64(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^4} + \frac{\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^4} - \frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\right.}{64}$$
$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^4} - \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x,\right.}{64c^4}$$
$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64bc^4} + \frac{3\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{64bc^4} - \frac{\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx\right.}{64bc^4}$$

Mathematica [A] time = 0.654168, size = 179, normalized size = 0.73

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{7a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a)
/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] - 3*Cosh[a/b]*Sinh
Integral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSin
h[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(7*a)/
b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^4)
```

Maple [A] time = 0.269, size = 238, normalized size = 1.

$$\frac{1}{128\,c^4b}e^{7\,\frac{a}{b}}\text{Ei}\left(1,7\,\text{Arcsinh}\,(cx)+7\,\frac{a}{b}\right)+\frac{1}{128\,c^4b}e^{5\,\frac{a}{b}}\text{Ei}\left(1,5\,\text{Arcsinh}\,(cx)+5\,\frac{a}{b}\right)-\frac{3}{128\,c^4b}e^{3\,\frac{a}{b}}\text{Ei}\left(1,3\,\text{Arcsinh}\,(cx)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/128/c^4/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/128/c^4/b*exp(5*a/b)*Ei
(1,5*arcsinh(c*x)+5*a/b)-3/128/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-
3/128/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128/c^4/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)+3/128/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/128/c^
4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/128/c^4/b*exp(-7*a/b)*Ei(1,-7
*arcsinh(c*x)-7*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b\operatorname{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\operatorname{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**3*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a), x)
```

3.365 $\int \frac{x^2(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$-\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{32bc^3}+\frac{\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}}{3}$$

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cos
h[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c^3) + (Cosh[(6*
a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) - Log[a + b*ArcS
inh[c*x]]/(16*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))
/b])/(32*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/
(16*b*c^3) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b
*c^3)
```

Rubi [A] time = 0.494725, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc^3}+\sinh$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[
(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^3) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3) - Log[a + b*ArcSinh[c*
x]]/(16*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32
*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c^3)
 - (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} + \frac{\cosh(4x)}{16(a+bx)} + \frac{\cosh(6x)}{32(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx, x, \sinh\right.}{32c^3}$$

$$= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{16bc^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Sub}}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\textbf{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\textbf{Chi}\left(\right.}{32}$$

Mathematica [A] time = 0.480728, size = 152, normalized size = 0.74

$$\frac{-\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 2\cosh\left(\frac{4a}{b}\right)\textbf{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{6a}{b}\right)\textbf{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])]) + 2*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] - 2*Log[a + b*ArcSinh[c*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/
b + ArcSinh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] -
 Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c^3)
```

Maple [A] time = 0.208, size = 199, normalized size = 1.

$$-\frac{\ln\left(a+b\text{Arcsinh}\left(cx\right)\right)}{16\,bc^3} - \frac{1}{64\,bc^3}e^{6\,\frac{a}{b}}\text{Ei}\left(1, 6\,\text{Arcsinh}\left(cx\right)+6\,\frac{a}{b}\right) - \frac{1}{32\,bc^3}e^{4\,\frac{a}{b}}\text{Ei}\left(1, 4\,\text{Arcsinh}\left(cx\right)+4\,\frac{a}{b}\right) + \frac{1}{64\,bc^3}e^{2\,\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] -1/16*ln(a+b*arcsinh(c*x))/b/c^3-1/64/c^3/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+
6*a/b)-1/32/c^3/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^3/b*exp(2*a/
b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2
*a/b)-1/32/c^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^3/b*exp(-6*
a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^2}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b\operatorname{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\operatorname{asinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a), x)
```

3.366 $\int \frac{x(1+c^2x^2)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=183

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^2}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16bc^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\,\mathrm{s}}{}\right.}{8bc^2}$$

```
[Out] -(CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^2) - (3*CoshIntegr
al[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^2) - (CoshIntegral[(5
*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^2) + (Cosh[a/b]*SinhIntegr
al[(a + b*ArcSinh[c*x])/b])/(8*b*c^2) + (3*Cosh[(3*a)/b]*SinhIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b*c^2) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*
ArcSinh[c*x]))/b])/(16*b*c^2)
```

Rubi [A] time = 0.417174, antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16bc^2}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^2}+\frac{\cosh\left(\frac{a}{b}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b*c^2) - (3*CoshIntegral[(
3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b*c^2) - (CoshIntegral[(5*a)/b
+ 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b*c^2) + (Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(8*b*c^2) + (3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSi
nh[c*x]])/(16*b*c^2) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]
])/(16*b*c^2)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)} + \frac{3\sinh(3x)}{16(a+bx)} + \frac{\sinh(5x)}{16(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^2} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^2} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\right.}{16}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^2} + \frac{\left(3\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \sinh\right.}{16c^2}$$
$$= -\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^2} - \frac{3\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^2} - \frac{\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(c\right.}{16bc^2}$$

Mathematica [A] time = 0.484674, size = 136, normalized size = 0.74

$$\frac{-2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 2\cosh\left(\right.}{16bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*a
)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSi
nh[c*x])])/(16*b*c^2)
```

Maple [A] time = 0.175, size = 178, normalized size = 1.

$$\frac{1}{32\,c^2b}e^{5\,\frac{a}{b}}\text{Ei}\left(1, 5\,\text{Arcsinh}\,(cx) + 5\,\frac{a}{b}\right) + \frac{3}{32\,c^2b}e^{3\,\frac{a}{b}}\text{Ei}\left(1, 3\,\text{Arcsinh}\,(cx) + 3\,\frac{a}{b}\right) + \frac{1}{16\,c^2b}e^{\frac{a}{b}}\text{Ei}\left(1, \text{Arcsinh}\,(cx) + \frac{a}{b}\right) - \frac{}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/32/c^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/32/c^2/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)+1/16/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-3/32/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^3+x\right)\sqrt{c^2x^2+1}}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}x}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a), x)
```

3.367 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=144

$$\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc}-\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc}-\frac{\sinh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4\ldots}{}\right)}{8bc\ldots}$$

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c) + (Cosh[(4
*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c) + (3*Log[a + b*Arc
Sinh[c*x]])/(8*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/
b])/(2*b*c) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b
*c)
```

Rubi [A] time = 0.286632, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5699, 3312, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc}-\frac{\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}-\sinh\left(\frac{4\ldots}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c) + (Cosh[(4*a
)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c) + (3*Log[a + b*ArcSinh
[c*x]])/(8*b*c) - (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2
*b*c) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c)
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c}$$

$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}\right.}{2c}$$

$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Subs}}{}$$

$$= \frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc}+\frac{3\log\left(a+b\sinh^{-1}\right.}{8bc}$$

Mathematica [A] time = 0.25348, size = 109, normalized size = 0.76

$$\frac{4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh}{8bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*CoshI
ntegral[4*(a/b + ArcSinh[c*x])] + 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcSinh[c*x])])/(8*b*c)
```

Maple [A] time = 0.144, size = 139, normalized size = 1.

$$\frac{3\ln(a+b\text{Arcsinh}(cx))}{8cb}-\frac{1}{16cb}e^{4\frac{a}{b}}\text{Ei}\left(1,4\text{Arcsinh}(cx)+4\frac{a}{b}\right)-\frac{1}{4cb}e^{2\frac{a}{b}}\text{Ei}\left(1,2\text{Arcsinh}(cx)+2\frac{a}{b}\right)-\frac{1}{4cb}e^{-2\frac{a}{b}}\text{Ei}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 3/8*ln(a+b*arcsinh(c*x))/b/c-1/16/c/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)
-1/4/c/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b*exp(-2*a/b)*Ei(1,-2*
arcsinh(c*x)-2*a/b)-1/16/c/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b\,\text{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b\,\text{arsinh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a), x)
```

3.368 $\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=138

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b}+$$

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b) - (CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b) + (5*Cosh[a/b]*SinhIntegral
[(a + b*ArcSinh[c*x])/b])/(4*b) + (Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*Arc
Sinh[c*x]))/b])/(4*b) + Unintegrable[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])), x]
```

Rubi [A] time = 0.853149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b) - (CoshIntegral[(3*a)
/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b) + (5*Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(4*b) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x
]])/(4*b) + Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \left(\frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)} + \frac{2c^2x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)} + \frac{c^4x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh\right.}\right.$$

$$= \left(2c^2\right)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx + c^4\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx + \int \frac{}{x\sqrt{1}}$$

$$= 2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx + \text{Subst}\left(\int\right.$$

$$= i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right) + \left(2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}\right.}{a+b}\right.$$

$$= -\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} + \frac{1}{4}\text{Subst}\left(\int \frac{\sinh(3}{a+b}\right.$$

$$= -\frac{2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} - \frac{1}{4}\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subs}$$

$$= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b} - \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4b} + \frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.}{4b}$$

Mathematica [A] time = 1.95463, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\text{Arcsinh}\,(cx)\right)}\left(c^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{bx\,\mathrm{arsinh}\,(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x*arcsinh(c*x) + a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x\,(a+b\,\mathrm{asinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x), x)
```

3.369 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=105

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)+\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b}-\frac{c\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b}$$

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b) + (3*c*Log
[a + b*ArcSinh[c*x]])/(2*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSi
nh[c*x]))/b])/(2*b) + Unintegrable[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[
c*x])), x]
```

Rubi [A] time = 0.604161, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b) + (3*c*Log[a
 + b*ArcSinh[c*x]])/(2*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSi
nh[c*x]])/(2*b) + Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
, x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\sinh^{-1}(cx))}\,dx = \int \left(\frac{2c^2}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))} + \frac{1}{x^2\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))} + \frac{c^4x^2}{\sqrt{1+c^2x^2}\,(a+b\sin\right.$$

$$= (2c^2)\int \frac{1}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx + c^4\int \frac{x^2}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx + \int \frac{}{x^2\sqrt{}}$$

$$= \frac{2c\log(a+b\sinh^{-1}(cx))}{b} + c\,\mathrm{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{1}{x^2\sqrt{1+c^2x^2}\,(a+}$$

$$= \frac{2c\log(a+b\sinh^{-1}(cx))}{b} - c\,\mathrm{Subst}\left(\int \left(\frac{1}{2(a+bx)} - \frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right) + \int \frac{}{x^2}$$

$$= \frac{3c\log(a+b\sinh^{-1}(cx))}{2b} + \frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{}{x^2\sqrt{1+c^2x^2}\,(a}$$

$$= \frac{3c\log(a+b\sinh^{-1}(cx))}{2b} + \frac{1}{2}\left(c\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) -$$

$$= \frac{c\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2b} + \frac{3c\log(a+b\sinh^{-1}(cx))}{2b} - \frac{c\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\right.}{2b}$$

Mathematica [A] time = 1.3949, size = 0, normalized size = 0.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^2(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.143, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(a+b\mathrm{Arcsinh}(cx))}\left(c^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{(b \operatorname{arsinh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{bx^2 \operatorname{arsinh}(cx)+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^2*arcsinh(c*x) + a*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b \operatorname{asinh}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\,\mathrm{arsinh}\,(cx)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

3.370 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.14518, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 4.59578, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.44, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b\text{Arcsinh}(cx))} \left(c^2x^2+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{(b \operatorname{arsinh}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{bx^3 \operatorname{arsinh}(cx) + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^3*arcsinh(c*x) + a*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

3.371 $\int \frac{(1+c^2x^2)^{3/2}}{x^4(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(c^2x^2+1)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.14133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.816342, size = 0, normalized size = 0.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b\text{Arcsinh}(cx))} \left(c^2x^2 + 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2 + 1\right)^{\frac{3}{2}}}{(b \operatorname{arsinh}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2 + 1\right)^{\frac{3}{2}}}{bx^4 \operatorname{arsinh}(cx) + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b*x^4*arcsinh(c*x) + a*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{(b\operatorname{arsinh}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

3.372 $\int \frac{x^3\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^4}-\frac{3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{256bc^4}-\frac{\sinh\left(\frac{9a}{b}\right)\text{Chi}\left(\frac{9}{\cdots}\right.}{256b\cdots}$$

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(128*b*c^4) + (CoshInteg
ral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(32*b*c^4) - (3*CoshIntegral
[(7*(a + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(256*b*c^4) - (CoshIntegral[(9*
(a + b*ArcSinh[c*x]))/b]*Sinh[(9*a)/b])/(256*b*c^4) - (3*Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(128*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*
(a + b*ArcSinh[c*x]))/b])/(32*b*c^4) + (3*Cosh[(7*a)/b]*SinhIntegral[(7*(a
+ b*ArcSinh[c*x]))/b])/(256*b*c^4) + (Cosh[(9*a)/b]*SinhIntegral[(9*(a + b*
ArcSinh[c*x]))/b])/(256*b*c^4)
```

Rubi [A] time = 0.576533, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128bc^4}+\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{32bc^4}-\frac{3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7a}{b}+7\sinh^{-1}(cx)\right)}{256bc^4}-\frac{\sinh\left(\frac{9a}{b}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(128*b*c^4) + (CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(32*b*c^4) - (3*CoshIntegral[(7*a)
/b + 7*ArcSinh[c*x]]*Sinh[(7*a)/b])/(256*b*c^4) - (CoshIntegral[(9*a)/b + 9
*ArcSinh[c*x]]*Sinh[(9*a)/b])/(256*b*c^4) - (3*Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(128*b*c^4) - (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSi
nh[c*x]])/(32*b*c^4) + (3*Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*
x]])/(256*b*c^4) + (Cosh[(9*a)/b]*SinhIntegral[(9*a)/b + 9*ArcSinh[c*x]])/(
256*b*c^4)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(x)}{128(a+bx)} - \frac{\sinh(3x)}{32(a+bx)} + \frac{3\sinh(7x)}{256(a+bx)} + \frac{\sinh(9x)}{256(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(9x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256c^4} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256c^4} - \frac{3\,\text{Subst}\left(\int \frac{\sin}{a+}\right.}{}$$

$$= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128c^4} - \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x, \text{si}\right.}{32c^4}$$

$$= \frac{3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{128bc^4} + \frac{\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{32bc^4} - \frac{3\text{Chi}\left(\frac{7a}{b}+7\sinh^{-1}(\right.}{256bc^4}$$

Mathematica [A] time = 0.969224, size = 180, normalized size = 0.73

$$\frac{6\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+8\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-3\sinh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] + 8*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] - 3*CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*
a)/b] - CoshIntegral[9*(a/b + ArcSinh[c*x])]*Sinh[(9*a)/b] - 6*Cosh[a/b]*Si
nhIntegral[a/b + ArcSinh[c*x]] - 8*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])] + 3*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + Cosh[(9
*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(256*b*c^4)
```

Maple [A] time = 0.35, size = 238, normalized size = 1.

$$\frac{1}{512\,c^4b}e^{9\,\frac{a}{b}}\text{Ei}\left(1,9\,\text{Arcsinh}\,(cx)+9\,\frac{a}{b}\right)+\frac{3}{512\,c^4b}e^{7\,\frac{a}{b}}\text{Ei}\left(1,7\,\text{Arcsinh}\,(cx)+7\,\frac{a}{b}\right)-\frac{1}{64\,c^4b}e^{3\,\frac{a}{b}}\text{Ei}\left(1,3\,\text{Arcsinh}\,(cx)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/512/c^4/b*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)+3/512/c^4/b*exp(7*a/b)*Ei
(1,7*arcsinh(c*x)+7*a/b)-1/64/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3
/256/c^4/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/256/c^4/b*exp(-a/b)*Ei(1,-arcs
inh(c*x)-a/b)+1/64/c^4/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-3/512/c^4/
b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/512/c^4/b*exp(-9*a/b)*Ei(1,-9*a
rcsinh(c*x)-9*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^3}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4x^7+2\,c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b\,\mathrm{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^3}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a), x)
```

3.373 $\int \frac{x^2(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=268

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc^3}+\frac{\cosh\left(\frac{8a}{b}\right)\text{Chi}}{128}$$

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cos
h[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cosh[(6*
a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c^3) + (Cosh[(8*a)/b]
*CoshIntegral[(8*(a + b*ArcSinh[c*x]))/b])/(128*b*c^3) - (5*Log[a + b*ArcSi
nh[c*x]])/(128*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x])
)/b])/(32*b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])
/(32*b*c^3) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*
b*c^3) - (Sinh[(8*a)/b]*SinhIntegral[(8*(a + b*ArcSinh[c*x]))/b])/(128*b*c^
3)
```

Rubi [A] time = 0.619348, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{32bc^3}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc^3}+\cosh$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[
(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3) + (Cosh[(8*a)/b]*CoshI
ntegral[(8*a)/b + 8*ArcSinh[c*x]])/(128*b*c^3) - (5*Log[a + b*ArcSinh[c*x]]
)/(128*b*c^3) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*
b*c^3) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(32*b*c^3)
- (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c^3) - (Sinh
[(8*a)/b]*SinhIntegral[(8*a)/b + 8*ArcSinh[c*x]])/(128*b*c^3)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{5}{128(a+bx)} - \frac{\cosh(2x)}{32(a+bx)} + \frac{\cosh(4x)}{32(a+bx)} + \frac{\cosh(6x)}{32(a+bx)} + \frac{\cosh(8x)}{128(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= -\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3} + \frac{\text{Subst}\left(\int \frac{\cosh(8x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128c^3} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh\right.}{32c^3}$$

$$= -\frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{128bc^3} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Sub}}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc^3} + \frac{\cosh\left(\frac{4a}{b}\right)\textbf{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{32bc^3} + \frac{\cosh\left(\frac{6a}{b}\right)\textbf{Chi}\left(\frac{6a}{b}\right.}{32b}$$

Mathematica [A] time = 0.866425, size = 197, normalized size = 0.74

$$\frac{-4\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 4\cosh\left(\frac{4a}{b}\right)\textbf{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 4\cosh\left(\frac{6a}{b}\right)\textbf{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 4*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + 4*Cosh[(6*a)/b]*CoshIntegral[6*(a/b +
ArcSinh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcSinh[c*x])] - 5*Log
[a + b*ArcSinh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])]
 - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*Sinh[(6*a)/b]*S
inhIntegral[6*(a/b + ArcSinh[c*x])] - Sinh[(8*a)/b]*SinhIntegral[8*(a/b + A
rcSinh[c*x])])/(128*b*c^3)
```

Maple [A] time = 0.316, size = 259, normalized size = 1.

$$-\frac{5\ln\left(a+b\text{Arcsinh}\left(cx\right)\right)}{128\,bc^3} - \frac{1}{256\,bc^3}e^{8\,\frac{a}{b}}\text{Ei}\left(1, 8\,\text{Arcsinh}\left(cx\right)+8\,\frac{a}{b}\right) - \frac{1}{64\,bc^3}e^{6\,\frac{a}{b}}\text{Ei}\left(1, 6\,\text{Arcsinh}\left(cx\right)+6\,\frac{a}{b}\right) - \frac{1}{64\,bc^3}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] -5/128*ln(a+b*arcsinh(c*x))/b/c^3-1/256/c^3/b*exp(8*a/b)*Ei(1,8*arcsinh(c*x
)+8*a/b)-1/64/c^3/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-1/64/c^3/b*exp(4*
a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64/c^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2
*a/b)+1/64/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-1/64/c^3/b*exp(-4*
a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c^3/b*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x
)-6*a/b)-1/256/c^3/b*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^2}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^6+2\,c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b\operatorname{arsinh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^2}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a), x)
```

3.374 $\int \frac{x(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64bc^2}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64bc^2}-\frac{\sinh\left(\frac{7a}{b}\right)\text{Chi}}{6}$$

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(64*b*c^2) - (9*CoshInt
egral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(64*b*c^2) - (5*CoshIntegr
al[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(64*b*c^2) - (CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b]*Sinh[(7*a)/b])/(64*b*c^2) + (5*Cosh[a/b]*SinhInte
gral[(a + b*ArcSinh[c*x])/b])/(64*b*c^2) + (9*Cosh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(64*b*c^2) + (5*Cosh[(5*a)/b]*SinhIntegral[(5*(a
 + b*ArcSinh[c*x]))/b])/(64*b*c^2) + (Cosh[(7*a)/b]*SinhIntegral[(7*(a + b*
ArcSinh[c*x]))/b])/(64*b*c^2)
```

Rubi [A] time = 0.490502, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5779, 5448, 3303, 3298, 3301}

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64bc^2}-\frac{9\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64bc^2}-\frac{5\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64bc^2}-\text{sinh}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(64*b*c^2) - (9*CoshIntegra
l[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(64*b*c^2) - (5*CoshIntegral[(5*
a)/b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(64*b*c^2) - (CoshIntegral[(7*a)/b +
7*ArcSinh[c*x]]*Sinh[(7*a)/b])/(64*b*c^2) + (5*Cosh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]])/(64*b*c^2) + (9*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcS
inh[c*x]])/(64*b*c^2) + (5*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c
*x]])/(64*b*c^2) + (Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(
64*b*c^2)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64(a+bx)} + \frac{9\sinh(3x)}{64(a+bx)} + \frac{5\sinh(5x)}{64(a+bx)} + \frac{\sinh(7x)}{64(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{5\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{5\,\text{Subst}\left(\int \frac{\sinh}{a+}\right.}{}$$

$$= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64c^2} + \frac{\left(9\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx, x\right.}{64c^2}$$

$$= -\frac{5\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64bc^2} - \frac{9\textbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{64bc^2} - \frac{5\textbf{Chi}\left(\frac{5a}{b}+5\sinh\right.}{64b}$$

Mathematica [A] time = 0.809432, size = 180, normalized size = 0.73

$$\frac{-5\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 9\sinh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 5\sinh\left(\frac{5a}{b}\right)\textbf{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \sinh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 9*CoshIntegral[3*(a/b + Ar
cSinh[c*x])]*Sinh[(3*a)/b] - 5*CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5
*a)/b] - CoshIntegral[7*(a/b + ArcSinh[c*x])]*Sinh[(7*a)/b] + 5*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] + 9*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 5*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] + Cosh[(
7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^2)
```

Maple [A] time = 0.262, size = 238, normalized size = 1.

$$\frac{1}{128\,c^2b}e^{7\,\frac{a}{b}}\text{Ei}\left(1,7\,\text{Arcsinh}\,(cx)+7\,\frac{a}{b}\right) + \frac{5}{128\,c^2b}e^{5\,\frac{a}{b}}\text{Ei}\left(1,5\,\text{Arcsinh}\,(cx)+5\,\frac{a}{b}\right) + \frac{9}{128\,c^2b}e^{3\,\frac{a}{b}}\text{Ei}\left(1,3\,\text{Arcsinh}\,(cx)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/128/c^2/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+5/128/c^2/b*exp(5*a/b)*Ei
(1,5*arcsinh(c*x)+5*a/b)+9/128/c^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+
5/128/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/128/c^2/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)-9/128/c^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-5/128/c^
2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/128/c^2/b*exp(-7*a/b)*Ei(1,-7
*arcsinh(c*x)-7*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}x}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(c^4x^5+2\,c^2x^3+x)\sqrt{c^2x^2+1}}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a), x)
```

3.375 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=206

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32bc}-\frac{15\sinh\left(\frac{2a}{b}\right)\cdots}{\cdots}$$

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(32*b*c) + (3*C
osh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(16*b*c) + (Cosh[(6*
a)/b]*CoshIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*b*c) + (5*Log[a + b*Arc
Sinh[c*x]])/(16*b*c) - (15*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(32*b*c) - (3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b
])/(16*b*c) - (Sinh[(6*a)/b]*SinhIntegral[(6*(a + b*ArcSinh[c*x]))/b])/(32*
b*c)
```

Rubi [A] time = 0.358841, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5699, 3312, 3303, 3298, 3301}

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{32bc}-\frac{15\,\text{s}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(32*b*c) + (3*Cos
h[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c) + (Cosh[(6*a)/b
]*CoshIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c) + (5*Log[a + b*ArcSinh[c
*x]])/(16*b*c) - (15*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/
(32*b*c) - (3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(16*b*c
) - (Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*x]])/(32*b*c)
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
```

```
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5}{16(a+bx)}+\frac{15\cosh(2x)}{32(a+bx)}+\frac{3\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c}$$
$$= \frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh\right.}{16c}$$
$$= \frac{5\log\left(a+b\sinh^{-1}(cx)\right)}{16bc}+\frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{32c}+\frac{\left(3\cosh\left(\frac{4a}{b}\right.\right.}{}$$
$$= \frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{32bc}+\frac{3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{16bc}+\frac{\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\right.}{3}$$

Mathematica [A] time = 0.541804, size = 153, normalized size = 0.74

$$\frac{15\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+6\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-15}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] + 6*Cosh[(4*a)/b]*Co
shIntegral[4*(a/b + ArcSinh[c*x])] + Cosh[(6*a)/b]*CoshIntegral[6*(a/b + Ar
cSinh[c*x])] + 10*Log[a + b*ArcSinh[c*x]] - 15*Sinh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcSinh[c*x])] - 6*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x]
)] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(32*b*c)
```

Maple [A] time = 0.225, size = 199, normalized size = 1.

$$\frac{5\ln\left(a+b\text{Arcsinh}\left(cx\right)\right)}{16\,cb}-\frac{1}{64\,cb}e^{6\,\frac{a}{b}}\text{Ei}\left(1,6\,\text{Arcsinh}\left(cx\right)+6\,\frac{a}{b}\right)-\frac{3}{32\,cb}e^{4\,\frac{a}{b}}\text{Ei}\left(1,4\,\text{Arcsinh}\left(cx\right)+4\,\frac{a}{b}\right)-\frac{15}{64\,cb}e^{2\,\frac{a}{b}}\text{Ei}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 5/16*ln(a+b*arcsinh(c*x))/b/c-1/64/c/b*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b
)-3/32/c/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-15/64/c/b*exp(2*a/b)*Ei(1,
2*arcsinh(c*x)+2*a/b)-15/64/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)-3/3
2/c/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)-1/64/c/b*exp(-6*a/b)*Ei(1,-6*
arcsinh(c*x)-6*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(c^4x^4+2\,c^2x^2+1)\sqrt{c^2x^2+1}}{b\operatorname{arsinh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a), x)
```

3.376 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=194

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)-\frac{11\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b}-\frac{7\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b}$$

```
[Out] (-11*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b) - (7*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b) - (CoshIntegral[(5*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b) + (11*Cosh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(8*b) + (7*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSi
nh[c*x]))/b])/(16*b) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))
/b])/(16*b) + Unintegrable[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 1.27963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-11*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b) - (7*CoshIntegral[(3
*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b) - (CoshIntegral[(5*a)/b + 5*A
rcSinh[c*x]]*Sinh[(5*a)/b])/(16*b) + (11*Cosh[a/b]*SinhIntegral[a/b + ArcSi
nh[c*x]])/(8*b) + (7*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/
(16*b) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b) + De
fer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\sinh^{-1}(cx))}\,dx = \int \left(\frac{1}{x\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))} + \frac{3c^2x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))} + \frac{3c^4x^3}{\sqrt{1+c^2x^2}\,(a+b\sinh\right.$$

$$= (3c^2)\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx + (3c^4)\int \frac{x^3}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx + c^6\int$$

$$= 3\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + 3\,\mathrm{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + \int \frac{}{x\sqrt{1}}$$

$$= -\left(i\,\mathrm{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3x)}{16(a+bx)} + \frac{i\sinh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)\right) + 3i\,\mathrm{Subst}\left(\int \right.$$

$$= -\frac{3\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{3\cosh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} + \frac{1}{16}\,\mathrm{Subst}\left(\int \frac{\sinh(}{a+}\right.$$

$$= -\frac{3\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b} + \frac{3\cosh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b} + \frac{1}{8}\left(5\cosh\left(\frac{a}{b}\right)\right)\mathrm{Sub}$$

$$= -\frac{11\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b} - \frac{7\mathbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16b} - \frac{\mathbf{Chi}\left(\frac{5a}{b}+5\sin\right.}{1}$$

Mathematica [A] time = 1.94295, size = 0, normalized size = 0.

$$\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.22, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\mathrm{Arcsinh}(cx))}\left(c^2x^2+1\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{bx\operatorname{arsinh}(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x*arcsinh(c*x) + a*
x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x)),x)
```

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x), x)
```

3.377 $\int \frac{(1+c^2x^2)^{5/2}}{x^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=158

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)+\frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b}+\frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c\right.}{b}\right.}{8b}$$

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/b + (c*Cosh[(4*a
)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b) + (15*c*Log[a + b*ArcS
inh[c*x]])/(8*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b
])/b - (c*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b) + U
nintegrable[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.966754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (c*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/b + (c*Cosh[(4*a)/
b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b) + (15*c*Log[a + b*ArcSinh[
c*x]])/(8*b) - (c*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/b -
 (c*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b) + Defer[Int
][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \left(\frac{3c^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)} + \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)} + \frac{3c^4x^2}{\sqrt{1+c^2x^2}\left(a+b\sin\right.}\right.$$

$$= \left(3c^2\right)\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx + \left(3c^4\right)\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx + c^6 \int$$

$$= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{b} + c\,\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) + (3c)\,\text{Subst}\left(\int \frac{\sinh^2}{a+}\right.$$

$$= \frac{3c\log\left(a+b\sinh^{-1}(cx)\right)}{b} + c\,\text{Subst}\left(\int \left(\frac{3}{8(a+bx)} - \frac{\cosh(2x)}{2(a+bx)} + \frac{\cosh(4x)}{8(a+bx)}\right) dx, x, \sinh^{-1}\right.$$

$$= \frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b} + \frac{1}{8}c\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right) - \frac{1}{2}c\,\text{Subst}\left(\int \frac{\cos}{a}\right.$$

$$= \frac{15c\log\left(a+b\sinh^{-1}(cx)\right)}{8b} - \frac{1}{2}\left(c\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= \frac{c\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b} + \frac{c\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8b} + \frac{15c\log\left(a+\right.}{8}$$

Mathematica [A] time = 1.30436, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.171, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\text{Arcsinh}\,(cx)\right)}\left(c^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{bx^2\operatorname{arsinh}(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^2*arcsinh(c*x) +
a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b \operatorname{arsinh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^2), x)
```

3.378 $\int \frac{(1+c^2x^2)^{5/2}}{x^3(a+b\sinh^{-1}(cx))} \, dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(c^2x^2+1)^{5/2}}{x^3\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.139597, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\,(a+b\sinh^{-1}(cx))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\,(a+b\sinh^{-1}(cx))} \, dx = \int \frac{(1+c^2x^2)^{5/2}}{x^3\,(a+b\sinh^{-1}(cx))} \, dx$$

Mathematica [A] time = 4.5686, size = 0, normalized size = 0.

$$\int \frac{(1+c^2x^2)^{5/2}}{x^3\,(a+b\sinh^{-1}(cx))} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.542, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b\mathrm{Arcsinh}\,(cx))} \left(c^2x^2+1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{bx^3\,\mathrm{arsinh}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^3*arcsinh(c*x) +
a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\mathrm{arsinh}\,(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^3), x)
```

3.379 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.140862, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.898547, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.429, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b\mathrm{Arcsinh}(cx))} \left(c^2x^2 + 1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2 + 1\right)^{\frac{5}{2}}}{(b\,\mathrm{arsinh}\,(cx) + a)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4x^4 + 2\,c^2x^2 + 1\right)\sqrt{c^2x^2 + 1}}{bx^4\,\mathrm{arsinh}\,(cx) + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b*x^4*arcsinh(c*x) +
a*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{(b\operatorname{arsinh}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)*x^4), x)
```

3.380 $\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

```
[Out] -CoshIntegral[2*ArcSinh[a*x]]/(2*a^5) + CoshIntegral[4*ArcSinh[a*x]]/(8*a^5
) + (3*Log[ArcSinh[a*x]])/(8*a^5)
```

Rubi [A] time = 0.161055, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5779, 3312, 3301}

$$-\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcSinh[a*x]]/(2*a^5) + CoshIntegral[4*ArcSinh[a*x]]/(8*a^5
) + (3*Log[ArcSinh[a*x]])/(8*a^5)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^5} \\
&= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}-\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^5} \\
&= \frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{8a^5}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^5} \\
&= -\frac{\textbf{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^5}+\frac{\textbf{Chi}\left(4\sinh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\sinh^{-1}(ax)\right)}{8a^5}
\end{aligned}
$$

Mathematica [A] time = 0.0710375, size = 31, normalized size = 0.76

$$\frac{-4\textbf{Chi}\left(2\sinh^{-1}(ax)\right)+\textbf{Chi}\left(4\sinh^{-1}(ax)\right)+3\log\left(\sinh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-4*CoshIntegral[2*ArcSinh[a*x]] + CoshIntegral[4*ArcSinh[a*x]] + 3*Log[Arc
Sinh[a*x]])/(8*a^5)
```

Maple [A] time = 0.052, size = 36, normalized size = 0.9

$$-\frac{\text{Chi}\left(2\,\text{Arcsinh}\left(ax\right)\right)}{2\,a^5}+\frac{\text{Chi}\left(4\,\text{Arcsinh}\left(ax\right)\right)}{8\,a^5}+\frac{3\,\ln\left(\text{Arcsinh}\left(ax\right)\right)}{8\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^5+1/8*Chi(4*arcsinh(a*x))/a^5+3/8*ln(arcsinh(a*x
))/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^4}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

3.381 $\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

```
[Out] (-3*SinhIntegral[ArcSinh[a*x]])/(4*a^4) + SinhIntegral[3*ArcSinh[a*x]]/(4*a
^4)
```

Rubi [A] time = 0.155289, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5779, 3312, 3298}

$$\frac{\text{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\text{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[ArcSinh[a*x]])/(4*a^4) + SinhIntegral[3*ArcSinh[a*x]]/(4*a
^4)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}-\frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{4a^4}\\
&= -\frac{3\textbf{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}+\frac{\textbf{Shi}\left(3\sinh^{-1}(ax)\right)}{4a^4}
\end{aligned}$$

Mathematica [A] time = 0.0774442, size = 22, normalized size = 0.81

$$\frac{\textbf{Shi}\left(3\sinh^{-1}(ax)\right)-3\textbf{Shi}\left(\sinh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[ArcSinh[a*x]] + SinhIntegral[3*ArcSinh[a*x]])/(4*a^4)
```

Maple [A] time = 0.043, size = 23, normalized size = 0.9

$$-\frac{3\,\text{Shi}\left(\text{Arcsinh}\left(ax\right)\right)-\text{Shi}\left(3\,\text{Arcsinh}\left(ax\right)\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(3*Shi(arcsinh(a*x))-Shi(3*arcsinh(a*x)))/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a^2x^2+1}\,\mathrm{asinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

3.382 $\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rubi [A] time = 0.144956, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5779, 3312, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^3}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Mathematica [A] time = 0.0682203, size = 22, normalized size = 0.81

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)-\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

Maple [A] time = 0.041, size = 24, normalized size = 0.9

$$\frac{\text{Chi}\left(2\,\text{Arcsinh}\left(ax\right)\right)}{2\,a^3}-\frac{\ln\left(\text{Arcsinh}\left(ax\right)\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

3.383 $\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rubi [A] time = 0.14396, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {5779, 3312, 3301}

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcSinh[a*x]]/(2*a^3) - Log[ArcSinh[a*x]]/(2*a^3)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}\\ &= -\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{2a^3}\\ &= \frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)}{2a^3}-\frac{\log\left(\sinh^{-1}(ax)\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.0232666, size = 22, normalized size = 0.81

$$\frac{\text{Chi}\left(2\sinh^{-1}(ax)\right)-\log\left(\sinh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Log[ArcSinh[a*x]])/(2*a^3)
```

Maple [A] time = 0., size = 24, normalized size = 0.9

$$\frac{\text{Chi}\left(2\,\text{Arcsinh}\left(ax\right)\right)}{2\,a^3}-\frac{\ln\left(\text{Arcsinh}\left(ax\right)\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*Chi(2*arcsinh(a*x))/a^3-1/2*ln(arcsinh(a*x))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\mathrm{asinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a^2x^2+1}\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

3.384 $\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

```
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Rubi [A] time = 0.0801654, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5779, 3298}

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0258395, size = 9, normalized size = 1.

$$\frac{\text{Shi}\left(\sinh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] SinhIntegral[ArcSinh[a*x]]/a^2
```

Maple [A] time = 0.034, size = 10, normalized size = 1.1

$$\frac{\text{Shi}\left(\text{Arcsinh}\left(ax\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] Shi(arcsinh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a^2x^2+1}\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 + 1)*arcsinh(a*x)), x)
```

3.385 $\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

```
[Out] Log[ArcSinh[a*x]]/a
```

Rubi [A] time = 0.0367984, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5673}

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]

[Out] Log[ArcSinh[a*x]]/a
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx=\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.0147082, size = 9, normalized size = 1.

$$\frac{\log\left(\sinh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Log[ArcSinh[a*x]]/a
```

Maple [A] time = 0.004, size = 10, normalized size = 1.1

$$\frac{\ln(\operatorname{Arcsinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] ln(arcsinh(a*x))/a
```

Maxima [A] time = 1.11884, size = 12, normalized size = 1.33

$$\frac{\log(\operatorname{arsinh}(ax))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(arcsinh(a*x))/a
```

Fricas [B] time = 2.03353, size = 50, normalized size = 5.56

$$\frac{\log\left(\log\left(ax + \sqrt{a^2x^2 + 1}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(log(a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] time = 0.532033, size = 7, normalized size = 0.78

$$\frac{\log\left(\operatorname{asinh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] log(asinh(a*x))/a
```

Giac [B] time = 1.32494, size = 30, normalized size = 3.33

$$\frac{\log\left(\left|\log\left(ax + \sqrt{a^2x^2 + 1}\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(log(a*x + sqrt(a^2*x^2 + 1))))/a
```

3.386 $\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a^2x^2+1}\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi [A] time = 0.0995991, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx=\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.05339, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Maple [A] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{1}{x\text{Arcsinh}\,(ax)}\frac{1}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2x^2+1}x\,\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a^2x^2+1}}{\left(a^2x^3+x\right)\text{arsinh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^3 + x)*arcsinh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a^2x^2+1}\,\text{asinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2x^2+1}x\,\mathrm{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x*arcsinh(a*x)), x)
```

3.387 $\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a^2x^2+1}\sinh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi [A] time = 0.0981286, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx=\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.131467, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{1+a^2x^2}\sinh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]), x]
```

Maple [A] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\operatorname{Arcsinh}\left(a x\right)}\frac{1}{\sqrt{a^{2} x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2} x^{2}+1}x^{2} \operatorname{arsinh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^{2} x^{2}+1}}{\left(a^{2} x^{4}+x^{2}\right) \operatorname{arsinh}\left(a x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)/((a^2*x^4 + x^2)*arcsinh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\sqrt{a^{2} x^{2}+1} \operatorname{asinh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/asinh(a*x)/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2x^2+1}x^2 \operatorname{arsinh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arcsinh(a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + 1)*x^2*arcsinh(a*x)), x)
```

3.388 $\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=183

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^6}+\frac{5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^6}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16bc^6}+\frac{5\cosh\left(\frac{a}{b}\right)\text{Shi}}{8b}$$

```
[Out] (-5*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b*c^6) + (5*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b*c^6) - (CoshIntegral[
(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b*c^6) + (5*Cosh[a/b]*SinhIn
tegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^6) - (5*Cosh[(3*a)/b]*SinhIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^6) + (Cosh[(5*a)/b]*SinhIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b*c^6)
```

Rubi [A] time = 0.459218, antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$-\frac{5\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^6}+\frac{5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16bc^6}-\frac{\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16bc^6}+\frac{5\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (-5*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b*c^6) + (5*CoshIntegral
[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b*c^6) - (CoshIntegral[(5*a)/
b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b*c^6) + (5*Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]])/(8*b*c^6) - (5*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*A
rcSinh[c*x]])/(16*b*c^6) + (Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[
c*x]])/(16*b*c^6)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^6}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3x)}{16(a+bx)} + \frac{i\sinh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^6} - \frac{5\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16c^6} + \frac{5\,\text{S}}{}$$
$$= \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^6} - \frac{\left(5\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh}{}\right.}{16c}$$
$$= -\frac{5\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^6} + \frac{5\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{16bc^6} - \frac{\text{Chi}\left(\right.}{}$$

Mathematica [A] time = 0.342057, size = 136, normalized size = 0.74

$$-\frac{10\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 5\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 10\,\text{co}}{16bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -(10*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b + A
rcSinh[c*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh[(5*
a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*Cosh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcSinh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + Arc
Sinh[c*x])])/(16*b*c^6)
```

Maple [A] time = 0.125, size = 178, normalized size = 1.

$$\frac{1}{32\,c^6 b}e^{5\,\frac{a}{b}}\text{Ei}\left(1, 5\,\text{Arcsinh}\,(cx) + 5\,\frac{a}{b}\right) - \frac{5}{32\,c^6 b}e^{3\,\frac{a}{b}}\text{Ei}\left(1, 3\,\text{Arcsinh}\,(cx) + 3\,\frac{a}{b}\right) + \frac{5}{16\,c^6 b}e^{\frac{a}{b}}\text{Ei}\left(1, \text{Arcsinh}\,(cx) + \frac{a}{b}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/32/c^6/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-5/32/c^6/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)+5/16/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/
b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+5/32/c^6/b*exp(-3*a/b)*Ei(1,-3*arcsinh(
c*x)-3*a/b)-1/32/c^6/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{c^2x^2+1}(b \operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{c^2x^2+1}x^5}{ac^2x^2+\left(bc^2x^2+b\right)\operatorname{arsinh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.389 $\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4}{}\right.}{8bc}$$

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c^5) + (Cosh
[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b])/(8*b*c^5) + (3*Log[a +
b*ArcSinh[c*x]])/(8*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[
c*x]))/b])/(2*b*c^5) - (Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))
/b])/(8*b*c^5)
```

Rubi [A] time = 0.40299, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^5}+\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5}-\sinh\left(\frac{4}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -(Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c^5) + (Cosh[(
4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^5) + (3*Log[a + b*Ar
cSinh[c*x]])/(8*b*c^5) + (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*
x]])/(2*b*c^5) - (Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*
b*c^5)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8(a+bx)} - \frac{\cosh(2x)}{2(a+bx)} + \frac{\cosh(4x)}{8(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^5}$$

$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8c^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2}{a+bx}\right.}{}$$

$$= \frac{3\log\left(a+b\sinh^{-1}(cx)\right)}{8bc^5} - \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c^5} + \frac{\text{co}}{}$$

$$= -\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^5} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^5} + \frac{3\log}{}$$

Mathematica [A] time = 0.218429, size = 109, normalized size = 0.76

$$-\frac{4\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sin}{8bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Cosh[(4*a)/b]*Cosh
Integral[4*(a/b + ArcSinh[c*x])] - 3*Log[a + b*ArcSinh[c*x]] - 4*Sinh[(2*a)
/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Sinh[(4*a)/b]*SinhIntegral[4*(a/
b + ArcSinh[c*x])])/(8*b*c^5)
```

Maple [A] time = 0.151, size = 139, normalized size = 1.

$$\frac{3\ln(a+b\mathrm{Arcsinh}(cx))}{8c^5b}-\frac{1}{16c^5b}\mathrm{e}^{4\frac{a}{b}}\mathrm{Ei}\left(1,4\,\mathrm{Arcsinh}(cx)+4\frac{a}{b}\right)+\frac{1}{4c^5b}\mathrm{e}^{2\frac{a}{b}}\mathrm{Ei}\left(1,2\,\mathrm{Arcsinh}(cx)+2\frac{a}{b}\right)+\frac{1}{4c^5b}\mathrm{e}^{-2\frac{a}{b}}\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 3/8*ln(a+b*arcsinh(c*x))/b/c^5-1/16/c^5/b*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*
a/b)+1/4/c^5/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/4/c^5/b*exp(-2*a/b)*
Ei(1,-2*arcsinh(c*x)-2*a/b)-1/16/c^5/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a
/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^4}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a+b\,\text{asinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\left(cx\right)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.390 $\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=121

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\,\text{si}\right.}{}\right.}{4bc^4}$$

```
[Out] (3*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b*c^4) - (CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b*c^4) - (3*Cosh[a/b]*SinhI
ntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^4) + (Cosh[(3*a)/b]*SinhIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(4*b*c^4)
```

Rubi [A] time = 0.38649, antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$\frac{3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^4}-\frac{\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^4}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^4}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Sh}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b*c^4) - (CoshIntegral[(3
*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b*c^4) - (3*Cosh[a/b]*SinhIntegra
l[a/b + ArcSinh[c*x]])/(4*b*c^4) + (Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcSinh[c*x]])/(4*b*c^4)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}\\
&= \frac{i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^4} - \frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^4}\\
&= -\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4c^4} + \frac{\cosh\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}\right.}{a+b}\right.}{4c^4}\\
&= \frac{3\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^4} - \frac{\mathbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)\sinh\left(\frac{3a}{b}\right)}{4bc^4} - \frac{3\cosh\left(\frac{a}{b}\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.215617, size = 92, normalized size = 0.76

$$
\frac{3\sinh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - \sinh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 3\cosh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + \cosh\left(\frac{3a}{b}\right)\mathbf{Sh}}{4bc^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (3*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - CoshIntegral[3*(a/b + ArcSi
nh[c*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + Co
sh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^4)
```

Maple [A] time = 0.095, size = 118, normalized size = 1.

$$\frac{1}{8c^4b}e^{3\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\,(cx)+3\,\frac{a}{b}\right)-\frac{3}{8c^4b}e^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right)+\frac{3}{8c^4b}e^{-\frac{a}{b}}\mathrm{Ei}\left(1,-\mathrm{Arcsinh}\,(cx)-\frac{a}{b}\right)-\frac{1}{8c^4b}e^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8/c^4/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^4/b*exp(a/b)*Ei(1,arc
sinh(c*x)+a/b)+3/8/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)-1/8/c^4/b*exp(-3
*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}x^3}{ac^2x^2+\left(bc^2x^2+b\right)\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.391 $\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - Log[a
+ b*ArcSinh[c*x]]/(2*b*c^3) - (Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/(2*b*c^3)
```

Rubi [A] time = 0.291869, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5779, 3312, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(2*b*c^3) - Log[a +
b*ArcSinh[c*x]]/(2*b*c^3) - (Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh
[c*x]])/(2*b*c^3)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}\\
&= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c^3}\\
&= -\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2c^3}-\frac{\text{si}}{}\\
&= \frac{\cosh\left(\frac{2a}{b}\right)\textbf{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}-\frac{\sinh\left(\frac{2a}{b}\right)\textbf{Shi}\left(\frac{2a}{b}\right.}{2bc}
\end{aligned}$$

Mathematica [A] time = 0.180165, size = 65, normalized size = 0.79

$$\frac{\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\log\left(a+b\sinh^{-1}(cx)\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] (Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c*x])] - Log[a + b*ArcSinh[c*x
]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^3)
```

Maple [A] time = 0.083, size = 79, normalized size = 1.

$$-\frac{\ln(a+b\text{Arcsinh}(cx))}{2c^3b}-\frac{1}{4c^3b}e^{2\frac{a}{b}}\text{Ei}\left(1,2\,\text{Arcsinh}(cx)+2\frac{a}{b}\right)-\frac{1}{4c^3b}e^{-2\frac{a}{b}}\text{Ei}\left(1,-2\,\text{Arcsinh}(cx)-2\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*ln(a+b*arcsinh(c*x))/b/c^3-1/4/c^3/b*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*
a/b)-1/4/c^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^2}{ac^2x^2+\left(bc^2x^2+b\right)\text{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+b\,\text{asinh}\,(cx))\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{c^2x^2+1}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.392 $\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)} dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc^2}$$

```
[Out] -((CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b*c^2)) + (Cosh[a/b]*Si
nhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c^2)
```

Rubi [A] time = 0.185191, antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {5779, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b*c^2)) + (Cosh[a/b]*SinhIn
tegral[a/b + ArcSinh[c*x]])/(b*c^2)
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,\right.}{c^2}$$

$$= -\frac{\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Mathematica [A] time = 0.11135, size = 46, normalized size = 0.85

$$-\frac{\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - \cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] -((CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]])/(b*c^2))
```

Maple [A] time = 0.034, size = 58, normalized size = 1.1

$$\frac{1}{2c^2b}e^{\frac{a}{b}}\text{Ei}\left(1, \text{Arcsinh}(cx)+\frac{a}{b}\right) - \frac{1}{2c^2b}e^{-\frac{a}{b}}\text{Ei}\left(1, -\text{Arcsinh}(cx)-\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x}{ac^2x^2+\left(bc^2x^2+b\right)\operatorname{arsinh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) + a)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c^2x^2+1}(b \operatorname{arsinh}(cx)+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.393 $\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=16

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Rubi [A] time = 0.0497027, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5673}

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Rule 5673

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Symbol] :> Simp[Log[a + b*ArcSinh[c*x]]/(b*c*Sqrt[d]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Mathematica [A] time = 0.0254949, size = 16, normalized size = 1.

$$\frac{\log\left(a+b\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Log[a + b*ArcSinh[c*x]]/(b*c)
```

Maple [A] time = 0.006, size = 17, normalized size = 1.1

$$\frac{\ln\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] ln(a+b*arcsinh(c*x))/b/c
```

Maxima [A] time = 1.08688, size = 22, normalized size = 1.38

$$\frac{\log\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(b*arcsinh(c*x) + a)/(b*c)
```

Fricas [A] time = 2.13455, size = 63, normalized size = 3.94

$$\frac{\log\left(b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(b*log(c*x + sqrt(c^2*x^2 + 1)) + a)/(b*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.41217, size = 39, normalized size = 2.44

$$\frac{\log\left(\left|b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(b*log(c*x + sqrt(c^2*x^2 + 1)) + a))/(b*c)
```

3.394 $\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.125666, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.69428, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\text{Arcsinh}\left(cx\right)\right)}\frac{1}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c^{2}x^{2}+1}(b\,\operatorname{arsinh}\left(cx\right)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{ac^{2}x^{3}+ax+\left(bc^{2}x^{3}+bx\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^3 + a*x + (b*c^2*x^3 + b*x)*arcsinh(c*x
)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\,\mathrm{asinh}\left(cx\right)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\left(cx\right)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x), x)
```

3.395 $\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.13306, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.1657, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(a + b\mathrm{Arcsinh}\left(cx\right)\right)} \frac{1}{\sqrt{c^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c^{2}x^{2}+1}(b\operatorname{arsinh}\left(cx\right)+a)x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{ac^{2}x^{4}+ax^{2}+\left(bc^{2}x^{4}+bx^{2}\right)\operatorname{arsinh}\left(cx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^2*x^4 + a*x^2 + (b*c^2*x^4 + b*x^2)*arcsinh
(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{asinh}(cx)) \sqrt{c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c^2x^2 + 1}(b \operatorname{arsinh}(cx) + a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)*x^2), x)
```

3.396 $\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^2}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.145122, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 2.05527, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{x^{2}}{a+b\mathrm{Arcsinh}\left(cx\right)}\left(c^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}\,x^{2}}{ac^{4}x^{4}+2\,ac^{2}x^{2}+\left(bc^{4}x^{4}+2\,bc^{2}x^{2}+b\right)\operatorname{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+b\operatorname{asinh}(cx))\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

3.397 $\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{x}{(c^2x^2+1)^{3/2}(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.101509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx = \int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A] time = 1.0656, size = 0, normalized size = 0.

$$\int \frac{x}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.121, size = 0, normalized size = 0.

$$\int \frac{x}{a + b\mathrm{Arcsinh}\,(cx)} \left(c^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}x}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^
2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\,\mathrm{asinh}\,(cx))\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

3.398 $\int \frac{1}{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{(c^2x^2+1)^{3/2}\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.0534405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx=\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A] time = 0.0632239, size = 0, normalized size = 0.

$$\int \frac{1}{(1+c^2x^2)^{3/2}\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\mathrm{Arcsinh}\left(cx\right)}\left(c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\mathrm{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*c^2*
x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

3.399 $\int \frac{1}{x(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.137728, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.45323, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.249, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(a+b\mathrm{Arcsinh}\,(cx))}\left(c^2x^2+1\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{ac^4x^5+2\,ac^2x^3+ax+\left(bc^4x^5+2\,bc^2x^3+bx\right)\mathrm{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^5 + 2*a*c^2*x^3 + a*x + (b*c^4*x^5 + 2*
b*c^2*x^3 + b*x)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x), x)
```

3.400 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.140516, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.08296, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.158, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(a+b\text{Arcsinh}\left(cx\right)\right)}\left(c^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{ac^{4}x^{6}+2\,ac^{2}x^{4}+ax^{2}+\left(bc^{4}x^{6}+2\,bc^{2}x^{4}+bx^{2}\right)\text{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a*c^4*x^6 + 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 +
2*b*c^2*x^4 + b*x^2)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\operatorname{asinh}(cx)\right)\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}(cx)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)*x^2), x)
```

3.401 $\int \frac{x^m(1+c^2x^2)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{5/2}x^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi [A] time = 0.131746, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx = \int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.882049, size = 0, normalized size = 0.

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x]), x]
```

Maple [A] time = 0.766, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{a+b\mathrm{Arcsinh}\left(cx\right)}\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}x^{m}}{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^{4}x^{4}+2\,c^{2}x^{2}+1\right)\sqrt{c^{2}x^{2}+1}x^{m}}{b\operatorname{arsinh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) +
a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^m}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a), x)
```

3.402 $\int \frac{x^m \left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{3/2}x^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi [A] time = 0.130641, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m \left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{x^m \left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.489895, size = 0, normalized size = 0.

$$\int \frac{x^m \left(1+c^2x^2\right)^{3/2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x]), x]
```

Maple [A] time = 0.683, size = 0, normalized size = 0.

$$\int \frac{x^m}{a + b\operatorname{Arcsinh}(cx)} \left(c^2x^2+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}} x^m}{b \operatorname{arsinh}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}} x^m}{b \operatorname{arsinh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^m}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a), x)
```

3.403 $\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\sqrt{c^2x^2+1}x^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Rubi [A] time = 0.113754, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]

[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.114576, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]), x]
```

Maple [A] time = 0.605, size = 0, normalized size = 0.

$$\int \frac{x^m}{a + b\text{Arcsinh}\,(cx)}\sqrt{c^2x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2 + 1}x^m}{b\,\text{arsinh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2 + 1}x^m}{b\,\text{arsinh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{c^2x^2+1}}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}x^m}{b\operatorname{arsinh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a), x)
```

3.404 $\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^m}{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.124043, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.332625, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.157, size = 0, normalized size = 0.

$$\int \frac{x^m}{a + b\mathrm{Arcsinh}\,(cx)} \frac{1}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}x^m}{ac^2x^2+\left(bc^2x^2+b\right)\mathrm{arsinh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 + b)*arcsinh(c*x) +
a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a + b\operatorname{asinh}(cx))\sqrt{c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{c^2x^2 + 1}(b\operatorname{arsinh}(cx) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

3.405 $\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^m}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.131909, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.587233, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{x^m}{a+b\text{Arcsinh}\,(cx)}\left(c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^m}{ac^4x^4+2\,ac^2x^2+\left(bc^4x^4+2\,bc^2x^2+b\right)\text{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^4*x^4 + 2*b*
c^2*x^2 + b)*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a+b\operatorname{asinh}(cx))\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

3.406 $\int \frac{(c+a^2cx^2)^3}{\sinh^{-1}(ax)^2} dx$

Optimal. Leaf size=94

$$-\frac{c^3\left(a^2x^2+1\right)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{63c^3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

```
[Out] -((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (35*c^3*SinhIntegral[ArcSin
h[a*x]])/(64*a) + (63*c^3*SinhIntegral[3*ArcSinh[a*x]])/(64*a) + (35*c^3*Si
nhIntegral[5*ArcSinh[a*x]])/(64*a) + (7*c^3*SinhIntegral[7*ArcSinh[a*x]])/(
64*a)
```

Rubi [A] time = 0.183676, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5696, 5779, 5448, 3298}

$$-\frac{c^3\left(a^2x^2+1\right)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\text{Shi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{63c^3\text{Shi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Shi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(7\sinh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```


```
[Out] -((c^3*(1 + a^2*x^2)^(7/2))/(a*ArcSinh[a*x])) + (35*c^3*SinhIntegral[ArcSin
h[a*x]])/(64*a) + (63*c^3*SinhIntegral[3*ArcSinh[a*x]])/(64*a) + (35*c^3*Si
nhIntegral[5*ArcSinh[a*x]])/(64*a) + (7*c^3*SinhIntegral[7*ArcSinh[a*x]])/(
64*a)
```


Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+a^2cx^2\right)^3}{\sinh^{-1}(ax)^2}\,dx &= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)}+\left(7ac^3\right)\int \frac{x\left(1+a^2x^2\right)^{5/2}}{\sinh^{-1}(ax)}\,dx\\
&= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)}+\frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}\\
&= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)}+\frac{\left(7c^3\right)\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64x}+\frac{9\sinh(3x)}{64x}+\frac{5\sinh(5x)}{64x}+\frac{\sinh(7x)}{64x}\right)dx,x,\sinh^{-1}(ax)\right)}{a}\\
&= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)}+\frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{64a}+\frac{\left(35c^3\right)\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}\right.}{64a}\\
&= -\frac{c^3\left(1+a^2x^2\right)^{7/2}}{a\sinh^{-1}(ax)}+\frac{35c^3\textbf{Shi}\left(\sinh^{-1}(ax)\right)}{64a}+\frac{63c^3\textbf{Shi}\left(3\sinh^{-1}(ax)\right)}{64a}+\frac{35c^3\textbf{Shi}\left(5\sinh^{-1}(ax)\right)}{64a}+\frac{7}{}\end{aligned}$$

Mathematica [A] time = 0.48972, size = 82, normalized size = 0.87

$$\frac{c^3\left(-64\left(a^2x^2+1\right)^{7/2}+35\sinh^{-1}(ax)\textbf{Shi}\left(\sinh^{-1}(ax)\right)+63\sinh^{-1}(ax)\textbf{Shi}\left(3\sinh^{-1}(ax)\right)+35\sinh^{-1}(ax)\textbf{Shi}\left(5\sinh^{-1}(a\right.\right.}{64a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^3/ArcSinh[a*x]^2,x]
```

```
[Out] (c^3*(-64*(1 + a^2*x^2)^(7/2) + 35*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 63*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 35*ArcSinh[a*x]*SinhIntegr
al[5*ArcSinh[a*x]] + 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]]))/(64*a*Ar
cSinh[a*x])
```

Maple [A] time = 0.052, size = 106, normalized size = 1.1

$$\frac{c^3}{64\,a\mathrm{Arcsinh}\,(ax)}\Big(35\,\mathrm{Shi}\,(\mathrm{Arcsinh}\,(ax))\,\mathrm{Arcsinh}\,(ax)+63\,\mathrm{Shi}\,(3\,\mathrm{Arcsinh}\,(ax))\,\mathrm{Arcsinh}\,(ax)+35\,\mathrm{Shi}\,(5\,\mathrm{Arcsinh}\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x)
```

```
[Out] 1/64/a*c^3*(35*Shi(arcsinh(a*x))*arcsinh(a*x)+63*Shi(3*arcsinh(a*x))*arcsin
h(a*x)+35*Shi(5*arcsinh(a*x))*arcsinh(a*x)+7*Shi(7*arcsinh(a*x))*arcsinh(a*
x)-21*cosh(3*arcsinh(a*x))-7*cosh(5*arcsinh(a*x))-cosh(7*arcsinh(a*x))-35*(
a^2*x^2+1)^(1/2))/arcsinh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^9c^3x^9+4\,a^7c^3x^7+6\,a^5c^3x^5+4\,a^3c^3x^3+ac^3x+\left(a^8c^3x^8+4\,a^6c^3x^6+6\,a^4c^3x^4+4\,a^2c^3x^2+c^3\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int\frac{7\,a^{10}c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x + (
a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*sqrt(a^2
*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2
+ 1))) + integrate((7*a^10*c^3*x^10 + 29*a^8*c^3*x^8 + 46*a^6*c^3*x^6 + 34*
a^4*c^3*x^4 + 11*a^2*c^3*x^2 + c^3 + (7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a
^4*c^3*x^4 + 4*a^2*c^3*x^2 - c^3)*(a^2*x^2 + 1) + 7*(2*a^9*c^3*x^9 + 7*a^7*
```

```
c^3*x^7 + 9*a^5*c^3*x^5 + 5*a^3*c^3*x^3 + a*c^3*x)*sqrt(a^2*x^2 + 1))/((a^4
*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 +
 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^6c^3x^6 + 3\,a^4c^3x^4 + 3\,a^2c^3x^2 + c^3}{\operatorname{arsinh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arcsinh(a*x)^2
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^3\left(\int\frac{3a^2x^2}{\operatorname{asinh}^2\left(ax\right)}\,dx + \int\frac{3a^4x^4}{\operatorname{asinh}^2\left(ax\right)}\,dx + \int\frac{a^6x^6}{\operatorname{asinh}^2\left(ax\right)}\,dx + \int\frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**3/asinh(a*x)**2,x)
```

```
[Out] c**3*(Integral(3*a**2*x**2/asinh(a*x)**2, x) + Integral(3*a**4*x**4/asinh(a
*x)**2, x) + Integral(a**6*x**6/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-
2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2cx^2 + c\right)^3}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^3/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^3/arcsinh(a*x)^2, x)
```

3.407 $\int \frac{(c+a^2cx^2)^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{c^2\left(a^2x^2+1\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\mathrm{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

```
[Out] -((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*SinhIntegral[ArcSinh
[a*x]])/(8*a) + (15*c^2*SinhIntegral[3*ArcSinh[a*x]])/(16*a) + (5*c^2*SinhI
ntegral[5*ArcSinh[a*x]])/(16*a)
```

Rubi [A] time = 0.169523, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5696, 5779, 5448, 3298}

$$-\frac{c^2\left(a^2x^2+1\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\mathrm{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\mathrm{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\mathrm{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((c^2*(1 + a^2*x^2)^(5/2))/(a*ArcSinh[a*x])) + (5*c^2*SinhIntegral[ArcSinh
[a*x]])/(8*a) + (15*c^2*SinhIntegral[3*ArcSinh[a*x]])/(16*a) + (5*c^2*SinhI
ntegral[5*ArcSinh[a*x]])/(16*a)
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
```

```
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+a^2cx^2\right)^2}{\sinh^{-1}(ax)^2}\,dx &= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)}+\left(5ac^2\right)\int \frac{x\left(1+a^2x^2\right)^{3/2}}{\sinh^{-1}(ax)}\,dx \\
&= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a} \\
&= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x}+\frac{3\sinh(3x)}{16x}+\frac{\sinh(5x)}{16x}\right)dx,x,\sinh^{-1}(ax)\right)}{a} \\
&= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{16a}+\frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh
\end{aligned}$$

$$= -\frac{c^2\left(1+a^2x^2\right)^{5/2}}{a\sinh^{-1}(ax)}+\frac{5c^2\textbf{Shi}\left(\sinh^{-1}(ax)\right)}{8a}+\frac{15c^2\textbf{Shi}\left(3\sinh^{-1}(ax)\right)}{16a}+\frac{5c^2\textbf{Shi}\left(5\sinh^{-1}(ax)\right)}{16a}$$

Mathematica [A] time = 0.382716, size = 69, normalized size = 0.9

$$\frac{c^2\left(-16\left(a^2x^2+1\right)^{5/2}+10\sinh^{-1}(ax)\textbf{Shi}\left(\sinh^{-1}(ax)\right)+15\sinh^{-1}(ax)\textbf{Shi}\left(3\sinh^{-1}(ax)\right)+5\sinh^{-1}(ax)\textbf{Shi}\left(5\sinh^{-1}(a\right.\right.}{16a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)^2/ArcSinh[a*x]^2,x]
```

```
[Out] (c^2*(-16*(1 + a^2*x^2)^(5/2) + 10*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]]
+ 15*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 5*ArcSinh[a*x]*SinhIntegra
l[5*ArcSinh[a*x]]))/(16*a*ArcSinh[a*x])
```

Maple [A] time = 0.033, size = 84, normalized size = 1.1

$$\frac{c^2}{16\,a\mathrm{Arcsinh}\,(ax)}\Big(10\,\mathrm{Shi}\,(\mathrm{Arcsinh}\,(ax))\,\mathrm{Arcsinh}\,(ax)+15\,\mathrm{Shi}\,(3\,\mathrm{Arcsinh}\,(ax))\,\mathrm{Arcsinh}\,(ax)+5\,\mathrm{Shi}\,(5\,\mathrm{Arcsinh}\,(ax))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

```
[Out] 1/16/a*c^2*(10*Shi(arcsinh(a*x))*arcsinh(a*x)+15*Shi(3*arcsinh(a*x))*arcsin
h(a*x)+5*Shi(5*arcsinh(a*x))*arcsinh(a*x)-10*(a^2*x^2+1)^(1/2)-5*cosh(3*arc
sinh(a*x))-cosh(5*arcsinh(a*x)))/arcsinh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^7c^2x^7+3\,a^5c^2x^5+3\,a^3c^2x^3+ac^2x+\left(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int\frac{5\,a^8c^2x^8+16\,a^6c^2x^6+18\,a^4c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x + (a^6*c^2*x^6 + 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*
x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^8*c^2*x^
8 + 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 + 8*a^2*c^2*x^2 + (5*a^6*c^2*x^6 + 9*a^
4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*(a^2*x^2 + 1) + c^2 + 5*(2*a^7*c^2*x^7 + 5
*a^5*c^2*x^5 + 4*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2*x^2 + 1))/((a^4*x^4 + (a^2
*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*lo
g(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4c^2x^4 + 2\,a^2c^2x^2 + c^2}{\operatorname{arsinh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c^2\left(\int \frac{2a^2x^2}{\operatorname{asinh}^2\left(ax\right)}\,dx + \int \frac{a^4x^4}{\operatorname{asinh}^2\left(ax\right)}\,dx + \int \frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] c**2*(Integral(2*a**2*x**2/asinh(a*x)**2, x) + Integral(a**4*x**4/asinh(a*x
)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2 + c\right)^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^2/arcsinh(a*x)^2, x)
```

3.408 $\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\mathbf{Shi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{3c\mathbf{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

```
[Out] -((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*SinhIntegral[ArcSinh[a*x
]])/(4*a) + (3*c*SinhIntegral[3*ArcSinh[a*x]])/(4*a)
```

Rubi [A] time = 0.135066, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5696, 5779, 5448, 3298}

$$-\frac{c\left(a^2x^2+1\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\mathbf{Shi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{3c\mathbf{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] -((c*(1 + a^2*x^2)^(3/2))/(a*ArcSinh[a*x])) + (3*c*SinhIntegral[ArcSinh[a*x
]])/(4*a) + (3*c*SinhIntegral[3*ArcSinh[a*x]])/(4*a)
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+a^2cx^2}{\sinh^{-1}(ax)^2}\,dx &= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)}+(3ac)\int \frac{x\sqrt{1+a^2x^2}}{\sinh^{-1}(ax)}\,dx\\
&= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{(3c)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{a}\\
&= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{(3c)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x}+\frac{\sinh(3x)}{4x}\right)dx,x,\sinh^{-1}(ax)\right)}{a}\\
&= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{(3c)\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\sinh^{-1}(ax)\right)}{4a}+\frac{(3c)\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx,x,\sinh^{-1}(a\right.}{4a}\\
&= -\frac{c\left(1+a^2x^2\right)^{3/2}}{a\sinh^{-1}(ax)}+\frac{3c\,\textbf{Shi}\left(\sinh^{-1}(ax)\right)}{4a}+\frac{3c\,\textbf{Shi}\left(3\sinh^{-1}(ax)\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.211704, size = 54, normalized size = 1.

$$\frac{c\left(-4\left(a^2x^2+1\right)^{3/2}+3\sinh^{-1}(ax)\textbf{Shi}\left(\sinh^{-1}(ax)\right)+3\sinh^{-1}(ax)\textbf{Shi}\left(3\sinh^{-1}(ax)\right)\right)}{4a\sinh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a^2*c*x^2)/ArcSinh[a*x]^2,x]
```

```
[Out] (c*(-4*(1 + a^2*x^2)^(3/2) + 3*ArcSinh[a*x]*SinhIntegral[ArcSinh[a*x]] + 3*
ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]]))/(4*a*ArcSinh[a*x])
```

Maple [A] time = 0.036, size = 60, normalized size = 1.1

$$\frac{c}{4\,a\mathrm{Arcsinh}\,(ax)}\left(3\,\mathrm{Shi}\,(\mathrm{Arcsinh}\,(ax))\,\mathrm{Arcsinh}\,(ax)+3\,\mathrm{Shi}\,(3\,\mathrm{Arcsinh}\,(ax))\,\mathrm{Arcsinh}\,(ax)-3\,\sqrt{a^2x^2+1}-\cosh\,(3\,\mathrm{Arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

```
[Out] 1/4/a*c*(3*Shi(arcsinh(a*x))*arcsinh(a*x)+3*Shi(3*arcsinh(a*x))*arcsinh(a*x
)-3*(a^2*x^2+1)^(1/2)-cosh(3*arcsinh(a*x)))/arcsinh(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^5cx^5+2\,a^3cx^3+acx+\left(a^4cx^4+2\,a^2cx^2+c\right)\sqrt{a^2x^2+1}}{\left(a^3x^2+\sqrt{a^2x^2+1}a^2x+a\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}+\int\frac{3\,a^6cx^6+7\,a^4cx^4+5\,a^2cx^2+\left(3\,a^4cx^4+2\,a^2cx^2-c\right)\left(a^2x^2\right.}{\left(a^4x^4+\left(a^2x^2+1\right)a^2x^2+2\,a^2x^2+2\left(a^3x^3+ax\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 + 2*a^2*c*x^2 + c)*sqrt(a^2*
x^2 + 1))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 +
 1))) + integrate((3*a^6*c*x^6 + 7*a^4*c*x^4 + 5*a^2*c*x^2 + (3*a^4*c*x^4 +
 2*a^2*c*x^2 - c)*(a^2*x^2 + 1) + 3*(2*a^5*c*x^5 + 3*a^3*c*x^3 + a*c*x)*sqr
t(a^2*x^2 + 1) + c)/((a^4*x^4 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*
x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2cx^2+c}{\mathrm{arsinh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$c\left(\int \frac{a^2x^2}{\operatorname{asinh}^2(ax)}\,dx + \int \frac{1}{\operatorname{asinh}^2(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)/asinh(a*x)**2,x)
```

```
[Out] c*(Integral(a**2*x**2/asinh(a*x)**2, x) + Integral(asinh(a*x)**(-2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^2cx^2 + c}{\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)/arcsinh(a*x)^2, x)
```

3.409 $\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{a\text{Unintegrable}\left(\frac{x}{(a^2x^2+1)^{3/2}\sinh^{-1}(ax)},x\right)}{c}-\frac{1}{ac\sqrt{a^2x^2+1}\sinh^{-1}(ax)}$$

```
[Out] -(1/(a*c*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])) - (a*Unintegrable[x/((1 + a^2*x^2
)^(3/2)*ArcSinh[a*x]), x])/c
```

Rubi [A] time = 0.101692, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])) - (a*Defer[Int][x/((1 + a^2*x^2)^
(3/2)*ArcSinh[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx=-\frac{1}{ac\sqrt{1+a^2x^2}\sinh^{-1}(ax)}-\frac{a\int \frac{x}{(1+a^2x^2)^{3/2}\sinh^{-1}(ax)}\,dx}{c}$$

Mathematica [A] time = 1.4745, size = 0, normalized size = 0.

$$\int \frac{1}{(c+a^2cx^2)\sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)*ArcSinh[a*x]^2), x]
```

Maple [A] time = 0.08, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right) \left(\operatorname{Arcsinh}\left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

```
[Out] int(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ax+\sqrt{a^{2}x^{2}+1}}{\left(a^{3}cx^{2}+\sqrt{a^{2}x^{2}+1}a^{2}cx+ac\right)\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)}-\int\frac{a^{4}x^{4}+\left(a^{2}x^{2}+1\right)^{2}+(2}{\left(a^{6}cx^{6}+3\,a^{4}cx^{4}+3\,a^{2}cx^{2}+\left(a^{4}cx^{4}+a^{2}cx^{2}\right)\left(a^{2}x^{2}+1\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^3*c*x^2 + sqrt(a^2*x^2 + 1)*a^2*c*x + a*c)*l
og(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^4*x^4 + (a^2*x^2 + 1)^2 + (2*a^
3*x^3 + a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^6*c*x^6 + 3*a^4*c*x^4 + 3*a^2*c*x^2
 + (a^4*c*x^4 + a^2*c*x^2)*(a^2*x^2 + 1) + 2*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c
*x)*sqrt(a^2*x^2 + 1) + c)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^{2} c x^{2}+c\right) \operatorname{arsinh}\left(a x\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^2x^2 \operatorname{asinh}^2(ax) + \operatorname{asinh}^2(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(a**2*x**2*asinh(a*x)**2 + asinh(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2 + c)\operatorname{arsinh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)*arcsinh(a*x)^2), x)
```

3.410 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3a\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)^{5/2}\sinh^{-1}(ax)},x\right)}{c^2}-\frac{1}{ac^2\left(a^2x^2+1\right)^{3/2}\sinh^{-1}(ax)}$$

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])) - (3*a*Unintegrable[x/((1 + a
^2*x^2)^(5/2)*ArcSinh[a*x]), x])/c^2
```

Rubi [A] time = 0.101213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(1 + a^2*x^2)^(3/2)*ArcSinh[a*x])) - (3*a*Defer[Int][x/((1 + a^2
*x^2)^(5/2)*ArcSinh[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2}\,dx=-\frac{1}{ac^2\left(1+a^2x^2\right)^{3/2}\sinh^{-1}(ax)}-\frac{(3a)\int \frac{x}{\left(1+a^2x^2\right)^{5/2}\sinh^{-1}(ax)}\,dx}{c^2}$$

Mathematica [A] time = 3.63577, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \sinh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^2*ArcSinh[a*x]^2), x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2 (\operatorname{Arcsinh}(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

```
[Out] int(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{ax+\sqrt{a^2x^2+1}}{\left(a^5c^2x^4+2\,a^3c^2x^2+ac^2+\left(a^4c^2x^3+a^2c^2x\right)\sqrt{a^2x^2+1}\right)\log\left(ax+\sqrt{a^2x^2+1}\right)}-\int\frac{}{\left(a^8c^2x^8+4\,a^6c^2x^6+6\,a^4c^2x^4+4\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a^2*x^2 + 1))/((a^5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2 + (a^4*c^2
*x^3 + a^2*c^2*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))) - integr
ate((3*a^4*x^4 + 2*a^2*x^2 + (3*a^2*x^2 + 1)*(a^2*x^2 + 1) + 3*(2*a^3*x^3 +
 a*x)*sqrt(a^2*x^2 + 1) - 1)/((a^8*c^2*x^8 + 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4
+ 4*a^2*c^2*x^2 + (a^6*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a^2*x^2 + 1)
 + c^2 + 2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*sqrt(a^2
*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^4+2\,a^2c^2x^2+c^2\right)\text{arsinh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arcsinh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{a^4x^4\,\text{asinh}^2\,(ax)+2a^2x^2\,\text{asinh}^2\,(ax)+\text{asinh}^2\,(ax)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**2/asinh(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*asinh(a*x)**2 + 2*a**2*x**2*asinh(a*x)**2 + asinh(a*x
)**2), x)/c**2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^2\text{arsinh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^2*arcsinh(a*x)^2), x)
```

3.411 $\int \frac{x^3\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^4}+\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+}{}\right.}{8b^2c^4}$$

```
[Out] -((x^3*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^4) + (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*
x]))/b])/(16*b^2*c^4)
```

Rubi [A] time = 0.71069, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5669, 5448, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16b^2c^4}+\frac{\sinh\left(\frac{a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(8*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c*x]])/(16*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*
ArcSinh[c*x]])/(16*b^2*c^4) + (Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(8*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*
b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2
*c^4)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
```

```
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(5c)\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(x)\sin}{a+bx}\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{5\,\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^4} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{si}\right.}{8bc^4}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^4} - \frac{\left(3\cosh\left(\frac{a}{b}\right)\right)}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c\right.}{16b^2c^4}$$

Mathematica [A] time = 0.668116, size = 175, normalized size = 0.82

$$-\frac{\frac{16bc^5x^5}{a+b\sinh^{-1}(cx)} + \frac{16bc^3x^3}{a+b\sinh^{-1}(cx)} + 2\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 3\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 5\cosh\left(\frac{5a}{b}\right)\textbf{Ch}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((16*b*c^3*x^3)/(a + b*ArcSinh[c*x]) + (16*b*c^5*x^5)/(a + b*ArcSinh[c*x])
 + 2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 3*Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcSinh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh
[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 3*Sinh[(3*a)/b]*Si
nhIntegral[3*(a/b + ArcSinh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a/b +
ArcSinh[c*x])])/(16*b^2*c^4)
```

Maple [B] time = 0.25, size = 633, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/32/c^4/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+1/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/32/c^4/b^2*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+1/16*(c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*ar
csinh(c*x))+1/16/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1/16/c^4/b^2*(arcs
inh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a
/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+1/32/c^4/b^2*(4*x^3*b*c
^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh
(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2
+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^4/b^2*(16*x^5*b*c^5+16*(c^2*x^2+1)^(
1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*arcsinh(c*x)*e
xp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)
-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^5+x^3\right)\left(c^2x^2+1\right)+\left(c^3x^6+cx^4\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x^2+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^4*x^6 + 11*c^2*x^4 + 3*x^2)*(c^
2*x^2 + 1) + (5*c^5*x^7 + 9*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*
x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c
^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*
x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c
^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^3}{b^2\operatorname{arsinh}(cx)^2+2ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}x^3}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^3/(b*arcsinh(c*x) + a)^2, x)
```

3.412 $\int \frac{x^2\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^3}-\frac{x^2\left(c^2x^2+1\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -((x^2*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(4*(a + b
*ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(2*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral
[(4*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3)
```

Rubi [A] time = 0.561542, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5777, 5669, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^3}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^3}-\frac{x^2\left(c^2x^2+1\right)}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(4*a)/b +
 4*ArcSinh[c*x]]*Sinh[(4*a)/b])/(2*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(
4*a)/b + 4*ArcSinh[c*x]])/(2*b^2*c^3)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(4c)\int \frac{x^3}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{4\,\text{Subst}\left(\int \frac{\cosh(x)\text{si}}{a+b}\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{4\,\text{Subst}\left(\int\left(-\frac{\sinh(2x)}{4(a+bx)}+\right.\right.}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{4a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} - \frac{\sinh\left(\frac{4a}{b}\right)\text{Su}}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\mathbf{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cosh\left(\frac{4a}{b}\right)\mathbf{Shi}\left(\frac{4a}{b}+4\sinh^{-}\right.}{2b^2c^3}$$

Mathematica [A] time = 0.309546, size = 82, normalized size = 0.88

$$\frac{-\frac{2bc^2x^2\left(c^2x^2+1\right)}{a+b\sinh^{-1}(cx)} - \sinh\left(\frac{4a}{b}\right)\mathbf{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{4a}{b}\right)\mathbf{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*c^2*x^2*(1 + c^2*x^2))/(a + b*ArcSinh[c*x]) - CoshIntegral[4*(a/b +
ArcSinh[c*x])]*Sinh[(4*a)/b] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c*x])])/(2*b^2*c^3)
```

Maple [B] time = 0.145, size = 248, normalized size = 2.7

$$\frac{1}{8\,c^3\,(a+b\text{Arcsinh}\,(cx))\,b} - \frac{1}{16\,c^3\,(a+b\text{Arcsinh}\,(cx))\,b}\left(8\,c^4x^4 - 8\,c^3x^3\sqrt{c^2x^2+1} + 8\,c^2x^2 - 4\,cx\sqrt{c^2x^2+1} + 1\right) + \frac{1}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/8/c^3/(a+b*arcsinh(c*x))/b-1/16*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*
c^2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+1/4/c^3/b^2*exp
(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/16/c^3/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^
(1/2)*x^3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-
4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/
b)*a+b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^4+x^2\right)\left(c^2x^2+1\right)+\left(c^3x^5+cx^3\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x^2+1\right)abc^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b
*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^
2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^3
*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^4*x^5 + 4*c^2*x^3 + x)*(c^2*x^2
+ 1) + (4*c^5*x^6 + 7*c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 +
(c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2
 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqr
t(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*
sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^2}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c^2x^2+1}}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}x^2}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

3.413 $\int \frac{x\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=149

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^2}-\frac{3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{4b^2c^2}$$

```
[Out] -((x*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(
a + b*ArcSinh[c*x])/b])/(4*b^2*c^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(4*b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[
c*x])/b])/(4*b^2*c^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x
]))/b])/(4*b^2*c^2)
```

Rubi [A] time = 0.411558, antiderivative size = 198, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5777, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^2}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}+\frac{3\sinh\left(\frac{a}{b}\right)\text{Sh}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2))/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(4*b^2*c^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c*x]])/(4*b^2*c^2) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x]
)/b])/(b^2*c^2) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b^2*c^2
) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2*c^2) -
(Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c^2)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
```

```
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(3c)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{b^2c^2} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(x)\sin}{a+bx}\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{\cosh\left(\frac{a}{b}\right)\text{Sul}}{}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2} - \frac{3\,\text{Sul}}{}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2} - \frac{\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2} - \frac{\left(3\,\text{co}\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}\right.}{4b^2c^2}$$

Mathematica [A] time = 0.362418, size = 126, normalized size = 0.85

$$-\frac{\frac{4bc^3x^3}{a+b\sinh^{-1}(cx)} - \cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 3\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) +}{4b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((4*b*c*x)/(a + b*ArcSinh[c*x]) + (4*b*c^3*x^3)/(a + b*ArcSinh[c*x]) - Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a
/b + ArcSinh[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*Sinh[(
3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b^2*c^2)
```

Maple [B] time = 0.135, size = 364, normalized size = 2.4

$$-\frac{1}{8\,bc^{2}\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}\left(4\,c^{3}x^{3}-4\,c^{2}x^{2}\sqrt{c^{2}x^{2}+1}+3\,cx-\sqrt{c^{2}x^{2}+1}\right)-\frac{3}{8\,c^{2}b^{2}}\mathrm{e}^{3\,\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\left(cx\right)+3\,\frac{a}{b}\right)-\frac{}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/
(a+b*arcsinh(c*x))-3/8/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/8*(c
*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-1/8/c^2/b^2*exp(a/b)*Ei(1,ar
csinh(c*x)+a/b)-1/8/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)
*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcs
inh(c*x))-1/8/c^2/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(
c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsin
h(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{2}x^{3}+x\right)\left(c^{2}x^{2}+1\right)+\left(c^{3}x^{4}+cx^{2}\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{}{abc^{5}x^{4}+\left(c^{2}x^{2}+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 + x)*(c^2*x^2 + 1) + (c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c
^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2
+ 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate((3*(c^2*x
^2 + 1)^(3/2)*c^3*x^3 + (6*c^4*x^4 + 5*c^2*x^2 + 1)*(c^2*x^2 + 1) + (3*c^5*
x^5 + 5*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}x}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x/(b*arcsinh(c*x) + a)^2, x)
```

3.414 $\int \frac{\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}-\frac{c^2x^2+1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -((1 + c^2*x^2)/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b
*ArcSinh[c*x]))/b])/(b^2*c)
```

Rubi [A] time = 0.180853, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5696, 5669, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}-\frac{c^2x^2+1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*a)/b + 2*Arc
Sinh[c*x]]*Sinh[(2*a)/b])/(b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2
*ArcSinh[c*x]])/(b^2*c)
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(2c)\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{2a}{b}\right)\text{Su}}{}$$

$$= -\frac{1+c^2x^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} + \frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-}\right.}{b^2c}$$

Mathematica [A] time = 0.16659, size = 73, normalized size = 0.86

$$\frac{-\frac{bc^2x^2+b}{a+b\sinh^{-1}(cx)} - \sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-((b + b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*
x])]*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b
^2*c)
```

Maple [B] time = 0.126, size = 192, normalized size = 2.3

$$-\frac{1}{2bc\left(a+b\text{Arcsinh}\,(cx)\right)} - \frac{1}{4bc\left(a+b\text{Arcsinh}\,(cx)\right)}\left(2c^2x^2 - 2cx\sqrt{c^2x^2+1}+1\right) + \frac{1}{2cb^2}e^{2\frac{a}{b}}\text{Ei}\left(1, 2\,\text{Arcsinh}\,(cx)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/2/b/c/(a+b*arcsinh(c*x))-1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c/(a+
b*arcsinh(c*x))/b+1/2/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b^2
*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-2*
arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*a
rcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^2+1\right)^2+\left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^4x^4+\left(c^2x^2+1\right)abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x
 + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^2*x^2 - 1)*(c^2*x
^2 + 1)^(3/2) + 2*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*x^2
+ 1)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^
2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^
2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(a+b\,\mathrm{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/(b*arcsinh(c*x) + a)^2, x)
```

3.415 $\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2}-\frac{c^2x^2+1}{bcx\left(a+b\sinh^{-1}(cx\right.}$$

```
[Out] -((1 + c^2*x^2)/(b*c*x*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a
+ b*ArcSinh[c*x])/b])/b^2 - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b]
)/b^2 - Unintegrable[1/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi [A] time = 0.229078, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a
+ b*ArcSinh[c*x])/b])/b^2 - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b]
)/b^2 - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc} + \frac{c\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{b^2} - \frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx\right.}}{bc}$$

$$= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\,\text{s}\right.}{b^2}$$

$$= -\frac{1+c^2x^2}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2} - $$

Mathematica [A] time = 10.3625, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.263, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\text{Arcsinh}\left(cx\right)\right)^2}\sqrt{c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{2}x^{2}+1\right)^{2}+\left(c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{3}+\sqrt{c^{2}x^{2}+1}abc^{2}x^{2}+abcx+\left(b^{2}c^{3}x^{3}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x^{2}+b^{2}cx\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{}{abc^{5}x^{6}+\left(c^{2}x^{2}+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c
^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*x^3 - 2*c
*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 - c^2*x^2 - 1)*(c^2*x^2 + 1) + (c^5*x^
5 + c^3*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 +
2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^
2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{b^{2}x\operatorname{arsinh}(cx)^{2}+2\,abx\operatorname{arsinh}(cx)+a^{2}x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) + a
^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^{2}x^{2}+1}}{x\left(a+b\operatorname{asinh}(cx)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\mathrm{arsinh}\,(cx)+a)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x), x)
```

3.416 $\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{c^2x^2+1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -((1 + c^2*x^2)/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Unintegrable[1/(x^3*(a
 + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi [A] time = 0.154345, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a +
 b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1+c^2x^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 2.67017, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.177, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\sqrt{c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{2}x^{2}+1\right)^{2}+\left(c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{4}+\sqrt{c^{2}x^{2}+1}abc^{2}x^{3}+abcx^{2}+\left(b^{2}c^{3}x^{4}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x^{3}+b^{2}cx^{2}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}-\int\frac{}{abc^{5}x^{7}+\left(c^{2}x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate((3*(c^2*x^2
 + 1)^(3/2)*c*x + 2*(2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^3*x^3 + c*x)*sqrt(c^
2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 + 2*a*b*c^3*x^5 + a*b*
c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b^2*c^3*x^5 + b^2*c*x^
3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x^2\operatorname{arsinh}(cx)^2+2\,abx^2\operatorname{arsinh}(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*x)
 + a^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}}{x^2\left(a+b\operatorname{asinh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**2*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}}{\left(b\operatorname{arsinh}(cx)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

3.417 $\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\sqrt{c^2x^2+1}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.127029, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 18.2912, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.385, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b\text{Arcsinh}(cx))^2} \sqrt{c^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^2+1\right)^2 + \left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^5 + \sqrt{c^2x^2+1}abc^2x^4 + abcx^3 + \left(b^2c^3x^5 + \sqrt{c^2x^2+1}b^2c^2x^4 + b^2cx^3\right)\log\left(cx + \sqrt{c^2x^2+1}\right)} - \int \frac{}{abc^5x^8 + \left(c^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((c^3*x^3 +
 4*c*x)*(c^2*x^2 + 1)^(3/2) + (2*c^4*x^4 + 7*c^2*x^2 + 3)*(c^2*x^2 + 1) + (
c^5*x^5 + 3*c^3*x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1
)*a*b*c^3*x^6 + 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^
2*c^3*x^6 + 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*
sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x^3\,\text{arsinh}\,(cx)^2+2\,abx^3\,\text{arsinh}\,(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*x)
 + a^2*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}}{x^3\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**3*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^3), x)
```

3.418 $\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\sqrt{c^2x^2+1}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.130909, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 3.51667, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+c^2x^2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 + c^2*x^2]/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.472, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b\text{Arcsinh}(cx))^2} \sqrt{c^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^2+1\right)^2 + \left(c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^6 + \sqrt{c^2x^2+1}abc^2x^5 + abcx^4 + \left(b^2c^3x^6 + \sqrt{c^2x^2+1}b^2c^2x^5 + b^2cx^4\right)\log\left(cx + \sqrt{c^2x^2+1}\right)} - \int \frac{}{abc^5x^9 + \left(c^2x^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2 + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(
c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2
*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) - integrate(((2*c^3*x^3
 + 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^4*x^4 + 5*c^2*x^2 + 2)*(c^2*x^2 + 1)
 + (2*c^5*x^5 + 5*c^3*x^3 + 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x
^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 +
 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2
*x^6)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{b^2x^4\,\text{arsinh}\,(cx)^2+2\,abx^4\,\text{arsinh}\,(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*x)
 + a^2*x^4), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{x^4\,(a+b\,\text{asinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/(x**4*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2x^2+1}}{(b\,\text{arsinh}\,(cx)+a)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

3.419 $\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=277

$$-\frac{3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{64b^2c^4}+\frac{7\cosh\left(\frac{7a}{b}\right)\mathrm{C}}{}$$

```
[Out] -((x^3*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(64*b^2*c^4) - (9*Cosh[(3*a)/b]*CoshIntegral[
(3*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (7*Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b^2*c^4) + (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*Ar
cSinh[c*x]))/b])/(64*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*Arc
Sinh[c*x]))/b])/(64*b^2*c^4) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcS
inh[c*x]))/b])/(64*b^2*c^4)
```

Rubi [A] time = 1.00638, antiderivative size = 273, normalized size of antiderivative = 0.99, number of steps used = 28, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^4}-\frac{9\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64b^2c^4}+\frac{5\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64b^2c^4}+\frac{7\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[a/b + ArcSinh[c*x]])/(64*b^2*c^4) - (9*Cosh[(3*a)/b]*CoshIntegral[(3*a
)/b + 3*ArcSinh[c*x]])/(64*b^2*c^4) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b
 + 5*ArcSinh[c*x]])/(64*b^2*c^4) + (7*Cosh[(7*a)/b]*CoshIntegral[(7*a)/b +
7*ArcSinh[c*x]])/(64*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x
]])/(64*b^2*c^4) + (9*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])
/(64*b^2*c^4) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(6
4*b^2*c^4) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64*b
^2*c^4)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx = -\frac{x^3(1+c^2x^2)^2}{bc(a+b\sinh^{-1}(cx))} + \frac{3\int \frac{x^2(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^4(1+c^2x^2)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\sinh^{-1}(cx))} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{7\,\text{Subst}\left(\int \frac{\cosh^3(}{a}\right.}{}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\sinh^{-1}(cx))} + \frac{3\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)} + \frac{\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{7}{}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\sinh^{-1}(cx))} - \frac{7\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^4} + \frac{7\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x\right.}{64bc^4}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\sinh^{-1}(cx))} + \frac{\left(21\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^4} - \frac{\left(3\cosh\left(\frac{a}{b}\right.\right.}{}$$

$$= -\frac{x^3(1+c^2x^2)^2}{bc(a+b\sinh^{-1}(cx))} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^4} - \frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}\right.}{64b^2c^4}$$

Mathematica [A] time = 0.800154, size = 399, normalized size = 1.44

$$\frac{-3\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 9\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 5a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (-64*b*c^3*x^3 - 128*b*c^5*x^5 - 64*b*c^7*x^7 - 3*(a + b*ArcSinh[c*x])*Cosh
[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/
b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + 5*a*Cosh[(5*a)/b]*CoshIntegral[5*
(a/b + ArcSinh[c*x])] + 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] + 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] +
 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] + 3*a*
```

```
Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 3*b*ArcSinh[c*x]*Sinh[a/b]*Sin
hIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[
c*x])] - 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 5*b*ArcSi
nh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Sinh[(7*a)
/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] - 7*b*ArcSinh[c*x]*Sinh[(7*a)/b]*S
inhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b^2*c^4*(a + b*ArcSinh[c*x]))
```

Maple [B] time = 0.369, size = 958, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2
*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^4/(a+b*arcsinh(c*x))/b-7/128/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-1/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))-5/128/c
^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+3/128*(4*c^3*x^3-4*c^2*x^2*(c^
2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+9/128/c^4/
b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/128*(c*x-(c^2*x^2+1)^(1/2))/c^4
/b/(a+b*arcsinh(c*x))+3/128/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+3/128/c
^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)
-a/b)*exp(-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+3/128/c^4/b
^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei
(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x
*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/128/c^4/b^2*(16*x^5*b*c^5+16
*(c^2*x^2+1)^(1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*
arcsinh(c*x)*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-
5*arcsinh(c*x)-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/1
28/c^4/b^2*(64*x^7*b*c^7+64*(c^2*x^2+1)^(1/2)*x^6*b*c^6+112*x^5*b*c^5+80*(c
^2*x^2+1)^(1/2)*x^4*b*c^4+56*x^3*b*c^3+24*(c^2*x^2+1)^(1/2)*x^2*b*c^2+7*arc
sinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x)-
7*a/b)*exp(-7*a/b)*a+7*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{4}x^{7}+2\,c^{2}x^{5}+x^{3}\right)\left(c^{2}x^{2}+1\right)+\left(c^{5}x^{8}+2\,c^{3}x^{6}+cx^{4}\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{(7}{abc^{5}x^{4}+\left(c^{2}x^{2}+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^7 + 2*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^5*x^8 + 2*c^3*x^6 + c*x^4)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((7*c^5*x^7 + 9*c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + (
14*c^6*x^8 + 27*c^4*x^6 + 16*c^2*x^4 + 3*x^2)*(c^2*x^2 + 1) + (7*c^7*x^9 +
18*c^5*x^7 + 15*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x
^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*
b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c
^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^{2}x^{5}+x^{3}\right)\sqrt{c^{2}x^{2}+1}}{b^{2}\operatorname{arsinh}\left(cx\right)^{2}+2\,ab\operatorname{arsinh}\left(cx\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{3}{2}}x^3}{(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^3/(b*arcsinh(c*x) + a)^2, x)
```

3.420 $\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=219

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^3}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\right.}{16b}$$

```
[Out] -((x^2*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*(a + b*
ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c^3) - (3*CoshIntegral[(6*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(
2*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*(
a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^3)
```

Rubi [A] time = 0.715753, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{16b^2c^3}-\cosh\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*a)/b
 + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*a)/b + 4*
ArcSinh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c^3) - (3*CoshIntegral[(6*a)/b + 6*ArcS
inh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b
 + 2*ArcSinh[c*x]])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*
ArcSinh[c*x]])/(4*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcS
inh[c*x]])/(16*b^2*c^3)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
```

```
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(6c)\int \frac{x^3\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{6\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)}{a+}\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\mathrm{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)} + \frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{6\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \mathrm{s}\right.}{4bc^3}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^3} - \frac{\left(9\cosh\left(\frac{2a}{b}\right)\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\mathrm{sin}}{4b^2c^3}$$

<u>Mathematica</u> [A] time = 0.730781, size = 306, normalized size = 1.4

$$-\frac{-\sinh\left(\frac{2a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 4\sinh\left(\frac{4a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(16*b*c^2*x^2 + 32*b*c^4*x^4 + 16*b*c^6*x^6 - (a + b*ArcSinh[c*x])*CoshInt
egral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 4*(a + b*ArcSinh[c*x])*CoshIn
tegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + Ar
cSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6*(a/b + ArcSinh
[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])
] + b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 4*a
*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 4*b*ArcSinh[c*x]*Cosh
[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhInte
gral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[
```

```
6*(a/b + ArcSinh[c*x])])/(16*b^2*c^3*(a + b*ArcSinh[c*x]))
```

Maple [B] time = 0.298, size = 704, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/16/c^3/(a+b*arcsinh(c*x))/b-1/64*(32*c^6*x^6-32*c^5*x^5*(c^2*x^2+1)^(1/2)
+48*c^4*x^4-32*c^3*x^3*(c^2*x^2+1)^(1/2)+18*c^2*x^2-6*c*x*(c^2*x^2+1)^(1/2)
+1)/c^3/(a+b*arcsinh(c*x))/b+3/32/c^3/b^2*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*
a/b)-1/32*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^2*x^2-4*c*x*(c^2*x^2+1
)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+1/8/c^3/b^2*exp(4*a/b)*Ei(1,4*arcsinh(c
*x)+4*a/b)+1/64*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x)
)/b-1/32/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)+1/64/c^3/b^2*(2*x^2*
b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-2*arcsinh(
c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*arcsinh(c
*x))-1/32/c^3/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^3*b*c^3+8*x^2*b*c^2+4*
b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a
/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/(a+b*arcsinh(c*x))-1/6
4/c^3/b^2*(32*x^6*b*c^6+32*(c^2*x^2+1)^(1/2)*x^5*b*c^5+48*x^4*b*c^4+32*(c^2
*x^2+1)^(1/2)*x^3*b*c^3+18*x^2*b*c^2+6*b*c*(c^2*x^2+1)^(1/2)*x+6*arcsinh(c*
x)*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b+6*exp(-6*a/b)*Ei(1,-6*arcsinh(
c*x)-6*a/b)*a+b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^4x^6+2\,c^2x^4+x^2\right)\left(c^2x^2+1\right)+\left(c^5x^7+2\,c^3x^5+cx^3\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x^2+1\right)abc^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^6 + 2*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^5*x^7 + 2*c^3*x^5 + c*x^3)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
```

```
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate(((6*c^5*x^6 + 7*c^3*x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(
6*c^6*x^7 + 11*c^4*x^5 + 6*c^2*x^3 + x)*(c^2*x^2 + 1) + 3*(2*c^7*x^8 + 5*c^
5*x^6 + 4*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcs
inh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^2}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(3/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

3.421 $\int \frac{x(1+c^2x^2)^{3/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{}\right.}{8b^2c^2}$$

```
[Out] -((x*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^2) + (9*Cosh[(3*a)/b]*CoshIntegral[(3*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a
+ b*ArcSinh[c*x]))/b])/(16*b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^2) - (9*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*
x]))/b])/(16*b^2*c^2)
```

Rubi [A] time = 0.732221, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5777, 5699, 3312, 3303, 3298, 3301, 5779, 5448}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^2}+\frac{9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16b^2c^2}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16b^2c^2}-\sinh\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^2)/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(8*b^2*c^2) + (9*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b +
 3*ArcSinh[c*x]])/(16*b^2*c^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*
ArcSinh[c*x]])/(16*b^2*c^2) - (Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(8*b^2*c^2) - (9*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*
b^2*c^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2
*c^2)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
```

```
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
```

```
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{1+c^2x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(5c)\int \frac{x^2\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{5\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\right.}{bc^2}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{5\,\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4bc^2} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \text{s}\right.}{16bc^2}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^2} + \frac{\left(3\cosh\left(\frac{a}{b}\right)\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^2} + \frac{9\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}\right.}{16b^2c^2}$$

Mathematica [A] time = 0.607108, size = 295, normalized size = 1.38

$$-\frac{-2\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 9\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 5a\,\text{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(16*b*c*x + 32*b*c^3*x^3 + 16*b*c^5*x^5 - 2*(a + b*ArcSinh[c*x])*Cosh[a/b]
*CoshIntegral[a/b + ArcSinh[c*x]] - 9*(a + b*ArcSinh[c*x])*Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c*x])] - 5*a*Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c*x])] - 5*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Arc
Sinh[c*x])] + 2*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 2*b*ArcSinh[
c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 9*a*Sinh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcSinh[c*x])] + 9*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral
[3*(a/b + ArcSinh[c*x])] + 5*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])] + 5*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])
])/(16*b^2*c^2*(a + b*ArcSinh[c*x]))
```

Maple [B] time = 0.233, size = 633, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-5/32/c^2/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-9/32/c^2/b^2*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/16*(c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*ar
csinh(c*x))-1/16/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/16/c^2/b^2*(arcs
inh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a
/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/32/c^2/b^2*(4*x^3*b*c
^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh
(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2
+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^2/b^2*(16*x^5*b*c^5+16*(c^2*x^2+1)^(
1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*arcsinh(c*x)*e
xp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)
-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^4x^5+2\,c^2x^3+x\right)\left(c^2x^2+1\right)+\left(c^5x^6+2\,c^3x^4+cx^2\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^4+\left(c^2x^2+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^5 + 2*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^5*x^6 + 2*c^3*x^4 + c*x^2)*s
qrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2
*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)
)) + integrate((5*(c^5*x^5 + c^3*x^3)*(c^2*x^2 + 1)^(3/2) + (10*c^6*x^6 + 1
7*c^4*x^4 + 8*c^2*x^2 + 1)*(c^2*x^2 + 1) + (5*c^7*x^7 + 12*c^5*x^5 + 9*c^3*
x^3 + 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 +
2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^
3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sq
rt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^3+x\right)\sqrt{c^2x^2+1}}{b^2\operatorname{arsinh}\left(cx\right)^2+2\,ab\operatorname{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsin
h(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x*(c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x/(b*arcsinh(c*x) + a)^2, x)
```

3.422 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\right.}{2b}$$

```
[Out] -((1 + c^2*x^2)^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(2*a)/b])/(b^2*c) - (CoshIntegral[(4*(a + b*ArcSinh[c*x
]))/b]*Sinh[(4*a)/b])/(2*b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*Arc
Sinh[c*x]))/b])/(b^2*c) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x
]))/b])/(2*b^2*c)
```

Rubi [A] time = 0.317802, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5696, 5779, 5448, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*a)/b + 2*A
rcSinh[c*x]]*Sinh[(2*a)/b])/(b^2*c) - (CoshIntegral[(4*a)/b + 4*ArcSinh[c*x
]]*Sinh[(4*a)/b])/(2*b^2*c) + (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSi
nh[c*x]])/(b^2*c) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/
(2*b^2*c)
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
```

```
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(4c)\int \frac{x\left(1+c^2x^2\right)}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{4\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{4\,\mathrm{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)} + \frac{\sinh(4x)}{8(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{2bc} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sin\right.}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc} + \frac{\cosh\left(\frac{4a}{b}\right)\mathrm{Su}}{}$$

$$= -\frac{\left(1+c^2x^2\right)^2}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c} - \frac{\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\sin}{2b^2c}$$

Mathematica [A] time = 0.479051, size = 122, normalized size = 0.82

$$\frac{-\frac{2b\left(c^2x^2+1\right)^2}{a+b\sinh^{-1}(cx)} - 2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - \sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sin\right.\right.}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-2*b*(1 + c^2*x^2)^2)/(a + b*ArcSinh[c*x]) - 2*CoshIntegral[2*(a/b + ArcS
inh[c*x])]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/
b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*S
inhIntegral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c)
```

Maple [B] time = 0.191, size = 420, normalized size = 2.8

$$-\frac{3}{8\,bc\,(a+b\mathrm{Arcsinh}\,(cx))} - \frac{1}{16\,bc\,(a+b\mathrm{Arcsinh}\,(cx))}\left(8\,c^4x^4 - 8\,c^3x^3\sqrt{c^2x^2+1} + 8\,c^2x^2 - 4\,cx\sqrt{c^2x^2+1} + 1\right) + \frac{1}{4\,cb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -3/8/b/c/(a+b*arcsinh(c*x))-1/16*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c
^2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c/(a+b*arcsinh(c*x))/b+1/4/c/b^2*exp(4*a/
b)*Ei(1,4*arcsinh(c*x)+4*a/b)-1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c/(
a+b*arcsinh(c*x))/b+1/2/c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/c/b
^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-
2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b
*arcsinh(c*x))-1/16/c/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^3*b*c^3+8*x^2*
b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*
exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/(a+b*arcsinh(c
*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^4x^4+\left(c^2x^2+1\right)abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c
^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
 + integrate(((4*c^4*x^4 + 3*c^2*x^2 - 1)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^5*x^
5 + 3*c^3*x^3 + c*x)*(c^2*x^2 + 1) + (4*c^6*x^6 + 9*c^4*x^4 + 6*c^2*x^2 + 1
)*sqrt(c^2*x^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x
^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 +
 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
+ 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2\,\mathrm{arsinh}\left(cx\right)^2+2\,ab\,\mathrm{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/(b*arcsinh(c*x) + a)^2, x)
```

3.423 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=174

$$-\frac{\text{Unintegrable}\left(\frac{c^2x^2+1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2}-\frac{9\sinh\left(\frac{a}{b}\right)\text{S}\ldots}{}$$

```
[Out] -((1 + c^2*x^2)^2/(b*c*x*(a + b*ArcSinh[c*x]))) + (9*Cosh[a/b]*CoshIntegral
[(a + b*ArcSinh[c*x])/b])/(4*b^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b
*ArcSinh[c*x]))/b])/(4*b^2) - (9*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x]
)/b])/(4*b^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(4*b^2) - Unintegrable[(1 + c^2*x^2)/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi [A] time = 0.437108, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x*(a + b*ArcSinh[c*x]))) + (9*Cosh[a/b]*CoshIntegral
[a/b + ArcSinh[c*x]])/(4*b^2) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*A
rcSinh[c*x]])/(4*b^2) - (9*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b
^2) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2) - De
fer[Int][(1 + c^2*x^2)/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\sinh^{-1}(cx))^2}\,dx = -\frac{(1+c^2x^2)^2}{bcx(a+b\sinh^{-1}(cx))} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc} + \frac{(3c)\int \frac{1+c^2x^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx(a+b\sinh^{-1}(cx))} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc}$$

$$= -\frac{(1+c^2x^2)^2}{bcx(a+b\sinh^{-1}(cx))} + \frac{3\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\int \frac{1+}{x^2(a+b\,s}$$

$$= -\frac{(1+c^2x^2)^2}{bcx(a+b\sinh^{-1}(cx))} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{4b} + \frac{9\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,\right.}{4b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx(a+b\sinh^{-1}(cx))} - \frac{\int \frac{1+c^2x^2}{x^2(a+b\sinh^{-1}(cx))}\,dx}{bc} + \frac{\left(9\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x,\right.}{4b}$$

$$= -\frac{(1+c^2x^2)^2}{bcx(a+b\sinh^{-1}(cx))} + \frac{9\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2} + \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\,\text{si}\right.}{4b^2}$$

Mathematica [A] time = 7.29065, size = 0, normalized size = 0.

$$\int \frac{(1+c^2x^2)^{3/2}}{x(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\text{Arcsinh}(cx))^2}(c^2x^2+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^3+\sqrt{c^2x^2+1}abc^2x^2+abcx+\left(b^2c^3x^3+\sqrt{c^2x^2+1}b^2c^2x^2+b^2cx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^6+\left(c^2x^2+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^2 + a*b*c*x + (b
^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*x)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate(((3*c^5*x^5 + c^3*x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (
6*c^6*x^6 + 7*c^4*x^4 - 1)*(c^2*x^2 + 1) + 3*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x
^4 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 +
 b^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqr
t(c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x\,\mathrm{arsinh}\left(cx\right)^2+2\,abx\,\mathrm{arsinh}\left(cx\right)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x*arcsinh(c*x)^2 + 2*a*b*x*arcsinh(c*x) +
 a^2*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x), x)
```

3.424 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{2\text{Unintegrable}\left(\frac{c^2x^2+1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{2c\text{Unintegrable}\left(\frac{c^2x^2+1}{x\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{\left(c^2x^2+1\right)^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Unintegrable[(1 + c^
2*x^2)/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (2*c*Unintegrable[(1 + c^2*x
^2)/(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi [A] time = 0.251591, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (2*c*Defer[Int][(1 + c^2*x^2)/
(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{\left(1+c^2x^2\right)^2}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1+c^2x^2}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{(2c)\int \frac{1+c^2x^2}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 4.11425, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.345, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^2}\left(c^2x^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^4+\sqrt{c^2x^2+1}abc^2x^3+abcx^2+\left(b^2c^3x^4+\sqrt{c^2x^2+1}b^2c^2x^3+b^2cx^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^7+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^3 + a*b*c*x^2 +
(b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*c*x^2)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((2*c^5*x^5 - c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2)
 + 2*(2*c^6*x^6 + c^4*x^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (2*c^7*x^7 + 3*c
```

```
^5*x^5 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^2\operatorname{arsinh}\left(cx\right)^2+2\,abx^2\operatorname{arsinh}\left(cx\right)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^2*arcsinh(c*x)^2 + 2*a*b*x^2*arcsinh(c*
x) + a^2*x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**2*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

3.425 $\int \frac{(1+c^2x^2)^{3/2}}{x^3(a+b\sinh^{-1}(cx))^2} \, dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{(c^2x^2+1)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.139466, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} \, dx = \int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$$

Mathematica [A] time = 14.0377, size = 0, normalized size = 0.

$$\int \frac{(1+c^2x^2)^{3/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.813, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b\mathrm{Arcsinh}(cx))^2} \left(c^2x^2+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^4x^4+2\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^5x^5+2\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^5+\sqrt{c^2x^2+1}abc^2x^4+abcx^3+\left(b^2c^3x^5+\sqrt{c^2x^2+1}b^2c^2x^4+b^2cx^3\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^8+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^4 + a*b*c*x^3 +
(b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*c*x^3)*log(c*x + sqrt(c^
2*x^2 + 1))) + integrate(((c^5*x^5 - 3*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2)
 + (2*c^6*x^6 - 3*c^4*x^4 - 8*c^2*x^2 - 3)*(c^2*x^2 + 1) + (c^7*x^7 - 3*c^3
*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6 +
 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*b
^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^3\operatorname{arsinh}\left(cx\right)^2+2\,abx^3\operatorname{arsinh}\left(cx\right)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^3*arcsinh(c*x)^2 + 2*a*b*x^3*arcsinh(c*
x) + a^2*x^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{x^3\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((c**2*x**2 + 1)**(3/2)/(x**3*(a + b*asinh(c*x))**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^3), x)
```

3.426 $\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{4\text{Unintegrable}\left(\frac{c^2x^2+1}{x^5\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{\left(c^2x^2+1\right)^2}{bcx^4\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^4*(a + b*ArcSinh[c*x]))) - (4*Unintegrable[(1 + c^
2*x^2)/(x^5*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi [A] time = 0.199351, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^2/(b*c*x^4*(a + b*ArcSinh[c*x]))) - (4*Defer[Int][(1 + c^2*
x^2)/(x^5*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{\left(1+c^2x^2\right)^2}{bcx^4\left(a+b\sinh^{-1}(cx)\right)}-\frac{4\int \frac{1+c^2x^2}{x^5\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 2.68651, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(3/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.45, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(a + b\,\mathrm{Arcsinh}\left(cx\right)\right)^{2}} \left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{4}x^{4}+2\,c^{2}x^{2}+1\right)\left(c^{2}x^{2}+1\right)+\left(c^{5}x^{5}+2\,c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{6}+\sqrt{c^{2}x^{2}+1}abc^{2}x^{5}+abcx^{4}+\left(b^{2}c^{3}x^{6}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x^{5}+b^{2}cx^{4}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}-\int\frac{}{abc^{5}x^{9}+\left(c^{2}x^{2}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^5*x^5 + 2*c^3*x^3 + c*x)*sqr
t(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*b*c^2*x^5 + a*b*c*x^4 +
(b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*c*x^4)*log(c*x + sqrt(c^
2*x^2 + 1))) - integrate((5*(c^3*x^3 + c*x)*(c^2*x^2 + 1)^(3/2) + 4*(2*c^4*
x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + 3*(c^5*x^5 + 2*c^3*x^3 + c*x)*sqrt(c^2
*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 + 2*a*b*c^3*x^7 + a*b*c
*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^2*c^3*x^7 + b^2*c*x^5
 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^4 \operatorname{arsinh}(cx)^2+2\,abx^4 \operatorname{arsinh}(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)/(b^2*x^4*arcsinh(c*x)^2 + 2*a*b*x^4*arcsinh(c*
x) + a^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(3/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{3}{2}}}{(b \operatorname{arsinh}(cx)+a)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(3/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

3.427 $\int \frac{x^3(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=277

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{128b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{32b^2c^4}+\frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7(a+b\sinh^{-1}(cx))}{b}\right)}{256b^2c^4}+\frac{9\cosh\left(\frac{9a}{b}\right)\text{C}}{2}$$

```
[Out] -((x^3*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(128*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral
[(3*(a + b*ArcSinh[c*x]))/b])/(32*b^2*c^4) + (21*Cosh[(7*a)/b]*CoshIntegral
[(7*(a + b*ArcSinh[c*x]))/b])/(256*b^2*c^4) + (9*Cosh[(9*a)/b]*CoshIntegral
[(9*(a + b*ArcSinh[c*x]))/b])/(256*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a
+ b*ArcSinh[c*x])/b])/(128*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(32*b^2*c^4) - (21*Sinh[(7*a)/b]*SinhIntegral[(7*(a +
 b*ArcSinh[c*x]))/b])/(256*b^2*c^4) - (9*Sinh[(9*a)/b]*SinhIntegral[(9*(a +
 b*ArcSinh[c*x]))/b])/(256*b^2*c^4)
```

Rubi [A] time = 1.21539, antiderivative size = 273, normalized size of antiderivative = 0.99, number of steps used = 34, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128b^2c^4}-\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{32b^2c^4}+\frac{21\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(\frac{7a}{b}+7\sinh^{-1}(cx)\right)}{256b^2c^4}+\frac{9\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^3*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshInte
gral[a/b + ArcSinh[c*x]])/(128*b^2*c^4) - (3*Cosh[(3*a)/b]*CoshIntegral[(3*
a)/b + 3*ArcSinh[c*x]])/(32*b^2*c^4) + (21*Cosh[(7*a)/b]*CoshIntegral[(7*a)
/b + 7*ArcSinh[c*x]])/(256*b^2*c^4) + (9*Cosh[(9*a)/b]*CoshIntegral[(9*a)/b
 + 9*ArcSinh[c*x]])/(256*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh
[c*x]])/(128*b^2*c^4) + (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c
*x]])/(32*b^2*c^4) - (21*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x
]])/(256*b^2*c^4) - (9*Sinh[(9*a)/b]*SinhIntegral[(9*a)/b + 9*ArcSinh[c*x]]
)/(256*b^2*c^4)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3 \left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\int \frac{x^2\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(9c)\int \frac{x^4\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^5(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{9\,\text{Subst}\left(\int \frac{\cosh^5(x)\,\ldots}{a+\ldots}\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{3\,\text{Subst}\left(\int\left(-\frac{5\cosh(x)}{64(a+bx)} + \frac{\cosh(3x)}{64(a+bx)} + \frac{3\cosh(5x)}{64(a+bx)} + \frac{\cosh(7x)}{64(a+bx)}\right)dx, x, \sinh\ldots\right.}{bc^4}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{9\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{256bc^4} + \frac{9\,\text{Subst}\left(\int \frac{\cosh(9x)}{a+bx}\,dx, x, \ldots\right.}{256bc^4}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(27\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{128bc^4} - \frac{\left(15\cosh\left(\frac{a}{b}\ldots\right.\right.}{}$$

$$= -\frac{x^3\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{128b^2c^4} - \frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}\ldots\right.}{32b^2c^4}$$

Mathematica [A] time = 1.33807, size = 408, normalized size = 1.47

$$-\frac{6\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 24\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 21a\,\text{co}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(256*b*c^3*x^3 + 768*b*c^5*x^5 + 768*b*c^7*x^7 + 256*b*c^9*x^9 + 6*(a + b*
ArcSinh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + 24*(a + b*ArcSin
h[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 21*a*Cosh[(7*a
)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 21*b*ArcSinh[c*x]*Cosh[(7*a)/b]
*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 9*a*Cosh[(9*a)/b]*CoshIntegral[9*(a
/b + ArcSinh[c*x])] - 9*b*ArcSinh[c*x]*Cosh[(9*a)/b]*CoshIntegral[9*(a/b +
```

```
ArcSinh[c*x])] - 6*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 6*b*ArcSi
nh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 24*a*Sinh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcSinh[c*x])] - 24*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcSinh[c*x])] + 21*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + Ar
cSinh[c*x])] + 21*b*ArcSinh[c*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSin
h[c*x])] + 9*a*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])] + 9*b*Arc
Sinh[c*x]*Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcSinh[c*x])])/(256*b^2*c^4*
(a + b*ArcSinh[c*x]))
```

Maple [B] time = 0.452, size = 1070, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/512*(256*c^9*x^9-256*c^8*x^8*(c^2*x^2+1)^(1/2)+576*c^7*x^7-448*c^6*x^6*(
c^2*x^2+1)^(1/2)+432*c^5*x^5-240*c^4*x^4*(c^2*x^2+1)^(1/2)+120*c^3*x^3-40*c
^2*x^2*(c^2*x^2+1)^(1/2)+9*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x))/b-
9/512/c^4/b^2*exp(9*a/b)*Ei(1,9*arcsinh(c*x)+9*a/b)-3/512*(64*c^7*x^7-64*c^
6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2*x^2+1)^(1/2)+56*c^3*x^3
-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/2))/c^4/(a+b*arcsinh(c*x
))/b-21/512/c^4/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)+1/64*(4*c^3*x^3-4
*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x)
)+3/64/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/256*(c*x-(c^2*x^2+1)
^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/256/c^4/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)+3/256/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-a
rcsinh(c*x)-a/b)*exp(-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+
1/64/c^4/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(
-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)*a+3*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-3/512/c^4/b^2*(64*x^
7*b*c^7+64*(c^2*x^2+1)^(1/2)*x^6*b*c^6+112*x^5*b*c^5+80*(c^2*x^2+1)^(1/2)*x
^4*b*c^4+56*x^3*b*c^3+24*(c^2*x^2+1)^(1/2)*x^2*b*c^2+7*arcsinh(c*x)*Ei(1,-7
*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b
)*a+7*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/512/c^4/b^2*(256*x^9*
b*c^9+256*(c^2*x^2+1)^(1/2)*x^8*b*c^8+576*x^7*b*c^7+448*(c^2*x^2+1)^(1/2)*x
^6*b*c^6+432*x^5*b*c^5+240*(c^2*x^2+1)^(1/2)*x^4*b*c^4+120*x^3*b*c^3+40*(c^
2*x^2+1)^(1/2)*x^2*b*c^2+9*arcsinh(c*x)*exp(-9*a/b)*Ei(1,-9*arcsinh(c*x)-9*
a/b)*b+9*exp(-9*a/b)*Ei(1,-9*arcsinh(c*x)-9*a/b)*a+9*x*b*c+(c^2*x^2+1)^(1/2
)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^9+3\,c^4x^7+3\,c^2x^5+x^3\right)\left(c^2x^2+1\right)+\left(c^7x^{10}+3\,c^5x^8+3\,c^3x^6+cx^4\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{\left(9\,c^7x^9+20\,c^5x^7+\cdots\right.}{abc^5x^4+\left(c^2x^2+1\right)abc^3x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^9 + 3*c^4*x^7 + 3*c^2*x^5 + x^3)*(c^2*x^2 + 1) + (c^7*x^10 + 3*c^5
*x^8 + 3*c^3*x^6 + c*x^4)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 +
1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*
log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((9*c^7*x^9 + 20*c^5*x^7 + 13*c^3
*x^5 + 2*c*x^3)*(c^2*x^2 + 1)^(3/2) + 3*(6*c^8*x^10 + 17*c^6*x^8 + 17*c^4*x
^6 + 7*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (9*c^9*x^11 + 31*c^7*x^9 + 39*c^5*x^7
 + 21*c^3*x^5 + 4*c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*
b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^
2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4x^7+2\,c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 + 2*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^3}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^3/(b*arcsinh(c*x) + a)^2, x)
```

3.428 $\int \frac{x^2(1+c^2x^2)^{5/2}}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=281

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(cx))}{b}\right)}{8b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2c^3}-\frac{\sinh\left(\frac{8a}{b}\right)\text{Chi}\left(\frac{8}{\ldots}\right.}{16b^2\ldots}$$

```
[Out] -((x^2*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a +
 b*ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*(a + b*
ArcSinh[c*x]))/b]*Sinh[(4*a)/b])/(8*b^2*c^3) - (3*CoshIntegral[(6*(a + b*Ar
cSinh[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c^3) - (CoshIntegral[(8*(a + b*ArcSi
nh[c*x]))/b]*Sinh[(8*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*(
a + b*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a +
 b*ArcSinh[c*x]))/b])/(8*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b
*ArcSinh[c*x]))/b])/(16*b^2*c^3) + (Cosh[(8*a)/b]*SinhIntegral[(8*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2*c^3)
```

Rubi [A] time = 1.07198, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5777, 5779, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{16b^2c^3}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8b^2c^3}-\frac{3\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{16b^2c^3}-\frac{\sinh\left(\frac{8}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x^2*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*a)/b
 + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(16*b^2*c^3) - (CoshIntegral[(4*a)/b + 4*
ArcSinh[c*x]]*Sinh[(4*a)/b])/(8*b^2*c^3) - (3*CoshIntegral[(6*a)/b + 6*ArcS
inh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3) - (CoshIntegral[(8*a)/b + 8*ArcSinh[c
*x]]*Sinh[(8*a)/b])/(16*b^2*c^3) - (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*
ArcSinh[c*x]])/(16*b^2*c^3) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSi
nh[c*x]])/(8*b^2*c^3) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c
*x]])/(16*b^2*c^3) + (Cosh[(8*a)/b]*SinhIntegral[(8*a)/b + 8*ArcSinh[c*x]])
/(16*b^2*c^3)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rubi steps

$$\int \frac{x^2\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\int \frac{x\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(8c)\int \frac{x^3\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{8\,\mathrm{Subst}\left(\int \frac{\cosh^5(x)\,\mathrm{s}}{a+b}\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2\,\mathrm{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} + \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh(6x)}{32(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{8\,\mathrm{S}}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(8x)}{a+bx}\,dx, x, \sinh\right.}{16bc^3}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^3} - \frac{\left(3\cosh\left(\frac{2}{b}\right.\right.}{}$$

$$= -\frac{x^2\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\sinh}{8b^2c^3}$$

Mathematica [A] time = 1.20148, size = 413, normalized size = 1.47

$$-\frac{-\sinh\left(\frac{2a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 2\sinh\left(\frac{4a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + 3a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(16*b*c^2*x^2 + 48*b*c^4*x^4 + 48*b*c^6*x^6 + 16*b*c^8*x^8 - (a + b*ArcSin
h[c*x])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 2*(a + b*ArcSi
nh[c*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshInteg
ral[6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6
*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + a*CoshIntegral[8*(a/b + ArcSinh[c*x]
)]*Sinh[(8*a)/b] + b*ArcSinh[c*x]*CoshIntegral[8*(a/b + ArcSinh[c*x])]*Sinh
```

```
[(8*a)/b] + a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + b*ArcSin
h[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - 2*a*Cosh[(4*a)/
b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 2*b*ArcSinh[c*x]*Cosh[(4*a)/b]*Si
nhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b
+ ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + Arc
Sinh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])] - b*ArcS
inh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b + ArcSinh[c*x])])/(16*b^2*c^3*(a
 + b*ArcSinh[c*x]))
```

Maple [B] time = 0.416, size = 1044, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 5/128/c^3/(a+b*arcsinh(c*x))/b-1/256*(128*c^8*x^8-128*c^7*x^7*(c^2*x^2+1)^(
1/2)+256*c^6*x^6-192*c^5*x^5*(c^2*x^2+1)^(1/2)+160*c^4*x^4-80*c^3*x^3*(c^2*
x^2+1)^(1/2)+32*c^2*x^2-8*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b
+1/32/c^3/b^2*exp(8*a/b)*Ei(1,8*arcsinh(c*x)+8*a/b)-1/64*(32*c^6*x^6-32*c^5
*x^5*(c^2*x^2+1)^(1/2)+48*c^4*x^4-32*c^3*x^3*(c^2*x^2+1)^(1/2)+18*c^2*x^2-6
*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+3/32/c^3/b^2*exp(6*a/b)*
Ei(1,6*arcsinh(c*x)+6*a/b)-1/64*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^
2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/(a+b*arcsinh(c*x))/b+1/16/c^3/b^2*exp(
4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/64*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1
)/c^3/(a+b*arcsinh(c*x))/b-1/32/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/
b)+1/64/c^3/b^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-
2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a
/b)*a+b)/(a+b*arcsinh(c*x))-1/64/c^3/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x
^3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsi
nh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)
/(a+b*arcsinh(c*x))-1/64/c^3/b^2*(32*x^6*b*c^6+32*(c^2*x^2+1)^(1/2)*x^5*b*c
^5+48*x^4*b*c^4+32*(c^2*x^2+1)^(1/2)*x^3*b*c^3+18*x^2*b*c^2+6*b*c*(c^2*x^2+
1)^(1/2)*x+6*arcsinh(c*x)*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*b+6*exp(-
6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a+b)/(a+b*arcsinh(c*x))-1/256/c^3/b^2*(1
28*x^8*b*c^8+128*(c^2*x^2+1)^(1/2)*x^7*b*c^7+256*x^6*b*c^6+192*(c^2*x^2+1)^
(1/2)*x^5*b*c^5+160*x^4*b*c^4+80*(c^2*x^2+1)^(1/2)*x^3*b*c^3+32*x^2*b*c^2+8
*b*c*(c^2*x^2+1)^(1/2)*x+8*arcsinh(c*x)*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*
a/b)*b+8*exp(-8*a/b)*Ei(1,-8*arcsinh(c*x)-8*a/b)*a+b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^8+3\,c^4x^6+3\,c^2x^4+x^2\right)\left(c^2x^2+1\right)+\left(c^7x^9+3\,c^5x^7+3\,c^3x^5+cx^3\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{\left(8\,c^7x^8+17\,c^5x^6\right.}{abc^5x^4+\left(c^2x^2+1\right)abc^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^8 + 3*c^4*x^6 + 3*c^2*x^4 + x^2)*(c^2*x^2 + 1) + (c^7*x^9 + 3*c^5*
x^7 + 3*c^3*x^5 + c*x^3)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1
)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*l
og(c*x + sqrt(c^2*x^2 + 1))) + integrate(((8*c^7*x^8 + 17*c^5*x^6 + 10*c^3*
x^4 + c*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(8*c^8*x^9 + 22*c^6*x^7 + 21*c^4*x^5 +
 8*c^2*x^3 + x)*(c^2*x^2 + 1) + (8*c^9*x^10 + 27*c^7*x^8 + 33*c^5*x^6 + 17*
c^3*x^4 + 3*c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*
x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*
b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4x^6+2\,c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{b^2\,\mathrm{arsinh}\left(cx\right)^2+2\,ab\,\mathrm{arsinh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^6 + 2*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2
+ 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^2}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^2/(b*arcsinh(c*x) + a)^2, x)
```

3.429 $\int \frac{x\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=275

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{64b^2c^2}+\frac{7\cosh\left(\frac{7a}{b}\right)\text{C}}{}$$

```
[Out] -((x*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegr
al[(a + b*ArcSinh[c*x])/b])/(64*b^2*c^2) + (27*Cosh[(3*a)/b]*CoshIntegral[(
3*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) + (25*Cosh[(5*a)/b]*CoshIntegral[(
5*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) + (7*Cosh[(7*a)/b]*CoshIntegral[(7
*(a + b*ArcSinh[c*x]))/b])/(64*b^2*c^2) - (5*Sinh[a/b]*SinhIntegral[(a + b*
ArcSinh[c*x])/b])/(64*b^2*c^2) - (27*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c*x]))/b])/(64*b^2*c^2) - (25*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*A
rcSinh[c*x]))/b])/(64*b^2*c^2) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*Ar
cSinh[c*x]))/b])/(64*b^2*c^2)
```

Rubi [A] time = 0.969514, antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 28, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {5777, 5699, 3312, 3303, 3298, 3301, 5779, 5448}

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^2}+\frac{27\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{64b^2c^2}+\frac{25\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{64b^2c^2}+\frac{7\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((x*(1 + c^2*x^2)^3)/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegr
al[a/b + ArcSinh[c*x]])/(64*b^2*c^2) + (27*Cosh[(3*a)/b]*CoshIntegral[(3*a)
/b + 3*ArcSinh[c*x]])/(64*b^2*c^2) + (25*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b
 + 5*ArcSinh[c*x]])/(64*b^2*c^2) + (7*Cosh[(7*a)/b]*CoshIntegral[(7*a)/b +
7*ArcSinh[c*x]])/(64*b^2*c^2) - (5*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x
]])/(64*b^2*c^2) - (27*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]]
)/(64*b^2*c^2) - (25*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/
(64*b^2*c^2) - (7*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64
*b^2*c^2)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\int \frac{\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{bc} + \frac{(7c)\int \frac{x^2\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{7\,\text{Subst}\left(\int \frac{\cosh^5(x)\sinh^2(x)}{a+bx}\,d\right.}{bc^2}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)} + \frac{5\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{7\,\text{S}}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^2} + \frac{7\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \sin\right.}{64bc^2}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{\left(35\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^2} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right.}{}$$

$$= -\frac{x\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{64b^2c^2} + \frac{27\cosh\left(\frac{3a}{b}\right)\textbf{Chi}\left(\frac{3a}{b}+3\sinh^{-1}\right.}{64b^2c^2}$$

Mathematica [A] time = 1.00757, size = 404, normalized size = 1.47

$$-\frac{-5\cosh\left(\frac{a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 27\cosh\left(\frac{3a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\textbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 25a\,\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(64*b*c*x + 192*b*c^3*x^3 + 192*b*c^5*x^5 + 64*b*c^7*x^7 - 5*(a + b*ArcSin
h[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - 27*(a + b*ArcSinh[c*x]
)*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 25*a*Cosh[(5*a)/b]*C
oshIntegral[5*(a/b + ArcSinh[c*x])] - 25*b*ArcSinh[c*x]*Cosh[(5*a)/b]*CoshI
ntegral[5*(a/b + ArcSinh[c*x])] - 7*a*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + A
rcSinh[c*x])] - 7*b*ArcSinh[c*x]*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSin
h[c*x])] + 5*a*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 5*b*ArcSinh[c*x
]*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 27*a*Sinh[(3*a)/b]*SinhInteg
ral[3*(a/b + ArcSinh[c*x])] + 27*b*ArcSinh[c*x]*Sinh[(3*a)/b]*SinhIntegral[
3*(a/b + ArcSinh[c*x])] + 25*a*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])] + 25*b*ArcSinh[c*x]*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x]
)] + 7*a*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])] + 7*b*ArcSinh[c
*x]*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b^2*c^2*(a + b*
ArcSinh[c*x]))
```

Maple [B] time = 0.346, size = 958, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -1/128*(64*c^7*x^7-64*c^6*x^6*(c^2*x^2+1)^(1/2)+112*c^5*x^5-80*c^4*x^4*(c^2
*x^2+1)^(1/2)+56*c^3*x^3-24*c^2*x^2*(c^2*x^2+1)^(1/2)+7*c*x-(c^2*x^2+1)^(1/
2))/c^2/(a+b*arcsinh(c*x))/b-7/128/c^2/b^2*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7
*a/b)-5/128*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*
(c^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-25/128/
c^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-9/128*(4*c^3*x^3-4*c^2*x^2*(c
^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-27/128/c^
2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/128*(c*x-(c^2*x^2+1)^(1/2))/c
^2/b/(a+b*arcsinh(c*x))-5/128/c^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/128
/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*
x)-a/b)*exp(-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-9/128/c^2
/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*
Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3
*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-5/128/c^2/b^2*(16*x^5*b*c^5+
16*(c^2*x^2+1)^(1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+
```

```
5*arcsinh(c*x)*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1
,-5*arcsinh(c*x)-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1
/128/c^2/b^2*(64*x^7*b*c^7+64*(c^2*x^2+1)^(1/2)*x^6*b*c^6+112*x^5*b*c^5+80*
(c^2*x^2+1)^(1/2)*x^4*b*c^4+56*x^3*b*c^3+24*(c^2*x^2+1)^(1/2)*x^2*b*c^2+7*a
rcsinh(c*x)*Ei(1,-7*arcsinh(c*x)-7*a/b)*exp(-7*a/b)*b+7*Ei(1,-7*arcsinh(c*x
)-7*a/b)*exp(-7*a/b)*a+7*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^7+3\,c^4x^5+3\,c^2x^3+x\right)\left(c^2x^2+1\right)+\left(c^7x^8+3\,c^5x^6+3\,c^3x^4+cx^2\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{7\left(c^7x^7+2\right.}{abc^5x^4+\left(c^2x^2+1\right)abc^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 + 3*c^4*x^5 + 3*c^2*x^3 + x)*(c^2*x^2 + 1) + (c^7*x^8 + 3*c^5*x^
6 + 3*c^3*x^4 + c*x^2)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*
a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log
(c*x + sqrt(c^2*x^2 + 1))) + integrate((7*(c^7*x^7 + 2*c^5*x^5 + c^3*x^3)*(
c^2*x^2 + 1)^(3/2) + (14*c^8*x^8 + 37*c^6*x^6 + 33*c^4*x^4 + 11*c^2*x^2 + 1
)*(c^2*x^2 + 1) + (7*c^9*x^9 + 23*c^7*x^7 + 27*c^5*x^5 + 13*c^3*x^3 + 2*c*x
)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x
^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2
*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^5+2\,c^2x^3+x\right)\sqrt{c^2x^2+1}}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 + 2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x/(b*arcsinh(c*x) + a)^2, x)
```

3.430 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=216

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c}-\frac{3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c}-\frac{3\sinh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c}+\frac{15\cosh\left(\frac{2}{b}\right.}{}$$

```
[Out] -((1 + c^2*x^2)^3/(b*c*(a + b*ArcSinh[c*x]))) - (15*CoshIntegral[(2*(a + b*
ArcSinh[c*x]))/b]*Sinh[(2*a)/b])/(16*b^2*c) - (3*CoshIntegral[(4*(a + b*Arc
Sinh[c*x]))/b]*Sinh[(4*a)/b])/(4*b^2*c) - (3*CoshIntegral[(6*(a + b*ArcSinh
[c*x]))/b]*Sinh[(6*a)/b])/(16*b^2*c) + (15*Cosh[(2*a)/b]*SinhIntegral[(2*(a
 + b*ArcSinh[c*x]))/b])/(16*b^2*c) + (3*Cosh[(4*a)/b]*SinhIntegral[(4*(a +
b*ArcSinh[c*x]))/b])/(4*b^2*c) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2*c)
```

Rubi [A] time = 0.446321, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5696, 5779, 5448, 3303, 3298, 3301}

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{16b^2c}-\frac{3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{4b^2c}-\frac{3\sinh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6a}{b}+6\sinh^{-1}(cx)\right)}{16b^2c}+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*(a + b*ArcSinh[c*x]))) - (15*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(16*b^2*c) - (3*CoshIntegral[(4*a)/b + 4*Arc
Sinh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c) - (3*CoshIntegral[(6*a)/b + 6*ArcSinh[c
*x]]*Sinh[(6*a)/b])/(16*b^2*c) + (15*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2
*ArcSinh[c*x]])/(16*b^2*c) + (3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcS
inh[c*x]])/(4*b^2*c) + (3*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcSinh[c*
x]])/(16*b^2*c)
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
```

```
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(6c)\int \frac{x\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{6\,\mathrm{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{6\,\mathrm{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}+\frac{\sinh(6x)}{32(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc}+\frac{3\,\mathrm{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\mathrm{s}\right.}{4bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(15\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{16bc}+\frac{\left(3\cosh\right(}{}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{15\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c}-\frac{3\mathrm{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)\mathrm{s}}{4b^2c}$$

Mathematica [A] time = 0.719435, size = 311, normalized size = 1.44

$$-\frac{15\sinh\left(\frac{2a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+12\sinh\left(\frac{4a}{b}\right)\left(a+b\sinh^{-1}(cx)\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(16*b + 48*b*c^2*x^2 + 48*b*c^4*x^4 + 16*b*c^6*x^6 + 15*(a + b*ArcSinh[c*x
])*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[(2*a)/b] + 12*(a + b*ArcSinh[c
*x])*CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] + 3*a*CoshIntegral[
6*(a/b + ArcSinh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcSinh[c*x]*CoshIntegral[6*(a/
b + ArcSinh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[(2*a)/b]*SinhIntegral[2*(a/b +
 ArcSinh[c*x])] - 15*b*ArcSinh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Arc
Sinh[c*x])] - 12*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 12*
b*ArcSinh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c*x])] - 3*a*Cos
h[(6*a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])] - 3*b*ArcSinh[c*x]*Cosh[(6*
```

```
a)/b]*SinhIntegral[6*(a/b + ArcSinh[c*x])])/(16*b^2*c*(a + b*ArcSinh[c*x]))
```

Maple [B] time = 0.301, size = 704, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] -5/16/b/c/(a+b*arcsinh(c*x))-1/64*(32*c^6*x^6-32*c^5*x^5*(c^2*x^2+1)^(1/2)+
48*c^4*x^4-32*c^3*x^3*(c^2*x^2+1)^(1/2)+18*c^2*x^2-6*c*x*(c^2*x^2+1)^(1/2)+
1)/c/(a+b*arcsinh(c*x))/b+3/32/c/b^2*exp(6*a/b)*Ei(1,6*arcsinh(c*x)+6*a/b)-
3/32*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8*c^2*x^2-4*c*x*(c^2*x^2+1)^(1/
2)+1)/c/(a+b*arcsinh(c*x))/b+3/8/c/b^2*exp(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b
)-15/64*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c/(a+b*arcsinh(c*x))/b+15/32/
c/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-15/64/c/b^2*(2*x^2*b*c^2+2*b*c*
(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*
b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)/(a+b*arcsinh(c*x))-3/32/c/
b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1
)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-
4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/(a+b*arcsinh(c*x))-1/64/c/b^2*(32*x^
6*b*c^6+32*(c^2*x^2+1)^(1/2)*x^5*b*c^5+48*x^4*b*c^4+32*(c^2*x^2+1)^(1/2)*x^
3*b*c^3+18*x^2*b*c^2+6*b*c*(c^2*x^2+1)^(1/2)*x+6*arcsinh(c*x)*exp(-6*a/b)*E
i(1,-6*arcsinh(c*x)-6*a/b)*b+6*exp(-6*a/b)*Ei(1,-6*arcsinh(c*x)-6*a/b)*a+b)
/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{6}x^{6}+3\,c^{4}x^{4}+3\,c^{2}x^{2}+1\right)\left(c^{2}x^{2}+1\right)+\left(c^{7}x^{7}+3\,c^{5}x^{5}+3\,c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{\left(6\,c^{6}x^{6}\right.}{abc^{4}x^{4}+\left(c^{2}x^{2}+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*
```

```
b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c
*x + sqrt(c^2*x^2 + 1))) + integrate(((6*c^6*x^6 + 11*c^4*x^4 + 4*c^2*x^2 -
 1)*(c^2*x^2 + 1)^(3/2) + 6*(2*c^7*x^7 + 5*c^5*x^5 + 4*c^3*x^3 + c*x)*(c^2*
x^2 + 1) + (6*c^8*x^8 + 19*c^6*x^6 + 21*c^4*x^4 + 9*c^2*x^2 + 1)*sqrt(c^2*x
^2 + 1))/(a*b*c^4*x^4 + (c^2*x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (
b^2*c^4*x^4 + (c^2*x^2 + 1)*b^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*
x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3
*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^{4}x^{4}+2\,c^{2}x^{2}+1\right)\sqrt{c^{2}x^{2}+1}}{b^{2}\operatorname{arsinh}\left(cx\right)^{2}+2\,ab\operatorname{arsinh}\left(cx\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*arcsinh(c*x)^2 +
2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/(b*arcsinh(c*x) + a)^2, x)
```

3.431 $\int \frac{(1+c^2x^2)^{5/2}}{x(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=232

$$-\frac{\text{Unintegrable}\left(\frac{(c^2x^2+1)^2}{x^2(a+b\sinh^{-1}(cx))},x\right)}{bc}+\frac{25\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2}+\frac{25\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{16b^2}+\frac{5\cosh\left(\frac{5a}{b}\right.}{}$$

```
[Out] -((1 + c^2*x^2)^3/(b*c*x*(a + b*ArcSinh[c*x]))) + (25*Cosh[a/b]*CoshIntegra
l[(a + b*ArcSinh[c*x])/b])/(8*b^2) + (25*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
 b*ArcSinh[c*x]))/b])/(16*b^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*Ar
cSinh[c*x]))/b])/(16*b^2) - (25*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/(8*b^2) - (25*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(16*b^2) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b
^2) - Unintegrable[(1 + c^2*x^2)^2/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi [A] time = 0.568258, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1+c^2x^2)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x*(a + b*ArcSinh[c*x]))) + (25*Cosh[a/b]*CoshIntegra
l[a/b + ArcSinh[c*x]])/(8*b^2) + (25*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3
*ArcSinh[c*x]])/(16*b^2) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcSin
h[c*x]])/(16*b^2) - (25*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b^2)
 - (25*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b^2) - (5*
Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2) - Defer[Int]
[(1 + c^2*x^2)^2/(x^2*(a + b*ArcSinh[c*x])), x]/(b*c)
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{\left(1+c^2x^2\right)^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc} + \frac{(5c)\int \frac{\left(1+c^2x^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\text{Subst}\left(\int \frac{\cosh^5(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\int \frac{\left(1+c^2x^2\right)^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8(a+bx)} + \frac{5\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16b} + \frac{25\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,\cdots\right.}{16b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} - \frac{\int \frac{\left(1+c^2x^2\right)^2}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc} + \frac{\left(25\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x\cdots\right.}{8b}$$

$$= -\frac{\left(1+c^2x^2\right)^3}{bcx\left(a+b\sinh^{-1}(cx)\right)} + \frac{25\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2} + \frac{25\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(\frac{3a}{b}+3\cdots\right.}{16b^2}$$

Mathematica [A] time = 9.04237, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.267, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\mathrm{Arcsinh}\,(cx)\right)^2}\left(c^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^6+3\,c^4x^4+3\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^7x^7+3\,c^5x^5+3\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^3+\sqrt{c^2x^2+1}abc^2x^2+abcx+\left(b^2c^3x^3+\sqrt{c^2x^2+1}b^2c^2x^2+b^2cx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^6+\left(c^2x^2+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^3 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^2 + a*b*c*x + (b^2*c^3*x^3 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^2 + b^2*c*
x)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((5*c^7*x^7 + 8*c^5*x^5 + c^3*
x^3 - 2*c*x)*(c^2*x^2 + 1)^(3/2) + (10*c^8*x^8 + 23*c^6*x^6 + 15*c^4*x^4 +
c^2*x^2 - 1)*(c^2*x^2 + 1) + 5*(c^9*x^9 + 3*c^7*x^7 + 3*c^5*x^5 + c^3*x^3)*
sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6 + (c^2*x^2 + 1)*a*b*c^3*x^4 + 2*a*b*c^3*x^4
 + a*b*c*x^2 + (b^2*c^5*x^6 + (c^2*x^2 + 1)*b^2*c^3*x^4 + 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 + b^2*c^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + 2*(a*b*c^4*x^5 + a*b*c^2*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x\operatorname{arsinh}\left(cx\right)^2+2\,abx\operatorname{arsinh}\left(cx\right)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x*arcsinh(c*x)^2
+ 2*a*b*x*arcsinh(c*x) + a^2*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2x^2+1)^{\frac{5}{2}}}{(b \operatorname{arsinh}(cx)+a)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x), x)
```

3.432 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{2\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^2}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}+\frac{4c\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{\left(c^2x^2+1\right)^3}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -((1 + c^2*x^2)^3/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Unintegrable[(1 + c^
2*x^2)^2/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (4*c*Unintegrable[(1 + c^2
*x^2)^2/(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi [A] time = 0.318361, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -((1 + c^2*x^2)^3/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][(1 + c^2*
x^2)^2/(x^3*(a + b*ArcSinh[c*x])), x])/(b*c) + (4*c*Defer[Int][(1 + c^2*x^2
)^2/(x*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{\left(1+c^2x^2\right)^3}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{\left(1+c^2x^2\right)^2}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}+\frac{(4c)\int \frac{\left(1+c^2x^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 3.2628, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\mathrm{Arcsinh}\,(cx)\right)^2}\left(c^2x^2+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^6+3\,c^4x^4+3\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^7x^7+3\,c^5x^5+3\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^4+\sqrt{c^2x^2+1}abc^2x^3+abcx^2+\left(b^2c^3x^4+\sqrt{c^2x^2+1}b^2c^2x^3+b^2cx^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^5x^7+\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^4 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^3 + a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^3 + b^2*
c*x^2)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((4*c^7*x^7 + 5*c^5*x^5 -
2*c^3*x^3 - 3*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(4*c^8*x^8 + 8*c^6*x^6 + 3*c^4*x
```

```
^4 - 2*c^2*x^2 - 1)*(c^2*x^2 + 1) + (4*c^9*x^9 + 11*c^7*x^7 + 9*c^5*x^5 + c
^3*x^3 - c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7 + (c^2*x^2 + 1)*a*b*c^3*x^5 +
 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 + (c^2*x^2 + 1)*b^2*c^3*x^5 + 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*sqrt(c^2*x^2 + 1))*l
og(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x^2\,\text{arsinh}\left(cx\right)^2+2\,abx^2\,\text{arsinh}\left(cx\right)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^2*arcsinh(c*x)^
2 + 2*a*b*x^2*arcsinh(c*x) + a^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")

[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^2), x)
```

3.433 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.143702, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 14.0174, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^3\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^3*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.807, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b \operatorname{Arcsinh}\left(cx\right)\right)^{2}} \left(c^{2}x^{2}+1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{6}x^{6}+3\,c^{4}x^{4}+3\,c^{2}x^{2}+1\right)\left(c^{2}x^{2}+1\right)+\left(c^{7}x^{7}+3\,c^{5}x^{5}+3\,c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}}{abc^{3}x^{5}+\sqrt{c^{2}x^{2}+1}abc^{2}x^{4}+abcx^{3}+\left(b^{2}c^{3}x^{5}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x^{4}+b^{2}cx^{3}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int \frac{}{abc^{5}x^{8}+\left(c^{2}x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^5 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^4 + a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^4 + b^2*
c*x^3)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((3*c^7*x^7 + 2*c^5*x^5 -
5*c^3*x^3 - 4*c*x)*(c^2*x^2 + 1)^(3/2) + 3*(2*c^8*x^8 + 3*c^6*x^6 - c^4*x^4
 - 3*c^2*x^2 - 1)*(c^2*x^2 + 1) + (3*c^9*x^9 + 7*c^7*x^7 + 3*c^5*x^5 - 3*c^
3*x^3 - 2*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^8 + (c^2*x^2 + 1)*a*b*c^3*x^6
+ 2*a*b*c^3*x^6 + a*b*c*x^4 + (b^2*c^5*x^8 + (c^2*x^2 + 1)*b^2*c^3*x^6 + 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 + b^2*c^2*x^5)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^7 + a*b*c^2*x^5)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^{4}x^{4}+2\,c^{2}x^{2}+1\right)\sqrt{c^{2}x^{2}+1}}{b^{2}x^{3}\operatorname{arsinh}\left(cx\right)^{2}+2\,abx^{3}\operatorname{arsinh}\left(cx\right)+a^{2}x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^3*arcsinh(c*x)^
2 + 2*a*b*x^3*arcsinh(c*x) + a^2*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**3/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^{2}x^{2}+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^3/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^3), x)
```

3.434 $\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.144383, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 2.93395, size = 0, normalized size = 0.

$$\int \frac{\left(1+c^2x^2\right)^{5/2}}{x^4\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[(1 + c^2*x^2)^(5/2)/(x^4*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.72, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b\text{Arcsinh}\,(cx))^2} \left(c^2x^2+1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^6+3\,c^4x^4+3\,c^2x^2+1\right)\left(c^2x^2+1\right)+\left(c^7x^7+3\,c^5x^5+3\,c^3x^3+cx\right)\sqrt{c^2x^2+1}}{abc^3x^6+\sqrt{c^2x^2+1}abc^2x^5+abcx^4+\left(b^2c^3x^6+\sqrt{c^2x^2+1}b^2c^2x^5+b^2cx^4\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int \frac{}{abc^5x^9+\left(c^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1) + (c^7*x^7 + 3*c^5*x^
5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^6 + sqrt(c^2*x^2 + 1)*a*
b*c^2*x^5 + a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c^2*x^2 + 1)*b^2*c^2*x^5 + b^2*
c*x^4)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c^7*x^7 - c^5*x^5 - 8*
c^3*x^3 - 5*c*x)*(c^2*x^2 + 1)^(3/2) + 2*(2*c^8*x^8 + c^6*x^6 - 6*c^4*x^4 -
 7*c^2*x^2 - 2)*(c^2*x^2 + 1) + (2*c^9*x^9 + 3*c^7*x^7 - 3*c^5*x^5 - 7*c^3*
x^3 - 3*c*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^9 + (c^2*x^2 + 1)*a*b*c^3*x^7 +
2*a*b*c^3*x^7 + a*b*c*x^5 + (b^2*c^5*x^9 + (c^2*x^2 + 1)*b^2*c^3*x^7 + 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 + b^2*c^2*x^6)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^8 + a*b*c^2*x^6)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}}{b^2x^4\operatorname{arsinh}\left(cx\right)^2+2\,abx^4\operatorname{arsinh}\left(cx\right)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)/(b^2*x^4*arcsinh(c*x)^
2 + 2*a*b*x^4*arcsinh(c*x) + a^2*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(5/2)/x**4/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(5/2)/x^4/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)/((b*arcsinh(c*x) + a)^2*x^4), x)
```

3.435 $\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=204

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^6}-\frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{16b^2c^6}-\frac{5\sinh\left(\frac{a}{b}\right)\text{Shi}}{8b^2}$$

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegral[(a + b*ArcSin
h[c*x])/b])/(8*b^2*c^6) - (15*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/(16*b^2*c^6) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c
*x]))/b])/(16*b^2*c^6) - (5*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])
/(8*b^2*c^6) + (15*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/
(16*b^2*c^6) - (5*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(
16*b^2*c^6)
```

Rubi [A] time = 0.480139, antiderivative size = 200, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^6}-\frac{15\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{16b^2c^6}+\frac{5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\sinh^{-1}(cx)\right)}{16b^2c^6}-5\sinh$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*Cosh[a/b]*CoshIntegral[a/b + ArcSinh
[c*x]])/(8*b^2*c^6) - (15*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(16*b^2*c^6) + (5*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcSinh[c*x]]
)/(16*b^2*c^6) - (5*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b^2*c^6)
 + (15*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b^2*c^6) -
 (5*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b^2*c^6)
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
```

```
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc^6}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)} - \frac{3\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \sinh^{-1}(\right.}{bc^6}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^6} + \frac{5\,\mathrm{Subst}\left(\int \frac{\cosh}{a+b}\right.}{}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(5\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^6} - \frac{(15}{}$$

$$= -\frac{x^5}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{5\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^6} - \frac{15\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}\right.}{16b^2c}$$

Mathematica [A] time = 0.330466, size = 158, normalized size = 0.77

$$\frac{5\left(2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + \cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 2\sinh\right.}{16b^2c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^5/(b*c*(a + b*ArcSinh[c*x]))) + (5*(2*Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Cosh[(5*
a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 2*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] + 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - Sin
h[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])]))/(16*b^2*c^6)
```

Maple [B] time = 0.179, size = 633, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c^2*x
^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))-5/32/c^6/b^2*
exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)+5/32*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)
^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*arcsinh(c*x))+15/32/c^6/b^2*exp(
3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-5/16*(c*x-(c^2*x^2+1)^(1/2))/c^6/b/(a+b*a
rcsinh(c*x))-5/16/c^6/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-5/16/c^6/b^2*(arc
sinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-
a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))+5/32/c^6/b^2*(4*x^3*b*
c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(c*x)*exp(-3*a/b)*Ei(1,-3*arcsin
h(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^
2+1)^(1/2)*b)/(a+b*arcsinh(c*x))-1/32/c^6/b^2*(16*x^5*b*c^5+16*(c^2*x^2+1)^
(1/2)*x^4*b*c^4+20*x^3*b*c^3+12*(c^2*x^2+1)^(1/2)*x^2*b*c^2+5*arcsinh(c*x)*
exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)*b+5*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x
)-5*a/b)*a+5*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^8 + cx^6 + \left(c^2x^7 + x^5\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x + \left(\left(c^2x^2+1\right)b^2c^2x + \left(b^2c^3x^2 + b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx + \sqrt{c^2x^2+1}\right) + \left(abc^3x^2 + abc\right)\sqrt{c^2x^2+1}} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^8 + c*x^6 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((5*c^5*x^9 + 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 + 4*c*x^5)*(c^2*x^2
 + 1) + 5*(2*c^4*x^8 + 3*c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(
3/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 +
1)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5
*x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1
)) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^5}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

3.436 $\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2b^2c^5}$$

```
[Out] -(x^4/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*(a + b*ArcSinh[c*x]))/
b]*Sinh[(2*a)/b])/(b^2*c^5) - (CoshIntegral[(4*(a + b*ArcSinh[c*x]))/b]*Sin
h[(4*a)/b])/(2*b^2*c^5) - (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(b^2*c^5) + (Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c*x]))/b]
)/(2*b^2*c^5)
```

Rubi [A] time = 0.395444, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5774, 5669, 5448, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^5}-\frac{\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^5}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^5}+\frac{\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{2b^2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^4/(b*c*(a + b*ArcSinh[c*x]))) + (CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]]
*Sinh[(2*a)/b])/(b^2*c^5) - (CoshIntegral[(4*a)/b + 4*ArcSinh[c*x]]*Sinh[(4
*a)/b])/(2*b^2*c^5) - (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]]
)/(b^2*c^5) + (Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(2*b^2
*c^5)
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\int \frac{x^3}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^5}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{4\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^5}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{2bc^5}-\frac{\text{Subst}\left(\int \frac{\sinh(2x}{a+bx}\right.}{}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^5}+\frac{\text{co}}{}$$

$$= -\frac{x^4}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^5}-\frac{\text{Chi}\left(\frac{4a}{b}+4\sinh\right.}{2b^2}$$

Mathematica [A] time = 0.279806, size = 117, normalized size = 0.83

$$\frac{-\frac{2bc^4x^4}{a+b\sinh^{-1}(cx)}+2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\text{si}\right.\right.}{2b^2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] ((-2*b*c^4*x^4)/(a + b*ArcSinh[c*x]) + 2*CoshIntegral[2*(a/b + ArcSinh[c*x]
)]*Sinh[(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*C
osh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcSinh[c*x])])/(2*b^2*c^5)
```

Maple [B] time = 0.195, size = 420, normalized size = 3.

$$-\frac{3}{8c^5(a+b\text{Arcsinh}(cx))b}-\frac{1}{16c^5(a+b\text{Arcsinh}(cx))b}\left(8c^4x^4-8c^3x^3\sqrt{c^2x^2+1}+8c^2x^2-4cx\sqrt{c^2x^2+1}+1\right)+\frac{}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -3/8/c^5/(a+b*arcsinh(c*x))/b-1/16*(8*c^4*x^4-8*c^3*x^3*(c^2*x^2+1)^(1/2)+8
*c^2*x^2-4*c*x*(c^2*x^2+1)^(1/2)+1)/c^5/(a+b*arcsinh(c*x))/b+1/4/c^5/b^2*ex
p(4*a/b)*Ei(1,4*arcsinh(c*x)+4*a/b)+1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+
1)/c^5/(a+b*arcsinh(c*x))/b-1/2/c^5/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/
b)+1/4/c^5/b^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2
*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/
b)*a+b)/(a+b*arcsinh(c*x))-1/16/c^5/b^2*(8*x^4*b*c^4+8*(c^2*x^2+1)^(1/2)*x^
3*b*c^3+8*x^2*b*c^2+4*b*c*(c^2*x^2+1)^(1/2)*x+4*arcsinh(c*x)*Ei(1,-4*arcsin
h(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arcsinh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/
(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^7 + cx^5 + \left(c^2x^6 + x^4\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x + \left(\left(c^2x^2+1\right)b^2c^2x + \left(b^2c^3x^2 + b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx + \sqrt{c^2x^2+1}\right) + \left(abc^3x^2 + abc\right)\sqrt{c^2x^2+1}} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^7 + c*x^5 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((4*c^5*x^8 + 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 + 3*c*x^4)*(c^2*x^2
+ 1) + 4*(2*c^4*x^7 + 3*c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^4}{a^2c^2x^2 + \left(b^2c^2x^2 + b^2\right)\operatorname{arsinh}\left(cx\right)^2 + a^2 + 2\left(abc^2x^2 + ab\right)\operatorname{arsinh}\left(cx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a + b\,\mathrm{asinh}\,(cx))^2\,\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{c^2x^2+1}(b\,\mathrm{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

3.437 $\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^4}-\frac{3\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(}{}\right.}{4b^2c^4}$$

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral[(a + b*ArcSin
h[c*x])/b])/(4*b^2*c^4) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(4*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/
(4*b^2*c^4) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4
*b^2*c^4)
```

Rubi [A] time = 0.366808, antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5774, 5669, 5448, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}-\frac{3\sinh\left(\frac{3a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) - (3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh
[c*x]])/(4*b^2*c^4) + (3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSinh[c*x
]])/(4*b^2*c^4) + (3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b^2*c^4
) - (3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2*c^4)
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\int\frac{x^2}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int\frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{3\,\text{Subst}\left(\int\left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^4}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\,\text{Subst}\left(\int\frac{\cosh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{4bc^4}+\frac{3\,\text{Subst}\left(\int\frac{\cosh(\ldots}{a+b\ldots}\right.}{\ldots}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\left(3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{4bc^4}+\frac{(3\ldots}{\ldots}$$

$$= -\frac{x^3}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^4}+\frac{3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}\ldots\right.}{4b^2c^4\ldots}$$

Mathematica [A] time = 0.260961, size = 113, normalized size = 0.8

$$\frac{3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Sh}\ldots\right.}{4b^2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^3/(b*c*(a + b*ArcSinh[c*x]))) + (3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcS
inh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]
*SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Sinh[c*x])]))/(4*b^2*c^4)
```

Maple [B] time = 0.128, size = 364, normalized size = 2.6

$$-\frac{1}{8\,bc^4\left(a+b\text{Arcsinh}\,(cx)\right)}\left(4\,c^3x^3-4\,c^2x^2\sqrt{c^2x^2+1}+3\,cx-\sqrt{c^2x^2+1}\right)-\frac{3}{8\,c^4b^2}e^{3\,\frac{a}{b}}\text{Ei}\left(1,3\,\text{Arcsinh}\,(cx)+3\,\frac{a}{b}\right)+\frac{\ldots}{8\,bc\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))/c^4/b/
(a+b*arcsinh(c*x))-3/8/c^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)+3/8*(c
*x-(c^2*x^2+1)^(1/2))/c^4/b/(a+b*arcsinh(c*x))+3/8/c^4/b^2*exp(a/b)*Ei(1,ar
csinh(c*x)+a/b)+3/8/c^4/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)
*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcs
inh(c*x))-1/8/c^4/b^2*(4*x^3*b*c^3+4*(c^2*x^2+1)^(1/2)*x^2*b*c^2+3*arcsinh(
c*x)*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*b+3*exp(-3*a/b)*Ei(1,-3*arcsin
h(c*x)-3*a/b)*a+3*x*b*c+(c^2*x^2+1)^(1/2)*b)/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^6+cx^4+\left(c^2x^5+x^3\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^6 + c*x^4 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((3*c^5*x^7 + 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 + 2*c*x^3)*(c^2*x^2
+ 1) + 3*(2*c^4*x^6 + 3*c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3
/2)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1
)^(3/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*
x^4 + 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^3}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + b\,\mathrm{asinh}\,(cx))^2\,\sqrt{c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{c^2x^2 + 1}(b\,\mathrm{arsinh}\,(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

3.438 $\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^3}-\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(x^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*(a + b*ArcSinh[c*x]))/
b]*Sinh[(2*a)/b])/(b^2*c^3) + (Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh
[c*x]))/b])/(b^2*c^3)
```

Rubi [A] time = 0.257786, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5774, 5669, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{b^2c^3}-\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(2*a)/b + 2*ArcSinh[c*x]]
*Sinh[(2*a)/b])/(b^2*c^3) + (Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh
[c*x]])/(b^2*c^3)
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\int \frac{x}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}-\frac{\text{si}}{}$$

$$= -\frac{x^2}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c^3}+\frac{\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}\right.}{b^2}$$

Mathematica [A] time = 0.35973, size = 70, normalized size = 0.89

$$\frac{-\frac{bc^2x^2}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c^2*x^2)/(a + b*ArcSinh[c*x])) - CoshIntegral[2*(a/b + ArcSinh[c*x])]
*Sinh[(2*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(b^2*c
^3)
```

Maple [B] time = 0.107, size = 192, normalized size = 2.4

$$\frac{1}{2\,c^3\,(a+b\text{Arcsinh}\,(cx))\,b}-\frac{1}{4\,c^3\,(a+b\text{Arcsinh}\,(cx))\,b}\left(2\,c^2x^2-2\,cx\sqrt{c^2x^2+1}+1\right)+\frac{1}{2\,c^3b^2}\text{e}^{2\,\frac{a}{b}}\text{Ei}\left(1,2\,\text{Arcsinh}\,(cx)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/c^3/(a+b*arcsinh(c*x))/b-1/4*(2*c^2*x^2-2*c*x*(c^2*x^2+1)^(1/2)+1)/c^3/
(a+b*arcsinh(c*x))/b+1/2/c^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(c*x)+2*a/b)-1/4/
c^3/b^2*(2*x^2*b*c^2+2*b*c*(c^2*x^2+1)^(1/2)*x+2*arcsinh(c*x)*exp(-2*a/b)*E
i(1,-2*arcsinh(c*x)-2*a/b)*b+2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*a+b)
/(a+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^5+cx^3+\left(c^2x^4+x^2\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^5 + c*x^3 + (c^2*x^4 + x^2)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c
^2*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*
log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + i
ntegrate((2*c^5*x^6 + 5*c^3*x^4 + 3*c*x^2 + (2*c^3*x^4 + c*x^2)*(c^2*x^2 +
1) + 2*(2*c^4*x^5 + 3*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*
a*b*c^3*x^2 + 2*(a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3
/2)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4
+ 2*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^4 + 2*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}\,x^2}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b\operatorname{asinh}(cx))^2 \sqrt{c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{c^2x^2 + 1}(b\operatorname{arsinh}(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

3.439 $\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(x/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
^2)
```

Rubi [A] time = 0.160922, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5774, 5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x/(b*c*(a + b*ArcSinh[c*x]))) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c^2) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c
^2)
```

Rule 5774

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m*(a + b*ArcSinh[c*x])^(n + 1))/
(b*c*Sqrt[d]*(n + 1)), x] - Dist[(f*m)/(b*c*Sqrt[d]*(n + 1)), Int[(f*x)^(m
- 1)*(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x
] && EqQ[e, c^2*d] && LtQ[n, -1] && GtQ[d, 0]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{b^2c^2}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{b^2c^2}-\frac{\text{s}}{}$$

$$= -\frac{x}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c^2}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(}{b}\right.}{b^2c^2}$$

Mathematica [A] time = 0.123444, size = 60, normalized size = 0.82

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\frac{bcx}{a+b\sinh^{-1}(cx)}}{b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] (-((b*c*x)/(a + b*ArcSinh[c*x])) + Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x
]] - Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c^2)
```

Maple [B] time = 0.066, size = 151, normalized size = 2.1

$$-\frac{1}{2\,bc^2\,(a+b\mathrm{Arcsinh}\,(cx))}\left(cx-\sqrt{c^2x^2+1}\right)-\frac{1}{2\,c^2b^2}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right)-\frac{1}{2\,c^2b^2\,(a+b\mathrm{Arcsinh}\,(cx))}\left(\mathrm{Arcsinh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(c*x-(c^2*x^2+1)^(1/2))/c^2/b/(a+b*arcsinh(c*x))-1/2/c^2/b^2*exp(a/b)*
Ei(1,arcsinh(c*x)+a/b)-1/2/c^2/b^2*(arcsinh(c*x)*Ei(1,-arcsinh(c*x)-a/b)*ex
p(-a/b)*b+Ei(1,-arcsinh(c*x)-a/b)*exp(-a/b)*a+x*b*c+(c^2*x^2+1)^(1/2)*b)/(a
+b*arcsinh(c*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^4+cx^2+\left(c^2x^3+x\right)\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^4 + c*x^2 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2
*x + ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*lo
g(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + int
egrate((c^5*x^5 + (c^2*x^2 + 1)*c^3*x^3 + 3*c^3*x^3 + 2*c*x + (2*c^4*x^4 +
3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^2 + 2*(a*b
*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^2 + 2*
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
```

```
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c*x
)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

3.440 $\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Rubi [A] time = 0.0447206, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5675}

$$-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Mathematica [A] time = 0.012315, size = 18, normalized size = 1.

$$-\frac{1}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(a + b*ArcSinh[c*x])))
```

Maple [A] time = 0.007, size = 19, normalized size = 1.1

$$-\frac{1}{bc\left(a+b\text{Arcsinh}\left(cx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/b/c/(a+b*arcsinh(c*x))
```

Maxima [A] time = 1.16412, size = 24, normalized size = 1.33

$$-\frac{1}{(b\,\text{arsinh}\,(cx)+a)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/((b*arcsinh(c*x) + a)*b*c)
```

Fricas [A] time = 2.24254, size = 66, normalized size = 3.67

$$-\frac{1}{b^2c\log\left(cx+\sqrt{c^2x^2+1}\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/(b^2*c*log(c*x + sqrt(c^2*x^2 + 1)) + a*b*c)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.35751, size = 41, normalized size = 2.28

$$-\frac{1}{\left(b\log\left(cx+\sqrt{c^2x^2+1}\right)+a\right)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/((b*log(c*x + sqrt(c^2*x^2 + 1)) + a)*b*c)
```

3.441 $\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{1}{bcx\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(1/(b*c*x*(a + b*ArcSinh[c*x]))) - Unintegrable[1/(x^2*(a + b*ArcSinh[c*x]
)), x]/(b*c)
```

Rubi [A] time = 0.146616, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x*(a + b*ArcSinh[c*x]))) - Defer[Int][1/(x^2*(a + b*ArcSinh[c*x]))
, x]/(b*c)
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bcx\left(a+b\sinh^{-1}(cx)\right)}-\frac{\int \frac{1}{x^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 5.12583, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\text{Arcsinh}\left(cx\right)\right)^{2}}\frac{1}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^{3}x^{3}+cx+\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(c^{2}x^{2}+1\right)abc^{2}x^{2}+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x^{2}+\left(b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{3}+abcx\right)\sqrt{c^{2}x^{2}+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x
^2 + 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate
((c^5*x^5 + c^3*x^3 + (c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (2*c^4*x^4 + 3*c^2*
x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^4 + 2*(a*b*c^4*x
^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^4 + 2*(b^2
*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^2*
c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2*a
*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^2x^3+a^2x+\left(b^2c^2x^3+b^2x\right)\operatorname{arsinh}\left(cx\right)^2+2\left(abc^2x^3+abx\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^3 + a^2*x + (b^2*c^2*x^3 + b^2*x)*arc
sinh(c*x)^2 + 2*(a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2*x), x)
```

3.442 $\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{2\text{Unintegrable}\left(\frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(1/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Unintegrable[1/(x^3*(a + b*ArcSinh
[c*x])), x])/(b*c)
```

Rubi [A] time = 0.148409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*x^2*(a + b*ArcSinh[c*x]))) - (2*Defer[Int][1/(x^3*(a + b*ArcSinh[c
*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bcx^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{2\int \frac{1}{x^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 1.19675, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(a+b \mathrm{Arcsinh}\left(c x\right)\right)^{2}} \frac{1}{\sqrt{c^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^{3} x^{3}+c x+\left(c^{2} x^{2}+1\right)^{\frac{3}{2}}}{\left(c^{2} x^{2}+1\right) a b c^{2} x^{3}+\left(\left(c^{2} x^{2}+1\right) b^{2} c^{2} x^{3}+\left(b^{2} c^{3} x^{4}+b^{2} c x^{2}\right) \sqrt{c^{2} x^{2}+1}\right) \log\left(c x+\sqrt{c^{2} x^{2}+1}\right)+\left(a b c^{3} x^{4}+a b c x^{2}\right) \sqrt{c^{2} x^{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x
^2 + 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x
+ sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integ
rate((2*c^5*x^5 + 3*c^3*x^3 + (2*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(
2*c^4*x^4 + 3*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^(3/2)*a*b*c^3*
x^5 + 2*(a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^
2*c^3*x^5 + 2*(b^2*c^4*x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^5*x^7 + 2*
b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
(a*b*c^5*x^7 + 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^2x^4+a^2x^2+\left(b^2c^2x^4+b^2x^2\right)\operatorname{arsinh}\left(cx\right)^2+2\left(abc^2x^4+abx^2\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^2*x^4 + a^2*x^2 + (b^2*c^2*x^4 + b^2*x^2)
*arcsinh(c*x)^2 + 2*(a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asinh(c*x))**2/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c^2x^2+1}(b\operatorname{arsinh}\left(cx\right)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsinh(c*x))^2/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

3.443 $\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^3}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.137439, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 7.69982, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^3/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.347, size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\left(c^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^{4}+\sqrt{c^{2}x^{2}+1}x^{3}}{\left(c^{2}x^{2}+1\right)abc^{2}x+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{2}+abc\right)\sqrt{c^{2}x^{2}+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*
b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((c^5*x^7 + 5*
c^3*x^5 + 4*c*x^3 + (c^3*x^5 + 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 + 7*c^2*
x^4 + 3*x^2)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^
(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c
^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3
+ b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 +
 b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*
a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^3}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(a+b\,\text{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.444 $\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{2\text{Unintegrable}\left(\frac{x}{\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{bc}-\frac{x^2}{bc\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(x^2/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) + (2*Unintegrable[x/((1 + c
^2*x^2)^2*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi [A] time = 0.200699, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^2/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) + (2*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/(b*c)
```

Rubi steps

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{x^2}{bc\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}+\frac{2\int \frac{x}{\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{bc}$$

Mathematica [A] time = 3.37974, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+b\text{Arcsinh}\,(cx))^2}\left(c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^3+\sqrt{c^2x^2+1}x^2}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*
b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate((3*c^3*x^4 +
(c^2*x^2 + 1)*c*x^2 + 3*c*x^2 + 2*(2*c^2*x^3 + x)*sqrt(c^2*x^2 + 1))/((a*b*
c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 + 2*a*b*c^4*x^3
 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^7*
x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 + a*b*c)*s
qrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^2}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.445 $\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{x}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0931618, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 5.08647, size = 0, normalized size = 0.

$$\int \frac{x}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.129, size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\mathrm{Arcsinh}(cx))^2}\left(c^2x^2+1\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^2+\sqrt{c^2x^2+1}x}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^
2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 +
 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^5 + (c^2
*x^2 + 1)*c^3*x^3 - c^3*x^3 - 2*c*x + (2*c^4*x^4 - c^2*x^2 - 1)*sqrt(c^2*x^
2 + 1))/((a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^6*x^5 +
 2*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*c^5*x^4 + b^2*c^3*x^2)*(c
^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^5 + 2*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 +
1) + (b^2*c^7*x^6 + 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^6 + 3*a*b*c^5*x^4 + 3*a*b*c^3*
x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}x}{a^2c^4x^4+2a^2c^2x^2+\left(b^2c^4x^4+2b^2c^2x^2+b^2\right)\mathrm{arsinh}(cx)^2+a^2+2\left(abc^4x^4+2abc^2x^2+ab\right)\mathrm{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 +
2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2
+ a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\operatorname{asinh}(cx))^2 (c^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.446 $\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{2c\text{Unintegrable}\left(\frac{x}{\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)}{b}-\frac{1}{bc\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(1/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) - (2*c*Unintegrable[x/((1 + c
^2*x^2)^2*(a + b*ArcSinh[c*x])), x])/b
```

Rubi [A] time = 0.110704, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))) - (2*c*Defer[Int][x/((1 + c^2
*x^2)^2*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bc\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}-\frac{(2c)\int \frac{x}{\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 2.33559, size = 0, normalized size = 0.

$$\int \frac{1}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\text{Arcsinh}\,(cx))^2}\left(c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1)*b^2*c^
2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
 + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^4*x^4 + c^2*x^
2 + (2*c^2*x^2 + 1)*(c^2*x^2 + 1) + 2*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) -
 1)/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2
 + 1)^(3/2) + 2*(b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^
2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x
 + sqrt(c^2*x^2 + 1)) + (a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)
*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\text{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4 + 2*
b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^2 +
a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\,\text{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.447 $\int \frac{1}{x(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.131955, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 7.47269, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.326, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\left(c^{2}x^{2}+1\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{c^{2}x^{2}+1}}{\left(c^{2}x^{2}+1\right)abc^{2}x^{2}+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x^{2}+\left(b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{3}+abcx\right)\sqrt{c^{2}x^{2}+1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^2 + ((c^2*x^2 + 1)*b^2*
c^2*x^2 + (b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
 + 1)) + (a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5
+ 3*c^3*x^3 + (3*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (6*c^4*x^4 + 5*c^2*x^2 +
1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^6*x^7 + 2*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b^2*c^5*x^6 +
 b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^7 + 2*b^2*c^4*x^5 + b^2*c^
2*x^3)*(c^2*x^2 + 1) + (b^2*c^7*x^8 + 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 + b^2*c
*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^8 + 3*a*
b*c^5*x^6 + 3*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^{2}x^{2}+1}}{a^{2}c^{4}x^{5}+2\,a^{2}c^{2}x^{3}+a^{2}x+\left(b^{2}c^{4}x^{5}+2\,b^{2}c^{2}x^{3}+b^{2}x\right)\mathrm{arsinh}\left(cx\right)^{2}+2\left(abc^{4}x^{5}+2\,abc^{2}x^{3}+abx\right)\mathrm{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^5 + 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4*
x^5 + 2*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^5 + 2*a*b*c^2*x^
3 + a*b*x)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{asinh}\left(cx\right)\right)^{2}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2*x), x)
```

3.448 $\int \frac{1}{x^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.130146, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 19.0271, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^2}\left(c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{c^2x^2+1}}{\left(c^2x^2+1\right)abc^2x^3+\left(\left(c^2x^2+1\right)b^2c^2x^3+\left(b^2c^3x^4+b^2cx^2\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^4+abcx^2\right)\sqrt{c^2x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)*a*b*c^2*x^3 + ((c^2*x^2 + 1)*b^2*
c^2*x^3 + (b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((4*c^5*
x^5 + 5*c^3*x^3 + (4*c^3*x^3 + 3*c*x)*(c^2*x^2 + 1) + c*x + 2*(4*c^4*x^4 +
4*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1))/((a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^2 + 1
)^(3/2) + 2*(a*b*c^6*x^8 + 2*a*b*c^4*x^6 + a*b*c^2*x^4)*(c^2*x^2 + 1) + ((b
^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^8 + 2*b^2*c^4*
x^6 + b^2*c^2*x^4)*(c^2*x^2 + 1) + (b^2*c^7*x^9 + 3*b^2*c^5*x^7 + 3*b^2*c^3
*x^5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^
7*x^9 + 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^4x^6+2\,a^2c^2x^4+a^2x^2+\left(b^2c^4x^6+2\,b^2c^2x^4+b^2x^2\right)\mathrm{arsinh}\left(cx\right)^2+2\left(abc^4x^6+2\,abc^2x^4+abx^2\right)\mathrm{arsinh}\left(cx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^4*x^6 + 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c^
4*x^6 + 2*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*x)^2 + 2*(a*b*c^4*x^6 + 2*a*b*c^
2*x^4 + a*b*x^2)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{asinh}(cx))^2 \left(c^2 x^2 + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2 x^2 + 1\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

3.449 $\int \frac{x^3}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^3}{(c^2x^2+1)^{5/2}\,(a+b\sinh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.136841, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx = \int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Mathematica [A] time = 13.8681, size = 0, normalized size = 0.

$$\int \frac{x^3}{(1+c^2x^2)^{5/2}\,(a+b\sinh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^3/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.547, size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\left(c^{2}x^{2}+1\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^{4}+\sqrt{c^{2}x^{2}+1}x^{3}}{\left(abc^{4}x^{3}+abc^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(\left(b^{2}c^{4}x^{3}+b^{2}c^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(b^{2}c^{5}x^{4}+2\,b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^4 + sqrt(c^2*x^2 + 1)*x^3)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^7 - 3*c^3*x^5 - 4
*c*x^3 + (c^3*x^5 - 2*c*x^3)*(c^2*x^2 + 1) + (2*c^4*x^6 - 5*c^2*x^4 - 3*x^2
)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2
+ 1)^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c
^2*x^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3
/2) + 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2
+ 1) + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^
7*x^6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^3}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^6x^6+3\,abc^4x^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+b\,\text{asinh}\,(cx))^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**3/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(c^2x^2+1\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.450 $\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^2}{\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.135928, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 5.63141, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.412, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b\text{Arcsinh}\,(cx))^2} \left(c^2x^2 + 1\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^3 + \sqrt{c^2x^2 + 1}x^2}{\left(abc^4x^3 + abc^2x\right)\left(c^2x^2 + 1\right) + \left(\left(b^2c^4x^3 + b^2c^2x\right)\left(c^2x^2 + 1\right) + \left(b^2c^5x^4 + 2\,b^2c^3x^2 + b^2c\right)\sqrt{c^2x^2 + 1}\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^3 + sqrt(c^2*x^2 + 1)*x^2)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) +
 ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 +
b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a
*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((2*c^5*x^6 - c^3*x^4 - 3
*c*x^2 + (2*c^3*x^4 - c*x^2)*(c^2*x^2 + 1) + 2*(2*c^4*x^5 - c^2*x^3 - x)*sq
rt(c^2*x^2 + 1))/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)
^(3/2) + 2*(a*b*c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x
^2 + 1) + ((b^2*c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2)
+ 2*(b^2*c^8*x^7 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1)
 + (b^2*c^9*x^8 + 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^
6 + 6*a*b*c^5*x^4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^2}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\left(cx\right)^2+a^2+2\left(abc^6x^6+3\,abc^4x^4+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**2/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.451 $\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{x}{(c^2x^2+1)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0920734, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 10.6859, size = 0, normalized size = 0.

$$\int \frac{x}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{x}{(a + b\mathrm{Arcsinh}\,(cx))^2} \left(c^2x^2 + 1\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx^2 + \sqrt{c^2x^2 + 1}x}{\left(abc^4x^3 + abc^2x\right)\left(c^2x^2 + 1\right) + \left(\left(b^2c^4x^3 + b^2c^2x\right)\left(c^2x^2 + 1\right) + \left(b^2c^5x^4 + 2\,b^2c^3x^2 + b^2c\right)\sqrt{c^2x^2 + 1}\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c^2*x^2 + 1)*x)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + (
(b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^
2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b
*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*x^5 + 3*(c^2*x^2 +
1)*c^3*x^3 + c^3*x^3 - 2*c*x + (6*c^4*x^4 + c^2*x^2 - 1)*sqrt(c^2*x^2 + 1))
/((a*b*c^7*x^6 + 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^8*x^7 + 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2*
c^7*x^6 + 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^8*x^7
 + 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^9*x^8
+ 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 + 4*b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))
*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^8 + 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
4 + 4*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\operatorname{arsinh}(cx)^2+a^2+2\left(abc^6x^6+3\,abc^4x^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 +
 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+b\operatorname{asinh}(cx))^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(c^2x^2+1\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.452 $\int \frac{1}{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=61

$$-\frac{4c\text{Unintegrable}\left(\frac{x}{(c^2x^2+1)^3(a+b\sinh^{-1}(cx))},x\right)}{b}-\frac{1}{bc\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(1/(b*c*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))) - (4*c*Unintegrable[x/((1 +
 c^2*x^2)^3*(a + b*ArcSinh[c*x])), x])/b
```

Rubi [A] time = 0.109984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(1/(b*c*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))) - (4*c*Defer[Int][x/((1 + c
^2*x^2)^3*(a + b*ArcSinh[c*x])), x])/b
```

Rubi steps

$$\int \frac{1}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{1}{bc\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}-\frac{(4c)\int \frac{x}{(1+c^2x^2)^3\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

Mathematica [A] time = 2.77519, size = 0, normalized size = 0.

$$\int \frac{1}{(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.182, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\left(c^{2}x^{2}+1\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{c^{2}x^{2}+1}}{\left(abc^{4}x^{3}+abc^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(\left(b^{2}c^{4}x^{3}+b^{2}c^{2}x\right)\left(c^{2}x^{2}+1\right)+\left(b^{2}c^{5}x^{4}+2\,b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1) + ((b^2
*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2 + b^2*c)
*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2*a*b*c^3
*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) - integrate((4*c^4*x^4 + 3*c^2*x^2 + (4*c^
2*x^2 + 1)*(c^2*x^2 + 1) + 4*(2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) - 1)/((a*b
*c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*c^7*x^
7 + 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 + a*b*c*x)*(c^2*x^2 + 1) + ((b^2*c^6*x^6
+ 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^7*x^7 + 3*b^2
*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*(c^2*x^2 + 1) + (b^2*c^8*x^8 + 4*b^2*c^
6*x^6 + 6*b^2*c^4*x^4 + 4*b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(c*x + s
qrt(c^2*x^2 + 1)) + (a*b*c^8*x^8 + 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 + 4*a*b*c^
```

```
2*x^2 + a*b)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\text{arsinh}\,(cx)^2+a^2+2\left(abc^6x^6+3\,abc^4x^4+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 + (
b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2
*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\,\text{asinh}\,(cx)\right)^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.453 $\int \frac{1}{x(1+c^2x^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.131227, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 12.4301, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.617, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+b\operatorname{Arcsinh}\left(cx\right)\right)^{2}}\left(c^{2}x^{2}+1\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx+\sqrt{c^{2}x^{2}+1}}{\left(abc^{4}x^{4}+abc^{2}x^{2}\right)\left(c^{2}x^{2}+1\right)+\left(\left(b^{2}c^{4}x^{4}+b^{2}c^{2}x^{2}\right)\left(c^{2}x^{2}+1\right)+\left(b^{2}c^{5}x^{5}+2\,b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b
^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^
2*c*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a
*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)) - integrate((5*c^5*x^5 + 5*c^3*x^3
 + (5*c^3*x^3 + 2*c*x)*(c^2*x^2 + 1) + (10*c^4*x^4 + 7*c^2*x^2 + 1)*sqrt(c^
2*x^2 + 1))/((a*b*c^7*x^8 + 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c^2*x^2 + 1)^(3/2
) + 2*(a*b*c^8*x^9 + 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2
+ 1) + ((b^2*c^7*x^8 + 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c^2*x^2 + 1)^(3/2) + 2
*(b^2*c^8*x^9 + 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1)
+ (b^2*c^9*x^10 + 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6 + 4*b^2*c^3*x^4 + b^2*c*x^2
)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^10 + 4*a*b*c
^7*x^8 + 6*a*b*c^5*x^6 + 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^6x^7+3\,a^2c^4x^5+3\,a^2c^2x^3+a^2x+\left(b^2c^6x^7+3\,b^2c^4x^5+3\,b^2c^2x^3+b^2x\right)\operatorname{arsinh}(cx)^2+2\left(abc^6x^7+3\,abc^4x^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^7 + 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 + a
^2*x + (b^2*c^6*x^7 + 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 + b^2*x)*arcsinh(c*x)^2
 + 2*(a*b*c^6*x^7 + 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 + a*b*x)*arcsinh(c*x)), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(a+b\operatorname{asinh}(cx))^2\left(c^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2*x), x)
```

3.454 $\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{1}{x^2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.13187, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 11.4198, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.7, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b\text{Arcsinh}(cx))^2} \left(c^2x^2 + 1\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cx + \sqrt{c^2x^2 + 1}}{\left(abc^4x^5 + abc^2x^3\right)\left(c^2x^2 + 1\right) + \left(\left(b^2c^4x^5 + b^2c^2x^3\right)\left(c^2x^2 + 1\right) + \left(b^2c^5x^6 + 2\,b^2c^3x^4 + b^2cx^2\right)\sqrt{c^2x^2 + 1}\right)\log\left(cx + \sqrt{c^2x^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima
")
```

```
[Out] -(c*x + sqrt(c^2*x^2 + 1))/((a*b*c^4*x^5 + a*b*c^2*x^3)*(c^2*x^2 + 1) + ((b
^2*c^4*x^5 + b^2*c^2*x^3)*(c^2*x^2 + 1) + (b^2*c^5*x^6 + 2*b^2*c^3*x^4 + b^
2*c*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^6 + 2
*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c^2*x^2 + 1)) - integrate((6*c^5*x^5 + 7*c^3
*x^3 + 3*(2*c^3*x^3 + c*x)*(c^2*x^2 + 1) + c*x + 2*(6*c^4*x^4 + 5*c^2*x^2 +
 1)*sqrt(c^2*x^2 + 1))/((a*b*c^7*x^9 + 2*a*b*c^5*x^7 + a*b*c^3*x^5)*(c^2*x^
2 + 1)^(3/2) + 2*(a*b*c^8*x^10 + 3*a*b*c^6*x^8 + 3*a*b*c^4*x^6 + a*b*c^2*x^
4)*(c^2*x^2 + 1) + ((b^2*c^7*x^9 + 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(c^2*x^2 +
1)^(3/2) + 2*(b^2*c^8*x^10 + 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6 + b^2*c^2*x^4)*(
c^2*x^2 + 1) + (b^2*c^9*x^11 + 4*b^2*c^7*x^9 + 6*b^2*c^5*x^7 + 4*b^2*c^3*x^
5 + b^2*c*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x
^11 + 4*a*b*c^7*x^9 + 6*a*b*c^5*x^7 + 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c^2*x
^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{c^2x^2+1}}{a^2c^6x^8+3\,a^2c^4x^6+3\,a^2c^2x^4+a^2x^2+\left(b^2c^6x^8+3\,b^2c^4x^6+3\,b^2c^2x^4+b^2x^2\right)\operatorname{arsinh}\left(cx\right)^2+2\left(abc^6x^8+3\,abc^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)/(a^2*c^6*x^8 + 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 + a
^2*x^2 + (b^2*c^6*x^8 + 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 + b^2*x^2)*arcsinh(c*
x)^2 + 2*(a*b*c^6*x^8 + 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 + a*b*x^2)*arcsinh(c*
x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2*x^2), x)
```

3.455 $\int \frac{x^m(1+c^2x^2)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{5/2}x^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [A] time = 0.127831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.962829, size = 0, normalized size = 0.

$$\int \frac{x^m\left(1+c^2x^2\right)^{5/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(5/2))/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.737, size = 0, normalized size = 0.

$$\int \frac{x^m}{(a + b\mathrm{Arcsinh}\,(cx))^2} \left(c^2x^2 + 1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^6x^6 + 3\,c^4x^4 + 3\,c^2x^2 + 1\right)\left(c^2x^2 + 1\right)x^m + \left(c^7x^7 + 3\,c^5x^5 + 3\,c^3x^3 + cx\right)\sqrt{c^2x^2 + 1}x^m}{abc^3x^2 + \sqrt{c^2x^2 + 1}abc^2x + abc + \left(b^2c^3x^2 + \sqrt{c^2x^2 + 1}b^2c^2x + b^2c\right)\log\left(cx + \sqrt{c^2x^2 + 1}\right)} + \int \frac{\left(c^7(m + 6)x^7 + c^5(3\,m + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 + 3*c^4*x^4 + 3*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^7*x^7 + 3*c^
5*x^5 + 3*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2
 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*
c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^7*(m + 6)*x^7 + c^5*(3*m +
 11)*x^5 + c^3*(3*m + 4)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^
8*(m + 6)*x^8 + c^6*(7*m + 30)*x^6 + 3*c^4*(3*m + 8)*x^4 + c^2*(5*m + 6)*x^
2 + m)*(c^2*x^2 + 1)*x^m + (c^9*(m + 6)*x^9 + c^7*(4*m + 19)*x^7 + 3*c^5*(2
*m + 7)*x^5 + c^3*(4*m + 9)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*
c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^
5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 +
b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x
^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4x^4+2\,c^2x^2+1\right)\sqrt{c^2x^2+1}x^m}{b^2\,\text{arsinh}\,(cx)^2+2\,ab\,\text{arsinh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 + 2*c^2*x^2 + 1)*sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^
2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2x^2+1\right)^{\frac{5}{2}}x^m}{\left(b\,\text{arsinh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(5/2)*x^m/(b*arcsinh(c*x) + a)^2, x)
```

3.456 $\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\left(c^2x^2+1\right)^{3/2}x^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [A] time = 0.127819, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]

[Out] Defer[Int][(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.538049, size = 0, normalized size = 0.

$$\int \frac{x^m\left(1+c^2x^2\right)^{3/2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(x^m*(1 + c^2*x^2)^(3/2))/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.673, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a+b\,\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{4}x^{4}+2\,c^{2}x^{2}+1\right)\left(c^{2}x^{2}+1\right)x^{m}+\left(c^{5}x^{5}+2\,c^{3}x^{3}+cx\right)\sqrt{c^{2}x^{2}+1}x^{m}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{\left(c^{5}(m+4)x^{5}+c^{3}(2\,m\right.}{abc^{5}x^{5}+\left(c^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^4*x^4 + 2*c^2*x^2 + 1)*(c^2*x^2 + 1)*x^m + (c^5*x^5 + 2*c^3*x^3 + c*x)
*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c
+ (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^
2 + 1))) + integrate(((c^5*(m + 4)*x^5 + c^3*(2*m + 3)*x^3 + c*(m - 1)*x)*(
c^2*x^2 + 1)^(3/2)*x^m + (2*c^6*(m + 4)*x^6 + c^4*(5*m + 12)*x^4 + 4*c^2*(m
 + 1)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^7*(m + 4)*x^7 + 3*c^5*(m + 3)*x^5 + 3
*c^3*(m + 2)*x^3 + c*(m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*
x^2 + 1)*a*b*c^3*x^3 + 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 +
1)*b^2*c^3*x^3 + 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sq
rt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^
2)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^m}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^2 + 1)^(3/2)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) +
 a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}x^m}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)^(3/2)*x^m/(b*arcsinh(c*x) + a)^2, x)
```

3.457 $\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\sqrt{c^2x^2+1}x^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [A] time = 0.113262, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.118895, size = 0, normalized size = 0.

$$\int \frac{x^m\sqrt{1+c^2x^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(x^m*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.591, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^2}\sqrt{c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^2x^2+1\right)^2x^m+\left(c^3x^3+cx\right)\sqrt{c^2x^2+1}\,x^m}{abc^3x^2+\sqrt{c^2x^2+1}\,abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}\,b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{\left(c^3(m+2)\right.}{abc^5x^5+\left(c^2x^2+1\right)abc^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^2*x^m + (c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^3*x^2
 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b
^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*(m + 2)*x
^3 + c*(m - 1)*x)*(c^2*x^2 + 1)^(3/2)*x^m + (2*c^4*(m + 2)*x^4 + c^2*(3*m +
 2)*x^2 + m)*(c^2*x^2 + 1)*x^m + (c^5*(m + 2)*x^5 + c^3*(2*m + 3)*x^3 + c*(
m + 1)*x)*sqrt(c^2*x^2 + 1)*x^m)/(a*b*c^5*x^5 + (c^2*x^2 + 1)*a*b*c^3*x^3 +
 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 + (c^2*x^2 + 1)*b^2*c^3*x^3 + 2*b^2
*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 + b^2*c^2*x^2)*sqrt(c^2*x^2 + 1))*log(c
*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*sqrt(c^2*x^2 + 1)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^m}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a
^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\sqrt{c^2x^2+1}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m*sqrt(c**2*x**2 + 1)/(a + b*asinh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c^2x^2+1}x^m}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)*x^m/(b*arcsinh(c*x) + a)^2, x)
```

3.458 $\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{m\text{Unintegrable}\left(\frac{x^{m-1}}{a+b\sinh^{-1}(cx)},x\right)}{bc}-\frac{x^m}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(x^m/(b*c*(a + b*ArcSinh[c*x]))) + (m*Unintegrable[x^(-1 + m)/(a + b*ArcSi
nh[c*x]), x])/(b*c)
```

Rubi [A] time = 0.152405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] -(x^m/(b*c*(a + b*ArcSinh[c*x]))) + (m*Defer[Int][x^(-1 + m)/(a + b*ArcSinh
[c*x]), x])/(b*c)
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=-\frac{x^m}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{m\int \frac{x^{-1+m}}{a+b\sinh^{-1}(cx)}\,dx}{bc}$$

Mathematica [A] time = 0.342317, size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^m/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.177, size = 0, normalized size = 0.

$$\int \frac{x^{m}}{\left(a+b\,\mathrm{Arcsinh}\left(cx\right)\right)^{2}}\frac{1}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}x^{m}+\left(c^{3}x^{3}+cx\right)x^{m}}{\left(c^{2}x^{2}+1\right)abc^{2}x+\left(\left(c^{2}x^{2}+1\right)b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}+b^{2}c\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+\left(abc^{3}x^{2}+abc\right)\sqrt{c^{2}x^{2}+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*x^m + (c^3*x^3 + c*x)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x +
 ((c^2*x^2 + 1)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integra
te(((c^3*m*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*m*x^4 + 3*c^2*m*x^
2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*m*x^5 + c^3*(2*m + 1)*x^3 + c*(m + 1)*x
)*x^m)/((c^2*x^2 + 1)^(3/2)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 + a*b*c^2*x^2)*(c^
2*x^2 + 1) + ((c^2*x^2 + 1)^(3/2)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 + b^2*c^2*x^
2)*(c^2*x^2 + 1) + (b^2*c^5*x^5 + 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 + 1
))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^5 + 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^m}{a^2c^2x^2+\left(b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^2x^2+ab\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 + b^2)*arcsinh(c
*x)^2 + a^2 + 2*(a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))**2*sqrt(c**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m}{\sqrt{c^2x^2+1}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)^2), x)
```

3.459 $\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^m}{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.140431, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.632856, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^m/((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{x^m}{(a+b\mathrm{Arcsinh}\,(cx))^2}\left(c^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cxx^m+\sqrt{c^2x^2+1}x^m}{\left(c^2x^2+1\right)abc^2x+\left(\left(c^2x^2+1\right)b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+abc\right)\sqrt{c^2x^2+1}}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((c^2*x^2 + 1)*a*b*c^2*x + ((c^2*x^2 + 1
)*b^2*c^2*x + (b^2*c^3*x^2 + b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m -
2)*x^3 + c*(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 2)*x^4 + c^2*(3*m - 2
)*x^2 + m)*sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 2)*x^5 + c^3*(2*m - 1)*x^3 + c
*(m + 1)*x)*x^m)/((a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(a*b*
c^6*x^6 + 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) + ((b^2*c^5*x^5 + b^2*
c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*c^6*x^6 + 2*b^2*c^4*x^4 + b^2*c^2*x^2
)*(c^2*x^2 + 1) + (b^2*c^7*x^7 + 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 + b^2*c*x)*s
qrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^7*x^7 + 3*a*b*c^5*x
^5 + 3*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2x^2+1}x^m}{a^2c^4x^4+2\,a^2c^2x^2+\left(b^2c^4x^4+2\,b^2c^2x^2+b^2\right)\mathrm{arsinh}\,(cx)^2+a^2+2\left(abc^4x^4+2\,abc^2x^2+ab\right)\mathrm{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 + 2*a^2*c^2*x^2 + (b^2*c^4*x^4
+ 2*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 + 2*a*b*c^2*x^
2 + a*b)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(a + b\operatorname{asinh}(cx))^2 \left(c^2x^2 + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(x**m/((a + b*asinh(c*x))**2*(c**2*x**2 + 1)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c^2x^2 + 1\right)^{\frac{3}{2}} (b\operatorname{arsinh}(cx) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.460 $\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{x^m}{\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.139888, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.84869, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[x^m/((1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.4, size = 0, normalized size = 0.

$$\int \frac{x^m}{(a + b\mathrm{Arcsinh}\,(cx))^2}\left(c^2x^2+1\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{cxx^m + \sqrt{c^2x^2+1}x^m}{\left(abc^4x^3 + abc^2x\right)\left(c^2x^2+1\right) + \left(\left(b^2c^4x^3 + b^2c^2x\right)\left(c^2x^2+1\right) + \left(b^2c^5x^4 + 2\,b^2c^3x^2 + b^2c\right)\sqrt{c^2x^2+1}\right)\log\left(cx + \sqrt{c^2x^2+1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x*x^m + sqrt(c^2*x^2 + 1)*x^m)/((a*b*c^4*x^3 + a*b*c^2*x)*(c^2*x^2 + 1)
 + ((b^2*c^4*x^3 + b^2*c^2*x)*(c^2*x^2 + 1) + (b^2*c^5*x^4 + 2*b^2*c^3*x^2
+ b^2*c)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*x^4 + 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c^2*x^2 + 1)) + integrate(((c^3*(m - 4)*x^3 + c*
(m - 1)*x)*(c^2*x^2 + 1)*x^m + (2*c^4*(m - 4)*x^4 + c^2*(3*m - 4)*x^2 + m)*
sqrt(c^2*x^2 + 1)*x^m + (c^5*(m - 4)*x^5 + c^3*(2*m - 3)*x^3 + c*(m + 1)*x)
*x^m)/((a*b*c^7*x^7 + 2*a*b*c^5*x^5 + a*b*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*
(a*b*c^8*x^8 + 3*a*b*c^6*x^6 + 3*a*b*c^4*x^4 + a*b*c^2*x^2)*(c^2*x^2 + 1) +
 ((b^2*c^7*x^7 + 2*b^2*c^5*x^5 + b^2*c^3*x^3)*(c^2*x^2 + 1)^(3/2) + 2*(b^2*
c^8*x^8 + 3*b^2*c^6*x^6 + 3*b^2*c^4*x^4 + b^2*c^2*x^2)*(c^2*x^2 + 1) + (b^2
*c^9*x^9 + 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 + 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c^
2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^9*x^9 + 4*a*b*c^7*x^7 + 6
*a*b*c^5*x^5 + 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2x^2+1}x^m}{a^2c^6x^6+3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6+3\,b^2c^4x^4+3\,b^2c^2x^2+b^2\right)\operatorname{arsinh}\left(cx\right)^2+a^2+2\left(abc^6x^6+3\,abc^4x^4+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 + 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 + b^2)*arcsinh(c*x)^2 + a^2
 + 2*(a*b*c^6*x^6 + 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 + a*b)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c**2*x**2+1)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(c^2x^2+1\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c^2*x^2+1)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((c^2*x^2 + 1)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.461 $\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

```
[Out] -1/(2*a*ArcSinh[a*x]^2)
```

Rubi [A] time = 0.0354877, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5675}

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

```
[Out] -1/(2*a*ArcSinh[a*x]^2)
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sinh^{-1}(ax)^2}$$

Mathematica [A] time = 0.0073274, size = 13, normalized size = 1.

$$-\frac{1}{2a\sinh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3),x]
```

```
[Out] -1/(2*a*ArcSinh[a*x]^2)
```

Maple [A] time = 0.008, size = 12, normalized size = 0.9

$$-\frac{1}{2\,a\left(\mathrm{Arcsinh}\left(ax\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2/a/arcsinh(a*x)^2
```

Maxima [A] time = 1.32081, size = 15, normalized size = 1.15

$$-\frac{1}{2\,a\,\mathrm{arsinh}\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/(a*arcsinh(a*x)^2)
```

Fricas [B] time = 2.6282, size = 55, normalized size = 4.23

$$-\frac{1}{2\,a\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

Sympy [A] time = 1.25273, size = 12, normalized size = 0.92

$$-\frac{1}{2a\operatorname{asinh}^2(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**3/(a**2*x**2+1)**(1/2),x)
```

```
[Out] -1/(2*a*asinh(a*x)**2)
```

Giac [B] time = 1.26494, size = 31, normalized size = 2.38

$$-\frac{1}{2\,a\log\left(ax+\sqrt{a^2x^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^3/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2/(a*log(a*x + sqrt(a^2*x^2 + 1))^2)
```

3.462 $\int \frac{x^3(d+c^2dx^2)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{3\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}de^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}de^{-\frac{6a}{b}}\operatorname{Erfi}}{16}$$

```
[Out] (-2*d*x^3*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^((2*
a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^4) + (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) + (d*Sqrt[(3*Pi)/2]*Er
fi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((6*a)/b)
)
```

Rubi [A] time = 1.30785, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5777, 5779, 5448, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}de^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}de^{-\frac{6a}{b}}\operatorname{Erfi}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^3*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^((2*
a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^4) + (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (3*d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b)) + (d*Sqrt[(3*Pi)/2]*Er
fi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((6*a)/b)
)
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3\left(d+c^2dx^2\right)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\int\frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc}+\frac{(12cd)\int\frac{x^4\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\,\mathrm{Subst}\left(\int\frac{\cosh^2(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^4}+\frac{(12d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(6d)\,\mathrm{Subst}\left(\int\left(-\frac{1}{8\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^4}+\frac{(12d)\,\mathrm{Su}}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(3d)\,\mathrm{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{8bc^4}+\frac{(3d)\,\mathrm{Subst}\left(\int\frac{\cosh(6x)}{\sqrt{a+bx}}\right.}{8bc^4}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(3d)\,\mathrm{Subst}\left(\int\frac{e^{-6x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^4}-\frac{(3d)\,\mathrm{Subst}\left(\int\frac{e^{-2x}}{\sqrt{a+bx}}\,dx,\right.}{16bc^4}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(3d)\,\mathrm{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^4}-\frac{(3d)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^3\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{de^{\frac{6a}{b}}\sqrt{\frac{3\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}$$

Mathematica [A] time = 0.841677, size = 232, normalized size = 0.91

$$\frac{de^{-\frac{6a}{b}}\left(\sqrt{6}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-3\sqrt{2}e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+3\sqrt{2}e\right.}{32bc^4\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcSinh[c
*x]))/b] - 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
 (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*
x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x]))/b] - 8*E^((6*a)/b)*Sinh
[2*ArcSinh[c*x]]^3))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int x^3 \left(c^2 d x^2 + d\right) (a + b\text{Arcsinh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 + d\right) x^3}{(b\,\text{arsinh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{c^2x^5}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)x^3}{\left(b\operatorname{arsinh}(cx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.463 $\int \frac{x^2(d+c^2dx^2)}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=335

$$\frac{\sqrt{\pi}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}de^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\cdots}\right)}{8b^{3/2}c^3}$$

```
[Out] (-2*d*x^2*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3) - (d*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(16*b^(3/2)*c^3) - (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) + (d*Sqrt[5*Pi]*Erfi[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*a)/b))
```

Rubi [A] time = 1.37859, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5777, 5779, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}de^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{\pi}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\cdots}\right)}{8b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```


```
[Out] (-2*d*x^2*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3) - (d*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/
2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/
Sqrt[b]])/(16*b^(3/2)*c^3) - (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcS
inh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) + (d*Sqrt[5*Pi]*Erfi[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((5*a)/b))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2\left(d+c^2dx^2\right)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(4d)\int\frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc}+\frac{(10cd)\int\frac{x^3\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(4d)\operatorname{Subst}\left(\int\frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^3}+\frac{(10d)\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(4d)\operatorname{Subst}\left(\int\left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^3}+\frac{(10d)\operatorname{Su}}{}$$

$$= -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(5d)\operatorname{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{8bc^3}+\frac{(5d)\operatorname{Subst}\left(\int\frac{\sinh(5x}{\sqrt{a+bx}}\right.}{8bc}$$

$$= -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(5d)\operatorname{Subst}\left(\int\frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{16bc^3}+\frac{(5d)\operatorname{Subst}\left(\int\frac{e^{-3x}}{\sqrt{a+bx}}\,d\right.}{16bc^3}$$

$$= -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{(5d)\operatorname{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^3}+\frac{(5d)\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{2dx^2\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{de^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}-\frac{de^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-$$

Mathematica [A] time = 0.695819, size = 436, normalized size = 1.3

$$de^{-5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(-2e^{\frac{6a}{b}+5\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\operatorname{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{5}e^{5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\operatorname{Gamm}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((5*a)/b) - E^((5*a)/b + 2*ArcSinh[c*x]) + 2*E^((5*a)/b + 4*ArcSinh[
c*x]) + 2*E^((5*a)/b + 6*ArcSinh[c*x]) - E^((5*a)/b + 8*ArcSinh[c*x]) - E^(
(5*a)/b + 10*ArcSinh[c*x]) - 2*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*x])*Sqrt[
-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + Sqrt[3
]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (
-3*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*
ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((8*a)/
b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c
*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c^3*E^(5*(a/b + ArcSinh[c*x]
))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.226, size = 0, normalized size = 0.

$$\int x^2 \left(c^2 d x^2 + d\right) \left(a + b\text{Arcsinh}\left(cx\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 + d\right) x^2}{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{x^2}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{c^2x^4}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*as
inh(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2dx^2+d)x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.464 $\int \frac{x\left(d+c^2dx^2\right)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{\pi}de^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+\frac{\sqrt{\pi}de^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.}{2b^{3/2}c^2}$$

```
[Out] (-2*d*x*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((4*a)/b
)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) + (d*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2*
b^(3/2)*c^2) + (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b
^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rubi [A] time = 0.766615, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5777, 5699, 3312, 3307, 2180, 2204, 2205, 5779, 5448}

$$\frac{\sqrt{\pi}de^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}+\frac{\sqrt{\pi}de^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b}}{}\right.}{2b^{3/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((4*a)/b
)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) + (d*
E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2*
b^(3/2)*c^2) + (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b
^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
```

```
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(d+c^2dx^2\right)}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\int \frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(8cd)\int \frac{x^2\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{(8d)\,\mathrm{Subst}\left(\int \frac{\cosh^2(
}{\sqrt{}}\right.}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{(8d)\,\mathrm{Sub}}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx,\right.}{bc^2}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2bc^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \mathrm{s}\right.}{2bc^2}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\,\mathrm{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c^2} + \frac{d\,\mathrm{Subst}\left(\int e^{\frac{2a}{b}-}\right.}{}$$

$$= -\frac{2dx\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2} +$$

Mathematica [A] time = 0.44492, size = 227, normalized size = 0.96

$$\frac{de^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2}, -\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right) + \sqrt{2}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2}, -\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right) - e^{\frac{4a}{b}}\left(\sqrt{2}e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d + c^2*d*x^2))/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b]
 + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a +
```

```
b*ArcSinh[c*x]))/b] - E^((4*a)/b)*(Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c
*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSin
h[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + 2*Sinh[2*ArcSinh[c*x]] + S
inh[4*ArcSinh[c*x]])))/(4*b*c^2*E^((4*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int x\left(c^2dx^2+d\right)(a+b\mathrm{Arcsinh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)x}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{x}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{c^2x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh(c*
x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c^2dx^2+d)x}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.465 $\int \frac{d+c^2dx^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{3\sqrt{\pi}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{\sqrt{3\pi}de^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (d*E^((3*a)/b
)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c)
 + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c*E^(a/
b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^
(3/2)*c*E^((3*a)/b))
```

Rubi [A] time = 0.52502, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{\sqrt{3\pi}de^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (3*d*E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (d*E^((3*a)/b
)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c)
 + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c*E^(a/
b)) + (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^
(3/2)*c*E^((3*a)/b))
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
```

```
1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{d+c^2dx^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6cd)\int \frac{x\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(6d)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2bc} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,d\right.}{2bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(3d)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{4bc} - \frac{(3d)\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,\right.}{4bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(3d)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{2b^2c} - \frac{(3d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} +$$

Mathematica [A] time = 0.872506, size = 295, normalized size = 1.29

$$\frac{de^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(3e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}e^{3\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c^2*d*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d*(-E^((3*a)/b) - 3*E^((3*a)/b + 2*ArcSinh[c*x]) - 3*E^((3*a)/b + 4*ArcSin
h[c*x]) - E^((3*a)/b + 6*ArcSinh[c*x]) + 3*E^((4*a)/b + 3*ArcSinh[c*x])*Sqr
```

```
t[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh
[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))
/b] + 3*E^((2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[
1/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt
[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(4*b*c*E^(3*(
a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \left(c^{2}d x^{2}+d\right) \left(a +b \mathrm{Arcsinh}\left(c x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^{2}dx^{2}+d}{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{c^2x^2}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{1}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*
asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh
(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2dx^2+d}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.466 $\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{2d\,\text{Unintegrable}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}},x\right)}{bc}+\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{}{bc}$$

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((2*a)/b
)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/b^(3/2) + (d*
Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2
*a)/b)) - (2*d*Unintegrable[1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x
]]), x])/(b*c)
```

Rubi [A] time = 0.796411, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d*(1 + c^2*x^2)^(3/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d*E^((2*a)/b
)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/b^(3/2) + (d*
Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2
*a)/b)) - (2*d*Defer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]
), x])/(b*c)
```

Rubi steps

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(2d)\int \frac{\sqrt{1+c^2x^2}}{x^2\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{(4cd)\int \frac{\sqrt{1+c^2x^2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{(2d)\int\Bigg(\frac{c^2}{\sqrt{1+c^2x^2}\sqrt{a+b}}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(4d)\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{(2d)\int}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{(2d)\int \frac{1}{x^2\sqrt{1+c^2x^2}\sqrt{a+}}}{bc}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x,\right.}{b}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2} + \frac{(2d)\,\mathrm{Subst}}{}$$

$$= -\frac{2d\left(1+c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx}}{\sqrt{b}}\right.}{b^{3/2}}$$

Mathematica [A] time = 4.07211, size = 0, normalized size = 0.

$$\int \frac{d+c^2dx^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Integrate[(d + c^2*d*x^2)/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A] time = 0.184, size = 0, normalized size = 0.

$$\int \frac{c^2 d x^2 + d}{x} (a + b\mathrm{Arcsinh}(cx))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2 d x^2 + d}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{c^2 x^2}{ax\sqrt{a + b \operatorname{asinh}(cx)} + bx\sqrt{a + b \operatorname{asinh}(cx)} \operatorname{asinh}(cx)} dx + \int \frac{1}{ax\sqrt{a + b \operatorname{asinh}(cx)} + bx\sqrt{a + b \operatorname{asinh}(cx)} \operatorname{asinh}(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d*(Integral(c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asinh(c*
x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2dx^2 + d}{(b\,\mathrm{arsinh}\,(cx) + a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

3.467 $\int \frac{x^3\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=474

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\operatorname{Erf}}{32}$$

```
[Out] (-2*d^2*x^3*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d^2*E^((
4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4
) - (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((6*a)/b)*Sqrt[(3*
Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E
^((4*a)/b)) - (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) + (d^2*Sqrt[(3*
Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^
((6*a)/b))
```

Rubi [A] time = 1.52709, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5777, 5779, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}d^2e^{\frac{8a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\operatorname{Erf}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d^2*E^((
4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4
) - (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[Pi/2]*Erf[(2*Sqrt[2]*Sqrt[
```

```
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((6*a)/b)*Sqrt[(3*
Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E
^((4*a)/b)) - (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi/2]*Erfi[(2*Sqrt[2]*Sqrt[
a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((8*a)/b)) + (d^2*Sqrt[(3*
Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^
((6*a)/b))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^3\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(6d^2\right)\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(16cd^2\right)\int \frac{x^4\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(6d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^4} + \frac{\left(16d^2\right)\text{Subst}}{}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(6d^2\right)\text{Subst}\left(\int\left(-\frac{1}{16\sqrt{a+bx}} - \frac{\cosh(2x)}{32\sqrt{a+bx}} + \frac{\cosh(4x)}{16\sqrt{a+bx}} + \frac{\cosh(6x)}{32\sqrt{a+bx}}\right)dx, x, \text{s}\right.}{bc^4}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(8x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc^4} - \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\right.}{16bc^4}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-8x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc^4} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{8x}}{\sqrt{a+bx}}\,dx, x, \text{s}\right.}{16bc^4}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c^4} + \frac{d^2\,\text{Subst}\left(\int e^{-\frac{8a}{b}}\right.}{}$$

$$= -\frac{2d^2x^3\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} - \frac{3d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} -$$

Mathematica [A] time = 0.982722, size = 462, normalized size = 0.97

$$\frac{d^2e^{-\frac{8a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2}, -\frac{8\left(a+b\sinh^{-1}(cx)\right)}{b}\right) + \sqrt{6}e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2}, -\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right) - 2e^{\frac{4a}{b}}\sqrt{\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-8*(a + b*ArcSinh
[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2,
```

```
(-6*(a + b*ArcSinh[c*x]))/b] - 2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b
)]*Gamma[1/2, (-4*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((
a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 3*Sqrt[2]
*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/
b] + 2*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c
*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a +
 b*ArcSinh[c*x]))/b] - Sqrt[2]*E^((16*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, (8*(a + b*ArcSinh[c*x]))/b] + 6*E^((8*a)/b)*Sinh[2*ArcSinh[c*x]] + 2*E
^((8*a)/b)*Sinh[4*ArcSinh[c*x]] - 2*E^((8*a)/b)*Sinh[6*ArcSinh[c*x]] - E^((
8*a)/b)*Sinh[8*ArcSinh[c*x]]))/(64*b*c^4*E^((8*a)/b)*Sqrt[a + b*ArcSinh[c*x
]])
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int x^{3} \left(c^{2} d x^{2}+d\right)^{2} \left(a+b \operatorname{Arcsinh}\left(c x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^{2} d x^{2}+d\right)^{2} x^{3}}{\left(b \operatorname{arsinh}\left(c x\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,dx+\int \frac{2c^2x^5}{a\sqrt{a+b\,\mathrm{asinh}\,(cx)}+b\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,\mathrm{asinh}\,(cx)}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**5/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2x^3}{\left(b\,\mathrm{arsinh}\,(cx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^3/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.468 $\int \frac{x^2(d+c^2dx^2)^2}{(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=457

$$\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{7\pi}d^2e^{\frac{7a}{b}}\text{Erf}\left(\right.}{64}$$

```
[Out] (-2*d^2*x^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (5*d^2*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3) - (d
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (d^2*E^((7*a)/b)*Sqrt[7*Pi]*Erf[(
Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (5*d^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^(a/b)) + (d^2
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c
^3*E^((3*a)/b)) + (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi[(Sqrt[7]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rubi [A] time = 1.77982, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5777, 5779, 5448, 3308, 2180, 2204, 2205}

$$\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{\sqrt{7\pi}d^2e^{\frac{7a}{b}}\text{Erf}\left(\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (5*d^2*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3) - (d
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*Ar
cSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (d^2*E^((7*a)/b)*Sqrt[7*Pi]*Erf[(
```

```
Sqrt[7]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (5*d^2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*b^(3/2)*c^3*E^(a/b)) + (d^2
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c
^3*E^((3*a)/b)) + (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])
/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqrt[7*Pi]*Erfi[(Sqrt[7]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((7*a)/b))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(4d^2\right)\int \frac{x\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(14cd^2\right)\int \frac{x^3\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc^3} + \frac{\left(14d^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(4d^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx)\right)}{bc^3}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{32bc^3} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{\sinh(}{\sqrt{a+}}\right.}{32b}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{e^{-7x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{64bc^3} - \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\right.}{64bc^3}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(7d^2\right)\text{Subst}\left(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{32b^2c^3} - \frac{\left(7d^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2d^2x^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} - \frac{d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}$$

Mathematica [A] time = 1.55672, size = 577, normalized size = 1.26

$$-\frac{d^2e^{-7\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(5e^{\frac{8a}{b}+7\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right) - \sqrt{7}e^{7\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamm}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -(d^2*(E^((7*a)/b) + 3*E^((7*a)/b + 2*ArcSinh[c*x]) + E^((7*a)/b + 4*ArcSin
h[c*x]) - 5*E^((7*a)/b + 6*ArcSinh[c*x]) - 5*E^((7*a)/b + 8*ArcSinh[c*x]) +
```

```
E^((7*a)/b + 10*ArcSinh[c*x]) + 3*E^((7*a)/b + 12*ArcSinh[c*x]) + E^((7*a)
/b + 14*ArcSinh[c*x]) + 5*E^((8*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c
*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] - Sqrt[7]*E^(7*ArcSinh[c*x])*Sqrt[-((a
+ b*ArcSinh[c*x])/b)]*Gamma[1/2, (-7*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E
^((2*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*
(a + b*ArcSinh[c*x]))/b] - Sqrt[3]*E^((4*a)/b + 7*ArcSinh[c*x])*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 5*E^((6*a)/b
 + 7*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c*x])/b)] - Sqrt[3]*E^((10*a)/b + 7*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*
x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b + 7*ArcS
inh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b] -
 Sqrt[7]*E^(7*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2,
 (7*(a + b*ArcSinh[c*x]))/b]))/(64*b*c^3*E^(7*(a/b + ArcSinh[c*x]))*Sqrt[a
+ b*ArcSinh[c*x]])
```

Maple [F] time = 0.366, size = 0, normalized size = 0.

$$\int x^2 \left(c^2 d x^2 + d\right)^2 (a + b\operatorname{Arcsinh}(cx))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 + d\right)^2 x^2}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{x^2}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int\frac{2c^2x^4}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*as
inh(c*x)), x) + Integral(2*c**2*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a +
 b*asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*asinh(c*x
)) + b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2dx^2+d\right)^2x^2}{\left(b\operatorname{arsinh}(cx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.469 $\int \frac{x\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=358

$$\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\sqrt{\pi}d^2e^{-\frac{4a}{b}}\operatorname{Erfi}\cdots$$

```
[Out] (-2*d^2*x*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) +
 (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (5*d^2*Sqrt[P
i/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((
2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rubi [A] time = 1.33021, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5777, 5699, 3312, 3307, 2180, 2204, 2205, 5779, 5448}

$$\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}d^2e^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\sqrt{\pi}d^2e^{-\frac{4a}{b}}\operatorname{Erfi}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2) +
 (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a
 + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt
[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (5*d^2*Sqrt[P
i/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((
```

```
2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(2d^2\right)\int \frac{\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(12cd^2\right)\int \frac{x^2\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc^2} + \frac{\left(12d^2\right)\text{Subst}\left(\int \frac{\cosh}{}\right.}{}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(2d^2\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx,x,\sinh^{-1}(cx)\right)}{bc^2} +$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4bc^2} - \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\right.}{8bc^2}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{8bc^2} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx,x,\text{s}\right.}{8bc^2}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{4b^2c^2} + \frac{d^2\,\text{Subst}\left(\int e^{-\frac{4a}{b}}\right.}{}$$

$$= -\frac{2d^2x\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}$$

Mathematica [A] time = 0.663044, size = 351, normalized size = 0.98

$$-\frac{d^2e^{-\frac{6a}{b}}\left(-\sqrt{6}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-8e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-5\sqrt{2}e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d + c^2*d*x^2)^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -(d^2*(-(Sqrt[6]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcS
inh[c*x]))/b]) - 8*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (
```

```
-4*(a + b*ArcSinh[c*x]))/b] - 5*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c
*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] + 5*Sqrt[2]*E^((8*a)/b)*Sq
rt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b] + 8*E^((10*a)
/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c*x]))/b] + Sqrt[
6]*E^((12*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (6*(a + b*ArcSinh[c*x])
)/b] + 10*E^((6*a)/b)*Sinh[2*ArcSinh[c*x]] + 8*E^((6*a)/b)*Sinh[4*ArcSinh[c
*x]] + 2*E^((6*a)/b)*Sinh[6*ArcSinh[c*x]]))/(32*b*c^2*E^((6*a)/b)*Sqrt[a +
b*ArcSinh[c*x]])
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int x \left(c^2 d x^2 + d\right)^2 (a + b\text{Arcsinh}(cx))^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 + d\right)^2 x}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{x}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,dx+\int \frac{2c^2x^3}{a\sqrt{a+b\operatorname{asinh}(cx)}+b\sqrt{a+b\operatorname{asinh}(cx)}\operatorname{asinh}(cx)}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c*x))*asinh
(c*x)), x) + Integral(2*c**2*x**3/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*
asinh(c*x))*asinh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2x}{\left(b\operatorname{arsinh}(cx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2*x/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.470 $\int \frac{\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{Er}}{8}$$

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (5*d^2*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c) - (5*d^2*E^
((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b
^(3/2)*c) - (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]
]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) + (5*d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2*Sqrt[5*Pi]*E
rfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((5*a)/b))
```

Rubi [A] time = 0.728844, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{5\sqrt{\pi}d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}-\frac{\sqrt{5\pi}d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}d^2e^{-\frac{a}{b}}\text{Er}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (5*d^2*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c) - (5*d^2*E^
((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b
^(3/2)*c) - (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]
]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) + (5*d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2*Sqrt[5*Pi]*E
rfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((5*a)/b))
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
```

```
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(10cd^2\right)\int \frac{x\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(10d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(10d^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(cx\right.}{bc}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{8bc} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sinh}{\sqrt{a}}\right.}{4}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{16bc} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+b}}\right.}{16b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(5d^2\right)\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{8b^2c} + \frac{\left(5d^2\right)\text{Subs}}{}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c} - \frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right.}{16b^{3/2}c}$$

Mathematica [A] time = 2.02187, size = 440, normalized size = 1.27

$$\frac{d^2e^{-5\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(10e^{\frac{6a}{b}+5\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2}, \frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{5}e^{5\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gam}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c^2*d*x^2)^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (d^2*(-E^((5*a)/b) - 5*E^((5*a)/b + 2*ArcSinh[c*x]) - 10*E^((5*a)/b + 4*Arc
Sinh[c*x]) - 10*E^((5*a)/b + 6*ArcSinh[c*x]) - 5*E^((5*a)/b + 8*ArcSinh[c*x
]) - E^((5*a)/b + 10*ArcSinh[c*x]) + 10*E^((6*a)/b + 5*ArcSinh[c*x])*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[5]*E^(5*ArcSinh[c*
x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b]
 + 5*Sqrt[3]*E^((2*a)/b + 5*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*G
amma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 10*E^((4*a)/b + 5*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 5*Sqr
t[3]*E^((8*a)/b + 5*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a
 + b*ArcSinh[c*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c*x]))*Sqrt[a/b +
ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b]))/(16*b*c*E^(5*(a/b +
ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \left(c^2 d x^2 + d\right)^2 \left(a + b \mathrm{Arcsinh}\left(cx\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2 d x^2 + d\right)^2}{\left(b \operatorname{arsinh}\left(cx\right) + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{2c^2x^2}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}\,dx+\int \frac{c^4x^4}{a\sqrt{a+b\,\text{asinh}\,(cx)}+b\sqrt{a+b\,\text{asinh}\,(cx)}\,\text{asinh}\,(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*sqrt(a + b*asinh(c*x)) + b*sqrt(a + b*asinh(c
*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asinh(c*x)) + b*sqr
t(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*sqrt(a + b*asinh(c*x))
+ b*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2}{\left(b\,\text{arsinh}\,(cx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.471 $\int \frac{(d+c^2dx^2)^2}{x(a+b\sinh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=374

$$-\frac{2d^2\text{Unintegrable}\left(\frac{1}{x^2\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}},x\right)}{bc}+\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}+\frac{\sqrt{2\pi}d^2e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-$$

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)) - (d^
2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
2*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*E^((4*a)/b)) - (d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) + (d^2*Sqrt[2*Pi]*Erfi[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2*a)/b)) - (2*d^2*Un
integrable[1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]), x])/(b*c)
```

Rubi [A] time = 1.44103, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d+c^2dx^2)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*(1 + c^2*x^2)^(5/2))/(b*c*x*Sqrt[a + b*ArcSinh[c*x]]) + (d^2*E^((4*
a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)) - (d^
2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(
2*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*
x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*E^((4*a)/b)) - (d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*
ArcSinh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) + (d^2*Sqrt[2*Pi]*Erfi[(Sq
rt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*E^((2*a)/b)) - (2*d^2*De
```

```
fer[Int][1/(x^2*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{\left(2d^2\right)\int \frac{\left(1+c^2x^2\right)^{3/2}}{x^2\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{bc} + \frac{\left(8cd^2\right)\int \frac{\left(1+c^2x^2\right)^{3/2}}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{b} - \frac{\left(2d^2\right)\int\left(\frac{}{\sqrt{1+c^2x^2}} \right.}{}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int\left(\frac{3}{8\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c\right.}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\sinh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c\right.}{b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\sinh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{2b}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2} + \frac{d^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c}{\sqrt{b}}\right.}{b^{3/2}}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c}{\sqrt{b}}\right.}{b^{3/2}}$$

$$= -\frac{2d^2\left(1+c^2x^2\right)^{5/2}}{bcx\sqrt{a+b\sinh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(cx}{\sqrt{b}}\right.}{2b^{3/2}}$$

Mathematica [A] time = 3.12948, size = 0, normalized size = 0.

$$\int \frac{\left(d+c^2dx^2\right)^2}{x\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Integrate[(d + c^2*d*x^2)^2/(x*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2}{x}(a+b\mathrm{Arcsinh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^2}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$d^2 \left(\int \frac{2c^2x^2}{ax\sqrt{a + b \operatorname{asinh}(cx)} + bx\sqrt{a + b \operatorname{asinh}(cx)} \operatorname{asinh}(cx)} \, dx + \int \frac{c^4x^4}{ax\sqrt{a + b \operatorname{asinh}(cx)} + bx\sqrt{a + b \operatorname{asinh}(cx)} \operatorname{asinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**2/x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(2*c**2*x**2/(a*x*sqrt(a + b*asinh(c*x)) + b*x*sqrt(a + b*asi
nh(c*x))*asinh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*asinh(c*x)) +
 b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*as
inh(c*x)) + b*x*sqrt(a + b*asinh(c*x))*asinh(c*x)), x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^2}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}} x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^2/x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^2/((b*arcsinh(c*x) + a)^(3/2)*x), x)
```

3.472 $\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=319

$$\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-$$

```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/8 + (x*(c + a^2*c*x^2)^(3/2)
*Sqrt[ArcSinh[a*x]])/4 + (c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(4*a*Sq
rt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]
])/(256*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi]*Sqrt[c + a^2*
c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*
x^2])
```

Rubi [A] time = 0.408109, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5684, 5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5779}

$$\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```


```
[Out] (3*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/8 + (x*(c + a^2*c*x^2)^(3/2)
*Sqrt[ArcSinh[a*x]])/4 + (c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(4*a*Sq
rt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]
])/(256*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi]*Sqrt[c + a^2*
c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (c*Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*
x^2])
```


Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx - \frac{\left(ac\sqrt{c+a^2\right.}{8}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{\left(3c\sqrt{c+a^2cx^2}\right)\int}{8\sqrt{1+a^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\sinh^{-}}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\sinh^{-}}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\sinh^{-}}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\sinh^{-}}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\sinh^{-}}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{3}{8}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}+\frac{c\sqrt{c+a^2cx^2}\sinh^{-}}{4a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.189342, size = 142, normalized size = 0.45

$$\frac{c\sqrt{a^2cx^2+c}\left(-\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-4\sinh^{-1}(ax)\right)-8\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{3}{2},-2\sinh^{-1}(ax)\right)+\sqrt{\sinh^{-}}\right.}{128a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(-(Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])
- 8*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[
a*x]]*(32*ArcSinh[a*x]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] - Gamma
```

```
[3/2, 4*ArcSinh[a*x]])))/(128*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\text{Arcsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\text{arsinh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x)), x)
```

3.473 $\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx$

Optimal. Leaf size=186

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2$$

```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh
[a*x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[
Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (Sqrt[Pi/2]*Sqrt[c
+ a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.168029, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2cx^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]
```


```
[Out] (x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh
[a*x]^(3/2))/(3*a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[
Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (Sqrt[Pi/2]*Sqrt[c
+ a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```


Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}}{4\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(x)\,\text{si}}{\sqrt{x}}\right.}{4a\sqrt{1+a}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\right.}{4a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} - \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\right.}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,\right.}{16a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx\right.}{8a\sqrt{1+a^2x^2}}$$

$$= \frac{1}{2}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh\right.}{16a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0999308, size = 104, normalized size = 0.56

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(-3\sqrt{2}\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2},2\sinh^{-1}(ax)\right)-3\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{3}{2},-2\sinh^{-1}(ax)\right)+16\sinh\right.}{48a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]],x]

[Out] (Sqrt[c*(1 + a^2*x^2)]*(16*ArcSinh[a*x]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*G
amma[3/2, -2*ArcSinh[a*x]] - 3*Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, 2*ArcS
inh[a*x]]))/(48*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\sqrt{\operatorname{Arcsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\left(a^2x^2 + 1\right)}\sqrt{\operatorname{asinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\sqrt{\operatorname{arsinh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x)), x)
```

3.474 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0771497, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\,\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\sqrt{\sinh^{-1}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}} = \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0378993, size = 42, normalized size = 1.

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}{3a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(3*a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.037, size = 36, normalized size = 0.9

$$\frac{2}{3a}\left(\mathrm{Arcsinh}\left(ax\right)\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}\frac{1}{\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/3*arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{arsinh}\left(ax\right)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.475 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{x\sqrt{\sinh^{-1}(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\,\text{Unintegrable}\left(\frac{x}{(a^2x^2+1)\sqrt{\sinh^{-1}(ax)}},x\right)}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Unint
egrable[x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0963678, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(c*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer
[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{x\sqrt{\sinh^{-1}(ax)}}{c\sqrt{c+a^2cx^2}}-\frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{(1+a^2x^2)\sqrt{\sinh^{-1}(ax)}}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.586713, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.205, size = 0, normalized size = 0.

$$\int \sqrt{\operatorname{Arcsinh}(ax)}\left(a^2cx^2+c\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{asinh}(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(asinh(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

3.476 $\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2x^2+1}\,\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)^2\sqrt{\sinh^{-1}(ax)}},x\right)}{6c^2\sqrt{a^2cx^2+c}}-\frac{a\sqrt{a^2x^2+1}\,\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)\sqrt{\sinh^{-1}(ax)}},x\right)}{3c^2\sqrt{a^2cx^2+c}}+\frac{2x\sqrt{\sinh^{-1}(a}}{3c^2\sqrt{a^2cx^2+}}$$

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[a*x]
])/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Unintegrable[x/((1 +
a^2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(6*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1
 + a^2*x^2]*Unintegrable[x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(3*c^2*S
qrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.198965, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[a*x]
])/(3*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^
2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(6*c^2*Sqrt[c + a^2*c*x^2]) - (a*Sqrt[1 +
 a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)*Sqrt[ArcSinh[a*x]]), x])/(3*c^2*Sqrt[
c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{x\sqrt{\sinh^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx}{3c} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}}$$

$$= \frac{x\sqrt{\sinh^{-1}(ax)}}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{2x\sqrt{\sinh^{-1}(ax)}}{3c^2\sqrt{c+a^2cx^2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{6c^2\sqrt{c+a^2cx^2}} - \frac{\left(a\sqrt{1+a^2x^2}\right)\int \frac{}{(1+}}{3c^2\sqrt{c+}}$$

Mathematica [A] time = 1.36561, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh^{-1}(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[a*x]]/(c + a^2*c*x^2)^(5/2), x]
```

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int \sqrt{\mathrm{Arcsinh}\,(ax)}\left(a^2cx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{arsinh}\,(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}(ax)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

3.477 $\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=449

$$\frac{3\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right.}{2048a\sqrt{a^2x^2+1}}$$

```
[Out] (-27*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(256*a*Sqrt[1 + a^2*x^2]) -
(9*a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*Sqrt[1 + a^2*x^2]) -
 (3*c*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*a) +
(3*c*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/8 + (x*(c + a^2*c*x^2)^(3/2)
*ArcSinh[a*x]^(3/2))/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/(20*a
*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[
a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqr
t[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (
3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*
Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.564976, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5684, 5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205, 5717, 5699}

$$\frac{3\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{2048a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right.}{2048a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-27*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(256*a*Sqrt[1 + a^2*x^2]) -
(9*a*c*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*Sqrt[1 + a^2*x^2]) -
 (3*c*(1 + a^2*x^2)^(3/2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(32*a) +
(3*c*x*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/8 + (x*(c + a^2*c*x^2)^(3/2)
*ArcSinh[a*x]^(3/2))/4 + (3*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2))/(20*a
*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[
a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c*Sqrt[Pi]*Sqr
t[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(2048*a*Sqrt[1 + a^2*x^2]) + (
```

```
3*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*
Sqrt[1 + a^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}\,dx=\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}\,dx-\frac{\left(3ac\sqrt{c+\right.}{}$$

$$=-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}+\frac{1}{4}x\left(c\right.$$

$$=-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32a}+\frac{3}{8}cx$$

$$=-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32a}+\frac{3}{8}cx$$

$$=\frac{9c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}\sqrt{c}}{32}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}}{}$$

$$=-\frac{27c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{256a\sqrt{1+a^2x^2}}-\frac{9acx^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{32\sqrt{1+a^2x^2}}-\frac{3c\left(1+a^2x^2\right)^{3/2}}{}$$

Mathematica [A] time = 0.319373, size = 186, normalized size = 0.41

$$\frac{c\sqrt{a^2cx^2+c}\left(-5\sqrt{\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{5}{2},4\sinh^{-1}(ax)\right)+5\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{5}{2},-4\sinh^{-1}(ax)\right)+60\sqrt{2\pi}\sqrt{\sinh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(384*ArcSinh[a*x]^3 - 480*ArcSinh[a*x]*Cosh[2*ArcSin
h[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]]
+ 60*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 5*Sqr
t[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]] - 5*Sqrt[ArcSinh[a*x]]*Gamma[5
/2, 4*ArcSinh[a*x]] + 640*ArcSinh[a*x]^2*Sinh[2*ArcSinh[a*x]]))/(2560*a*Sqr
t[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \left(\mathrm{Arcsinh}\left(ax\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}(ax)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2), x)
```

3.478 $\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=271

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2c}$$

```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(16*a*Sqrt[1 + a^2*x^2]) - (3*a
*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*Sqrt[1 + a^2*x^2]) + (x*Sqr
t[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^
(5/2))/(5*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c +
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.272977, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```


```
[Out] (-3*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(16*a*Sqrt[1 + a^2*x^2]) - (3*a
*x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*Sqrt[1 + a^2*x^2]) + (x*Sqr
t[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^
(5/2))/(5*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*Sqrt[Pi/2]*Sqrt[c +
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```


Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
```

```
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\int\frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(3a\sqrt{c+a^2cx^2}\right)\int x\sqrt{\sinh}}{4\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)}{5a\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)}{5a\sqrt{1+a^2x^2}}$$

$$= -\frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)}{5a\sqrt{1+a^2x^2}}$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax$$

$$= -\frac{3\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{16a\sqrt{1+a^2x^2}} - \frac{3ax^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax$$

Mathematica [A] time = 0.271102, size = 126, normalized size = 0.46

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(15\sqrt{2\pi}\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+15\sqrt{2\pi}\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+8\sqrt{\sinh^{-1}(ax)}\left(4\sinh^{-1}(ax)\left(4\sinh^{-1}(\right.\right.\right.}{640a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 15*
Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(-15*Cos
h[2*ArcSinh[a*x]] + 4*ArcSinh[a*x]*(4*ArcSinh[a*x] + 5*Sinh[2*ArcSinh[a*x]]
))))/(640*a*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (\mathrm{Arcsinh}\,(ax))^{\frac{3}{2}}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,\mathrm{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\,\operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2), x)
```

3.479 $\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0742584, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0415843, size = 42, normalized size = 1.

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{5/2}}{5a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(5/2))/(5*a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.039, size = 36, normalized size = 0.9

$$\frac{2}{5\,a}\left(\mathrm{Arcsinh}\left(ax\right)\right)^{\frac{5}{2}}\sqrt{a^2x^2+1}\frac{1}{\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/5*arcsinh(a*x)^(5/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.480 $\int \frac{\sinh^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{x\sinh^{-1}(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}-\frac{3a\sqrt{a^2x^2+1}\text{Unintegrable}\left(\frac{x\sqrt{\sinh^{-1}(ax)}}{a^2x^2+1},x\right)}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] (x*ArcSinh[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*a*Sqrt[1 + a^2*x^2]*Uni
ntegrable[(x*Sqrt[ArcSinh[a*x]])/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] time = 0.0931448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*Sqrt[ArcSinh[a*x]])/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{x\sinh^{-1}(ax)^{3/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(3a\sqrt{1+a^2x^2}\right)\int \frac{x\sqrt{\sinh^{-1}(ax)}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.636408, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^{3/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.172, size = 0, normalized size = 0.

$$\int (\mathrm{Arcsinh}\,(ax))^{\frac{3}{2}} \left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.481 $\int \left(c+a^2cx^2\right)^{3/2} \sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=514

$$\frac{15\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right.}{16384a\sqrt{a^2x^2+1}}$$

```
[Out] (225*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/512 + (15*c*x*(1 + a^2*x^2
)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/256 - (45*c*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(256*a*Sqrt[1 + a^2*x^2]) - (15*a*c*x^2*Sqrt[c + a^2*c*x
^2]*ArcSinh[a*x]^(3/2))/(32*Sqrt[1 + a^2*x^2]) - (5*c*(1 + a^2*x^2)^(3/2)*S
qrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(32*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(5/2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2))/4 + (3
*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(28*a*Sqrt[1 + a^2*x^2]) + (15*c
*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 +
a^2*x^2]) + (15*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[
2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi/2]*Sqrt[
c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.764722, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {5684, 5682, 5675, 5663, 5758, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5717, 5779}

$$\frac{15\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{16384a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\pi}c\sqrt{a^2cx^2+c}\operatorname{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right.}{16384a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/512 + (15*c*x*(1 + a^2*x^2
)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/256 - (45*c*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(256*a*Sqrt[1 + a^2*x^2]) - (15*a*c*x^2*Sqrt[c + a^2*c*x
^2]*ArcSinh[a*x]^(3/2))/(32*Sqrt[1 + a^2*x^2]) - (5*c*(1 + a^2*x^2)^(3/2)*S
qrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))/(32*a) + (3*c*x*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(5/2))/8 + (x*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2))/4 + (3
*c*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(28*a*Sqrt[1 + a^2*x^2]) + (15*c
```

```
*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 +
a^2*x^2]) + (15*c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(256*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[
2*Sqrt[ArcSinh[a*x]]])/(16384*a*Sqrt[1 + a^2*x^2]) - (15*c*Sqrt[Pi/2]*Sqrt[
c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
```

```
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}+\frac{1}{4}(3c)\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}\,dx-\frac{\left(5ac\sqrt{c+\right.}{}$$

$$= -\frac{5c\left(1+a^2x^2\right)^{3/2}\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{32a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}+\frac{1}{4}x\left(c\right.$$

$$= \frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{15acx^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{32\sqrt{1+a^2x^2}}-\frac{5c\left(\right.}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{15ac}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{}}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{}}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{}}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{}}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{}}{}$$

$$= \frac{225}{512}cx\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}+\frac{15}{256}cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}-\frac{45c\sqrt{}}{}$$

Mathematica [A] time = 0.344886, size = 201, normalized size = 0.39

$$\frac{c\sqrt{a^2cx^2+c}\left(-7\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},4\sinh^{-1}(ax)\right)-7\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{7}{2},-4\sinh^{-1}(ax)\right)+420\sqrt{2\pi}\sqrt{\sin\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(1536*ArcSinh[a*x]^4 - 4480*ArcSinh[a*x]^2*Cosh[2*Ar
cSinh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] -
 7*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]] - 7*Sqrt[ArcSinh[a*x]]*G
amma[7/2, 4*ArcSinh[a*x]] + 3360*ArcSinh[a*x]*Sinh[2*ArcSinh[a*x]] + 3584*A
rcSinh[a*x]^3*Sinh[2*ArcSinh[a*x]]))/(14336*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSin
h[a*x]])
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \left(\operatorname{Arcsinh}\left(a x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)*asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2cx^2 + c\right)^{\frac{3}{2}} \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2), x)
```

3.482 $\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=298

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a$$

```
[Out] (15*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/32 - (5*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(16*a*Sqrt[1 + a^2*x^2]) - (5*a*x^2*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(3/2))/(8*Sqrt[1 + a^2*x^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[
a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*
x^2]) + (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])
/(256*a*Sqrt[1 + a^2*x^2]) - (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.300659, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5682, 5675, 5663, 5758, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{256a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2x^2+1}}+\frac{1}{2}x\sqrt{a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```


```
[Out] (15*x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/32 - (5*Sqrt[c + a^2*c*x^2]*A
rcSinh[a*x]^(3/2))/(16*a*Sqrt[1 + a^2*x^2]) - (5*a*x^2*Sqrt[c + a^2*c*x^2]*
ArcSinh[a*x]^(3/2))/(8*Sqrt[1 + a^2*x^2]) + (x*Sqrt[c + a^2*c*x^2]*ArcSinh[
a*x]^(5/2))/2 + (Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[1 + a^2*
x^2]) + (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])
/(256*a*Sqrt[1 + a^2*x^2]) - (15*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(256*a*Sqrt[1 + a^2*x^2])
```


Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
```

```
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}\,dx = \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{1+a^2x^2}}\,dx}{2\sqrt{1+a^2x^2}} - \frac{\left(5a\sqrt{c+a^2cx^2}\right)\int x\sinh}{4\sqrt{1+a^2x^2}}$$

$$= -\frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2} + \frac{\sqrt{c+a^2cx^2}\sinh^{-1}(a}{7a\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{8\sqrt{1+a^2x^2}} + \frac{1}{2}x\sqrt{c+a^2cx^2}\sinh^{-1}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(a}{8\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(a}{8\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(a}{8\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(a}{8\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(a}{8\sqrt{1+a^2x^2}}$$

$$= \frac{15}{32}x\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)} - \frac{5\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}{16a\sqrt{1+a^2x^2}} - \frac{5ax^2\sqrt{c+a^2cx^2}\sinh^{-1}(a}{8\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.290943, size = 135, normalized size = 0.45

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(105\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-105\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+8\sqrt{\sinh^{-1}(ax)}\left(64\sinh^{-1}(ax)^3+7\left(\right.\right.\right.}{3584a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 10
5*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 8*Sqrt[ArcSinh[a*x]]*(64*Ar
```

```
cSinh[a*x]^3 - 140*ArcSinh[a*x]*Cosh[2*ArcSinh[a*x]] + 7*(15 + 16*ArcSinh[a
*x]^2)*Sinh[2*ArcSinh[a*x]])))/(3584*a*Sqrt[1 + a^2*x^2])
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int (\text{Arcsinh}\,(ax))^{\frac{5}{2}}\sqrt{a^2cx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,\text{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2), x)
```

3.483 $\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}} \, dx$

Optimal. Leaf size=42

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0720049, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{\sinh^{-1}(ax)^{5/2}}{\sqrt{1+a^2x^2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{2\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0361538, size = 42, normalized size = 1.

$$\frac{2\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{7/2}}{7a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(7/2))/(7*a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.036, size = 36, normalized size = 0.9

$$\frac{2}{7a}(\mathrm{Arcsinh}(ax))^{\frac{7}{2}}\sqrt{a^2x^2+1}\frac{1}{\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/7*arcsinh(a*x)^(7/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.484 $\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{x\sinh^{-1}(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}-\frac{5a\sqrt{a^2x^2+1}\,\text{Unintegrable}\left(\frac{x\sinh^{-1}(ax)^{3/2}}{a^2x^2+1},x\right)}{2c\sqrt{a^2cx^2+c}}$$

```
[Out] (x*ArcSinh[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (5*a*Sqrt[1 + a^2*x^2]*Uni
ntegrable[(x*ArcSinh[a*x]^(3/2))/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] time = 0.0894574, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (5*a*Sqrt[1 + a^2*x^2]*Def
er[Int][(x*ArcSinh[a*x]^(3/2))/(1 + a^2*x^2), x])/(2*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{x\sinh^{-1}(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}-\frac{\left(5a\sqrt{1+a^2x^2}\right)\int \frac{x\sinh^{-1}(ax)^{3/2}}{1+a^2x^2}\,dx}{2c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.611235, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^{5/2}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2), x]
```

Maple [A] time = 0.177, size = 0, normalized size = 0.

$$\int (\mathrm{Arcsinh}\,(ax))^{\frac{5}{2}} \left(a^2cx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{\frac{5}{2}}}{(a^2cx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

3.485 $\int \left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} \, dx$

Optimal. Leaf size=309

$$\frac{\sqrt{\pi} a^3 \sqrt{a^2+x^2} \operatorname{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}} + \frac{\sqrt{\frac{\pi}{2}} a^3 \sqrt{a^2+x^2} \operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}} - \frac{\sqrt{\pi} a^3 \sqrt{a^2+x^2} \operatorname{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}} - \sqrt{\frac{\pi}{2}}$$

```
[Out] (3*a^2*x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/8 + (x*(a^2 + x^2)^(3/2)*Sqrt[
ArcSinh[x/a]])/4 + (a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(4*Sqrt[1 + x^2
/a^2]) + (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(256*Sqrt
[1 + x^2/a^2]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[A
rcSinh[x/a]]])/(256*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Er
fi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rubi [A] time = 0.360075, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5684, 5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205, 5779}

$$\frac{\sqrt{\pi} a^3 \sqrt{a^2+x^2} \operatorname{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}} + \frac{\sqrt{\frac{\pi}{2}} a^3 \sqrt{a^2+x^2} \operatorname{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}} - \frac{\sqrt{\pi} a^3 \sqrt{a^2+x^2} \operatorname{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x^2}{a^2}+1}} - \sqrt{\frac{\pi}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/8 + (x*(a^2 + x^2)^(3/2)*Sqrt[
ArcSinh[x/a]])/4 + (a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2))/(4*Sqrt[1 + x^2
/a^2]) + (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(256*Sqrt
[1 + x^2/a^2]) + (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[A
rcSinh[x/a]]])/(256*Sqrt[1 + x^2/a^2]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Er
fi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
```

```
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(a^2+x^2\right)^{3/2} \sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx = \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}\left(3a^2\right)\int \sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx - \frac{\left(a\sqrt{a^2+x^2}\right) \cdots}{8\sqrt{1 \cdots}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} - \frac{\left(3a\sqrt{a^2+x^2}\right)\int \frac{x \cdots}{\sqrt{\sinh \cdots}}}{16\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right) \cdots}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right) \cdots}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right) \cdots}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right) \cdots}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right) \cdots}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{3}{8}a^2x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a^3\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right) \cdots}{4\sqrt{1+\frac{x^2}{a^2}}}$$

Mathematica [A] time = 0.174533, size = 156, normalized size = 0.5

$$\frac{a^3\sqrt{a^2+x^2}\left(-\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},-4\sinh^{-1}\left(\frac{x}{a}\right)\right) - 8\sqrt{2}\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},-2\sinh^{-1}\left(\frac{x}{a}\right)\right) + \sqrt{\sinh^{-1} \cdots}\right.}{128\sqrt{\frac{x^2}{a^2}+1}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + x^2)^(3/2)*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(-(Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -4*ArcSinh[x/a]]) -
8*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma[3/2, -2*ArcSinh[x/a]] + Sqrt[ArcSinh[x/
a]]*(32*ArcSinh[x/a]^(3/2) - 8*Sqrt[2]*Gamma[3/2, 2*ArcSinh[x/a]] - Gamma[3
/2, 4*ArcSinh[x/a]])))/(128*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{Arcsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2+x^2\right)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*sqrt(arcsinh(x/a)), x)
```

3.486 $\int \sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{}$$

```
[Out] (x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^
(3/2))/(3*Sqrt[1 + x^2/a^2]) + (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*E
rfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```

Rubi [A] time = 0.148164, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5682, 5675, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}-\frac{\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+x^2}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```



```
[Out] (x*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^
(3/2))/(3*Sqrt[1 + x^2/a^2]) + (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2]) - (a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*E
rfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(16*Sqrt[1 + x^2/a^2])
```


Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\,dx = \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{\sqrt{a^2+x^2}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\sqrt{a^2+x^2}\int \frac{x}{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{4a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(}{\sqrt{x}}\right.}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx,\right.}{4\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx,\right.}{8\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\right.}{16\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{\left(a\sqrt{a^2+x^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx,x,\right.}{8\sqrt{1+\frac{x^2}{a^2}}}$$

$$= \frac{1}{2}x\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1+\frac{x^2}{a^2}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2+x^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right.}\right.}{16\sqrt{1+\frac{x^2}{a^2}}}$$

Mathematica [A] time = 0.0872034, size = 110, normalized size = 0.62

$$\frac{a\sqrt{a^2+x^2}\left(-3\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},2\sinh^{-1}\left(\frac{x}{a}\right)\right)-3\sqrt{2}\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{3}{2},-2\sinh^{-1}\left(\frac{x}{a}\right)\right)+16\sinh^{-1}\left(\right.\right.}{48\sqrt{\frac{x^2}{a^2}+1}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]],x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(16*ArcSinh[x/a]^2 - 3*Sqrt[2]*Sqrt[-ArcSinh[x/a]]*Gamma
[3/2, -2*ArcSinh[x/a]] - 3*Sqrt[2]*Sqrt[ArcSinh[x/a]]*Gamma[3/2, 2*ArcSinh[
x/a]]))/(48*Sqrt[1 + x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{Arcsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

```
[Out] int((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2+x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + x^2}\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(1/2)*asinh(x/a)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + x**2)*sqrt(asinh(x/a)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + x^2}\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(1/2)*arcsinh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2 + x^2)*sqrt(arcsinh(x/a)), x)
```

3.487 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Rubi [A] time = 0.0622118, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5677, 5675}

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\,\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx = \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2+x^2}}$$
$$= \frac{2a\sqrt{1+\frac{x^2}{a^2}}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.0280797, size = 39, normalized size = 1.

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(3/2))/(3*Sqrt[a^2 + x^2])
```

Maple [A] time = 0.043, size = 34, normalized size = 0.9

$$\frac{2a}{3}\left(\mathrm{Arcsinh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}}\sqrt{\frac{a^2+x^2}{a^2}}\frac{1}{\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x)
```

```
[Out] 2/3*arcsinh(x/a)^(3/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

Fricas [A] time = 2.54614, size = 54, normalized size = 1.38

$$\frac{2}{3}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*log((x + sqrt(a^2 + x^2))/a)^(3/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(asinh(x/a))/sqrt(a**2 + x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(arcsinh(x/a))/sqrt(a^2 + x^2), x)
```

3.488 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\,\text{Unintegrable}\left(\frac{x}{\left(\frac{x^2}{a^2}+1\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(a^2*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Unintegra
ble[x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi [A] time = 0.0767944, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(a^2*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int
][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx=\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2+x^2}}-\frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.339169, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(3/2), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \sqrt{\mathrm{Arcsinh}\left(\frac{x}{a}\right)}\left(a^2+x^2\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

```
[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\mathrm{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{asinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(asinh(x/a))/(a**2 + x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(3/2), x)
```

3.489 $\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{\frac{x^2}{a^2}+1}\text{Unintegrable}\left(\frac{x}{\left(\frac{x^2}{a^2}+1\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2+x^2}}-\frac{\sqrt{\frac{x^2}{a^2}+1}\text{Unintegrable}\left(\frac{x}{\left(\frac{x^2}{a^2}+1\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2+x^2}}+\frac{2x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}}+\frac{x\sqrt{\text{si}}}{3a^2\left(a\right.}$$

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(3*a^2*(a^2 + x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[x/a]])
/(3*a^4*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Unintegrable[x/((1 + x^2/a^2)
^2*Sqrt[ArcSinh[x/a]]), x])/(6*a^5*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Un
integrable[x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(3*a^5*Sqrt[a^2 + x^2]
)
```

Rubi [A] time = 0.157542, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[ArcSinh[x/a]])/(3*a^2*(a^2 + x^2)^(3/2)) + (2*x*Sqrt[ArcSinh[x/a]])
/(3*a^4*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defer[Int][x/((1 + x^2/a^2)^2
*Sqrt[ArcSinh[x/a]]), x])/(6*a^5*Sqrt[a^2 + x^2]) - (Sqrt[1 + x^2/a^2]*Defe
r[Int][x/((1 + x^2/a^2)*Sqrt[ArcSinh[x/a]]), x])/(3*a^5*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx = \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}} + \frac{2\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{3/2}}\,dx}{3a^2} - \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}}$$

$$= \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2+x^2\right)^{3/2}} + \frac{2x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)^2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2+x^2}} - \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{x}{\left(1+\frac{x^2}{a^2}\right)\sqrt{\sinh^{-1}\left(\frac{x}{a}\right.}}}{3a^5\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.922676, size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcSinh[x/a]]/(a^2 + x^2)^(5/2), x]
```

Maple [A] time = 0.218, size = 0, normalized size = 0.

$$\int \sqrt{\mathrm{Arcsinh}\left(\frac{x}{a}\right)}\left(a^2+x^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

```
[Out] int(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(1/2)/(a**2+x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{arsinh}\left(\frac{x}{a}\right)}}{\left(a^2+x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(1/2)/(a^2+x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arcsinh(x/a))/(a^2 + x^2)^(5/2), x)
```

3.490 $\int \left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2}\, dx$

Optimal. Leaf size=433

$$\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}$$

```
[Out] (-27*a^3*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(256*Sqrt[1 + x^2/a^2]) - (9*a
*x^2*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(32*Sqrt[1 + x^2/a^2]) - (3*(a^2 +
 x^2)^(5/2)*Sqrt[ArcSinh[x/a]])/(32*a*Sqrt[1 + x^2/a^2]) + (3*a^2*x*Sqrt[a^
2 + x^2]*ArcSinh[x/a]^(3/2))/8 + (x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4
 + (3*a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(20*Sqrt[1 + x^2/a^2]) + (3*a
^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(2048*Sqrt[1 + x^2/a
^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(
64*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[ArcSinh
[x/a]]])/(2048*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[
Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rubi [A] time = 0.547982, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5684, 5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205, 5717, 5699}

$$\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2+x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\pi}a^3\sqrt{a^2+x^2}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-27*a^3*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(256*Sqrt[1 + x^2/a^2]) - (9*a
*x^2*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(32*Sqrt[1 + x^2/a^2]) - (3*(a^2 +
 x^2)^(5/2)*Sqrt[ArcSinh[x/a]])/(32*a*Sqrt[1 + x^2/a^2]) + (3*a^2*x*Sqrt[a^
2 + x^2]*ArcSinh[x/a]^(3/2))/8 + (x*(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2))/4
 + (3*a^3*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(20*Sqrt[1 + x^2/a^2]) + (3*a
^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erf[2*Sqrt[ArcSinh[x/a]]])/(2048*Sqrt[1 + x^2/a
^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(
64*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi]*Sqrt[a^2 + x^2]*Erfi[2*Sqrt[ArcSinh
[x/a]]])/(2048*Sqrt[1 + x^2/a^2]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[
```

```
Sqrt[2]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rubi steps

$$\int \left(a^2+x^2\right)^{3/2} \sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx = \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}\left(3a^2\right)\int \sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} dx - \frac{\left(3a\sqrt{a^2+x^2}\right)}{}$$

$$= -\frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}x\left(a^2+x^2\right)^{3/2}\sinh^{-1}\left(\right.$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right.$$

$$= -\frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}} + \frac{3}{8}a^2x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right.$$

$$= \frac{9a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\right.}}{32a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\right.}}{32a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\right.}}{32a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{27a^3\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{1+\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{1+\frac{x^2}{a^2}}} - \frac{3\left(a^2+x^2\right)^{5/2}\sqrt{\sinh^{-1}\left(\right.}}{32a\sqrt{1+\frac{x^2}{a^2}}}$$

Mathematica [A] time = 0.300583, size = 210, normalized size = 0.48

$$\frac{a^3\sqrt{a^2+x^2}\left(-5\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{5}{2},4\sinh^{-1}\left(\frac{x}{a}\right)\right)+5\sqrt{-\sinh^{-1}\left(\frac{x}{a}\right)}\text{Gamma}\left(\frac{5}{2},-4\sinh^{-1}\left(\frac{x}{a}\right)\right)+60\sqrt{2\pi}\sqrt{\sinh^{-1}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + x^2)^(3/2)*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a^3*Sqrt[a^2 + x^2]*(384*ArcSinh[x/a]^3 - 480*ArcSinh[x/a]*Cosh[2*ArcSinh[
x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] +
60*Sqrt[2*Pi]*Sqrt[ArcSinh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 5*Sqrt[
-ArcSinh[x/a]]*Gamma[5/2, -4*ArcSinh[x/a]] - 5*Sqrt[ArcSinh[x/a]]*Gamma[5/2
, 4*ArcSinh[x/a]] + 640*ArcSinh[x/a]^2*Sinh[2*ArcSinh[x/a]]))/(2560*Sqrt[1
+ x^2/a^2]*Sqrt[ArcSinh[x/a]])
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \left(\operatorname{Arcsinh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

```
[Out] int((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2+x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+x**2)**(3/2)*asinh(x/a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + x^2\right)^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+x^2)^(3/2)*arcsinh(x/a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 + x^2)^(3/2)*arcsinh(x/a)^(3/2), x)
```

3.491 $\int \sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}\,dx$

Optimal. Leaf size=259

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+}$$

```
[Out] (-3*a*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(16*Sqrt[1 + x^2/a^2]) - (3*x^2*S
qrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(8*a*Sqrt[1 + x^2/a^2]) + (x*Sqrt[a^2 +
x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt
[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(64*Sqrt[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rubi [A] time = 0.275563, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5682, 5675, 5663, 5779, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a\sqrt{a^2+x^2}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x^2}{a^2}+1}}+\frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x^2}{a^2}+1}}+\frac{1}{2}x\sqrt{a^2+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (-3*a*Sqrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(16*Sqrt[1 + x^2/a^2]) - (3*x^2*S
qrt[a^2 + x^2]*Sqrt[ArcSinh[x/a]])/(8*a*Sqrt[1 + x^2/a^2]) + (x*Sqrt[a^2 +
x^2]*ArcSinh[x/a]^(3/2))/2 + (a*Sqrt[a^2 + x^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt
[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[x
/a]]])/(64*Sqrt[1 + x^2/a^2]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 + x^2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[x/a]]])/(64*Sqrt[1 + x^2/a^2])
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
```

```
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}dx = \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{\sqrt{a^2+x^2}\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}}dx}{2\sqrt{1+\frac{x^2}{a^2}}} - \frac{\left(3\sqrt{a^2+x^2}\right)\int x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}d}{4a\sqrt{1+\frac{x^2}{a^2}}}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{(3}{}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} + \frac{(3}{}$$

$$= -\frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1+\frac{x^2}{a^2}}} - \frac{(3}{}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a}{}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a}{}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a}{}$$

$$= -\frac{3a\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{1+\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2+x^2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{1+\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2+x^2}\sinh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{a}{}$$

Mathematica [A] time = 0.24483, size = 133, normalized size = 0.51

$$\frac{a\sqrt{a^2+x^2}\left(15\sqrt{2\pi}\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\right)+8\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}\left(16\sinh^{-1}\left(\frac{x}{a}\right)^2+20\sinh\left(2\,\text{s}\right.\right.\right.}{640\sqrt{\frac{x^2}{a^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + x^2]*ArcSinh[x/a]^(3/2),x]
```

```
[Out] (a*Sqrt[a^2 + x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 15*Sqrt
[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[x/a]]] + 8*Sqrt[ArcSinh[x/a]]*(16*ArcSinh[
x/a]^2 - 15*Cosh[2*ArcSinh[x/a]] + 20*ArcSinh[x/a]*Sinh[2*ArcSinh[x/a]])))/
(640*Sqrt[1 + x^2/a^2])
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int \left(\mathrm{Arcsinh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}} \sqrt{a^2 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

```
[Out] int(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + x^2}\, \mathrm{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)*(a**2+x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a^2 + x^2}\,\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)*(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2 + x^2)*arcsinh(x/a)^(3/2), x)
```

3.492 $\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Rubi [A] time = 0.0639839, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5677, 5675}

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] && !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2+x^2}}\,dx = \frac{\sqrt{1+\frac{x^2}{a^2}}\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1+\frac{x^2}{a^2}}}\,dx}{\sqrt{a^2+x^2}} = \frac{2a\sqrt{1+\frac{x^2}{a^2}}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.029956, size = 39, normalized size = 1.

$$\frac{2a\sqrt{\frac{x^2}{a^2}+1}\sinh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[x/a]^(3/2)/Sqrt[a^2 + x^2],x]
```

```
[Out] (2*a*Sqrt[1 + x^2/a^2]*ArcSinh[x/a]^(5/2))/(5*Sqrt[a^2 + x^2])
```

Maple [A] time = 0.039, size = 34, normalized size = 0.9

$$\frac{2a}{5}\left(\mathrm{Arcsinh}\left(\frac{x}{a}\right)\right)^{\frac{5}{2}}\sqrt{\frac{a^2+x^2}{a^2}}\frac{1}{\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x)
```

```
[Out] 2/5*arcsinh(x/a)^(5/2)*a/(a^2+x^2)^(1/2)*((a^2+x^2)/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

Fricas [A] time = 2.48141, size = 54, normalized size = 1.38

$$\frac{2}{5}\log\left(\frac{x+\sqrt{a^2+x^2}}{a}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*log((x + sqrt(a^2 + x^2))/a)^(5/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] Integral(asinh(x/a)**(3/2)/sqrt(a**2 + x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(x/a)^(3/2)/sqrt(a^2 + x^2), x)
```

3.493 $\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{3\sqrt{\frac{x^2}{a^2}+1}\text{Unintegrable}\left(\frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{\frac{x^2}{a^2}+1},x\right)}{2a^3\sqrt{a^2+x^2}}$$

```
[Out] (x*ArcSinh[x/a]^(3/2))/(a^2*Sqrt[a^2 + x^2]) - (3*Sqrt[1 + x^2/a^2]*Uninteg
rable[(x*Sqrt[ArcSinh[x/a]])/(1 + x^2/a^2), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi [A] time = 0.0756793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

```
[Out] (x*ArcSinh[x/a]^(3/2))/(a^2*Sqrt[a^2 + x^2]) - (3*Sqrt[1 + x^2/a^2]*Defer[I
nt][(x*Sqrt[ArcSinh[x/a]])/(1 + x^2/a^2), x])/(2*a^3*Sqrt[a^2 + x^2])
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx=\frac{x\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2+x^2}}-\frac{\left(3\sqrt{1+\frac{x^2}{a^2}}\right)\int \frac{x\sqrt{\sinh^{-1}\left(\frac{x}{a}\right)}}{1+\frac{x^2}{a^2}}\,dx}{2a^3\sqrt{a^2+x^2}}$$

Mathematica [A] time = 0.370795, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2+x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2),x]
```

```
[Out] Integrate[ArcSinh[x/a]^(3/2)/(a^2 + x^2)^(3/2), x]
```

Maple [A] time = 0.161, size = 0, normalized size = 0.

$$\int \left(\mathrm{Arcsinh}\left(\frac{x}{a}\right)\right)^{\frac{3}{2}} \left(a^2+x^2\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

```
[Out] int(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x/a)**(3/2)/(a**2+x**2)**(3/2),x)
```

```
[Out] Integral(asinh(x/a)**(3/2)/(a**2 + x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x/a)^(3/2)/(a^2+x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(x/a)^(3/2)/(a^2 + x^2)^(3/2), x)
```

3.494 $\int \frac{x}{\sqrt{1+x^2}\sqrt{\sinh^{-1}(x)}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(x)}\right)-\frac{1}{2}\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(x)}\right)$$

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcSinh[x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[x]]])/2
```

Rubi [A] time = 0.0799689, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5779, 3308, 2180, 2204, 2205}

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}\left(\sqrt{\sinh^{-1}(x)}\right)-\frac{1}{2}\sqrt{\pi}\text{Erf}\left(\sqrt{\sinh^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[ArcSinh[x]]])/2 + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[x]]])/2
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x^2}\sqrt{\sinh^{-1}(x)}}\,dx &= \text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(x)\right)\\ &= -\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\sinh^{-1}(x)}\right) + \text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\sinh^{-1}(x)}\right)\\ &= -\frac{1}{2}\sqrt{\pi}\,\text{erf}\left(\sqrt{\sinh^{-1}(x)}\right) + \frac{1}{2}\sqrt{\pi}\,\text{erfi}\left(\sqrt{\sinh^{-1}(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.068194, size = 34, normalized size = 1.03

$$\frac{1}{2}\left(\frac{\sqrt{-\sinh^{-1}(x)}\,\text{Gamma}\left(\frac{1}{2}, -\sinh^{-1}(x)\right)}{\sqrt{\sinh^{-1}(x)}} + \text{Gamma}\left(\frac{1}{2}, \sinh^{-1}(x)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[ArcSinh[x]]),x]
```

```
[Out] ((Sqrt[-ArcSinh[x]]*Gamma[1/2, -ArcSinh[x]])/Sqrt[ArcSinh[x]] + Gamma[1/2,
ArcSinh[x]])/2
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{x^2+1}}\frac{1}{\sqrt{\operatorname{Arcsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

```
[Out] int(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{arsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{asinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/2)/asinh(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x**2 + 1)*sqrt(asinh(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+1}\sqrt{\operatorname{arsinh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/arcsinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^2 + 1)*sqrt(arcsinh(x))), x)
```

3.495 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=396

$$\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{6}\sqrt{\sinh^{-1}\right.}{64a\sqrt{a^2x^2+1}}$$

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*a*Sqrt[1 + a^2*x^2]) + (3
*c^2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 +
 a^2*x^2]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (
15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64
*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]*Sq
rt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.30802, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5702, 5699, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{6}}c^2\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{6}\sqrt{\sinh^{-1}\right.}{64a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (5*c^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(8*a*Sqrt[1 + a^2*x^2]) + (3
*c^2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 +
 a^2*x^2]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2]) + (
15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64
*a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[Pi/6]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]*Sq
rt[ArcSinh[a*x]]])/(64*a*Sqrt[1 + a^2*x^2])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\int \frac{\left(1+a^2x^2\right)^{5/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh^6(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \left(\frac{5}{16\sqrt{x}}+\frac{15\cosh(2x)}{32\sqrt{x}}+\frac{3\cosh(4x)}{16\sqrt{x}}+\frac{\cosh(6x)}{32\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{32a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2\right.}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{64a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx^2}\right)\operatorname{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{\left(c^2\sqrt{c+a^2cx\right.}{}$$

$$= \frac{5c^2\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{8a\sqrt{1+a^2x^2}} + \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\operatorname{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{64a\sqrt{1+a^2x^2}} + \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\operatorname{erf}\left(\sqrt{\right.}{64a\sqrt{1+a^2}}$$

Mathematica [A] time = 0.321456, size = 197, normalized size = 0.5

$$\frac{c^2\sqrt{a^2cx^2+c}\left(-45\sqrt{2}\sqrt{\sinh^{-1}(ax)}\operatorname{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)-18\sqrt{\sinh^{-1}(ax)}\operatorname{Gamma}\left(\frac{1}{2},4\sinh^{-1}(ax)\right)-\sqrt{6}\sqrt{\sinh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(240*ArcSinh[a*x] + Sqrt[6]*Sqrt[-ArcSinh[a*x]]*Ga
mma[1/2, -6*ArcSinh[a*x]] + 18*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*
x]] + 45*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - 45*Sqrt[
2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] - 18*Sqrt[ArcSinh[a*x]]*Ga
mma[1/2, 4*ArcSinh[a*x]] - Sqrt[6]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[
```

```
a*x]]]))/(384*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{5}{2}} \frac{1}{\sqrt{\operatorname{Arcsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arcsinh(a*x)), x)
```

3.496 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=264

$$\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+$$

```
[Out] (3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*S
qrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a*Sqrt[1 + a^2*x
^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(
4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSin
h[a*x]]])/(32*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.226588, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5702, 5699, 3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{a^2x^2+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```


```
[Out] (3*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*S
qrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(32*a*Sqrt[1 + a^2*x
^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(
4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSin
h[a*x]]])/(32*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2])
```


Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
```

```
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\left(c\sqrt{c+a^2cx^2}\right)\int \frac{\left(1+a^2x^2\right)^{3/2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}+\frac{\cosh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right.}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{S}}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right.}{}$$

$$= \frac{3c\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{4a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{32a\sqrt{1+a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{s}\right.}}{4a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.184342, size = 141, normalized size = 0.53

$$\frac{c\sqrt{a^2cx^2+c}\left(\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)+4\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+\sqrt{\sinh^{-1}\right.}{32a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (c*Sqrt[c + a^2*c*x^2]*(Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] + 4
*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[ArcSinh[a*x
]]*(24*Sqrt[ArcSinh[a*x]] - 4*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] - Gamma[1/
2, 4*ArcSinh[a*x]])))/(32*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \frac{1}{\sqrt{\operatorname{Arcsinh}\left(a x \right)}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c \left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}}}{\sqrt{\operatorname{asinh}\left(a x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arcsinh(a*x)), x)
```

3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\sinh^{-1}(ax)}} dx$

Optimal. Leaf size=156

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2x^2+1}}$$

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^
2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*
a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.163635, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5702, 5699, 3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}+\frac{\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[1 + a^2*x^2]) + (Sqrt[Pi/2
]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^
2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(4*
a*Sqrt[1 + a^2*x^2])
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
```

```
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\sqrt{c+a^2cx^2}\int \frac{\sqrt{1+a^2x^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a\sqrt{1+a^2x^2}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\right.}{4a\sqrt{1+a}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{2a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int\right.}{2a\sqrt{}}$$

$$= \frac{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh\right.}{4a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.0966667, size = 101, normalized size = 0.65

$$\frac{\sqrt{c\left(a^2x^2+1\right)}\left(-\sqrt{2}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+\sqrt{2}\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+8\sinh^{-1}(\right.}{8a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[c*(1 + a^2*x^2)]*(8*ArcSinh[a*x] + Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[
1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x
]]))/(8*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,\frac{1}{\sqrt{\operatorname{Arcsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(asinh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/sqrt(arcsinh(a*x)), x)
```

3.498 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0724067, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= \frac{2\sqrt{1+a^2x^2}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.0420324, size = 40, normalized size = 1.

$$\frac{2\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}{a\sqrt{a^2cx^2+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] (2*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(a*Sqrt[c + a^2*c*x^2])
```

Maple [A] time = 0.036, size = 36, normalized size = 0.9

$$2\,\frac{\sqrt{\mathrm{Arcsinh}\,(ax)}\sqrt{a^2x^2+1}}{a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] 2*arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\mathrm{arsinh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(1/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(asinh(a*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arcsinh(a*x))), x)
```

3.499 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi [A] time = 0.0418289, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Defer[Int][1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 0.720636, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \left(a^{2} c\,x^{2}+c\right)^{-\frac{3}{2}} \frac{1}{\sqrt{\operatorname{Arcsinh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arsinh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(asinh(a*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*sqrt(arcsinh(a*x))), x)
```

3.500 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\sinh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi [A] time = 0.0424377, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Defer[Int][1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx=\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Mathematica [A] time = 1.59789, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]), x]
```

Maple [A] time = 0.24, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{-\frac{5}{2}} \frac{1}{\sqrt{\operatorname{Arcsinh}\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} \sqrt{\operatorname{arsinh}\left(a x\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2cx^2 + c)^{\frac{5}{2}}\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arcsinh(a*x))), x)
```

3.501 $\int \frac{(c+a^2cx^2)^{5/2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{6}\sqrt{\sinh\cdots}\right.}{16a\sqrt{a^2x^2+1}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(5/2))/(a*Sqrt[ArcSinh[a*x]]) - (3*c^
2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2
*x^2]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2*x^2]) + (1
5*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*
a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]
*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.301731, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{a^2x^2+1}}-\frac{15\sqrt{\frac{\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{16a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{3\pi}{2}}c^2\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{6}\sqrt{\sinh\cdots}\right.}{16a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(5/2))/(a*Sqrt[ArcSinh[a*x]]) - (3*c^
2*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2
*x^2]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*
x]]])/(16*a*Sqrt[1 + a^2*x^2]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Er
f[Sqrt[6]*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2]) + (3*c^2*Sqrt[Pi]*S
qrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(8*a*Sqrt[1 + a^2*x^2]) + (1
5*c^2*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(16*
a*Sqrt[1 + a^2*x^2]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[6]
```

```
*Sqrt[ArcSinh[a*x]]])/(16*a*Sqrt[1 + a^2*x^2])
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12ac^2\sqrt{c+a^2cx^2}\right)\int \frac{x\left(1+a^2x^2\right)^2}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}} + \frac{\sinh(6x)}{32\sqrt{x}}\right) dx, x, \text{si}\right.}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2\right.}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{16a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2cx^2\right.}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} + \frac{\left(3c^2\sqrt{c+a^2\right.}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{5/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{3c^2\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{8a\sqrt{1+a^2x^2}} - \frac{15c^2\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{\right.}{16a\sqrt{1+a^2x}}$$

Mathematica [A] time = 1.06555, size = 399, normalized size = 1.02

$$\frac{c^2\sqrt{a^2cx^2+c}\,e^{-6\sinh^{-1}(ax)}\left(\sqrt{6}e^{6\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\,\text{Gamma}\left(\frac{1}{2},-6\sinh^{-1}(ax)\right)+12e^{6\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\,\text{Gamm}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(5/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (c^2*Sqrt[c + a^2*c*x^2]*(-1 - 6*E^(2*ArcSinh[a*x]) + E^(4*ArcSinh[a*x]) -
52*E^(6*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) - 6*E^(10*ArcSinh[a*x]) - E^(12*
ArcSinh[a*x]) - 64*a^2*E^(6*ArcSinh[a*x])*x^2 - 16*E^(6*ArcSinh[a*x])*Sqrt[
2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + 16*E^(6*ArcSinh[
a*x])*Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] + Sqrt
[6]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -6*ArcSinh[a*x]] + 12
*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[
2]*E^(6*ArcSinh[a*x])*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqr
t[2]*E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*ArcSinh[a*x]] + 12*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + Sqrt[6]*
E^(6*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 6*ArcSinh[a*x]]))/(32*a*E^
(6*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \left(\operatorname{Arcsinh}\left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^{2} c x^{2} + c\right)^{\frac{5}{2}}}{\operatorname{arsinh}\left(a x\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(5/2)/arcsinh(a*x)^(3/2), x)
```

3.502 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]) - (c*Sq
rt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2
]) - (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*
Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*
x]]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.208331, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5696, 5779, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}-\frac{\sqrt{\frac{\pi}{2}}c\sqrt{a^2cx^2+c}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\mathrm{Erfi}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(a*Sqrt[ArcSinh[a*x]]) - (c*Sq
rt[Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(4*a*Sqrt[1 + a^2*x^2
]) - (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*
Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh[a*
x]]])/(4*a*Sqrt[1 + a^2*x^2]) + (c*Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt
[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x^2])
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
```

```
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x\left(1+a^2x^2\right)}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right)dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}} + \frac{\left(2c\sqrt{c+a^2c\right.}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{2a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{S}}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{\left(c\sqrt{c+a^2cx\right.}{}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{4a\sqrt{1+a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\text{s}\right.}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.366563, size = 225, normalized size = 0.88

$$-\frac{c\sqrt{a^2cx^2+c}e^{-4\sinh^{-1}(ax)}\left(-2e^{4\sinh^{-1}(ax)}\sqrt{-\sinh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4\sinh^{-1}(ax)\right) - 2e^{4\sinh^{-1}(ax)}\sqrt{\sinh^{-1}(ax)}\text{Gamma}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) + 1
6*a^2*E^(4*ArcSinh[a*x])*x^2 + 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi]*Sqrt[ArcSinh
[a*x]]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 4*E^(4*ArcSinh[a*x])*Sqrt[2*Pi]*Sq
```

```
rt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]] - 2*E^(4*ArcSinh[a*x])*Sq
rt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - 2*E^(4*ArcSinh[a*x])*Sqrt[A
rcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]]))/(8*a*E^(4*ArcSinh[a*x])*Sqrt[1 +
a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} (\operatorname{Arcsinh}(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Integral((c*(a**2*x**2 + 1))**(3/2)/asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(3/2), x)
```

3.503 $\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[P
i/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x
^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a
*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.124301, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5696, 5669, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{a^2x^2+1}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}{a\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[P
i/2]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a*Sqrt[1 + a^2*x
^2]) + (Sqrt[Pi/2]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(a
*Sqrt[1 + a^2*x^2])
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{3/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sqrt{\sinh^{-1}(ax)}}\,dx}{\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} + \frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{c+a^2cx^2}\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}} \cdots\right.}{a\sqrt{1+a \cdots}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{\left(2\sqrt{c+a^2cx^2}\right)\text{Su}\cdots}{\cdots}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{a\sqrt{\sinh^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{a\sqrt{1+a^2x^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c+a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\sinh \cdots}\right.}{a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.185339, size = 115, normalized size = 0.76

$$-\frac{\sqrt{a^2cx^2+c}\left(4a^2x^2+\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)-\sqrt{2\pi}\sqrt{\sinh^{-1}(ax)}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)+4\right)}{2a\sqrt{a^2x^2+1}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(3/2),x]
```

```
[Out] -(Sqrt[c + a^2*c*x^2]*(4 + 4*a^2*x^2 + Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erf[Sq
rt[2]*Sqrt[ArcSinh[a*x]]] - Sqrt[2*Pi]*Sqrt[ArcSinh[a*x]]*Erfi[Sqrt[2]*Sqrt
[ArcSinh[a*x]]]))/(2*a*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2 + c}\,(\mathrm{Arcsinh}\,(ax))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2 + c}}{\mathrm{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c\left(a^2x^2 + 1\right)}}{\mathrm{asinh}^{\frac{3}{2}}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(c*(a**2*x**2 + 1))/asinh(a*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(3/2), x)
```

3.504 $\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Rubi [A] time = 0.0710817, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{3/2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{c+a^2cx^2}\sqrt{\sinh^{-1}(ax)}}$$

Mathematica [A] time = 0.0365253, size = 40, normalized size = 1.

$$-\frac{2\sqrt{a^2x^2+1}}{a\sqrt{a^2cx^2+c}\sqrt{\sinh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[c + a^2*c*x^2]*Sqrt[ArcSinh[a*x]])
```

Maple [A] time = 0.039, size = 36, normalized size = 0.9

$$-2\,\frac{\sqrt{a^2x^2+1}}{\sqrt{\mathrm{Arcsinh}\,(ax)}\,a\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -2/arcsinh(a*x)^(1/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,\mathrm{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

Fricas [A] time = 2.58406, size = 131, normalized size = 3.28

$$-\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{\left(a^3cx^2+ac\right)\sqrt{\log\left(ax+\sqrt{a^2x^2+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*sqrt(log(a*x +
sqrt(a^2*x^2 + 1))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*(a**2*x**2 + 1))*asinh(a*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(3/2)), x)
```

3.505 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{4a\sqrt{a^2x^2+1}\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)^2\sqrt{\sinh^{-1}(ax)}},x\right)}{c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\left(a^2cx^2+c\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]) - (4*a*
Sqrt[1 + a^2*x^2]*Unintegrable[x/((1 + a^2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/
(c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.089694, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcSinh[a*x]]) - (4*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*Sqrt[ArcSinh[a*x]]), x])/(c
*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{3/2}\sqrt{\sinh^{-1}(ax)}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sqrt{\sinh^{-1}(ax)}}\,dx}{c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.710224, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}} \left(\operatorname{Arcsinh}\left(a x \right)\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(3/2)), x)
```

3.506 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8a\sqrt{a^2x^2+1}\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)^3\sqrt{\sinh^{-1}(ax)}},x\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{a\left(a^2cx^2+c\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]) - (8*a*
Sqrt[1 + a^2*x^2]*Unintegrable[x/((1 + a^2*x^2)^3*Sqrt[ArcSinh[a*x]]), x])/
(c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0884567, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcSinh[a*x]]) - (8*a*
Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*Sqrt[ArcSinh[a*x]]), x])/(c
^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{a\left(c+a^2cx^2\right)^{5/2}\sqrt{\sinh^{-1}(ax)}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3\sqrt{\sinh^{-1}(ax)}}\,dx}{c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.46663, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)), x]
```

Maple [A] time = 0.238, size = 0, normalized size = 0.

$$\int \left(a^{2} c x^{2}+c\right)^{-\frac{5}{2}} \left(\operatorname{Arcsinh}\left(a x\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{\frac{5}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(3/2)), x)
```

3.507 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=296

$$\frac{2\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}c\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(a\right.}{3a\sqrt{a^2x^2+1}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) - (16
*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*c*Sqrt[
Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
+ (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh
[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
```

Rubi [A] time = 0.376707, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5696, 5777, 5699, 3312, 3307, 2180, 2204, 2205, 5779, 5448}

$$\frac{2\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\text{Erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}c\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{\pi}c\sqrt{a^2cx^2+c}\,\text{Erfi}\left(2\sqrt{\sinh^{-1}(a\right.}{3a\sqrt{a^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(c + a^2*c*x^2)^(3/2))/(3*a*ArcSinh[a*x]^(3/2)) - (16
*c*x*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*c*Sqrt[
Pi]*Sqrt[c + a^2*c*x^2]*Erf[2*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
+ (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[Pi]*Sqrt[c + a^2*c*x^2]*Erfi[2*Sqrt[ArcSinh
[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*c*Sqrt[2*Pi]*Sqrt[c + a^2*c*x^2]*Erfi
[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2])
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
```

```
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
[e, c^2*d] && LtQ[n, -1]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^
p*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (-Dist[(f*m*d^IntPart[p
]*(d + e*x^2)^FracPart[p])/(b*c*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*
x)^(m - 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - D
ist[(c*(m + 2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(b*f*(n + 1)*(1
+ c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p - 1/2)*(a + b*A
rcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[2*p, 0]
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{\left(8ac\sqrt{c+a^2cx^2}\right)\int \frac{x\left(1+a^2x^2\right)}{\sinh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\int \frac{\sqrt{1+a^2x^2}}{\sqrt{\sinh^{-1}(ax)}}\,dx}{3\sqrt{1+a^2x^2}} + -$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{x}}\,dx\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(16c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}}+\right.\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx,\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx,x,\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8c\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx,x,\right.}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\left(c+a^2cx^2\right)^{3/2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{16cx\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2c\sqrt{\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(2\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

Mathematica [A] time = 0.396522, size = 262, normalized size = 0.89

$$-\frac{c\sqrt{a^2cx^2+c}\,e^{-4\sinh^{-1}(ax)}\left(16e^{4\sinh^{-1}(ax)}\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-4\sinh^{-1}(ax)\right)+16\sqrt{2}e^{4\sinh^{-1}(ax)}\left(-\sinh^{-1}(ax)\right)^{3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + a^2*c*x^2)^(3/2)/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -(c*Sqrt[c + a^2*c*x^2]*(1 + 14*E^(4*ArcSinh[a*x]) + E^(8*ArcSinh[a*x]) + 1
6*a^2*E^(4*ArcSinh[a*x])*x^2 - 8*ArcSinh[a*x] + 8*E^(8*ArcSinh[a*x])*ArcSin
```

```
h[a*x] + 64*a*E^(4*ArcSinh[a*x])*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + 16*E^(4
*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] + 16*Sqrt[
2]*E^(4*ArcSinh[a*x])*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + 1
6*Sqrt[2]*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]
+ 16*E^(4*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]]))/(24
*a*E^(4*ArcSinh[a*x])*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int \left(a^2cx^2+c\right)^{\frac{3}{2}} (\text{Arcsinh}\,(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\text{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 + c)^(3/2)/arcsinh(a*x)^(5/2), x)
```

3.508 $\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{5/2}} dx$

Optimal. Leaf size=182

$$\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{8x\sqrt{a^2cx^2+c}}{3\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2\cdots}}{3a\sinh^{-1}(a\cdots}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x*
Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[c + a^2*c*
x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*Sqrt[2*P
i]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*
x^2])
```

Rubi [A] time = 0.114611, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5696, 5665, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\text{Erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}+\frac{2\sqrt{2\pi}\sqrt{a^2cx^2+c}\,\text{Erfi}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{a^2x^2+1}}-\frac{8x\sqrt{a^2cx^2+c}}{3\sqrt{\sinh^{-1}(ax)}}-\frac{2\sqrt{a^2x^2+1}\sqrt{a^2\cdots}}{3a\sinh^{-1}(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*Sqrt[c + a^2*c*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x*
Sqrt[c + a^2*c*x^2])/(3*Sqrt[ArcSinh[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[c + a^2*c*
x^2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*x^2]) + (2*Sqrt[2*P
i]*Sqrt[c + a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a*Sqrt[1 + a^2*
x^2])
```

Rule 5696

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(Sqrt[1 + c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^(n +
 1))/(b*c*(n + 1)), x] - Dist[(c*(2*p + 1)*d^IntPart[p]*(d + e*x^2)^FracPar
t[p])/(b*(n + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ
```

```
[e, c^2*d] && LtQ[n, -1]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{c+a^2cx^2}}{\sinh^{-1}(ax)^{5/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} + \frac{\left(4a\sqrt{c+a^2cx^2}\right)\int \frac{x}{\sinh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(4\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \sinh^{-1}(ax)\right)}{3a\sqrt{1+a^2x^2}} + $$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{\left(8\sqrt{c+a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{1+a^2x^2}}$$

$$= -\frac{2\sqrt{1+a^2x^2}\sqrt{c+a^2cx^2}}{3a\sinh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c+a^2cx^2}}{3\sqrt{\sinh^{-1}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c+a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\sinh^{-1}(ax)}\right)}{3a\sqrt{1+a^2x^2}} + \frac{2\sqrt{2\pi}\sqrt{c}}{}$$

Mathematica [A] time = 0.145474, size = 122, normalized size = 0.67

$$-\frac{2\sqrt{a^2cx^2+c}\left(\sqrt{2}\left(-\sinh^{-1}(ax)\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-2\sinh^{-1}(ax)\right)+\sqrt{2}\sinh^{-1}(ax)^{3/2}\text{Gamma}\left(\frac{1}{2},2\sinh^{-1}(ax)\right)+a^2x^2\right.}{3a\sqrt{a^2x^2+1}\sinh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + a^2*c*x^2]/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c + a^2*c*x^2]*(1 + a^2*x^2 + 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
 + Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*ArcS
inh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]))/(3*a*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^(3/2))
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int \sqrt{a^2cx^2+c}\,(\text{Arcsinh}\,(ax))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*c*x**2+c)**(1/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a^2cx^2+c}}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*c*x^2+c)^(1/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*c*x^2 + c)/arcsinh(a*x)^(5/2), x)
```

3.509 $\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

Rubi [A] time = 0.070311, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5677, 5675}

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

Rule 5677

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSinh[c*x])
^n/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*
d] &&  !GtQ[d, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{5/2}}\,dx = \frac{\sqrt{1+a^2x^2}\int \frac{1}{\sqrt{1+a^2x^2}\sinh^{-1}(ax)^{5/2}}\,dx}{\sqrt{c+a^2cx^2}}$$
$$= -\frac{2\sqrt{1+a^2x^2}}{3a\sqrt{c+a^2cx^2}\sinh^{-1}(ax)^{3/2}}$$

Mathematica [A] time = 0.039566, size = 42, normalized size = 1.

$$-\frac{2\sqrt{a^2x^2+1}}{3a\sqrt{a^2cx^2+c}\sinh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[c + a^2*c*x^2]*ArcSinh[a*x]^(3/2))
```

Maple [A] time = 0.042, size = 36, normalized size = 0.9

$$-\frac{2}{3a}\sqrt{a^2x^2+1}\,(\mathrm{Arcsinh}\,(ax))^{-\frac{3}{2}}\frac{1}{\sqrt{c\left(a^2x^2+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -2/3/arcsinh(a*x)^(3/2)/a/(c*(a^2*x^2+1))^(1/2)*(a^2*x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^2cx^2+c}\,\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

Fricas [A] time = 2.49659, size = 134, normalized size = 3.19

$$-\frac{2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}{3\left(a^3cx^2+ac\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/((a^3*c*x^2 + a*c)*log(a*x + sqr
t(a^2*x^2 + 1))^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a^2cx^2+c}\,\mathrm{arsinh}\,(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*c*x^2 + c)*arcsinh(a*x)^(5/2)), x)
```

3.510 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{4a\sqrt{a^2x^2+1}\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)^2\sinh^{-1}(ax)^{3/2}},x\right)}{3c\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\left(a^2cx^2+c\right)^{3/2}\sinh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)) - (4*
a*Sqrt[1 + a^2*x^2]*Unintegrable[x/((1 + a^2*x^2)^2*ArcSinh[a*x]^(3/2)), x]
)/(3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0913288, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(3/2)) - (4*
a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^2*ArcSinh[a*x]^(3/2)), x])/
(3*c*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{3/2}}-\frac{\left(4a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^2\sinh^{-1}(ax)^{3/2}}\,dx}{3c\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 0.702234, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcSinh[a*x]^(5/2)), x]
```

Maple [A] time = 0.174, size = 0, normalized size = 0.

$$\int \left(a^{2} c \,x^{2}+c \right)^{-\frac{3}{2}} \left(\operatorname{Arcsinh}\left(a x \right)\right)^{-\frac{5}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(3/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(3/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(3/2)*arcsinh(a*x)^(5/2)), x)
```

3.511 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{8a\sqrt{a^2x^2+1}\text{Unintegrable}\left(\frac{x}{\left(a^2x^2+1\right)^3\sinh^{-1}(ax)^{3/2}},x\right)}{3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}}{3a\left(a^2cx^2+c\right)^{5/2}\sinh^{-1}(ax)^{3/2}}$$

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)) - (8*
a*Sqrt[1 + a^2*x^2]*Unintegrable[x/((1 + a^2*x^2)^3*ArcSinh[a*x]^(3/2)), x]
)/(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] time = 0.0905415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*(c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(3/2)) - (8*
a*Sqrt[1 + a^2*x^2]*Defer[Int][x/((1 + a^2*x^2)^3*ArcSinh[a*x]^(3/2)), x])/
(3*c^2*Sqrt[c + a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{3/2}}-\frac{\left(8a\sqrt{1+a^2x^2}\right)\int \frac{x}{\left(1+a^2x^2\right)^3\sinh^{-1}(ax)^{3/2}}\,dx}{3c^2\sqrt{c+a^2cx^2}}$$

Mathematica [A] time = 1.4787, size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sinh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcSinh[a*x]^(5/2)), x]
```

Maple [A] time = 0.228, size = 0, normalized size = 0.

$$\int \left(a^{2} c\,x^{2}+c\right)^{-\frac{5}{2}} \left(\operatorname{Arcsinh}\left(a x\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} \operatorname{arsinh}\left(a x\right)^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*c*x**2+c)**(5/2)/asinh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*c*x^2+c)^(5/2)/arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 + c)^(5/2)*arcsinh(a*x)^(5/2)), x)
```

3.512 $\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=235

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2dx^2+}}{}$$

```
[Out] -(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c^3*(1 + n)*Sqrt[1
 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4
*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]
*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3
*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.454066, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5782, 5779, 5448, 3307, 2181}

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{c^2x^2+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{c^2dx^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c^3*(1 + n)*Sqrt[1
 + c^2*x^2]) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4
*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]
*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c^3
*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d+c^2dx^2}\int x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^2(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^3\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{8}(a+bx)^n+\frac{1}{8}(a+bx)^n\cosh(4x)\right)dx,x,\sinh^{-1}(cx\right.}{c^3\sqrt{1+c^2x^2}}$$
$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(4\right.}{8c^3\sqrt{1+c^2x^2}}$$
$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-4x}(a+bx)^n\,dx\right.}{16c^3\sqrt{1+c^2x^2}}$$
$$= -\frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{1+c^2x^2}} + \frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx\right.}{c^3}$$

Mathematica [A] time = 0.945789, size = 170, normalized size = 0.72

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-e^{\frac{8}{b}}\right.\right.}{64c^3\sqrt{d\left(c^2x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b +
 b*n) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] -
 E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c
*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c^3*Sqr
t[d*(1 + c^2*x^2)])
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int x^2\left(a+b\text{Arcsinh}\left(cx\right)\right)^n\sqrt{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^nx^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

3.513 $\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=355

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sin\cdots\right.}{}$$

```
[Out] (3^(-1 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcSinh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
-((a + b*ArcSinh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) + (E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (a + b*ArcSinh[c*x])/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + (3^(-1 - n)*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((
a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.473072, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5782, 5779, 5448, 3308, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{c^2x^2+1}}+\frac{e^{-\frac{a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sin\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (3^(-1 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcSinh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
-((a + b*ArcSinh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) + (E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (a + b*ArcSinh[c*x])/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + (3^(-1 - n)*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*c^2*Sqrt[1 + c^2*x^2]*((
a + b*ArcSinh[c*x])/b)^n)
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d+c^2dx^2}\int x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$
$$=\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^2\sqrt{1+c^2x^2}}$$
$$=\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{4}(a+bx)^n\sinh(x)+\frac{1}{4}(a+bx)^n\sinh(3x)\right)dx,x,\sinh^{-1}\right.}{c^2\sqrt{1+c^2x^2}}$$
$$=\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\sinh(x)\,dx,x,\sinh^{-1}(cx)\right)}{4c^2\sqrt{1+c^2x^2}}+\frac{\sqrt{d+c^2dx^2}\,\text{Subst}}{}$$
$$=-\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{8c^2\sqrt{1+c^2x^2}}-\frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\right.}{8c}$$
$$=\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+b\sinh}{b}\right.}{8c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 0.923611, size = 229, normalized size = 0.65

$$\frac{de^{-\frac{3a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\left(3^{-n}e^{\frac{6a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\text{Gamma}\left(n+1,\frac{3(a}{}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((3*E^((4*a)/b)*Gamma[1 + n, a/
b + ArcSinh[c*x]])/(a/b + ArcSinh[c*x])^n + (Gamma[1 + n, (-3*(a + b*ArcSin
h[c*x]))/b]/3^n + 3*E^((2*a)/b)*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + (
E^((6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSin
h[c*x]))/b])/(3^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n))/(-((a + b*ArcSinh[c*x
])/b))^n))/(24*c^2*E^((3*a)/b)*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int x\left(a+b\text{Arcsinh}\,(cx)\right)^n\sqrt{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c^2dx^2+d}(b\operatorname{arsinh}(cx)+a)^n x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{d\left(c^2x^2+1\right)}\left(a+b\operatorname{asinh}(cx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n*x, x)
```

3.514 $\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=235

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}\,(a}{}$$

```
[Out] (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[1 +
c^2*x^2]) + (2^(-3 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
 b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2
*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.298012, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5702, 5699, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{c^2dx^2+d}\,(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```


```
[Out] (Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[1 +
c^2*x^2]) + (2^(-3 - n)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
 b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2
*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```


Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx = \frac{\sqrt{d+c^2dx^2}\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\,dx, x, \sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx, x, \sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(2x)\,d\right.}{2c\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{\sqrt{d+c^2dx^2}\,\text{Subst}\left(\int e^{-2x}(a+bx)^n\,dx, x, \text{s}\right.}{4c\sqrt{1+c^2x^2}}$$
$$= \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{1+c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(\right.}{c\sqrt{1}}$$

Mathematica [A] time = 0.581176, size = 160, normalized size = 0.68

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-2^{-n}e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^{-n}\text{Gamma}\left(n+1,\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)+2^{-n}e^{-\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)\right.}{8c\sqrt{d\left(c^2x^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((4*a + 4*b*ArcSinh[c*x])/(b +
b*n) + Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b
*ArcSinh[c*x])/b))^n) - (E^((2*a)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/
b])/(2^n*(a/b + ArcSinh[c*x])^n)))/(8*c*Sqrt[d*(1 + c^2*x^2)])
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^n\sqrt{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\left(c^2x^2 + 1\right)}\,(a + b \operatorname{asinh}(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c^2dx^2 + d}(b \operatorname{arsinh}(cx) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

3.515 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=198

$$d\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x\sqrt{c^2dx^2+d}},x\right)+\frac{de^{-\frac{a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sin}{}\right.}{2\sqrt{c^2dx^2+d}}$$

```
[Out] (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c
*x])/b)])/(2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (
d*E^(a/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcS
inh[c*x])/b])/(2*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + d*Uninte
grable[(a + b*ArcSinh[c*x])^n/(x*Sqrt[d + c^2*d*x^2]), x]
```

Rubi [A] time = 0.141787, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A] time = 0.208022, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{Arcsinh}\,(cx))^n}{x}\sqrt{c^2dx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^2x^2+1\right)}\,(a+b\,\mathrm{asinh}\,(cx))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2+d}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x, x)
```

3.516 $\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=83

$$d\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{c^2dx^2+d}},x\right)+\frac{cd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^{n+1}}{b(n+1)\sqrt{c^2dx^2+d}}$$

```
[Out] (c*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(b*(1 + n)*Sqrt[d + c^
2*d*x^2]) + d*Unintegrable[(a + b*ArcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2])
, x]
```

Rubi [A] time = 0.146752, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A] time = 0.207683, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^n}{x^2}\sqrt{c^2dx^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^2dx^2 + d}(b\,\text{arsinh}\,(cx) + a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c^2dx^2 + d}(b\,\text{arsinh}\,(cx) + a)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,(a+b\operatorname{asinh}(cx))^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*(c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))**n/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c^{2}dx^{2}+d}\,(b\operatorname{arsinh}(cx)+a)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*(c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

3.517 $\int x^2 \left(d+c^2 d x^2\right)^{3/2} \left(a+b \sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=616

$$\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{c^2x^2+1}}+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{c^2d}}{}$$

```
[Out] -(d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c^3*(1 + n)*Sqr
t[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt
[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - 2*n)*d*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(
c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-7 -
 n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*Ar
cSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])
/b))^n) + (2^(-7 - n)*d*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a +
b*ArcSinh[c*x])/b)^n) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 +
 c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.832672, antiderivative size = 616, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5782, 5779, 5448, 3307, 2181}

$$\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{c^2x^2+1}}+\frac{d2^{-2n-7}e^{-\frac{4a}{b}}\sqrt{c^2d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -(d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c^3*(1 + n)*Sqr
t[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt
[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - 2*n)*d*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b])/(
```

```
c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-7 -
 n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*Ar
cSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])
/b))^n) + (2^(-7 - n)*d*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a +
b*ArcSinh[c*x])/b)^n) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 +
 c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\sinh^2(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^3\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{1}{16}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2x)+\frac{1}{16}(a+bx\right.\right.}{c^3\sqrt{1+c^2x^2}}$$

$$=-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}-\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\,c\right.}{32c^3\sqrt{1+c^2}}$$

$$=-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-6x}(a+b\right.}{64c^3\sqrt{1+c^2x^2}}$$

$$=-\frac{d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}\left(a+b\,s\right.}{}$$

Mathematica [A] time = 3.08736, size = 429, normalized size = 0.7

$$-\frac{d^22^{-2n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^ne^{\frac{6a}{b}}\left(b(n+1)e^{\frac{6a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\text{Gamma}\left(n+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-(
2^n*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))
/b]) - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b] + 2^n*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + A
rcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b] - 2^n*3^(1 + n)*b*
```

```
E^((8*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*Ar
cSinh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/
b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^n*E^((6*a)/b)*(2^(3 + n)
*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^2))^n + b*E^((6
*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh
[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])^2/b^2))^n))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int x^{2} \left(c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(a+b \mathrm{Arcsinh}\left(c x\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)

[Out] int(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")

[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(c^{2} d x^{4} + d x^{2}\right)} \sqrt{c^{2} d x^{2} + d} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
```

```
[Out] integral((c^2*d*x^4 + d*x^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{3}{2}} (b \operatorname{arsinh}(cx) + a)^n x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

3.518 $\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=542

$$\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\,(}{}$$

```
[Out] (5^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*c^2*E^(a/b)*Sqrt[1 + c^2
*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(16*c^2*Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*S
qrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x
]))/b])/(32*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.630958, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5782, 5779, 5448, 3308, 2181}

$$\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{c^2x^2+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{c^2dx^2+d}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```


```
[Out] (5^(-1 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*c^2*E^(a/b)*Sqrt[1 + c^2
*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a +
```

```
b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(16*c^2*Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a
+ b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(32*3^n*c^2*S
qrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x
]))/b])/(32*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\sinh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{8}(a+bx)^n\sinh(x)+\frac{3}{16}(a+bx)^n\sinh(3x)+\frac{1}{16}(\right.\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh(5x)\,dx,x,\sinh^{-1}(cx)\right)}{16c^2\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2\right.}{}$$

$$=-\frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-5x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{32c^2\sqrt{1+c^2x^2}}+\frac{\left(d\sqrt{d+c^2dx^2\right.}{}$$

$$=\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5(a}{}\right.}{32c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 1.74323, size = 390, normalized size = 0.72

$$\frac{d^2 15^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\left(3^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^n\text{Gamma}\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (15^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2*15^(1 + n)*E^(
(6*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Ga
mma[1 + n, a/b + ArcSinh[c*x]] + 3*(a/b + ArcSinh[c*x])^n*(3^n*(-((a + b*Ar
cSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b] + 5^(1 + n)
*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcS
inh[c*x]))/b] + 2*3^n*5^(1 + n)*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))
^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(-((a +
b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 3^n*E^
((10*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (5*(a + b*ArcSinh
[c*x]))/b])))/(32*c^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x
```

```
])^2/b^2))^(2*n))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int x\left(c^2dx^2+d\right)^{\frac{3}{2}}(a+b\text{Arcsinh}\,(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2dx^3+dx\right)\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^3 + d*x)*sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

3.519 $\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=420

$$\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+}}{}$$

```
[Out] (3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt[
1 + c^2*x^2]) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-3 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*
Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*d*E^((2*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh
[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(3 + n))*c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.416662, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5702, 5699, 3312, 3307, 2181}

$$\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2dx^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```


```
[Out] (3*d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt[
1 + c^2*x^2]) + (d*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n,
(-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt[1 + c^2*x^2
]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-3 - n)*d*Sqrt[d + c^2*d*x^2]*(a + b
*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*
Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*d*E^((2*a)/b
)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh
[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
```

```
h[c*x]))/b])/(2^(2*(3 + n))*c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^4(x)\,dx,x,\sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{3}{8}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)+\frac{1}{8}(a+bx)^n\cosh\right.\right.}{c\sqrt{1+c^2x^2}}$$

$$= \frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh\right.}{8c\sqrt{1+c^2x^2}}$$

$$= \frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}} + \frac{\left(d\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-4x}(a+bx)^n\right.}{16c\sqrt{1+c^2x^2}}$$

$$= \frac{3d\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{1+c^2x^2}} + \frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(c\right.}{c}$$

Mathematica [A] time = 1.44425, size = 287, normalized size = 0.68

$$\frac{d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{4\left(a+b\sinh^{-1}(cx)\right)}{b}\right)-e^{\frac{8a}{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*((-8*(a + b*ArcSinh[c*x]))/(b
 + b*n) + 8*((4*a + 4*b*ArcSinh[c*x])/(b + b*n) + Gamma[1 + n, (-2*(a + b*A
rcSinh[c*x]))/b]/(2^n*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n) - (E^((2*a
)/b)*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(2^n*(a/b + ArcSinh[c*x])^n)
) + ((a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] - E^(
(8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x])
)/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^n)))/(64*c*Sqrt[d +
c^2*d*x^2])
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int \left(c^2 d x^2 + d\right)^{\frac{3}{2}} \left(a + b \operatorname{Arcsinh}\left(cx\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2 d x^2 + d\right)^{\frac{3}{2}} \left(b \operatorname{arsinh}\left(cx\right) + a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2 d x^2 + d\right)^{\frac{3}{2}} \left(b \operatorname{arsinh}\left(cx\right) + a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n, x)
```

3.520 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=389

$$d^2\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x\sqrt{c^2dx^2+d}},x\right)+\frac{d^2 3^{-n-1}e^{-\frac{3a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+\right.}{8\sqrt{c^2dx^2+d}}$$

```
[Out] (3^(-1 - n)*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(
a + b*ArcSinh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSi
nh[c*x])/b))^n) + (5*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c*x])/b)])/(8*E^(a/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*Ar
cSinh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^
n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])/(8*Sqrt[d + c^2*d*x^2]*((a + b*ArcS
inh[c*x])/b)^n) + (3^(-1 - n)*d^2*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(8*Sqrt[d + c^2*d*x^2
]*((a + b*ArcSinh[c*x])/b)^n) + d^2*Unintegrable[(a + b*ArcSinh[c*x])^n/(x*
Sqrt[d + c^2*d*x^2]), x]
```

Rubi [A] time = 0.152764, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Defer[Int][((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A] time = 0.243552, size = 0, normalized size = 0.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int \frac{(a+b\mathrm{Arcsinh}\,(cx))^n}{x}\left(c^2dx^2+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
```

```
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

3.521 $\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=272

$$d^2\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{c^2dx^2+d}},x\right)+\frac{cd^2 2^{-n-3}e^{-\frac{2a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+\right.}{\sqrt{c^2dx^2+d}}$$

```
[Out] (3*c*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(2*b*(1 + n)*Sqrt[
d + c^2*d*x^2]) + (2^(-3 - n)*c*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^
n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a)/b)*Sqrt[d + c^2*d*x^
2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-3 - n)*c*d^2*E^((2*a)/b)*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(
Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + d^2*Unintegrable[(a + b*A
rcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x^2]), x]
```

Rubi [A] time = 0.153782, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Defer[Int][((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A] time = 0.617824, size = 0, normalized size = 0.

$$\int \frac{\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [A] time = 0.163, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^n}{x^2}\left(c^2dx^2 + d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

```
[Out] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2dx^2 + d\right)^{\frac{3}{2}}(b\,\text{arsinh}\,(cx) + a)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

3.522 $\int x^2 \left(d+c^2 d x^2\right)^{5/2} \left(a+b \sinh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=816

result too large to display

```
[Out] (-5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[1 + c^2*x^2]) + (2^(-11 - 3*n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
h[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((8*a)/b)*Sqrt[
1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*3^(-1 - n)*d^2*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*
x]))/b])/(c^3*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n)
+ (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1
 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*d^2*E^((2*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x
]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d^2*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(4 + n))*c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^
n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2
]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])/(c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.974681, antiderivative size = 816, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5782, 5779, 5448, 3307, 2181}

$$\frac{2^{-3n-11} d^2 e^{-\frac{8a}{b}} \sqrt{c^2 d x^2+d} \left(a+b \sinh^{-1}(cx)\right)^n \text{Gamma}\left(n+1,-\frac{8\left(a+b \sinh^{-1}(cx)\right)}{b}\right) \left(-\frac{a+b \sinh^{-1}(cx)}{b}\right)^{-n}}{c^3 \sqrt{c^2 x^2+1}}+\frac{2^{-n-7} 3^{-n-1} d^2 e^{-\frac{6a}{b}} \sqrt{\ }}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[1 + c^2*x^2]) + (2^(-11 - 3*n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSin
```

```
h[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((8*a)/b)*Sqrt[
1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*3^(-1 - n)*d^2*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*
x]))/b])/(c^3*E^((6*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n)
+ (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a +
b*ArcSinh[c*x])/b))^n) - (2^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[1
 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-7 - n)*d^2*E^((2*a)/b)*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x
]))/b])/(c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (d^2*E^((4*a)/
b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSin
h[c*x]))/b])/(2^(2*(4 + n))*c^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^
n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcS
inh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b])/(c^3*Sqrt[1 + c^2*x^2
]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])/(c
^3*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
```

```
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x^2\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int x^2\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\sinh^2(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^3\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5}{128}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2x)+\frac{1}{32}(a-\right.\right.}{c}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+b\right.}{128c^3\sqrt{1}}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-8x}(a\right.}{256c^3\sqrt{1+}}$$

$$=-\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 6.97482, size = 667, normalized size = 0.82

$$-\frac{d^3 2^{-3n-11}3^{-n-1}e^{-\frac{8a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(e^{\frac{8a}{b}}\left(b\left(-3^{n+1}\right)4^{n+2}(n+1)e^{\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^n\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] -((2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(-
(3^(1 + n)*b*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcSinh
[c*x]))/b]) - 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[
1 + n, (-6*(a + b*ArcSinh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1
+ n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/b] + 3^(
1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*Gamma[1 + n,
(-2*(a + b*ArcSinh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*3^(1 + n)*a*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*ArcSinh[c*x]*(-((a
 + b*ArcSinh[c*x])^2/b^2))^n - 3^(1 + n)*4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-
((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b] + 2^(3
 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
 + n, (4*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b] + 4^(2 + n)*b*E^((6
*a)/b)*n*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcSinh[c*x])
)/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (
8*(a + b*ArcSinh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-((a + b*ArcSinh[c*
x])/b))^n*Gamma[1 + n, (8*(a + b*ArcSinh[c*x]))/b])))/(b*c^3*E^((8*a)/b)*(1
 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^n))
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int x^{2} \left(c^{2} d x^{2}+d\right)^{\frac{5}{2}} \left(a +b \mathrm{Arcsinh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{n} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^4d^2x^6 + 2\,c^2d^2x^4 + d^2x^2\right)\sqrt{c^2dx^2 + d}(b\,\text{arsinh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas
")
```

```
[Out] integral((c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c^2*d*x^2 + d)*(b*arc
sinh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx) + a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x^2, x)
```

3.523 $\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=745

$$\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{c^2x^2+1}}+\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}}{}$$

```
[Out] (7^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcSinh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b
*ArcSinh[c*x])/b))^n) + (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gam
ma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)*Sqrt[1 + c
^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3^(1 - n)*d^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(128*c^2
*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5*d^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/
b)])/(128*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5
*d^2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b
*ArcSinh[c*x])/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) +
 (3^(1 - n)*d^2*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*Ar
cSinh[c*x])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) + (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])/(128
*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.785146, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5782, 5779, 5448, 3308, 2181}

$$\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{7\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{c^2x^2+1}}+\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{c^2dx^2+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (7^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcSinh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b
```

```
*ArcSinh[c*x])/b))^n) + (d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gam
ma[1 + n, (-5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)*Sqrt[1 + c
^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3^(1 - n)*d^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(128*c^2
*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5*d^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/
b)])/(128*c^2*E^(a/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (5
*d^2*E^(a/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b
*ArcSinh[c*x])/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) +
 (3^(1 - n)*d^2*E^((3*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamm
a[1 + n, (3*(a + b*ArcSinh[c*x]))/b])/(128*c^2*Sqrt[1 + c^2*x^2]*((a + b*Ar
cSinh[c*x])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x
])^n*Gamma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(128*5^n*c^2*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) + (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])/(128
*c^2*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5782

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_), x_Symbol] :> Dist[(d^IntPart[p]*(d + e*x^2)^FracPart[p])/(1 + c^2*
x^2)^FracPart[p], Int[x^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -
1] && IGtQ[m, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int x\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int x\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\sinh(x)\,dx,x,\sinh^{-1}(cx)\right)}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{64}(a+bx)^n\sinh(x)+\frac{9}{64}(a+bx)^n\sinh(3x)+\frac{5}{64}(\right.\right.}{c^2\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh(7x)\,dx,x,\sinh^{-1}(cx)\right)}{64c^2\sqrt{1+c^2x^2}}+\frac{\left(5d^2\sqrt{d+\right.}{}$$

$$=-\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-7x}(a+bx)^n\,dx,x,\sinh^{-1}(cx)\right)}{128c^2\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx\right.}{}$$

$$=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7(a+}{}\right.}{128c^2\sqrt{1+c^2x^2}}$$

Mathematica [A] time = 2.92995, size = 685, normalized size = 0.92

$$\frac{d^3105^{-n-1}e^{-\frac{7a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(5^{n+2}21^{n+1}e^{\frac{8a}{b}}\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{(}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (105^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(5^(2 + n)*21^(1
 + n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])
^2/b^2))^n*Gamma[1 + n, a/b + ArcSinh[c*x]] + 15^(1 + n)*(-((a + b*ArcSinh[
c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-7*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)
*(5*21^(1 + n)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a +
b*ArcSinh[c*x]))/b] + 9*35^(1 + n)*E^((2*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b] + 5^(2 + n)*21^(1 + n)*
E^((4*a)/b)*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*Arc
Sinh[c*x])/b)] + 35^(1 + n)*E^((8*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*Ar
cSinh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] + 8*35^(1 +
n)*E^((8*a)/b)*(-((a + b*ArcSinh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/
b^2))^n*Gamma[1 + n, (3*(a + b*ArcSinh[c*x]))/b] - 3^(2 + n)*7^(1 + n)*E^((
10*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 +
 n, (5*(a + b*ArcSinh[c*x]))/b] + 8*21^(1 + n)*E^((10*a)/b)*(-((a + b*ArcSi
nh[c*x])/b))^(2*n)*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (5*(a + b
*ArcSinh[c*x]))/b] + 15^(1 + n)*E^((12*a)/b)*(a/b + ArcSinh[c*x])^n*(-((a +
 b*ArcSinh[c*x])/b))^(3*n)*Gamma[1 + n, (7*(a + b*ArcSinh[c*x]))/b])))/(128
*c^2*E^((7*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n*(-((a +
b*ArcSinh[c*x])^2/b^2))^(2*n))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int x\left(c^{2}d x^{2}+d\right)^{\frac{5}{2}}\left(a+b\,\mathrm{Arcsinh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c^{2}d x^{2}+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{n}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c^4d^2x^5 + 2\,c^2d^2x^3 + d^2x\right)\sqrt{c^2dx^2 + d}(b\,\mathrm{arsinh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x)*sqrt(c^2*d*x^2 + d)*(b*arcsi
nh(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2 + d\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx) + a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n*x, x)
```

3.524 $\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=632

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{3d^2 2^{-2n-7}e^{-\frac{4a}{b}}}{}$$

```
[Out] (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sq
rt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c*E^((6*a)/b)*Sqr
t[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3*2^(-7 - 2*n)*d^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/
b])/(c*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (15*2
^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(
a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh
[c*x])/b))^n) - (15*2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) - (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/
(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d
^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(
a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rubi [A] time = 0.599918, antiderivative size = 632, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5702, 5699, 3312, 3307, 2181}

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{c\sqrt{c^2x^2+1}}+\frac{3d^2 2^{-2n-7}e^{-\frac{4a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```


```
[Out] (5*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(16*b*c*(1 + n)*Sq
rt[1 + c^2*x^2]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*Ar
cSinh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcSinh[c*x]))/b])/(c*E^((6*a)/b)*Sqr
t[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (3*2^(-7 - 2*n)*d^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcSinh[c*x]))/
```

```
b])/(c*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (15*2
^(-7 - n)*d^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(
a + b*ArcSinh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(-((a + b*ArcSinh
[c*x])/b))^n) - (15*2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^
2]*((a + b*ArcSinh[c*x])/b)^n) - (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b])/
(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d
^2*E^((6*a)/b)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (6*(
a + b*ArcSinh[c*x]))/b])/(c*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])/b)^n)
```

Rule 5702

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(d^(p - 1/2)*Sqrt[d + e*x^2])/Sqrt[1 + c^2*x^2], Int[(1 +
 c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x]
&& EqQ[e, c^2*d] && IGtQ[2*p, 0] &&  !(IntegerQ[p] || GtQ[d, 0])
```

Rule 5699

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[d^p/c, Subst[Int[(a + b*x)^n*Cosh[x]^(2*p + 1), x], x, Ar
cSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*
p, 0] && (IntegerQ[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
```

```
ntegerQ[m]
```

Rubi steps

$$\int \left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^6(x)\,dx,x,\sinh^{-1}(cx)\right)}{c\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{16}(a+bx)^n+\frac{15}{32}(a+bx)^n\cosh(2x)+\frac{3}{16}(a+bx)\right.\right.}{c\sqrt{1+c^2x^2}}$$

$$=\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\right.}{32c\sqrt{1+c^2}}$$

$$=\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\text{Subst}\left(\int e^{-6x}(a+b\right.}{64c\sqrt{1+c^2x^2}}$$

$$=\frac{5d^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{1+c^2x^2}}+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d+c^2dx^2}\left(a+b\right.}{}$$

Mathematica [A] time = 5.30577, size = 529, normalized size = 0.84

$$\frac{d^32^{-2n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(5b2^n3^{n+2}(n+1)e^{\frac{4a}{b}}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\sinh\right.}{b^2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*(2^n*
b*(1 + n)*(a/b + ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1
+ n, (-6*(a + b*ArcSinh[c*x]))/b] + 3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b +
ArcSinh[c*x])^(2*n)*(-((a + b*ArcSinh[c*x])/b))^n*Gamma[1 + n, (-4*(a + b*A
rcSinh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x
])^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]
))/b] - E^((6*a)/b)*(5*2^n*3^(2 + n)*b*E^((2*a)/b)*(1 + n)*(-((a + b*ArcSin
```

```
h[c*x])/b))^n*(-((a + b*ArcSinh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a + b*ArcS
inh[c*x]))/b] + 3^(2 + n)*b*E^((4*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((
a + b*ArcSinh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcSinh[c*x]))/b] + 2^
n*(-5*2^(3 + n)*3^(1 + n)*(a + b*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])^2/b^
2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcSinh[c*x])^n*(-((a + b*ArcSinh[
c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcSinh[c*x]))/b]))))/(b*c*E^((6*a)/
b)*(1 + n)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])^2/b^2))^(2*n))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left(c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b \operatorname{Arcsinh}\left(c x\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

```
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^{2} d x^{2}+d\right)^{\frac{5}{2}} \left(b \operatorname{arsinh}\left(c x\right)+a\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(c^{4} d^{2} x^{4}+2 \, c^{2} d^{2} x^{2}+d^{2}\right) \sqrt{c^{2} d x^{2}+d} \left(b \operatorname{arsinh}\left(c x\right)+a\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n, x)
```

3.525 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=755

$$d^3\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x\sqrt{c^2dx^2+d}},x\right)+\frac{d^3 5^{-n-1}e^{-\frac{5a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1\right.}{32\sqrt{c^2dx^2+d}}$$

```
[Out] (5^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-5*(
a + b*ArcSinh[c*x]))/b])/(32*E^((5*a)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcS
inh[c*x])/b))^n) - (5*3^(-1 - n)*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])
^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(32*E^((3*a)/b)*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c*x]))/b])/(8*3^n*E^((3*a)/b)*Sq
rt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (11*d^3*Sqrt[1 + c^2*x^2
]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c*x])/b)])/(16*E^(a/
b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (11*d^3*E^(a/b)*Sqr
t[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (a + b*ArcSinh[c*x])/b])
/(16*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (5*3^(-1 - n)*d^3*E^
((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcSinh[c*x]))/b])/(32*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) + (d^
3*E^((3*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (3*(a +
 b*ArcSinh[c*x]))/b])/(8*3^n*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n
) + (5^(-1 - n)*d^3*E^((5*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Ga
mma[1 + n, (5*(a + b*ArcSinh[c*x]))/b])/(32*Sqrt[d + c^2*d*x^2]*((a + b*Arc
Sinh[c*x])/b)^n) + d^3*Unintegrable[(a + b*ArcSinh[c*x])^n/(x*Sqrt[d + c^2*
d*x^2]), x]
```

Rubi [A] time = 0.155439, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Defer[Int][((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx=\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Mathematica [A] time = 0.270226, size = 0, normalized size = 0.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x, x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\mathrm{Arcsinh}\,(cx)\right)^n}{x}\left(c^2dx^2+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

```
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arsinh}\,(cx)+a\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^4d^2x^4+2\,c^2d^2x^2+d^2\right)\sqrt{c^2dx^2+d}(b\,\text{arsinh}\,(cx)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arsinh}\,(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x, x)
```

3.526 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=454

$$d^3\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^n}{x^2\sqrt{c^2dx^2+d}},x\right)+\frac{cd^32^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^n\left(-\frac{a+b\sinh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(\right.}{\sqrt{c^2dx^2+d}}$$

```
[Out] (15*c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(1 + n))/(8*b*(1 + n)*Sqrt
[d + c^2*d*x^2]) + (c*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1
+ n, (-4*(a + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*E^((4*a)/b)*Sqrt[d + c^2*
d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) + (2^(-2 - n)*c*d^3*Sqrt[1 + c^2*x^2]
*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c*x]))/b])/(E^((2*a
)/b)*Sqrt[d + c^2*d*x^2]*(-((a + b*ArcSinh[c*x])/b))^n) - (2^(-2 - n)*c*d^3
*E^((2*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (2*(a +
b*ArcSinh[c*x]))/b])/(Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[c*x])/b)^n) - (c*
d^3*E^((4*a)/b)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^n*Gamma[1 + n, (4*(a
 + b*ArcSinh[c*x]))/b])/(2^(2*(3 + n))*Sqrt[d + c^2*d*x^2]*((a + b*ArcSinh[
c*x])/b)^n) + d^3*Unintegrable[(a + b*ArcSinh[c*x])^n/(x^2*Sqrt[d + c^2*d*x
^2]), x]
```

Rubi [A] time = 0.155804, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Defer[Int][((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Rubi steps

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx=\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Mathematica [A] time = 0.638545, size = 0, normalized size = 0.

$$\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2,x]
```

```
[Out] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x])^n)/x^2, x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\mathrm{Arcsinh}\,(cx)\right)^n}{x^2}\left(c^2dx^2+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

```
[Out] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arsinh}\,(cx)+a\right)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="maxima
")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^4d^2x^4+2\,c^2d^2x^2+d^2\right)\sqrt{c^2dx^2+d}\left(b\,\mathrm{arsinh}\,(cx)+a\right)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2)*sqrt(c^2*d*x^2 + d)*(b*arcsinh
(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^n/x^2, x)
```

3.527 $\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{x^m \sinh^{-1}(ax)^n}{\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Rubi [A] time = 0.103665, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Defer[Int][(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Rubi steps

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 0.477464, size = 0, normalized size = 0.

$$\int \frac{x^m \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] Integrate[(x^m*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2], x]
```

Maple [A] time = 0.114, size = 0, normalized size = 0.

$$\int x^{m} \left(\operatorname{Arcsinh}\left(ax\right)\right)^{n} \frac{1}{\sqrt{a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m} \operatorname{arsinh}\left(ax\right)^{n}}{\sqrt{a^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^{m} \operatorname{arsinh}\left(ax\right)^{n}}{\sqrt{a^{2}x^{2}+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**m*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

3.528 $\int \frac{x^3 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=113

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,\right.}{8a^4}$$

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*Gamma[1 + n, ArcSinh[a*x]])/(8*a^4) + (3^(-1 - n)*Gamma[1 + n
, 3*ArcSinh[a*x]])/(8*a^4)
```

Rubi [A] time = 0.252861, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5779, 3312, 3308, 2181}

$$\frac{3^{-n-1}\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-3\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,\right.}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```


```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^4*(-ArcSinh[
a*x])^n) - (3*Gamma[1 + n, ArcSinh[a*x]])/(8*a^4) + (3^(-1 - n)*Gamma[1 + n
, 3*ArcSinh[a*x]])/(8*a^4)
```


Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n\sinh^3(x)\,dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{i\,\text{Subst}\left(\int\left(\frac{3}{4}ix^n\sinh(x)-\frac{1}{4}ix^n\sinh(3x)\right)dx,x,\sinh^{-1}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int x^n\sinh(3x)\,dx,x,\sinh^{-1}(ax)\right)}{4a^4}-\frac{3\,\text{Subst}\left(\int x^n\sinh(x)\,dx,x,\sinh^{-1}(ax)\right)}{4a^4}\\
&= -\frac{\text{Subst}\left(\int e^{-3x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{8a^4}+\frac{\text{Subst}\left(\int e^{3x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{8a^4}+\frac{3\,\text{Subst}\left(\int e^{-x}x^n\,dx,\right.}{8a^4}\\
&= \frac{3^{-1-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-3\sinh^{-1}(ax)\right)}{8a^4}-\frac{3\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(\right.}{8a^4}
\end{aligned}$$

Mathematica [A] time = 0.189915, size = 100, normalized size = 0.88

$$\frac{3^{-n-1}\left(-\sinh^{-1}(ax)\right)^{-n}\left(\left(-\sinh^{-1}(ax)\right)^n\left(\text{Gamma}\left(n+1,3\sinh^{-1}(ax)\right)-3^{n+2}\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)\right)+\sinh^{-1}(a\right.}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (3^(-1 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]] - 3^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(3^(2 + n)*Gamm
a[1 + n, ArcSinh[a*x]]) + Gamma[1 + n, 3*ArcSinh[a*x]])))/(8*a^4*(-ArcSinh[
a*x])^n)
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int x^3 (\text{Arcsinh}(ax))^n \frac{1}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

3.529 $\int \frac{x^2 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} \, dx$

Optimal. Leaf size=80

$$\frac{2^{-n-3} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^3} - \frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^3} - \frac{\sinh^{-1}(a}{2a^3(n+}$$

```
[Out] -ArcSinh[a*x]^(1 + n)/(2*a^3*(1 + n)) + (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1
+ n, -2*ArcSinh[a*x]])/(a^3*(-ArcSinh[a*x])^n) - (2^(-3 - n)*Gamma[1 + n, 2
*ArcSinh[a*x]])/a^3
```

Rubi [A] time = 0.193296, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5779, 3312, 3307, 2181}

$$\frac{2^{-n-3} \sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)}{a^3} - \frac{2^{-n-3}\text{Gamma}\left(n+1,2\sinh^{-1}(ax)\right)}{a^3} - \frac{\sinh^{-1}(a}{2a^3(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] -ArcSinh[a*x]^(1 + n)/(2*a^3*(1 + n)) + (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1
+ n, -2*ArcSinh[a*x]])/(a^3*(-ArcSinh[a*x])^n) - (2^(-3 - n)*Gamma[1 + n, 2
*ArcSinh[a*x]])/a^3
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n \sinh^2(x)\,dx, x, \sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{x^n}{2}-\frac{1}{2}x^n\cosh(2x)\right)dx, x, \sinh^{-1}(ax)\right)}{a^3}\\
&= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{\text{Subst}\left(\int x^n\cosh(2x)\,dx, x, \sinh^{-1}(ax)\right)}{2a^3}\\
&= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}+\frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \sinh^{-1}(ax)\right)}{4a^3}\\
&= -\frac{\sinh^{-1}(ax)^{1+n}}{2a^3(1+n)}+\frac{2^{-3-n}\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-2\sinh^{-1}(ax)\right)}{a^3}-\frac{2^{-3-n}\Gamma\left(1+n,2
\end{aligned}$$

Mathematica [A] time = 0.206218, size = 86, normalized size = 1.08

$$\frac{2^{-n-3}\left(-\sinh^{-1}(ax)\right)^{-n}\left((n+1)\sinh^{-1}(ax)^n\text{Gamma}\left(n+1,-2\sinh^{-1}(ax)\right)-\left(-\sinh^{-1}(ax)\right)^n\left((n+1)\text{Gamma}\left(n+1,2\right.\right.\right.}{a^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] (2^(-3 - n)*((1 + n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] - (-ArcSi
nh[a*x])^n*(2^(2 + n)*ArcSinh[a*x]^(1 + n) + (1 + n)*Gamma[1 + n, 2*ArcSinh
```

```
[a*x]])))/(a^3*(1 + n)*(-ArcSinh[a*x])^n)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int x^{2} \left(\operatorname{Arcsinh}\left(a x\right)\right)^{n} \frac{1}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2} \operatorname{arsinh}\left(a x\right)^{n}}{\sqrt{a^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^{2} \operatorname{arsinh}\left(a x\right)^{n}}{\sqrt{a^{2} x^{2}+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

3.530 $\int \frac{x \sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}} dx$

Optimal. Leaf size=49

$$\frac{\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a^2*(-ArcSinh[a*x])^n) + Ga
mma[1 + n, ArcSinh[a*x]]/(2*a^2)
```

Rubi [A] time = 0.117022, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5779, 3308, 2181}

$$\frac{\sinh^{-1}(ax)^n \left(-\sinh^{-1}(ax)\right)^{-n} \text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```


```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a^2*(-ArcSinh[a*x])^n) + Ga
mma[1 + n, ArcSinh[a*x]]/(2*a^2)
```


Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int x^n\sinh(x)\,dx,x,\sinh^{-1}(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int e^{-x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{2a^2}+\frac{\text{Subst}\left(\int e^{x}x^n\,dx,x,\sinh^{-1}(ax)\right)}{2a^2}\\
&= \frac{\left(-\sinh^{-1}(ax)\right)^{-n}\sinh^{-1}(ax)^n\Gamma\left(1+n,-\sinh^{-1}(ax)\right)}{2a^2}+\frac{\Gamma\left(1+n,\sinh^{-1}(ax)\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0716987, size = 43, normalized size = 0.88

$$\frac{\sinh^{-1}(ax)^n\left(-\sinh^{-1}(ax)\right)^{-n}\text{Gamma}\left(n+1,-\sinh^{-1}(ax)\right)+\text{Gamma}\left(n+1,\sinh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSinh[a*x]^n)/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[1 +
 n, ArcSinh[a*x]])/(2*a^2)
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x\left(\text{Arcsinh}\left(ax\right)\right)^n\frac{1}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{asinh}^n(ax)}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*asinh(a*x)**n/sqrt(a**2*x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^n/sqrt(a^2*x^2 + 1), x)
```

3.531 $\int \frac{\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Rubi [A] time = 0.0380282, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5675}

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{\sqrt{1+a^2x^2}}\,dx = \frac{\sinh^{-1}(ax)^{1+n}}{a(1+n)}$$

Mathematica [A] time = 0.0098283, size = 17, normalized size = 1.

$$\frac{\sinh^{-1}(ax)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x]^n/Sqrt[1 + a^2*x^2],x]
```

```
[Out] ArcSinh[a*x]^(1 + n)/(a*(1 + n))
```

Maple [A] time = 0.005, size = 18, normalized size = 1.1

$$\frac{(\mathrm{Arcsinh}\,(ax))^{1+n}}{a\,(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x)
```

```
[Out] arcsinh(a*x)^(1+n)/a/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52692, size = 212, normalized size = 12.47

$$\frac{\cosh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)\log\left(ax+\sqrt{a^2x^2+1}\right)+\log\left(ax+\sqrt{a^2x^2+1}\right)\sinh\left(n\log\left(\log\left(ax+\sqrt{a^2x^2+1}\right)\right)\right)}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(n*log(log(a*x + sqrt(a^2*x^2 + 1))))*log(a*x + sqrt(a^2*x^2 + 1)) + l
og(a*x + sqrt(a^2*x^2 + 1))*sinh(n*log(log(a*x + sqrt(a^2*x^2 + 1)))))/(a*n
 + a)
```

Sympy [A] time = 1.05296, size = 34, normalized size = 2.

$$\begin{cases} \tilde{\infty} x & \text{for } a = 0 \wedge n = -1 \\ 0^{n} x & \text{for } a = 0 \\ \frac{\log{\left(\operatorname{asinh}{\left(a x \right)} \right)}}{a} & \text{for } n = -1 \\ \frac{\operatorname{asinh}{\left(a x \right)} \operatorname{asinh}^{n}{\left(a x \right)}}{a n + a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(n, -1)), (0**n*x, Eq(a, 0)), (log(asinh(a*x
))/a, Eq(n, -1)), (asinh(a*x)*asinh(a*x)**n/(a*n + a), True))
```

Giac [A] time = 1.39808, size = 39, normalized size = 2.29

$$\frac{\log\left(ax + \sqrt{a^2x^2 + 1}\right)^{n+1}}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] log(a*x + sqrt(a^2*x^2 + 1))^(n + 1)/(a*(n + 1))
```

3.532 $\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sinh^{-1}(ax)^n}{x\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Rubi [A] time = 0.102227, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx = \int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 5.66551, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^n}{x\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Integrate[ArcSinh[a*x]^n/(x*Sqrt[1 + a^2*x^2]), x]
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{(\mathrm{Arcsinh}(ax))^n}{x} \frac{1}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{arsinh}(ax)^n}{a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^3 + x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^n(ax)}{x\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**n/(x*sqrt(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x), x)
```

3.533 $\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sinh^{-1}(ax)^n}{x^2\sqrt{a^2x^2+1}},x\right)$$

```
[Out] Unintegrable[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Rubi [A] time = 0.101782, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx = \int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Mathematica [A] time = 1.8322, size = 0, normalized size = 0.

$$\int \frac{\sinh^{-1}(ax)^n}{x^2\sqrt{1+a^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]),x]
```

```
[Out] Integrate[ArcSinh[a*x]^n/(x^2*Sqrt[1 + a^2*x^2]), x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{Arcsinh}(ax))^n}{x^2} \frac{1}{\sqrt{a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^n}{\sqrt{a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^n}{a^2x^4+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a^2*x^2 + 1)*arcsinh(a*x)^n/(a^2*x^4 + x^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asinh}^{n}(ax)}{x^{2}\sqrt{a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x)**n/x**2/(a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(asinh(a*x)**n/(x**2*sqrt(a**2*x**2 + 1)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsinh}(ax)^{n}}{\sqrt{a^{2}x^{2}+1}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x)^n/x^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/(sqrt(a^2*x^2 + 1)*x^2), x)
```

3.534 $\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=416

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{2id^2\left(c^2x^2+1\right)\sqrt{d+i}}{}$$

```
[Out] (((-2*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2]
- (3*b*c*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2]
) - (((2*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 +
c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
 c^2*x^2]) + (3*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/8 - (c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/4 + (((2*I)/3)*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x]))/c + (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*
ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.59734, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5821, 5682, 5675, 30, 5717, 5742, 5758}

$$-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{2id^2\left(c^2x^2+1\right)\sqrt{d+i}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (((-2*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2]
- (3*b*c*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2]
) - (((2*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 +
c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
 c^2*x^2]) + (3*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/8 - (c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*
x]))/4 + (((2*I)/3)*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x]))/c + (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*
ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+2icd^2x\sqrt{1-\right.}{\sqrt{1+c}}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} + \frac{\left(2icd^2\sqrt{\right.}{}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right) - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2x}{}$$

$$= -\frac{2ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} - \frac{2ibc^2d^2}{}$$

Mathematica [A] time = 1.42174, size = 361, normalized size = 0.87

$$\frac{48ad^2\sqrt{c^2x^2+1}\left(-6c^3x^3+16ic^2x^2+9cx+16i\right)\sqrt{d+icdx}\sqrt{f-icfx}+720ad^{5/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*(16*I + 9*c
*x + (16*I)*c^2*x^2 - 6*c^3*x^3) + 720*a*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 144*b*
d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^2 - Cosh[2*ArcSinh[
c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]) - (64*I)*b*d^2*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*
ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]) + 9*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*
ArcSinh[c*x]]))/(1152*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{5}{2}}\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)\sqrt{f-icfx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(bc^2d^2x^2 - 2i\,bcd^2x - bd^2\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) - \left(ac^2d^2x^2 - 2i\,acd^2x - ad^2\right)\sqrt{i\,cdx +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.535 $\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=304

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{id\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-}$$

```
[Out] ((-I/3)*b*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c
*d*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) - ((I/9)*
b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (d*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (d*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2])
```

Rubi [A] time = 0.342716, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 5821, 5682, 5675, 30, 5717}

$$\frac{d\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{id\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```


```
[Out] ((-I/3)*b*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c
*d*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) - ((I/9)*
b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (d*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + ((I/3)*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (d*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+icdx\sqrt{1+\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}} + \frac{\left(icd\sqrt{d\right.}{}$$

$$= \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right) + \frac{id\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

$$= -\frac{ibdx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} - \frac{ibc^2dx^3\sqrt{d}}{9}$$

Mathematica [A] time = 1.54343, size = 273, normalized size = 0.9

$$\frac{12ad\left(2ic^2x^2+3cx+2i\right)\sqrt{d+icdx}\sqrt{f-icfx}+36ad^{3/2}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+\frac{9bd\sqrt{d+icdx}\sqrt{f-icfx}(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*I + 3*c*x + (2*I)*c^2*x^2) +
 36*a*d^(3/2)*Sqrt[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]] + (9*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]^
2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 + c
^2*x^2] - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSinh
[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]]))
/Sqrt[1 + c^2*x^2])/(72*c)
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{3}{2}}\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)\sqrt{f-icfx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((i\,bcdx + bd)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + (i\,acdx + ad)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c*d*x + b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + s
qrt(c^2*x^2 + 1)) + (I*a*c*d*x + a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.536 $\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}$$

```
[Out] -(b*c*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + (x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.195725, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5712, 5682, 5675, 30}

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + (x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 + (Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
```

```
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right) + \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \cdots}{2\sqrt{1+c^2x^2}}$$

$$= -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right.$$

Mathematica [A] time = 0.567566, size = 233, normalized size = 1.59

$$\frac{1}{2}ax\sqrt{id(cx-i)}\sqrt{-if(cx+i)} + \frac{a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{id(cx-i)}\sqrt{-if(cx+i)}\right)}{2c} - \frac{b\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\sqrt{-d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*x*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)])/2 + (a*Sqrt[d]*Sqrt[f]*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(
2*c) - (b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
 c^2*x^2))]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*A
rcSinh[c*x]])))/(8*c*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2])
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\,\sqrt{d+icdx}\sqrt{f-icfx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1)
) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\,(icx+1)}\sqrt{-f\,(icx-1)}\,(a+b\operatorname{asinh}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(I*c*x + 1))*sqrt(-f*(I*c*x - 1))*(a + b*asinh(c*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.537 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=158

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{ibfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] (I*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (I*f*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x])
```

Rubi [A] time = 0.297835, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5712, 5821, 5675, 5717, 8}

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{ibfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] (I*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (I*f*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
```

```
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}-\frac{icfx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(ibf\sqrt{1+\right.}{\sqrt{d+icd}}$$

$$= \frac{ibfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{2bc\sqrt{d+icdx}\sqrt{f-}}$$

Mathematica [A] time = 0.587778, size = 227, normalized size = 1.44

$$\frac{2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{c^2x^2+1}\right)+2a\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-2ib\sqrt{c^2x^2+1}}{2cd\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) -
(2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
+ b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[f
]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/(2*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\sqrt{f-icfx}\frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\sqrt{icdx+d}\sqrt{-icfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)-i\sqrt{icdx+d}\sqrt{-icfx+f}a}{cdx-id},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2
+ 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*d*x - I*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-f\,(icx-1)}\,(a+b\operatorname{asinh}(cx))}{\sqrt{d\,(icx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-f*(I*c*x - 1))*(a + b*asinh(c*x))/sqrt(d*(I*c*x + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.538 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf^2\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] ((2*I)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*f^2*(1 + c^2*x^2
)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.40068, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675}

$$-\frac{f^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bf^2\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*I)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*f^2*(1 + c^2*x^2
)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
```

```
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}} - \frac{f^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,d}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

Mathematica [A] time = 1.43535, size = 283, normalized size = 1.56

$$-\frac{-\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{cx-i} + 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + \frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(c\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] -((-4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) + 2*a*Sqrt[d]*Sqrt[
f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]
/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x
^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(
Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(2*c*d^2)
```

Maple [F] time = 0.308, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\sqrt{f-icfx}\,(d+icdx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{icdx+d}\sqrt{-icfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\sqrt{-icfx+f}a}{c^2d^2x^2-2icd^2x-d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-f\,(icx-1)}\,(a+b\,\text{asinh}\,(cx))}{(d\,(icx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-f*(I*c*x - 1))*(a + b*asinh(c*x))/(d*(I*c*x + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.539 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{if^3(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibf^3\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^3\left(c^2x^2+1\right)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] (((2*I)/3)*b*f^3*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 - I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*f^3*(1 + c^2*x^2)^(5/
2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.306451, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 651, 5819, 12, 627, 43}

$$\frac{if^3(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibf^3\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bf^3\left(c^2x^2+1\right)^{5/2}\log(-cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*f^3*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 - I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*f^3*(1 + c^2*x^2)^(5/
2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int \frac{if^3(1-icx)^3}{3c\left(1+c^2x^2\right)^2}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{(1-icx)^3}{\left(1+c^2x^2\right)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{1-icx}{(1+icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(ibf^3\left(1+c^2x^2\right)^{5/2}\right)\int \left(-\frac{2}{(-i+cx)^2}+ \ldots\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ibf^3\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if^3(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} \ldots$$

Mathematica [A] time = 0.457875, size = 141, normalized size = 0.75

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(-(cx+i)\left(a\sqrt{c^2x^2+1}+bcx-ib\right)-b(cx+i)\sqrt{c^2x^2+1}\sinh^{-1}(cx)+b(cx-i)^2\log(d+icdx)\right)}{3cd^3(cx-i)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-((I + c*x)*((-I)*b + b*c*x + a*Sqrt[
1 + c^2*x^2])) - b*(I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + b*(-I + c*x)^
2*Log[d + I*c*d*x]))/(3*c*d^3*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\sqrt{f-icfx}\,(d+icdx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.13449, size = 1332, normalized size = 7.12

$$-\frac{24\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx+f}bcx+12\left(bc^2x^2+2ibcx-b\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+2\left(3c^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(24*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 12*
(b*c^2*x^2 + 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + 2*(3*c^4*d^3*x^3 - 3*I*c^3*d^3*x^2 + 3*c^2*d^3*x - 3*I
*c*d^3)*sqrt(b^2*f/(c^2*d^5))*log(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*x + 4*I*
b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(3*I*c^9*
d^3*x^4 + 6*c^8*d^3*x^3 + 3*I*c^7*d^3*x^2 + 6*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^
5)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b)) - 2*(3*c^4*d^3*x^
3 - 3*I*c^3*d^3*x^2 + 3*c^2*d^3*x - 3*I*c*d^3)*sqrt(b^2*f/(c^2*d^5))*log(1/
3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*
x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*d^3*x^4 - 6*c^8*d^3*x^3 - 3*I*c^7*d
^3*x^2 - 6*c^6*d^3*x)*sqrt(b^2*f/(c^2*d^5)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2
 + 16*b*c*x - 16*I*b)) + 3*(4*a*c^2*x^2 + 8*I*a*c*x - 4*a)*sqrt(I*c*d*x + d
```

```
)*sqrt(-I*c*f*x + f))/(12*c^4*d^3*x^3 - 12*I*c^3*d^3*x^2 + 12*c^2*d^3*x - 1
2*I*c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.540 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=459

$$\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{id\left(c^2x^2+1\right)(d+ic$$

```
[Out] ((-I/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2)
- (5*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(
3/2)) - (((2*I)/15)*b*c^2*d*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
 + c^2*x^2)^(3/2) - (b*c^3*d*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(
16*(1 + c^2*x^2)^(3/2)) - ((I/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) + ((I/5)*d*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*d*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(
3/2))
```

Rubi [A] time = 0.441307, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 5821, 5684, 5682, 5675, 30, 14, 5717, 194}

$$\frac{3dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3d(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{id\left(c^2x^2+1\right)(d+ic$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] ((-I/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2)
- (5*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(
3/2)) - (((2*I)/15)*b*c^2*d*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
 + c^2*x^2)^(3/2) - (b*c^3*d*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(
16*(1 + c^2*x^2)^(3/2)) - ((I/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)*(a + b*ArcSinh[c*x]))/4 + (3*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) + ((I/5)*d*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*d*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(
```

```
3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{id(d+icdx)^{3/2}(f}{}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{3dx(d+icdx)^{3/2}}{}$$

$$= -\frac{ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}} - \frac{5bcdx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}$$

Mathematica [A] time = 1.6868, size = 683, normalized size = 1.49

$$\frac{3600ad^{5/2}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+1920iac^4d^2fx^4\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+240}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((-1200*I)*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (1920*I)*a*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d^2*f*x
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (3840*I)*a*c^2*d^2
*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d
^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1920*I)*a
*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800
*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d^2*f*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(5/2)*f^(3/2
)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]] - (200*I)*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Ar
```

```
cSinh[c*x]] + 60*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(
(10*I)*Cosh[3*ArcSinh[c*x]] + (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((4*I)*Sqrt[1
+ c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) - (24*I)*b*d^2
*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[1
 + c^2*x^2])
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{5}{2}} \left(f-icfx\right)^{\frac{3}{2}} \left(a+b\operatorname{Arcsinh}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(i\,bc^3d^2fx^3+bc^2d^2fx^2+i\,bcd^2fx+bd^2f\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(i\,ac^3d^2fx^3+ac^2d^2f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((I*b*c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 + I*b*c*d^2*f*x + b*d^2*f)*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^3*
d^2*f*x^3 + a*c^2*d^2*f*x^2 + I*a*c*d^2*f*x + a*d^2*f)*sqrt(I*c*d*x + d)*sq
rt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.541 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=247

$$\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f-ic$$

```
[Out] (-5*b*c*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3/2
)) - (b*c^3*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^
(3/2)) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/4
 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1
 + c^2*x^2)) + (3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*
x])^2)/(16*b*c*(1 + c^2*x^2)^(3/2))
```

Rubi [A] time = 0.25224, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 5684, 5682, 5675, 30, 14}

$$\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}+\frac{1}{4}x(d+icdx)^{3/2}(f-ic$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```



```
[Out] (-5*b*c*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3/2
)) - (b*c^3*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^
(3/2)) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/4
 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1
 + c^2*x^2)) + (3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*
x])^2)/(16*b*c*(1 + c^2*x^2)^(3/2))
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{\left(3(d+icdx)^{3/2}(f-\right.}{}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right) + \frac{3x(d+icdx)^{3/2}(f-}{8}$$

$$= -\frac{5bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}} - \frac{bc^3x^4(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}} +$$

Mathematica [A] time = 0.878913, size = 352, normalized size = 1.43

$$\frac{48ad^{3/2}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+32ac^3dfx^3\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+80acdfx\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (80*a*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32*a*
c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 24*b*d*
f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 16*b*d*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - b*d*f*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 48*a*d^(3/2)*f^(3/2)*Sqrt[1 + c^2*x^2
]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 4*b*
d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(8*Sinh[2*ArcSinh[c*x]
] + Sinh[4*ArcSinh[c*x]]))/(128*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{3}{2}}\left(f-icfx\right)^{\frac{3}{2}}(a+b\mathrm{Arcsinh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc^2dfx^2 + bdf\right)\sqrt{icdx + d}\sqrt{-icfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(ac^2dfx^2 + adf\right)\sqrt{icdx + d}\sqrt{-icfx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^2*d*f*x^2 + b*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d*f*x^2 + a*d*f)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.542 $\int \sqrt{d+icdx}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=304

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{if\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-}$$

```
[Out] ((I/3)*b*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c*
f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + ((I/9)*b
*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (f*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c
^2*x^2])
```

Rubi [A] time = 0.340541, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 5821, 5682, 5675, 30, 5717}

$$\frac{f\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}-\frac{if\left(c^2x^2+1\right)\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] ((I/3)*b*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] - (b*c*
f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(4*Sqrt[1 + c^2*x^2]) + ((I/9)*b
*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] + (f*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/2 - ((I/3)*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c
^2*x^2])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
+ e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (f-icfx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-icfx\sqrt{1+\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} - \frac{\left(icf\sqrt{d\right.}{}$$

$$= \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right) - \frac{if\sqrt{d+icdx}\sqrt{f-icfx}}{}$$

$$= \frac{ibfx\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{ibc^2fx^3\sqrt{d}}{9\sqrt{}}$$

Mathematica [A] time = 1.54905, size = 273, normalized size = 0.9

$$\frac{12af\left(-2ic^2x^2+3cx-2i\right)\sqrt{d+icdx}\sqrt{f-icfx}+36a\sqrt{d}f^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+\frac{9bf\sqrt{d+icdx}\sqrt{f-icf}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (12*a*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*I + 3*c*x - (2*I)*c^2*x^2)
+ 36*a*Sqrt[d]*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]] + (9*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(2*ArcSinh[c*x]
^2 - Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]))/Sqrt[1 +
c^2*x^2] + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(9*c*x - 3*ArcSin
h[c*x]*(3*Sqrt[1 + c^2*x^2] + Cosh[3*ArcSinh[c*x]]) + Sinh[3*ArcSinh[c*x]])
)/Sqrt[1 + c^2*x^2])/(72*c)
```

Maple [F] time = 0.259, size = 0, normalized size = 0.

$$\int \left(f-icfx\right)^{\frac{3}{2}}\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)\sqrt{d+icdx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-i\,bcfx+bf\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(-i\,acfx+af\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((-I*b*c*f*x + b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (-I*a*c*f*x + a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.543 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=266

$$\frac{3f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{\cdots}}{4\sqrt{d+icdx}\cdots}$$

```
[Out] ((2*I)*b*f^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
b*c*f^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((
2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]) - (f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]) + (3*f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b
*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rubi [A] time = 0.464772, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{3f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcf^2x^2\sqrt{\cdots}}{4\sqrt{d+icdx}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((2*I)*b*f^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
b*c*f^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((
2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]) - (f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]) + (3*f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b
*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
+ e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{2icf^2x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{c^2f^2x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2icf^2\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(c^2f\right.}{}$$

$$= -\frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f^2\sqrt{1}}{2}$$

$$= \frac{2ibf^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] time = 1.05654, size = 344, normalized size = 1.29

$$\frac{12a\sqrt{d}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) - 16iaf\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} - 4acfx\sqrt{c^2x^2+1}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((16*I)*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*f*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*f*(4*I + c*x)*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Co
sh[2*ArcSinh[c*x]] + 12*a*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*d*Sqrt[1 + c^2*x^
2])
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\left(f-icfx\right)^{\frac{3}{2}}\frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(bcfx+i\,bf\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(acfx+i\,af\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}{cdx-i\,d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
))/(c*d*x - I*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.544 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{3f^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] ((-I)*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) + ((4*I)*f^3*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*
d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*f^3*(1 + c^2*x^2)^(3/2
)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 (4*b*f^3*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

Rubi [A] time = 0.474088, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675, 5717, 8}

$$-\frac{3f^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{if^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4if^3(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-I)*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) + ((4*I)*f^3*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*
d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x
]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*f^3*(1 + c^2*x^2)^(3/2
)*(a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 (4*b*f^3*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c
*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{4i\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}} - \frac{3f^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{icf^3x\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x^2}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3\left(1+c^2x^2\right)^{3/2}\right)\int \frac{a+b\sinh^{-}}{\sqrt{1+c^2}}}{(d+icdx)^{3/2}(f-icfx)^{3}}$$

$$= \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3}{}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3}{}$$

$$= -\frac{ibf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3}{}$$

Mathematica [A] time = 2.26867, size = 514, normalized size = 1.81

$$-\frac{6af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}}+\frac{2af(5+icx)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(cx-i)}-\frac{bf\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\left(\log\left(c^2x^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((2*a*f*(5 + I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^2*(-I + c*x)) -
 (6*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]])/d^(3/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-
4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))
)/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) +
 (2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]]
 - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[
1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\left(f-icfx\right)^{\frac{3}{2}}(d+icdx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ibcfx - bf\right)\sqrt{icdx + d}\sqrt{-icfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(iacfx - af\right)\sqrt{icdx + d}\sqrt{-icfx + f}}{c^2d^2x^2 - 2icd^2x - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c*f*x - b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (I*a*c*f*x - a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.545 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=364

$$-\frac{2if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(c}{c(d+icdx)^{5/2}(f}$$

```
[Out] (((4*I)/3)*b*f^4*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(1 - I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((2*
I)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^4*(1 + c^
2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.383732, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 669, 653, 215, 5819, 627, 43, 31, 5675}

$$-\frac{2if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(c}{c(d+icdx)^{5/2}(f}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```


```
[Out] (((4*I)/3)*b*f^4*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(1 - I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((2*
I)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^4*(1 + c^
2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
```

```
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.}{c(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2if^4(1-icx)\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.}{c(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= -\frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} -$$

$$= \frac{4ibf^4\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^4(1-}{3}$$

Mathematica [A] time = 5.68736, size = 706, normalized size = 1.94

$$\frac{12af^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{5/2}} - \frac{16af(2cx-i)\sqrt{d+icdx}\sqrt{f-icfx}}{d^3(cx-i)^2} - \frac{bf\sqrt{d+icdx}\sqrt{f-icfx}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\left(2\sinh\left(\frac{1}{2}\,\mathrm{s}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((-16*a*f*(-I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(d^3*(-I + c*x)
^2) + (12*a*f^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]])/d^(5/2) - (b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSi
nh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I
)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (7*I)*Log[
1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(
8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*Log[1 + c^2*x^2])) + 2*(4 -
(4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]]
 + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh
[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[Ar
cSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])^4) + ((2*I)*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/
2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] -
 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*
x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log
[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1
 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*(4 + (2 + Sqrt[1 + c^2*x^
2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[
c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F] time = 0.292, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx)) \left(f - icfx\right)^{\frac{3}{2}} (d + icdx)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bcfx+ibf\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(acfx+iaf\right)\sqrt{icdx+d}\sqrt{-icfx+f}}{c^3d^3x^3-3ic^2d^3x^2-3cd^3x+id^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*f*x + I*b*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*f*x + I*a*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.546 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=344

$$\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{16\left(c^2x^2+1\right)^2}+\frac{5(d+icdx)^{5/2}(f-ic}{32bc(}$$

```
[Out] (-25*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^2)^(5/
2)) - (5*b*c^3*x^4*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^
2)^(5/2)) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*Sqrt[1 + c^2*x^2])/(
36*c) + (x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/6
+ (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16*(1
 + c^2*x^2)^2) + (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSin
h[c*x]))/(24*(1 + c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*(1 + c^2*x^2)^(5/2))
```

Rubi [A] time = 0.303869, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5712, 5684, 5682, 5675, 30, 14, 261}

$$\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{16\left(c^2x^2+1\right)^2}+\frac{5(d+icdx)^{5/2}(f-ic}{32bc(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (-25*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^2)^(5/
2)) - (5*b*c^3*x^4*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(96*(1 + c^2*x^
2)^(5/2)) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*Sqrt[1 + c^2*x^2])/(
36*c) + (x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/6
+ (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16*(1
 + c^2*x^2)^2) + (5*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSin
h[c*x]))/(24*(1 + c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a
 + b*ArcSinh[c*x])^2)/(32*b*c*(1 + c^2*x^2)^(5/2))
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
```

```
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\left(1+c^2x^2\right)^{5/2}}$$

$$= \frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{\left(5(d+icdx)^{5/2}(f\right.}{}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5}$$

$$= -\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}}{36c}+\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5}$$

$$= -\frac{25bcx^2(d+icdx)^{5/2}(f-icfx)^{5/2}}{96\left(1+c^2x^2\right)^{5/2}}-\frac{5bc^3x^4(d+icdx)^{5/2}(f-icfx)^{5}}{96\left(1+c^2x^2\right)^{5/2}}$$

Mathematica [A] time = 1.15647, size = 481, normalized size = 1.4

$$\frac{384ac^5d^2f^2x^5\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+1248ac^3d^2f^2x^3\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+1584acd^2f^2x\sqrt{c^2x^2+1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (1584*a*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] +
 1248*a*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^
2] + 384*a*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + 360*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 -
270*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 27
*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] - 2*b*d
^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSinh[c*x]] + 720*a*d^(
5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]] + 12*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x]*(45*Sinh[2*ArcSinh[c*x]] + 9*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcS
```

```
inh[c*x]]))/(2304*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{5}{2}} \left(f-icfx\right)^{\frac{5}{2}} (a+b\mathrm{Arcsinh}\,(cx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bc^4d^2f^2x^4+2\,bc^2d^2f^2x^2+bd^2f^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^4d^2f^2x^4+2\,ac^2d^2f^2x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((b*c^4*d^2*f^2*x^4 + 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^4*d^2*f^2*x^4
+ 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.547 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=459

$$\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}-\frac{if\left(c^2x^2+1\right)(d+ic}{}$$

```
[Out] ((I/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) -
 (5*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3
/2)) + (((2*I)/15)*b*c^2*f*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
+ c^2*x^2)^(3/2) - (b*c^3*f*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
6*(1 + c^2*x^2)^(3/2)) + ((I/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)*(a + b*ArcSinh[c*x]))/4 + (3*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) - ((I/5)*f*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*f*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(3
/2))
```

Rubi [A] time = 0.425982, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 5821, 5684, 5682, 5675, 30, 14, 5717, 194}

$$\frac{3fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{8\left(c^2x^2+1\right)}+\frac{3f(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\left(c^2x^2+1\right)^{3/2}}-\frac{if\left(c^2x^2+1\right)(d+ic}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```



```
[Out] ((I/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1 + c^2*x^2)^(3/2) -
 (5*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(16*(1 + c^2*x^2)^(3
/2)) + (((2*I)/15)*b*c^2*f*x^3*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
+ c^2*x^2)^(3/2) - (b*c^3*f*x^4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/(1
6*(1 + c^2*x^2)^(3/2)) + ((I/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2))/(1 + c^2*x^2)^(3/2) + (f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)*(a + b*ArcSinh[c*x]))/4 + (3*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)) - ((I/5)*f*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/c + (3*f*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(16*b*c*(1 + c^2*x^2)^(3
```

```
/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-i\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)-\frac{if(d+icdx)^{3/2}(f}{}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3fx(d+icdx)^{3/2}}{}$$

$$= \frac{ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/2}}-\frac{5bcfx^2(d+icdx)^{3/2}(f-icfx)^{3/2}}{16\left(1+c^2x^2\right)^{3/2}}+$$

Mathematica [A] time = 1.59681, size = 683, normalized size = 1.49

$$\frac{3600ad^{3/2}f^{5/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-1920iac^4df^2x^4\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+240}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((1200*I)*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (1920*I)*a*d*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 6000*a*c*d*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (3840*I)*a*c^2*d*f^
2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2400*a*c^3*d*
f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1920*I)*a*
c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1800*
b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 1200*b*d*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 75*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 3600*a*d^(3/2)*f^(5/2)
*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]] + (200*I)*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*Arc
```

```
Sinh[c*x]] + 60*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((
-10*I)*Cosh[3*ArcSinh[c*x]] - (2*I)*Cosh[5*ArcSinh[c*x]] + 5*((-4*I)*Sqrt[1
 + c^2*x^2] + 8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])) + (24*I)*b*d*
f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]])/(9600*c*Sqrt[
1 + c^2*x^2])
```

Maple [F] time = 0.253, size = 0, normalized size = 0.

$$\int (d + icdx)^{\frac{3}{2}} \left(f - icfx\right)^{\frac{5}{2}} (a + b\mathrm{Arcsinh}\,(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(-i\,bc^3df^2x^3 + bc^2df^2x^2 - i\,bcdf^2x + bdf^2\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(-i\,ac^3df^2x^3 + ac^2d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="fricas")
```

```
[Out] integral((-I*b*c^3*d*f^2*x^3 + b*c^2*d*f^2*x^2 - I*b*c*d*f^2*x + b*d*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^
3*d*f^2*x^3 + a*c^2*d*f^2*x^2 - I*a*c*d*f^2*x + a*d*f^2)*sqrt(I*c*d*x + d)*
sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x)),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.548 $\int \sqrt{d+icdx}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx$

Optimal. Leaf size=416

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{2if^2\left(c^2x^2+1\right)\sqrt{d+i}}{}$$

```
[Out] (((2*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] -
 (3*b*c*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c
^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
c^2*x^2]) + (3*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/8 - (c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/4 - (((2*I)/3)*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/c + (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*A
rcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.571439, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5821, 5682, 5675, 30, 5717, 5742, 5758}

$$-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)+\frac{5f^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}-\frac{2if^2\left(c^2x^2+1\right)\sqrt{d+i}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] (((2*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c^2*x^2] -
 (3*b*c*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/Sqrt[1 + c
^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(16*Sqrt[1 +
c^2*x^2]) + (3*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/8 - (c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x
]))/4 - (((2*I)/3)*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/c + (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*A
rcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (f-icfx)^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)-2icf^2x\sqrt{1}\right.}{\sqrt{1+c}}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)dx}{\sqrt{1+c^2x^2}} - \frac{\left(2icf^2\right.}{}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-}$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{4\sqrt{1+c^2x^2}} + \frac{2ibc^2f^2x}{}$$

$$= \frac{2ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}}{3\sqrt{1+c^2x^2}} - \frac{3bcf^2x^2\sqrt{d+icdx}\sqrt{f-icfx}}{16\sqrt{1+c^2x^2}} + \frac{2ibc^2f^2}{}$$

Mathematica [A] time = 1.18699, size = 565, normalized size = 1.36

$$\frac{-288ac^3f^2x^3\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-768iac^2f^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+432acf^2x\sqrt{c^2x^2+1}\sqrt{d+ic}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((576*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (768*I)*a*f^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 432*a*c*f^2*x*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (768*I)*a*c^2*f^2*x^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 288*a*c^3*f^2*x^3*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 360*b*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 - 144*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]*Cosh[2*ArcSinh[c*x]] + 9*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[4*ArcSinh[c*x]] + 720*a*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x
+ Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (64*I)*b*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 12*b*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-48*I)*Sqrt[1 + c^2*x^2] - (16*I)*Co
sh[3*ArcSinh[c*x]] + 24*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]))/(11
52*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int \left(f - icfx\right)^{\frac{5}{2}} (a + b\text{Arcsinh}\,(cx)) \sqrt{d + icdx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(bc^2f^2x^2+2i\,bcf^2x-bf^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(ac^2f^2x^2+2i\,acf^2x-af^2\right)\sqrt{i\,cdx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) - (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError
```

3.549 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=381

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{11if^3\left(c\right.}{3c}$$

```
[Out] (((11*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) - ((I/9)*b*c^2*f^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```

Rubi [A] time = 0.627529, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{11if^3\left(c\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```


```
[Out] (((11*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) - ((I/9)*b*c^2*f^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
)
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{3icf^3x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{3c^2f^3x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{ic^3f^3x}{}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3icf^3\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(3c^2\right.}{}$$

$$= -\frac{3if^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3f^3x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{icf^3}{}$$

$$= \frac{3ibf^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3\left(\right.}{3}$$

$$= \frac{11ibf^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{ibc^2f^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{11if^3}{}$$

Mathematica [A] time = 1.63091, size = 465, normalized size = 1.22

$$\frac{24iac^2f^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} - 108acf^2x\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} - 264iaf^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((264*I)*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (8*I)*b*c^3*f^2*x^
3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (264*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*Sqrt[1 + c^2*x^2] + (24*I)*a*c^2*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f
```

```
 - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*ArcSinh[c*x]^2 + 27*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcS
inh[c*x]] - 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(5*
I + 2*c*x)*Sqrt[1 + c^2*x^2] - I*Cosh[3*ArcSinh[c*x]]) + 180*a*Sqrt[d]*f^(5
/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]])/(72*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\left(f-icfx\right)^{\frac{5}{2}}\frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(i\,bc^2f^2x^2-2\,bcf^2x-i\,bf^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(i\,ac^2f^2x^2-2\,acf^2x-i\,af^2\right)\sqrt{i\,cc}}{cdx-i\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*f^2*x^2 - 2*b*c*f^2*x - I*b*f^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*f^2*x^2 - 2*a*c*f^2*x
 - I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.550 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=518

$$\frac{5if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sin\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/}}$$

```
[Out] (((-3*I)/2)*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b*c*f^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) + (5*b*f^4*(1 - I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)
/(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((2*I)*f^4*(1 - I*c*x)^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)) + (((15*I)/2)*f^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + (((5*I)/2)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(
a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*f^4*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^4*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.41613, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 669, 671, 641, 215, 5819, 627, 43, 5675}

$$\frac{5if^4(1-icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{15if^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2if^4(1-icx)^3\left(c^2x^2+1\right)\left(a+b\sin\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] (((-3*I)/2)*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b*c*f^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) + (5*b*f^4*(1 - I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)
/(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((2*I)*f^4*(1 - I*c*x)^3*(
1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/
2)) + (((15*I)/2)*f^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + (((5*I)/2)*f^4*(1 - I*c*x)*(1 + c^2*x^2)^2*(
```

```
a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*f^4*
(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^4*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
```

```
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{2if^4(1-icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15if^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx\right.}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2if^4(1-icx)^3\left(1+\right.}{c(d+icdx}$$

$$= -\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f}$$

$$= -\frac{15ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bf^4\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f}$$

$$= -\frac{3ibf^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcf^4x^2\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bf^4(1-icx)^2\left(1+c^2\right.}{4c(d+icdx)^{3/2}(f-ic}$$

Mathematica [A] time = 3.84289, size = 779, normalized size = 1.5

$$\frac{4af^2\left(c^2x^2+7icx+24\right)\sqrt{d+icdx}\sqrt{f-icfx}}{d^2(cx-i)} - \frac{60af^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{d^{3/2}} + \frac{bf^2\sqrt{d+icdx}\sqrt{f-icfx}\left(2\sinh^{-1}(cx)\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\left(-8\sqrt{c^2x^2+1}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((4*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 + (7*I)*c*x + c^2*x^2))/(
d^2*(-I + c*x)) - (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]])/d^(3/2) - (4*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSinh[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) +
 ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*
Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Si
nh[ArcSinh[c*x]/2])) + (16*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(Arc
Sinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*Arc
Tan[Coth[ArcSinh[c*x]/2]] - I*Log[1 + c^2*x^2])*((-I)*Cosh[ArcSinh[c*x]/2]
+ Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]*(I*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcS
inh[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(d^2*Sqrt[1
```

```
+ c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I
*Sinh[ArcSinh[c*x]/2]) - (Cosh[2*ArcSinh[c*x]] + 8*((2*I)*c*x + (4*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^
2*x^2] + I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((8*I)*(1 + Sqrt[1
+ c^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(d^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh
[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Maple [F] time = 0.292, size = 0, normalized size = 0.

$$\int (a + b\operatorname{Arcsinh}(cx)) \left(f - icfx\right)^{\frac{5}{2}} (d + icdx)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bc^2f^2x^2 + 2i\,bcf^2x - bf^2\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(ac^2f^2x^2 + 2i\,acf^2x - af^2\right)\sqrt{i\,cdx + \cdots}}{c^2d^2x^2 - 2i\,cd^2x - d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*f^2*x^2 + 2*I*b*c*f^2*x - b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*f^2*x^2 + 2*I*a*c*f^2*x -
 a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x -
d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.551 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=472

$$-\frac{5if^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{10if^5(1-icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-icx)^4\left(c^2x^2+1\right)\left(a+b\right.}{3c(d+icdx)^{5/2}(f-icfx}$$

```
[Out] (I*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (((8*I)/3)*b*f^5*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f
- I*c*f*x)^(5/2)) - (5*b*f^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^5*(1 - I*c*x)^4*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (
((10*I)/3)*f^5*(1 - I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((5*I)*f^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*f^5*(1 + c^2*x
^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (28*b*f^5*(1 + c^2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.444465, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 669, 641, 215, 5819, 627, 43, 5675}

$$-\frac{5if^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{10if^5(1-icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^5(1-icx)^4\left(c^2x^2+1\right)\left(a+b\right.}{3c(d+icdx)^{5/2}(f-icfx}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```


```
[Out] (I*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
 (((8*I)/3)*b*f^5*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f
- I*c*f*x)^(5/2)) - (5*b*f^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^5*(1 - I*c*x)^4*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (
((10*I)/3)*f^5*(1 - I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d +
I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((5*I)*f^5*(1 + c^2*x^2)^3*(a + b*Arc
Sinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*f^5*(1 + c^2*x
^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*
```

```
c*f*x)^(5/2)) + (28*b*f^5*(1 + c^2*x^2)^(5/2)*Log[I - c*x])/(3*c*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^5\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^5(1-icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{10if^5(1-icx)^2\left(1+c^2x^2\right)^2\left(a+b\right.}{3c(d+icdx)^{5/2}(f-icfx\ldots}$$

$$= \frac{5ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{10if\ldots}{\ldots}$$

$$= \frac{5ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-icx)^4\left(1\ldots\right.}{3c(d+ic\ldots}$$

$$= \frac{5ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^5(1-icx)^4\left(1\ldots\right.}{3c(d+ic\ldots}$$

$$= \frac{ibf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibf^5\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bf^5\left(1+c^2\ldots\right.}{2c(d+icdx\ldots}$$

Mathematica [B] time = 7.29367, size = 1005, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (((-4*I)*a*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-23 - (34*I)*c*x + 3*c^
2*x^2))/(d^3*(-I + c*x)^2) + (60*a*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/d^(5/2) - (2*b*f^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*Ar
cSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[Ar
cSinh[c*x]/2]] + (7*I)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[
Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 21*L
og[1 + c^2*x^2])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*A
rcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2*x^2] + Sqrt[1 + c^2*x^2]*(ArcS
inh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 7
*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(d^3*(I + c*x)*(Cosh[ArcSinh[c*x
]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((2*I)*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*Arc
Sinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - (I/2)*Log[1
+ c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[C
oth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((2 + Sqrt[1 + c^2*x^2])
*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[ArcSinh[c*x]/2]] + (I
/2)*(4 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/
(d^3*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + (b*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[
c*x]/2])*(2*(4 + (6*I)*c*x - 6*c^2*x^2 + 52*(-I + c*x)*ArcTan[Coth[ArcSinh[
c*x]/2]] + 13*(1 + I*c*x)*Log[1 + c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2]) + 18*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])^3 + ArcSinh[c*x]*((-24*I)*Cosh[ArcSinh[c*x]/2] - (35*I)*Cosh[(3*A
rcSinh[c*x])/2] + (3*I)*Cosh[(5*ArcSinh[c*x])/2] - 24*Sinh[ArcSinh[c*x]/2]
+ 35*Sinh[(3*ArcSinh[c*x])/2] + 3*Sinh[(5*ArcSinh[c*x])/2])))/(d^3*(I + c*x
)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx)) \left(f - icfx\right)^{\frac{5}{2}} (d + icdx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-i\,bc^2f^2x^2+2\,bcf^2x+i\,bf^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(-i\,ac^2f^2x^2+2\,acf^2x+i\,af^2\right)\sqrt{}}{c^3d^3x^3-3i\,c^2d^3x^2-3\,cd^3x+i\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*f^2*x^2 + 2*b*c*f^2*x + I*b*f^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*f^2*x^2 + 2*a*c*f^2
*x + I*a*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*
d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2),x, algorit
hm="giac")

[Out] Exception raised: AttributeError
```

3.552 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=381

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11id^3\left(\cdots\right.}{3\cdots}$$

```
[Out] (((-11*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + ((I/9)*b*c^2*d^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/
(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^3*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

Rubi [A] time = 0.629372, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11id^3\left(\cdots\right.}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] (((-11*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + ((I/9)*b*c^2*d^3*x^3*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/
(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^3*Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{3icd^3x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}-\frac{3c^2d^3x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}-\frac{ic^3d^3\cdots}{\cdots}\right.}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{\left(3icd^3\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{\left(3c\cdots\right.}{\cdots}$$

$$= \frac{3id^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x\cdots}{\cdots}$$

$$= -\frac{3ibd^3x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11i\cdots}{\cdots}$$

$$= -\frac{11ibd^3x\sqrt{1+c^2x^2}}{3\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{3bcd^3x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{ibc^2d^3x^3\sqrt{1+c^2x^2}}{9\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11\cdots}{\cdots}$$

Mathematica [A] time = 1.66686, size = 465, normalized size = 1.22

$$\frac{-24iac^2d^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-108acd^2x\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+264iad^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-264*I)*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (8*I)*b*c^3*d^2*x
^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (264*I)*a*d^2*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 108*a*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*Sqrt[1 + c^2*x^2] - (24*I)*a*c^2*d^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[
```

```
f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 90*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*ArcSinh[c*x]^2 + 27*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*Arc
Sinh[c*x]] - 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(9*(-
5*I + 2*c*x)*Sqrt[1 + c^2*x^2] + I*Cosh[3*ArcSinh[c*x]]) + 180*a*d^(5/2)*Sq
rt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]])/(72*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.293, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))\,(d + icdx)^{\frac{5}{2}}\,\frac{1}{\sqrt{f - icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-i\,bc^2d^2x^2 - 2\,bcd^2x + i\,bd^2\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right) + \left(-i\,ac^2d^2x^2 - 2\,acd^2x + i\,ad^2\right)\sqrt{i\,c}}{cfx + i\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c^2*d^2*x^2 - 2*b*c*d^2*x + I*b*d^2)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a*c^2*d^2*x^2 - 2*a*c*d^2
*x + I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.553 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=266

$$\frac{3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c}}{4\sqrt{d+icdx}}$$

```
[Out] ((-2*I)*b*d^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
(b*c*d^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
(2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]) + (3*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rubi [A] time = 0.465965, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5712, 5821, 5675, 5717, 8, 5758, 30}

$$\frac{3d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c}}{4\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*b*d^2*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
(b*c*d^2*x^2*Sqrt[1 + c^2*x^2])/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (
(2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]) + (3*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*
b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{2icd^2x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} - \frac{c^2d^2x^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2icd^2\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(c^2d^2 \right.}{}$$

$$= \frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d^2\sqrt{1+}}{2bc}$$

$$= -\frac{2ibd^2x\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+c^2x^2}}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(c\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] time = 1.02759, size = 344, normalized size = 1.29

$$12ad^{3/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+16iad\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-4acdx\sqrt{c^2x^2+1}\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-16*I)*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a*c*d*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*b*d*(-4*I + c*x)*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + 6*b*d*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] + 12*a*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(8*c*f*Sqrt[1 + c^2*
x^2])
```

Maple [F] time = 0.288, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(cx))(d+icdx)^{\frac{3}{2}}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(bcdx-i\,bd)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+(acdx-i\,ad)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}}{cfx+i\,f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
))/(c*f*x + I*f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.554 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=158

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{ibdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] ((-I)*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (I*d
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x])
```

Rubi [A] time = 0.290386, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5712, 5821, 5675, 5717, 8}

$$\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{ibdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-I)*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (I*d
*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{icdx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(ibd\sqrt{1+c^2\right.}{\sqrt{d+icdx}\sqrt{}}$$

$$= -\frac{ibdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}\left(a+b\sin\right.}{2bc\sqrt{d+icdx}\sqrt{f}}$$

Mathematica [A] time = 0.583145, size = 227, normalized size = 1.44

$$\frac{-2i\sqrt{d+icdx}\sqrt{f-icfx}\left(bcx-a\sqrt{c^2x^2+1}\right)+2a\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+2ib\sqrt{c^2x^2}}{2cf\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-2*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(b*c*x - a*Sqrt[1 + c^2*x^2]) +
 (2*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]
 + b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2 + 2*a*Sqrt[d]*Sqrt[
f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]])/(2*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.298, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\sqrt{d+icdx}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\sqrt{icdx+d}\sqrt{-icfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+i\sqrt{icdx+d}\sqrt{-icfx+f}a}{cfx+if},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c*f*x + I*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{d\,(icx+1)}\,(a+b\,\text{asinh}\,(cx))}{\sqrt{-f\,(icx-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(I*c*x + 1))*(a + b*asinh(c*x))/sqrt(-f*(I*c*x - 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

3.555 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rubi [A] time = 0.169565, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {5712, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] time = 0.438732, size = 113, normalized size = 1.92

$$\frac{a\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}+\frac{b\sqrt{c^2x^2+1}\sinh^{-1}(cx)^2}{2c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(2*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (a*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
)/(c*Sqrt[d]*Sqrt[f])
```

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\frac{1}{\sqrt{d+icdx}}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{icdx+d}\sqrt{-icfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\sqrt{-icfx+f}a}{c^2dfx^2+df},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1
)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f*x^2 + d*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{asinh}(cx)}{\sqrt{d(icx+1)}\sqrt{-f(icx-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(I*c*x + 1))*sqrt(-f*(I*c*x - 1))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.556 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=111

$$\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (b*f*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.241307, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 637, 5819, 12, 627, 31}

$$\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bf\left(c^2x^2+1\right)^{3/2}\log(-cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (b*f*(1 + c^2*x^2)^(3/2)*Log[I - c*x])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx &= \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{f(i+cx)}{c\left(1+c^2x^2\right)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i+cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i+cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\\
&= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bf\left(1+c^2x^2\right)^{3/2}\log(i-cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.397476, size = 113, normalized size = 1.02

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a\sqrt{c^2x^2+1}+b\sqrt{c^2x^2+1}\sinh^{-1}(cx)+b(-cx+i)\log(d+icdx)\right)}{cd^2f(cx-i)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a*Sqrt[1 + c^2*x^2] + b*Sqrt[1 + c^2*
x^2]*ArcSinh[c*x] + b*(I - c*x)*Log[d + I*c*d*x]))/(c*d^2*f*(-I + c*x)*Sqrt
[1 + c^2*x^2])
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\,(d+icdx)^{-\frac{3}{2}}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31198, size = 1011, normalized size = 9.11

$$\frac{2\sqrt{icdx+d}\sqrt{-icfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(c^2d^2fx-icd^2f\right)\sqrt{\frac{b^2}{c^2d^3f}}\log\left(\frac{\left(-2ibc^6x^2-4bc^5x+4ibc^4\right)\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx}}{16bc^3x^3-16i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
+ (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(((-2*I*b*c^6*x^2 - 4*
b*c^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
 + 2*(I*c^9*d^2*f*x^4 + 2*c^8*d^2*f*x^3 + I*c^7*d^2*f*x^2 + 2*c^6*d^2*f*x)*
sqrt(b^2/(c^2*d^3*f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b))
 - (c^2*d^2*f*x - I*c*d^2*f)*sqrt(b^2/(c^2*d^3*f))*log(((-2*I*b*c^6*x^2 - 4
*b*c^5*x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
) + 2*(-I*c^9*d^2*f*x^4 - 2*c^8*d^2*f*x^3 - I*c^7*d^2*f*x^2 - 2*c^6*d^2*f*x
)*sqrt(b^2/(c^2*d^3*f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b
)) + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d^2*f*x - I*c*d^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{asinh}(cx)}{(d(icx+1))^{\frac{3}{2}}\sqrt{-f(icx-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/((d*(I*c*x + 1))**(3/2)*sqrt(-f*(I*c*x - 1))),
x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.557 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=295

$$\frac{f^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^2\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] ((I/3)*b*f^2*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*b*f
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*f^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.337132, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 653, 191, 5819, 627, 44, 203, 260}

$$\frac{f^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2if^2(1-icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf^2\left(c^2x^2+1\right)^{5/2}}{3c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] ((I/3)*b*f^2*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 - I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*b*f
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*f^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(\right.\right.}{}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2ib\right.}{3}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2\left(\right.}{6c\left(\right.}$$

$$= \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf^2\left(\right.}{6c\left(\right.}$$

$$= \frac{ibf^2\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x}{}$$

$$= \frac{ibf^2\left(1+c^2x^2\right)^{5/2}}{3c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2if^2(1-icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x}{}$$

Mathematica [A] time = 0.495634, size = 143, normalized size = 0.48

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left((cx-2i)\left(a\sqrt{c^2x^2+1}+bcx-ib\right)+b(cx-2i)\sqrt{c^2x^2+1}\sinh^{-1}(cx)-b(cx-i)^2\log(d+icdx)\right)}{3cd^3f(cx-i)^2\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-2*I + c*x)*((-I)*b + b*c*x + a*Sqrt
[1 + c^2*x^2]) + b*(-2*I + c*x)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - b*(-I + c*
x)^2*Log[d + I*c*d*x]))/(3*c*d^3*f*(-I + c*x)^2*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\,(d+icdx)^{-\frac{5}{2}}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.91807, size = 1386, normalized size = 4.7

$$-\frac{12\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}bcx-12\left(bc^2x^2-i\,bcx+2\,b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)-2\left(3\,c^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(12*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 12*
(b*c^2*x^2 - I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) - 2*(3*c^4*d^3*f*x^3 - 3*I*c^3*d^3*f*x^2 + 3*c^2*d^3*f*x
```

```
 - 3*I*c*d^3*f)*sqrt(b^2/(c^2*d^5*f))*log(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*
x + 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(
3*I*c^9*d^3*f*x^4 + 6*c^8*d^3*f*x^3 + 3*I*c^7*d^3*f*x^2 + 6*c^6*d^3*f*x)*sq
rt(b^2/(c^2*d^5*f)))/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b)) +
 2*(3*c^4*d^3*f*x^3 - 3*I*c^3*d^3*f*x^2 + 3*c^2*d^3*f*x - 3*I*c*d^3*f)*sqrt
(b^2/(c^2*d^5*f))*log(1/3*(3*(-2*I*b*c^6*x^2 - 4*b*c^5*x + 4*I*b*c^4)*sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*d^3*f*x^4 -
 6*c^8*d^3*f*x^3 - 3*I*c^7*d^3*f*x^2 - 6*c^6*d^3*f*x)*sqrt(b^2/(c^2*d^5*f))
)/(16*b*c^3*x^3 - 16*I*b*c^2*x^2 + 16*b*c*x - 16*I*b)) - 3*(4*a*c^2*x^2 - 4
*I*a*c*x + 8*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(12*c^4*d^3*f*x^3 - 1
2*I*c^3*d^3*f*x^2 + 12*c^2*d^3*f*x - 12*I*c*d^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.558 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=517

$$-\frac{5id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15id^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sin\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] (((3*I)/2)*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) + (b*c*d^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2)) + (5*b*d^4*(1 + I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)/
(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((2*I)*d^4*(1 + I*c*x)^3*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)) - (((15*I)/2)*d^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (((5*I)/2)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a
 + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*d^4*(
1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^4*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.422032, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 669, 671, 641, 215, 5819, 627, 43, 5675}

$$-\frac{5id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{15id^4\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sin\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (((3*I)/2)*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) + (b*c*d^4*x^2*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2)) + (5*b*d^4*(1 + I*c*x)^2*(1 + c^2*x^2)^(3/2))/(4*c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (15*b*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]^2)/
(4*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((2*I)*d^4*(1 + I*c*x)^3*(1
 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2
)) - (((15*I)/2)*d^4*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - (((5*I)/2)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a
```

```
 + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (15*d^4*(
1 + c^2*x^2)^(3/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(2*c*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^4*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
```

```
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{15id^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}\right.}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2id^4(1+icx)^3\left(1+\right.}{c(d+icdx}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f}$$

$$= \frac{15ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{15bd^4\left(1+c^2x^2\right)^{3/2}}{4c(d+icdx)^{3/2}(f}$$

$$= \frac{3ibd^4x\left(1+c^2x^2\right)^{3/2}}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{bcd^4x^2\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{5bd^4(1+icx)^2\left(1+c^2\right.}{4c(d+icdx)^{3/2}(f-ic}$$

Mathematica [A] time = 3.88668, size = 781, normalized size = 1.51

$$\frac{4ad^2\left(c^2x^2-7icx+24\right)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(cx+i)} - \frac{60ad^{5/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}} + \frac{4bd^2\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((4*a*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(24 - (7*I)*c*x + c^2*x^2))/(
f^2*(I + c*x)) - (60*a*d^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]])/f^(3/2) + (4*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*
ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcT
an[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Si
nh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])) + (16*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSi
nh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTa
n[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sin
h[ArcSinh[c*x]/2]) + ArcSinh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSin
h[c*x]/2] - (-2 + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 +
```

```
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(-10*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*S
inh[ArcSinh[c*x]/2]) + (16*c*x + 32*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Cosh[2
*ArcSinh[c*x]] + (8*I)*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[Arc
Sinh[c*x]/2]) + 2*ArcSinh[c*x]*(Sinh[ArcSinh[c*x]/2]*(8 - 8*Sqrt[1 + c^2*x^
2] - I*Sinh[2*ArcSinh[c*x]]) + Cosh[ArcSinh[c*x]/2]*((-8*I)*(1 + Sqrt[1 + c
^2*x^2]) + Sinh[2*ArcSinh[c*x]]))))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(8*c)
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\,(d+icdx)^{\frac{5}{2}}\left(f-icfx\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bc^2d^2x^2-2i\,bcd^2x-bd^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(ac^2d^2x^2-2i\,acd^2x-ad^2\right)\sqrt{i\,cdx+d}}{c^2f^2x^2+2i\,cf^2x-f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c^2*d^2*x^2 - 2*I*b*c*d^2*x - b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a*c^2*d^2*x^2 - 2*I*a*c*d^2*x -
 a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x -
f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.559 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{3d^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] (I*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 ((4*I)*d^3*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*d^3*(1 + c^2*x^2)^(3/2)*(
a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4
*b*d^3*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2))
```

Rubi [A] time = 0.477803, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675, 5717, 8}

$$-\frac{3d^3\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] (I*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) -
 ((4*I)*d^3*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (3*d^3*(1 + c^2*x^2)^(3/2)*(
a + b*ArcSinh[c*x])^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4
*b*d^3*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*
x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{4i\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}-\frac{3d^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}-\frac{icd^3x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int\frac{\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{\left(3d^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{a+b\sinh^{-1}(c}{\sqrt{1+c^2x^2}}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(1+\right.}{c(d}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(1+\right.}{c(d}$$

$$= \frac{ibd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4id^3(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{id^3\left(1+\right.}{c(d}$$

Mathematica [A] time = 2.35144, size = 515, normalized size = 1.82

$$\frac{-\frac{6ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{3/2}}+\frac{2ad(5-icx)\sqrt{d+icdx}\sqrt{f-icfx}}{f^2(cx+i)}+\frac{bd\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\left(4\tan^{-1}\left(\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*a*d*(5 - I*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^2*(I + c*x)) -
(6*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]])/f^(3/2) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cos
h[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2
]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/
(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (
2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c
*x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (c*x - 4*ArcTan[Coth[ArcSinh[c*x]/2]] +
 I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]) + ArcS
inh[c*x]*((-I)*(2 + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[c*x]/2] - (-2 + Sqrt[1
+ c^2*x^2])*Sinh[ArcSinh[c*x]/2])))/(f^2*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c)
```

Maple [F] time = 0.3, size = 0, normalized size = 0.

$$\int (a+b\operatorname{Arcsinh}(cx))(d+icdx)^{\frac{3}{2}}\left(f-icfx\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-i\,bcdx - bd)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + (-i\,acdx - ad)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}{c^2f^2x^2 + 2i\,cf^2x - f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(((-I*b*c*d*x - b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
 sqrt(c^2*x^2 + 1)) + (-I*a*c*d*x - a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.560 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] ((-2*I)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*d^2*(1 + c^2*x^
2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.399947, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 5833, 637, 5819, 12, 627, 31, 5675}

$$-\frac{d^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2bd^2\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```


```
[Out] ((-2*I)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^2)/(2*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b*d^2*(1 + c^2*x^
2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
```

```
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(d^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,d}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] time = 1.20919, size = 285, normalized size = 1.58

$$\frac{\frac{4a\sqrt{d+icdx}\sqrt{f-icfx}}{cx+i} - 2a\sqrt{d}\sqrt{f}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + \frac{b\sqrt{d+icdx}\sqrt{f-icfx}\left(2\left(\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((4*a*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 2*a*Sqrt[d]*Sqrt[f]*
Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + (b*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh
[ArcSinh[c*x]/2]) + 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])
*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh
[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(2*c*f^2)
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\sqrt{d+icdx}\,\left(f-icfx\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{icdx+d}\sqrt{-icfx+f}b\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+d}\sqrt{-icfx+f}a}{c^2f^2x^2+2icf^2x-f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 +
1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*f^2*x^2 + 2*I*c*f^2*x -
f^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\,(icx+1)}\,(a+b\operatorname{asinh}(cx))}{\left(-f\,(icx-1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(d*(I*c*x + 1))*(a + b*asinh(c*x))/(-f*(I*c*x - 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.561 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] -((d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))) - (b*d*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.248849, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 637, 5819, 12, 627, 31}

$$-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{bd\left(c^2x^2+1\right)^{3/2}\log(cx+i)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] -((d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))) - (b*d*(1 + c^2*x^2)^(3/2)*Log[I + c*x])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{d(i-cx)}{c\left(1+c^2x^2\right)}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(bd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{i-cx}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(bd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{1}{-i-cx}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bd\left(1+c^2x^2\right)^{3/2}\log(i+cx)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] time = 0.403276, size = 94, normalized size = 0.84

$$\frac{\sqrt{f-icfx}\left(iacx+a-ib\sqrt{c^2x^2+1}\log(d(-1+icx))+(b+ibcx)\sinh^{-1}(cx)\right)}{cf^2(cx+i)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*(a + I*a*c*x + (b + I*b*c*x)*ArcSinh[c*x] - I*b*Sqrt[1 +
 c^2*x^2]*Log[d*(-1 + I*c*x)]))/(c*f^2*(I + c*x)*Sqrt[d + I*c*d*x])
```

Maple [F] time = 0.247, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(cx))\left(f-icfx\right)^{-\frac{3}{2}}\frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42846, size = 1008, normalized size = 9.

$$\frac{2\sqrt{icdx+d}\sqrt{-icfx+f}\,b\log\left(cx+\sqrt{c^2x^2+1}\right)-\left(c^2df^2x+icdf^2\right)\sqrt{\frac{b^2}{c^2df^3}}\log\left(\frac{\left(2ibc^6x^2-4bc^5x-4ibc^4\right)\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx+f\cdots}}{16bc^3x^3+16ibc^2\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*log(c*x + sqrt(c^2*x^2 + 1))
- (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(((2*I*b*c^6*x^2 - 4*b
*c^5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
+ 2*(I*c^9*d*f^2*x^4 - 2*c^8*d*f^2*x^3 + I*c^7*d*f^2*x^2 - 2*c^6*d*f^2*x)*s
qrt(b^2/(c^2*d*f^3)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b))
+ (c^2*d*f^2*x + I*c*d*f^2)*sqrt(b^2/(c^2*d*f^3))*log(((2*I*b*c^6*x^2 - 4*b
*c^5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
+ 2*(-I*c^9*d*f^2*x^4 + 2*c^8*d*f^2*x^3 - I*c^7*d*f^2*x^2 + 2*c^6*d*f^2*x)*
sqrt(b^2/(c^2*d*f^3)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b))
 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a)/(c^2*d*f^2*x + I*c*d*f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{d(icx+1)}\left(-f(icx-1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(I*c*x + 1))*(-f*(I*c*x - 1))**(3/2)),
x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.562 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) - (b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2*c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.205475, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {5712, 5687, 260}

$$\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b\left(c^2x^2+1\right)^{3/2}\log\left(c^2x^2+1\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) - (b*(1 + c^2*x^2)^(3/2)*Log[1 + c^2*x^2])/(2*c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
```

```
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$
$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{b\left(1+c^2x^2\right)^{3/2}\log\left(1+c^2x^2\right)}{2c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

Mathematica [A] time = 0.465136, size = 118, normalized size = 1.15

$$\frac{i\sqrt{f-icfx}\left(2acx-b\sqrt{c^2x^2+1}\log(d(-1+icx))-b\sqrt{c^2x^2+1}\log(d+icdx)+2bcx\sinh^{-1}(cx)\right)}{2cdf^2(cx+i)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((I/2)*Sqrt[f - I*c*f*x]*(2*a*c*x + 2*b*c*x*ArcSinh[c*x] - b*Sqrt[1 + c^2*x
^2]*Log[d*(-1 + I*c*x)] - b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(c*d*f^2*(
I + c*x)*Sqrt[d + I*c*d*x])
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\,(d+icdx)^{-\frac{3}{2}}\left(f-icfx\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [A] time = 1.22233, size = 112, normalized size = 1.09

$$-\frac{bc\sqrt{\frac{1}{c^4df}}\log\left(x^2+\frac{1}{c^2}\right)}{2\,df}+\frac{bx\,\mathrm{arsinh}\,(cx)}{\sqrt{c^2dfx^2+df}df}+\frac{ax}{\sqrt{c^2dfx^2+df}df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] -1/2*b*c*sqrt(1/(c^4*d*f))*log(x^2 + 1/c^2)/(d*f) + b*x*arcsinh(c*x)/(sqrt(
c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(c^2*d*f*x^2 + d*f)*d*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\sqrt{icdx+d}\sqrt{-icfx+f}bx\log\left(cx+\sqrt{c^2x^2+1}\right)+4\sqrt{icdx+d}\sqrt{-icfx+f}ax+\left(c^2d^2f^2x^2+d^2f^2\right)\sqrt{\frac{b^2}{c^2d^3f^3}}\log\left(\frac{bc^2x^4+\sqrt{c}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/4*(4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*x*log(c*x + sqrt(c^2*x^2 + 1)
) + 4*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*x + (c^2*d^2*f^2*x^2 + d^2*f^2
)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^4 + sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x +
 d)*sqrt(-I*c*f*x + f)*c*d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^
4 + 2*b*c^2*x^2 + b)) - (c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))
*log((b*c^2*x^4 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*
d*f*x^2*sqrt(b^2/(c^2*d^3*f^3)) + b*x^2)/(b*c^4*x^4 + 2*b*c^2*x^2 + b)) - 2
*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^2*d^3*f^3))*log((b*c^2*x^3 + sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3
```

```
*f^3)) + b*x)/(b*c^2*x^2 + b)) + 2*(c^2*d^2*f^2*x^2 + d^2*f^2)*sqrt(b^2/(c^
2*d^3*f^3))*log((b*c^2*x^3 - sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f)*c*d*f*x*sqrt(b^2/(c^2*d^3*f^3)) + b*x)/(b*c^2*x^2 + b)) + 4*(c^2*d
^2*f^2*x^2 + d^2*f^2)*integral(-sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I
*c*f*x + f)*b*c*x/(c^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2
*d^2*f^2*x^2 + d^2*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.563 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{2fx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf\left(c^2x^2+1\right)^{5/2}}{6c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf}{3}$$

```
[Out] ((I/6)*b*f*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) + (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/6)*b*f*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*f*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rubi [A] time = 0.306403, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 639, 191, 5819, 627, 44, 203, 260}

$$\frac{2fx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f(cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibf\left(c^2x^2+1\right)^{5/2}}{6c(-cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bf}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```


```
[Out] ((I/6)*b*f*(1 + c^2*x^2)^(5/2))/(c*(I - c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) + (f*(I + c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/6)*b*f*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*f*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+\right.\right.}{}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bf\left(1+\right.\right.}{3(d+}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf\left(1+\right.}{3c(d+}$$

$$= \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bf\left(1+\right.}{3c(d+}$$

$$= \frac{ibf\left(1+c^2x^2\right)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1\right.}{3(d}$$

$$= \frac{ibf\left(1+c^2x^2\right)^{5/2}}{6c(i-cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f(i+cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2fx\left(1\right.}{3(d}$$

Mathematica [A] time = 0.607269, size = 201, normalized size = 0.71

$$\frac{\sqrt{f-icfx}\left(8iac^2x^2+8acx+4ia-5ibcx\sqrt{c^2x^2+1}\log(d+icdx)+3b(-1-icx)\sqrt{c^2x^2+1}\log(d(-1+icx))-5b\sqrt{c^2x^2+1}\right.}{12d^2f^2\left(c^3x^2+c\right)\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a + 8*a*c*x + (8*I)*a*c^2*x^2 + 2*b*Sqrt[1 + c^2*
x^2] + 4*b*(I + 2*c*x + (2*I)*c^2*x^2)*ArcSinh[c*x] + 3*b*(-1 - I*c*x)*Sqrt
[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] - 5*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]
```

```
- (5*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*d^2*f^2*Sqrt[d + I*c
*d*x]*(c + c^3*x^2))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))\,(icdx + d)^{-\frac{5}{2}}\,\bigl(f - icfx\bigr)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(48*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 96*
(2*b*c^2*x^2 - 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) - 4*(9*c^4*d^3*f^2*x^3 - 9*I*c^3*d^3*f^2*x^2 + 9*c^2*d
```

```
^3*f^2*x - 9*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((4*I*sqrt(c^2*x^2 + 1
)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) +
4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b)) +
 4*(15*c^4*d^3*f^2*x^3 - 15*I*c^3*d^3*f^2*x^2 + 15*c^2*d^3*f^2*x - 15*I*c*d
^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) - 4*I*b*c^2*x^3 - 4*
I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^2 + 4*b*c*x - 4*I*b)) + 4*(9*c^4*d^3*f^2*
x^3 - 9*I*c^3*d^3*f^2*x^2 + 9*c^2*d^3*f^2*x - 9*I*c*d^3*f^2)*sqrt(b^2/(c^2*
d^5*f^3))*log((-4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x^3 +
 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b)) - 4*(15*c^4*d^3*f^2*x^3 - 15*I*c^3*d^3*f
^2*x^2 + 15*c^2*d^3*f^2*x - 15*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((-4
*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^
2/(c^2*d^5*f^3)) - 4*I*b*c^2*x^3 - 4*I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^2 +
4*b*c*x - 4*I*b)) + 4*(24*c^4*d^3*f^2*x^3 - 24*I*c^3*d^3*f^2*x^2 + 24*c^2*d
^3*f^2*x - 24*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log((sqrt(c^2*x^2 + 1)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d^5*f^3)) + b*c
^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 4*(24*c^4*d^3*f^2*x^3 - 24*I*c^3*d^3*f^2*x
^2 + 24*c^2*d^3*f^2*x - 24*I*c*d^3*f^2)*sqrt(b^2/(c^2*d^5*f^3))*log(-(sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f*x*sqrt(b^2/(c^2*d
^5*f^3)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 3*(64*a*c^2*x^2 - 64*I*a*c*x
 + 32*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 3*(96*c^4*d^3*f^2*x^3 - 96*
I*c^3*d^3*f^2*x^2 + 96*c^2*d^3*f^2*x - 96*I*c*d^3*f^2)*integral(-1/6*sqrt(c
^2*x^2 + 1)*(4*b*c*x + I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)/(c^4*d^3*f
^2*x^4 + 2*c^2*d^3*f^2*x^2 + d^3*f^2), x))/(96*c^4*d^3*f^2*x^3 - 96*I*c^3*d
^3*f^2*x^2 + 96*c^2*d^3*f^2*x - 96*I*c*d^3*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.564 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=470

$$\frac{5id^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{10id^5(1+icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^5(1+icx)^4\left(c^2x^2+1\right)\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] ((-I)*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) + (((8*I)/3)*b*d^5*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (5*b*d^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^5*(1 + I*c*x)^4*(1 +
c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (((10*I)/3)*d^5*(1 + I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((5*I)*d^5*(1 + c^2*x^2)^3*(a + b*
ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*d^5*(1 + c^
2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (28*b*d^5*(1 + c^2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.438858, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 669, 641, 215, 5819, 627, 43, 5675}

$$\frac{5id^5\left(c^2x^2+1\right)^3\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{10id^5(1+icx)^2\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^5(1+icx)^4\left(c^2x^2+1\right)\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```


```
[Out] ((-I)*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)
) + (((8*I)/3)*b*d^5*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (5*b*d^5*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^5*(1 + I*c*x)^4*(1 +
c^2*x^2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (((10*I)/3)*d^5*(1 + I*c*x)^2*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((5*I)*d^5*(1 + c^2*x^2)^3*(a + b*
ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5*d^5*(1 + c^
2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f -
```

```
 I*c*f*x)^(5/2)) + (28*b*d^5*(1 + c^2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^5\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^5(1+icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id^5(1+icx)^2\left(1+c^2x^2\right)^2\left(a+b\right.}{3c(d+icdx)^{5/2}(f-icfx}$$

$$= -\frac{5ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{10id}{}$$

$$= -\frac{5ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4\left(1+\right.}{3c(d+icd}$$

$$= -\frac{5ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^5(1+icx)^4\left(1+\right.}{3c(d+icd}$$

$$= -\frac{ibd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{8ibd^5\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{5bd^5\left(1+c^2x\right.}{2c(d+icdx}$$

Mathematica [B] time = 8.66659, size = 1331, normalized size = 2.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a*d^2)/f^3 + (((8*I)/3)*a*
d^2)/(f^3*(I + c*x)^2) - (28*a*d^2)/(3*f^3*(I + c*x)))))/c + (5*a*d^(5/2)*Lo
g[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(
c*f^(5/2)) - ((I/6)*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]
*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])
*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
 + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x]
- 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[
1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqr
t[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2
]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqr
t[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^4) + (b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqr
t[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Co
sh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]
*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*Arc
Sinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2
]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTa
n[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2])
)/(6*c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSin
h[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/12)*b*d^2*Sqrt[I*((-I)*d + c*d
*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]
/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (35*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[S
qrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] + 27*A
rcSinh[c*x]^2 + (156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*
x^2]]) - I*(3*(-I + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*
ArcSinh[c*x] + 18*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (
3*I)*(I + ArcSinh[c*x])*Cosh[2*ArcSinh[c*x]] + 52*Log[Sqrt[1 + c^2*x^2]] +
Sqrt[1 + c^2*x^2]*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcT
an[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]
)))/(c*f^3*(-I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4)
```

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Arcsinh}\left(cx\right)\right)\left(d+icdx\right)^{\frac{5}{2}}\left(f-icfx\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(i\,bc^2d^2x^2+2\,bcd^2x-i\,bd^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(i\,ac^2d^2x^2+2\,acd^2x-i\,ad^2\right)\sqrt{i\,cdx}}{c^3f^3x^3+3i\,c^2f^3x^2-3\,cf^3x-i\,f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((I*b*c^2*d^2*x^2 + 2*b*c*d^2*x - I*b*d^2)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a*c^2*d^2*x^2 + 2*a*c*d^2*x
 - I*a*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^
3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.565 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{2id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)\left(a\right.}{c(d+icdx)^{5/2}(f-i}$$

```
[Out] (((4*I)/3)*b*d^4*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^4*(1 + I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((2*
I)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^4*(1 + c^
2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.39221, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5712, 669, 653, 215, 5819, 627, 43, 31, 5675}

$$\frac{2id^4(1+icx)\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^4(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\sinh^{-1}(cx)\left(a\right.}{c(d+icdx)^{5/2}(f-i}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((4*I)/3)*b*d^4*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - (b*d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]^2)/(2*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^4*(1 + I*c*x)^3*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((2*
I)*d^4*(1 + I*c*x)*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(c*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x]*(a + b*Ar
cSinh[c*x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^4*(1 + c^
2*x^2)^(5/2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
```

```
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(m +
p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2id^4(1+icx)\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.}{c(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2id^4(1+icx)\left(1+c^2x^2\right)^2\left(a+b\,\mathrm{s}\right.}{c(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= -\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \cdots$$

$$= -\frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \cdots$$

$$= \frac{4ibd^4\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^4\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)^2}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^4(1+ic\cdots}{3c(\cdots}$$

Mathematica [A] time = 5.63159, size = 706, normalized size = 1.95

$$\frac{12ad^{3/2}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{f^{5/2}} - \frac{16ad(2cx+i)\sqrt{d+icdx}\sqrt{f-icfx}}{f^3(cx+i)^2} + \frac{bd\sqrt{d+icdx}\sqrt{f-icfx}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\left(-2i\sinh\left(\frac{1}{2}\,\mathrm{s}\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-16*a*d*(I + 2*c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(f^3*(I + c*x)^2
) + (12*a*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]])/f^(5/2) - ((2*I)*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[A
rcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSi
nh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + (I/2)*Log[1 + c^2*x^2])) + Cosh[
ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + ((
3*I)/2)*Log[1 + c^2*x^2]) + 2*(2 + (2*I)*ArcSinh[c*x] + (4*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + Log[1 + c^2*x^2] + (Sqrt[1 + c^2*x^2]*((2*I)*ArcSinh[c*x]
 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[1 + c^2*x^2]))/2)*Sinh[ArcSinh[
c*x]/2]))/(f^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
4) + (b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSi
nh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 7*Log[1 + c^2*x^2]) + Cosh[
ArcSinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[
Tanh[ArcSinh[c*x]/2]] + 21*Log[1 + c^2*x^2]) - (2*I)*(4 + (4*I)*ArcSinh[c*x
] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[1 + c^2
*x^2] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*Ar
cTan[Tanh[ArcSinh[c*x]/2]] + 7*Log[1 + c^2*x^2]))*Sinh[ArcSinh[c*x]/2]))/(f
^3*(1 + I*c*x)*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4))/(12*c)
```

Maple [F] time = 0.291, size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{Arcsinh}\,(cx))\,(d+icdx)^{\frac{3}{2}}\left(f-icfx\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bcdx - i\,bd)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right) + (acdx - i\,ad)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}}{c^3f^3x^3 + 3i\,c^2f^3x^2 - 3\,cf^3x - i\,f^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(((b*c*d*x - I*b*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + (a*c*d*x - I*a*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
)/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x - I*f^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.566 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{id^3(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibd^3\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3\left(c^2x^2+1\right)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] (((2*I)/3)*b*d^3*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*d^3*(1 + c^2*x^2)^(5/
2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.306474, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5712, 651, 5819, 12, 627, 43}

$$-\frac{id^3(1+icx)^3\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibd^3\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^3\left(c^2x^2+1\right)^{5/2}\log(cx+i)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (((2*I)/3)*b*d^3*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + I*c*x)^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*
x]))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*d^3*(1 + c^2*x^2)^(5/
2)*Log[I + c*x])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{5/2}\right)\int -\frac{id^3(1+icx)^3}{3c\left(1+c^2x^2\right)^2}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{(1+icx)^3}{\left(1+c^2x^2\right)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{1+icx}{(1-icx)^2}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(ibd^3\left(1+c^2x^2\right)^{5/2}\right)\int \left(-\frac{2}{(i+cx)^2} - \frac{i}{i+c\cdots}\right.}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ibd^3\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id^3(1+icx)^3\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + $$

Mathematica [A] time = 0.418799, size = 131, normalized size = 0.71

$$-\frac{id\sqrt{f-icfx}\left((cx-i)\left(acx-ia+b\sqrt{c^2x^2+1}\right)-b(cx+i)\sqrt{c^2x^2+1}\log(d(-1+icx))+b(cx-i)^2\sinh^{-1}(cx)\right)}{3cf^3(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x]))/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-I/3)*d*Sqrt[f - I*c*f*x]*((-I + c*x)*((-I)*a + a*c*x + b*Sqrt[1 + c^2*x^
2]) + b*(-I + c*x)^2*ArcSinh[c*x] - b*(I + c*x)*Sqrt[1 + c^2*x^2]*Log[d*(-1
 + I*c*x)]))/(c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))\sqrt{d+icdx}\left(f-icfx\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73191, size = 1330, normalized size = 7.19

$$-\frac{24\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,bcx+12\left(bc^2x^2-2i\,bcx-b\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)-2\left(3\,c^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(24*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x + 12*
(b*c^2*x^2 - 2*I*b*c*x - b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) - 2*(3*c^4*f^3*x^3 + 3*I*c^3*f^3*x^2 + 3*c^2*f^3*x + 3*I
*c*f^3)*sqrt(b^2*d/(c^2*f^5))*log(1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x - 4*I*b
*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(3*I*c^9*f
^3*x^4 - 6*c^8*f^3*x^3 + 3*I*c^7*f^3*x^2 - 6*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5
)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b)) + 2*(3*c^4*f^3*x^3
 + 3*I*c^3*f^3*x^2 + 3*c^2*f^3*x + 3*I*c*f^3)*sqrt(b^2*d/(c^2*f^5))*log(1/3
*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x
+ d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*f^3*x^4 + 6*c^8*f^3*x^3 - 3*I*c^7*f^3
*x^2 + 6*c^6*f^3*x)*sqrt(b^2*d/(c^2*f^5)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 +
 16*b*c*x + 16*I*b)) + 3*(4*a*c^2*x^2 - 8*I*a*c*x - 4*a)*sqrt(I*c*d*x + d)*
```

```
sqrt(-I*c*f*x + f))/(12*c^4*f^3*x^3 + 12*I*c^3*f^3*x^2 + 12*c^2*f^3*x + 12*
I*c*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.567 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=294

$$\frac{d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-$$

```
[Out] ((I/3)*b*d^2*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*b*d
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*d^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.332912, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 653, 191, 5819, 627, 44, 203, 260}

$$\frac{d^2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2id^2(1+icx)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd^2\left(c^2x^2+1\right)^{5/2}}{3c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```


```
[Out] ((I/3)*b*d^2*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + I*c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
))/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)^2*(a
+ b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*b*d
^2*(1 + c^2*x^2)^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b*d^2*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(6*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2))
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*
(d + e*x)*(a + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[(e^2*(p + 2))/(c*(p +
 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{(bc}{}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{(2i}{}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2}{6}$$

$$= -\frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd^2}{6}$$

$$= \frac{ibd^2\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x}{3}$$

$$= \frac{ibd^2\left(1+c^2x^2\right)^{5/2}}{3c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2id^2(1+icx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^2x}{3}$$

Mathematica [A] time = 0.438501, size = 139, normalized size = 0.47

$$\frac{\sqrt{f-icfx}\left((cx+2i)\left(iacx+a+ib\sqrt{c^2x^2+1}\right)+b(1-icx)\sqrt{c^2x^2+1}\log(d(-1+icx))+ib\left(c^2x^2+icx+2\right)\sinh^{-1}(cx)\right)}{3cf^3(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((2*I + c*x)*(a + I*a*c*x + I*b*Sqrt[1 + c^2*x^2]) + I*b
*(2 + I*c*x + c^2*x^2)*ArcSinh[c*x] + b*(1 - I*c*x)*Sqrt[1 + c^2*x^2]*Log[d
*(-1 + I*c*x)]))/(3*c*f^3*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F] time = 0.249, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}(cx)) \left(f - icfx\right)^{-\frac{5}{2}} \frac{1}{\sqrt{d + icdx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.92681, size = 1384, normalized size = 4.71

$$-\frac{12\sqrt{c^2x^2+1}\sqrt{icdx+d}\sqrt{-icfx+f}\,bcx - 12\left(bc^2x^2 + ibcx + 2b\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx + \sqrt{c^2x^2+1}\right) + 2\left(3c^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(12*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 12*
(b*c^2*x^2 + I*b*c*x + 2*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x +
sqrt(c^2*x^2 + 1)) + 2*(3*c^4*d*f^3*x^3 + 3*I*c^3*d*f^3*x^2 + 3*c^2*d*f^3*x
```

```
 + 3*I*c*d*f^3)*sqrt(b^2/(c^2*d*f^5))*log(1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x
 - 4*I*b*c^4)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(3
*I*c^9*d*f^3*x^4 - 6*c^8*d*f^3*x^3 + 3*I*c^7*d*f^3*x^2 - 6*c^6*d*f^3*x)*sqr
t(b^2/(c^2*d*f^5)))/(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b)) -
2*(3*c^4*d*f^3*x^3 + 3*I*c^3*d*f^3*x^2 + 3*c^2*d*f^3*x + 3*I*c*d*f^3)*sqrt(
b^2/(c^2*d*f^5))*log(1/3*(3*(2*I*b*c^6*x^2 - 4*b*c^5*x - 4*I*b*c^4)*sqrt(c^
2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) + 2*(-3*I*c^9*d*f^3*x^4 + 6
*c^8*d*f^3*x^3 - 3*I*c^7*d*f^3*x^2 + 6*c^6*d*f^3*x)*sqrt(b^2/(c^2*d*f^5)))/
(16*b*c^3*x^3 + 16*I*b*c^2*x^2 + 16*b*c*x + 16*I*b)) - 3*(4*a*c^2*x^2 + 4*I
*a*c*x + 8*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(12*c^4*d*f^3*x^3 + 12*
I*c^3*d*f^3*x^2 + 12*c^2*d*f^3*x + 12*I*c*d*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.568 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=282

$$\frac{2dx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd\left(c^2x^2+1\right)^{5/2}}{6c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd\,(}{3c}$$

```
[Out] ((I/6)*b*d*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) - (d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/6)*b*d*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*d*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rubi [A] time = 0.298819, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5712, 639, 191, 5819, 627, 44, 203, 260}

$$\frac{2dx\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{d(-cx+i)\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{ibd\left(c^2x^2+1\right)^{5/2}}{6c(cx+i)(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bd\,(}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*b*d*(1 + c^2*x^2)^(5/2))/(c*(I + c*x)*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2)) - (d*(I - c*x)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*
x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/6)*b*d*(1 + c^2*x^2)
^(5/2)*ArcTan[c*x])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b*d*(1 +
 c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
+ e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (
e_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p
, x]}, Dist[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[Dist[1/Sqrt[1 + c^2*x
^2], u, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] &&
IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m,
3])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1\right.\right.}{}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(bd\left(1\right.\right.}{3(d}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd\left(1+\right.}{3c(d}$$

$$= -\frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bd\left(1+\right.}{3c(d}$$

$$= \frac{ibd\left(1+c^2x^2\right)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+\right.}{3(d}$$

$$= \frac{ibd\left(1+c^2x^2\right)^{5/2}}{6c(i+cx)(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{d(i-cx)\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2dx\left(1+\right.}{3(d}$$

Mathematica [A] time = 0.600859, size = 202, normalized size = 0.72

$$\frac{\sqrt{f-icfx}\left(8iac^2x^2-8acx+4ia-3ibcx\sqrt{c^2x^2+1}\log(d+icdx)+5b(1-icx)\sqrt{c^2x^2+1}\log(d(-1+icx))+3b\sqrt{c^2x^2+1}\,\mathrm{l}\right.}{12cdf^3(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[f - I*c*f*x]*((4*I)*a - 8*a*c*x + (8*I)*a*c^2*x^2 - 2*b*Sqrt[1 + c^2*
x^2] + (4*I)*b*(1 + (2*I)*c*x + 2*c^2*x^2)*ArcSinh[c*x] + 5*b*(1 - I*c*x)*S
qrt[1 + c^2*x^2]*Log[d*(-1 + I*c*x)] + 3*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*
```

```
x] - (3*I)*b*c*x*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x]))/(12*c*d*f^3*(I + c*x)
^2*Sqrt[d + I*c*d*x])
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))\,(d + icdx)^{-\frac{3}{2}}\left(f - icfx\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*(48*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x - 96*
(2*b*c^2*x^2 + 2*I*b*c*x + b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) - 4*(15*c^4*d^2*f^3*x^3 + 15*I*c^3*d^2*f^3*x^2 + 15*c^
```

```
2*d^2*f^3*x + 15*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((4*I*sqrt(c^2*x^2
 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)
) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b
)) + 4*(9*c^4*d^2*f^3*x^3 + 9*I*c^3*d^2*f^3*x^2 + 9*c^2*d^2*f^3*x + 9*I*c*d
^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) - 4*I*b*c^2*x^3 - 4*
I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^2 + 4*b*c*x - 4*I*b)) + 4*(15*c^4*d^2*f^3
*x^3 + 15*I*c^3*d^2*f^3*x^2 + 15*c^2*d^2*f^3*x + 15*I*c*d^2*f^3)*sqrt(b^2/(
c^2*d^3*f^5))*log((-4*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x +
 f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + 4*I*b*c^2*x^3 + 4*I*b*x)/(4*b*c^3*x
^3 + 4*I*b*c^2*x^2 + 4*b*c*x + 4*I*b)) - 4*(9*c^4*d^2*f^3*x^3 + 9*I*c^3*d^2
*f^3*x^2 + 9*c^2*d^2*f^3*x + 9*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((-4
*I*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^
2/(c^2*d^3*f^5)) - 4*I*b*c^2*x^3 - 4*I*b*x)/(4*b*c^3*x^3 - 4*I*b*c^2*x^2 +
4*b*c*x - 4*I*b)) + 4*(24*c^4*d^2*f^3*x^3 + 24*I*c^3*d^2*f^3*x^2 + 24*c^2*d
^2*f^3*x + 24*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log((sqrt(c^2*x^2 + 1)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d^3*f^5)) + b*c
^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 4*(24*c^4*d^2*f^3*x^3 + 24*I*c^3*d^2*f^3*x
^2 + 24*c^2*d^2*f^3*x + 24*I*c*d^2*f^3)*sqrt(b^2/(c^2*d^3*f^5))*log(-(sqrt(
c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d*f^2*x*sqrt(b^2/(c^2*d
^3*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 3*(64*a*c^2*x^2 + 64*I*a*c*x
 + 32*a)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 3*(96*c^4*d^2*f^3*x^3 + 96*
I*c^3*d^2*f^3*x^2 + 96*c^2*d^2*f^3*x + 96*I*c*d^2*f^3)*integral(-1/6*sqrt(c
^2*x^2 + 1)*(4*b*c*x - I*b)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)/(c^4*d^2*f
^3*x^4 + 2*c^2*d^2*f^3*x^2 + d^2*f^3), x))/(96*c^4*d^2*f^3*x^3 + 96*I*c^3*d
^2*f^3*x^2 + 96*c^2*d^2*f^3*x + 96*I*c*d^2*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.569 $\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b\left(c^2x^2+1\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b\left(c^2x^2+1\right)^{5/2}\log\left(c\right.}{3c(d+icdx)^{5/2}(f-i}$$

```
[Out] (b*(1 + c^2*x^2)^(3/2))/(6*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (x*
(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - (b*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.244513, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5712, 5690, 5687, 260, 261}

$$\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b\left(c^2x^2+1\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b\left(c^2x^2+1\right)^{5/2}\log\left(c\right.}{3c(d+icdx)^{5/2}(f-i}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (b*(1 + c^2*x^2)^(3/2))/(6*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (x*
(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5/2)*(f
 - I*c*f*x)^(5/2)) - (b*(1 + c^2*x^2)^(5/2)*Log[1 + c^2*x^2])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(bc\left(1+c^2x^2\right)^{\ldots}\right.}{3(d+icdx)^{\ldots}}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x\left(1+c^2x^2\right)^2\left(a+b\ldots\right.}{3(d+icdx)^{5/2}(f-\ldots}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}}{6c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x\left(1+c^2x^2\right)^2\left(a+b\ldots\right.}{3(d+icdx)^{5/2}(f-\ldots}$$

Mathematica [A] time = 0.553006, size = 193, normalized size = 0.95

$$\frac{i\sqrt{f-icfx}\left(4ac^3x^3+6acx-2bc^2x^2\sqrt{c^2x^2+1}\log(d+icdx)-2b\left(c^2x^2+1\right)^{3/2}\log(d(-1+icx))-2b\sqrt{c^2x^2+1}\log(d+icd\right.}{6cd^2f^3(cx-i)(cx+i)^2\sqrt{d+icdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/6)*Sqrt[f - I*c*f*x]*(6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 + c^2*x^2] + 2*b
*c*x*(3 + 2*c^2*x^2)*ArcSinh[c*x] - 2*b*(1 + c^2*x^2)^(3/2)*Log[d*(-1 + I*c
*x)] - 2*b*Sqrt[1 + c^2*x^2]*Log[d + I*c*d*x] - 2*b*c^2*x^2*Sqrt[1 + c^2*x^
2]*Log[d + I*c*d*x]))/(c*d^2*f^3*(-I + c*x)*(I + c*x)^2*Sqrt[d + I*c*d*x])
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\,(d+icdx)^{-\frac{5}{2}}\left(f-icfx\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [A] time = 1.24843, size = 215, normalized size = 1.06

$$\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}f^{\frac{5}{2}}x^2+c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}-\frac{2\,\log\left(c^2x^2+1\right)}{c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}\right)+\frac{1}{3}\,b\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}}df}+\frac{2\,x}{\sqrt{c^2dfx^2+df}d^2f^2}\right)\text{arsinh}\,(cx)+\frac{1}{3}\,a\left(\frac{}{\left(c^2df\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^2
+ 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f) + 2*
```

```
x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a*(x/((c^2*d*f*x^2
+ d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="fricas")
```

```
[Out] -1/6*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b*c*x^2 - 2*(2
*b*c^2*x^3 + 3*b*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1)) - (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*
d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^4 + b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + (c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d
^5*f^5))*log(-(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2
*f^2*x^2*sqrt(b^2/(c^2*d^5*f^5)) - b*c^2*x^4 - b*x^2)/(b*c^4*x^4 + 2*b*c^2*
x^2 + b)) + 2*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2
*d^5*f^5))*log((sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^
2*f^2*x*sqrt(b^2/(c^2*d^5*f^5)) + b*c^2*x^3 + b*x)/(b*c^2*x^2 + b)) - 2*(c^
4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)*sqrt(b^2/(c^2*d^5*f^5))*log(-(
sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*c*d^2*f^2*x*sqrt(b^2
/(c^2*d^5*f^5)) - b*c^2*x^3 - b*x)/(b*c^2*x^2 + b)) - 2*(2*a*c^2*x^3 + 3*a*
x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f) - 6*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^
3*x^2 + d^3*f^3)*integral(-2/3*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*
c*f*x + f)*b*c*x/(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.570 $\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=680

$$\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4ibc^2d^2x^3\sqrt{d+icd}}{}$$

```
[Out] (((8*I)/9)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*d^2*x^3*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])/32 + (((4*I)/27)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*(1 + c^2*x^2))/c - (15*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) - (((4*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*
x^2]) - (((4*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*d^2*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*d^2*x^3*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*d^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c +
 (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*c
*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 1.10763, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43, 5742, 5758}

$$\frac{bc^3d^2x^4\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2-\frac{4ibc^2d^2x^3\sqrt{d+icd}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((8*I)/9)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*d^2*x^3*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x])/32 + (((4*I)/27)*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*(1 + c^2*x^2))/c - (15*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Ar
cSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) - (((4*I)/3)*b*d^2*x*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*d^2*x^2
```

```
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*
x^2]) - (((4*I)/9)*b*c^2*d^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*d^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*d^2*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*d^2*x^3*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 + (((2*I)/3)*d^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c +
 (5*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*c
*Sqrt[1 + c^2*x^2])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
```

```
:> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int (d+icdx)^{5/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2 + 2icd^2x\right.}{\sqrt{1}}$$

$$= \frac{\left(d^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}} + \frac{(2ic}{}$$

$$= \frac{1}{2}d^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{1}{4}c^2d^2x^3\sqrt{d+icdx}\sqrt{}$$

$$= -\frac{4ibd^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{bcd^2x^2\sqrt{d+icdx}}{}$$

$$= \frac{1}{4}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} - \frac{4ib}{}$$

$$= \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b}{}$$

$$= \frac{8ib^2d^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2d^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c$$

Mathematica [A] time = 2.43894, size = 890, normalized size = 1.31

$$\frac{-1728a^2c^3d^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^3 + 4608ia^2c^2d^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^2 - 6912iabcd^2\sqrt{icxd+d}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-6912*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (4608*I)*a^2*d
^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (6912*I)*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*d^2*x*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (4608*I)*a^2*c^2*d^2
*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*d
```

```
^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*d^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (256*I)*b^2*d^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*d^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*d^(5/2)*Sqrt[f]*Sqr
t[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]] + 864*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x
]] - (768*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x
]] - 27*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] +
12*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-576*I)*b*c*x +
 (576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] + (192*I)*a*Cosh[
3*ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] - (
64*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (48*I)*b*Sqrt[1 + c^2*
x^2] + (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh
[4*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.288, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{5}{2}} (a+b\operatorname{Arcsinh}(cx))^2 \sqrt{f-icfx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2c^2d^2x^2 - 2i\,b^2cd^2x - b^2d^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2 - 2\left(abc^2d^2x^2 - 2i\,abcd^2x - abd^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - 2*(a*b*c^2*d^2*x^2 - 2*I
*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) - (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.571 $\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=508

$$-\frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-}}{3\sqrt{c}}$$

```
[Out] (((4*I)/9)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*d*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x])/4 + (((2*I)/27)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (((2*I)/3)*b*d*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*d*x^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
- (((2*I)/9)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh
[c*x]))/Sqrt[1 + c^2*x^2] + (d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x])^2)/2 + ((I/3)*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/c + (d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.634031, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43}

$$-\frac{2ibc^2dx^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{bcdx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}-\frac{2ibdx\sqrt{d+icdx}\sqrt{f-}}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((4*I)/9)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*d*x*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x])/4 + (((2*I)/27)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (((2*I)/3)*b*d*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*d*x^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
- (((2*I)/9)*b*c^2*d*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh
[c*x]))/Sqrt[1 + c^2*x^2] + (d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b
*ArcSinh[c*x])^2)/2 + ((I/3)*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/c + (d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a
```

```
 + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+icdx)^{3/2}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int (d+icdx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \left(d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2 + icdx\sqrt{1}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2 dx}{\sqrt{1+c^2x^2}} + \frac{(icd}{}$$

$$= \frac{1}{2}dx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{id\sqrt{d+icdx}\sqrt{f-ic}}{}$$

$$= -\frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{bcdx^2\sqrt{d+icdx}\sqrt{}}{2}$$

$$= \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{2ibdx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh\right.}{3\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2d\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}} -$$

$$= \frac{4ib^2d\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2dx\sqrt{d+icdx}\sqrt{f-icfx} + \frac{2ib^2d\sqrt{d}}{}$$

Mathematica [A] time = 1.81415, size = 705, normalized size = 1.39

$$\frac{108a^2d^{3/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+72ia^2c^2dx^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+72ia^2d\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-108*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (72*I)*a^2*d*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (108*I)*b^2*d*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*d*x*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*d*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*d*Sqrt[d + I*c*d*x]*Sqrt
```

```
[f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]
*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cos
h[3*ArcSinh[c*x]] + 108*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
 Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*d*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a + (3*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh
[3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*d*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*d*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((-9*I)*b*c*x + (9*I
)*a*Sqrt[1 + c^2*x^2] + (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c
*x]] - I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (d + icdx)^{\frac{3}{2}} (a + b\text{Arcsinh}(cx))^2 \sqrt{f - icfx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ib^2cdx + b^2d\right)\sqrt{icdx + d}\sqrt{-icfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 - 2\left(-iabcdx - abd\right)\sqrt{icdx + d}\sqrt{-icfx + f}\log\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c*d*x + b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 - 2*(-I*a*b*c*d*x - a*b*d)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*d*x + a^2*d)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.572 $\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\right.$$

```
[Out] (b^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (b^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.352603, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {5712, 5682, 5675, 5661, 321, 215}

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{c^2x^2+1}}-\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (b^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (b^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) - (b*c*x^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2]) + (x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/2 + (Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int}{2\sqrt{1+c^2x}}$$

$$= -\frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}} + \frac{1}{2}x\sqrt{d+icdx}\sqrt{f-icfx}$$

$$= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{bcx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}} - \frac{bcx^2\sqrt{}}{}$$

Mathematica [A] time = 1.10848, size = 352, normalized size = 1.44

$$\frac{12a^2cx\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} + 12a^2\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right) + 6b\sqrt{d+icdx}\sqrt{f}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (12*a^2*c*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 6*a*b*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + 12*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c
^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]]
+ 3*b^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - 6*b*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(b*Cosh[2*ArcSinh[c*x]] - 2*a*
Sinh[2*ArcSinh[c*x]]) + 6*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x
]^2*(2*a + b*Sinh[2*ArcSinh[c*x]]))/(24*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))^2\sqrt{d+icdx}\sqrt{f-icfx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{icdx+d}\sqrt{-icfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\sqrt{icdx+d}\sqrt{-icfx+f}ab\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2 +
 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)*(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)*(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.573 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=259

$$\frac{2iabfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ib^2f\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+$$

```
[Out] ((2*I)*a*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (
(2*I)*b^2*f*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)
*b^2*f*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) - (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rubi [A] time = 0.507903, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {5712, 5821, 5675, 5717, 5653, 261}

$$\frac{2iabfx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{if\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{2ib^2f\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```


```
[Out] ((2*I)*a*b*f*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (
(2*I)*b^2*f*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)
*b^2*f*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) - (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) + (f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(icf\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2ibf\sqrt{1\right.}{}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{f\sqrt{1+c^2x^2}\left(a+b\sin\right.}{3bc\sqrt{d+icdx}\sqrt{f}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{if\left(1+c^2x^2\right)\left(a+b\sinh\right.}{c\sqrt{d+icdx}\sqrt{f-ic}}$$

$$= \frac{2iabfx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2f\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2fx\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} -$$

Mathematica [A] time = 1.14369, size = 315, normalized size = 1.22

$$\frac{-3i\sqrt{d+icdx}\sqrt{f-icfx}\left(a^2\sqrt{c^2x^2+1}-2abcx+2b^2\sqrt{c^2x^2+1}\right)+3a^2\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*
x^2] + 2*b^2*Sqrt[1 + c^2*x^2]) + (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a - I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*
x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3
*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\,\sqrt{f-icfx}\,\frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2-2i\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}ab\log\left(cx+\sqrt{c^2x^2+1}\right)-i\,\sqrt{i\,cd}}{cdx-i\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^
2 + 1))^2 - 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2
*x^2 + 1)) - I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*d*x - I*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-f\,(icx-1)}\,(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{d\,(icx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-f*(I*c*x - 1))*(a + b*asinh(c*x))**2/sqrt(d*(I*c*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.574 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=544

$$-\frac{4b^2f^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4b^2f^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^{3/2}\text{Poly}}{c(d+icdx)^{3/2}(}$$

```
[Out] ((2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) + (2*f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSi
nh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2
)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
nh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*f^2*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-
I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*f^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 1.00538, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675}

$$-\frac{4b^2f^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{4b^2f^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^{3/2}\text{Poly}}{c(d+icdx)^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```


```
[Out] ((2*I)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) + (2*f^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSi
nh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^2*(1 + c^2*x^2
)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSi
```

```
nh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*f^2*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-
I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*f^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*
(f - I*c*f*x)^(3/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcS
inh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{2i\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}-\frac{f^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^2+cf^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(f^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \left(\frac{if^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}+\frac{c}{}\right.}{(d+icdx)^{3/2}(f-icfx)^{3}}$$

$$= -\frac{f^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2f^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} -$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1\right.}{3}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^2\left(1\right.}{3}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2\left(\right.}{}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2\left(\right.}{}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2\left(\right.}{}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2f^2\left(\right.}{}$$

Mathematica [A] time = 4.15369, size = 594, normalized size = 1.09

$$b^2\sqrt{d+icdx}\sqrt{f-icfx}\left(24\left(\cosh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)\right)\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+\sinh^{-1}(cx)^3\left(-\left(\cosh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\tfrac{1}{2}\sinh^{-1}(cx)\right)\right)\right)-(6-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(-I + c*x) - 3*a^2*Sqrt[d]*Sqr
t[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (
3*a*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(ArcSinh[c*x]*((-4*I)*Cosh[ArcSin
h[c*x]/2] - 4*Sinh[ArcSinh[c*x]/2]) + ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2]
+ I*Sinh[ArcSinh[c*x]/2]) + 2*((4*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + Log[1 +
 c^2*x^2])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*((-6 + 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Si
nh[ArcSinh[c*x]/2]) - ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2]) + (12*I)*Pi*(Log[1 - I/E^ArcSinh[c*x]] + 2*Log[1 + E^ArcSinh[c*x]
] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(C
osh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 24*PolyLog[2, I/E^ArcSinh[c
*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 6*ArcSinh[c*x]*(Pi -
 (4*I)*Log[1 - I/E^ArcSinh[c*x]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/
2])))/(3*c*d^2)
```

Maple [F] time = 0.272, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \sqrt{f - icfx}\,(d + icdx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}ab\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{i\,cdx\cdot}}{c^2d^2x^2-2i\,cd^2x-d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d^2*x^2 - 2*I*c*d^2
*x - d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-f\,(icx-1)}\,(a+b\,\text{asinh}\,(cx))^2}{(d\,(icx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-f*(I*c*x - 1))*(a + b*asinh(c*x))**2/(d*(I*c*x + 1))**(3/2),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.575 $\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=518

$$\frac{4b^2f^3\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{\sinh^{-1}\right.}{3c(d+icdx)^{5/2}(f-}$$

```
[Out] -(f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b^2*f^3*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 + (I
/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*f^3
*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])
/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*f^3*(1 + c^2*x^2)^(5/2)
*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*f^3*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (4*b^2*f^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh
[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 1.14856, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {5712, 5833, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{4b^2f^3\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{f^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bf^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{\sinh^{-1}\right.}{3c(d+icdx)^{5/2}(f-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] -(f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b^2*f^3*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 + (I
/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*f^3
*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])
/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*f^3*(1 + c^2*x^2)^(5/2)
*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c
```

```
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*f^3*(1 + c^2*x^2)^(5/2)*(a
 + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) + (4*b^2*f^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh
[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

```
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(-\frac{2f^3\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}}+\frac{if^3\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(if^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2f^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(if^3\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(x)}\,dx,x,\sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cf^3\left(1+c^2x^2\right)^{5/2}\right.}{(}$$

$$= -\frac{\left(f^3\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}+\frac{ix}{2}\right)dx,x,\sinh^{-1}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^3\left(1\right.\right.}{}$$

$$= -\frac{if^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bf^3\left(1+c^2x^2\right.}{}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\cot}{3c(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{f^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4ib^2f^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 8.06206, size = 783, normalized size = 1.51

$$\frac{ib^2(cx+i)\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\sqrt{-df\left(c^2x^2+1\right)}\left(4\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)-(1-i)\sinh^{-1}(cx)^2-\frac{2\left(\sinh^{-1}(cx)-2i\right)\sin}{cx-i}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-2*I)/3)*a^2)/(d^3*(-I + c
*x)^2) - a^2/(3*d^3*(-I + c*x))))/c + ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*S
qrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] -
I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*Ar
cTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/
2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqr
t[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSi
nh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Co
th[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*
d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2]
 + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x
)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*
x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I
)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 +
E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSi
nh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[Arc
Sinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + A
rcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh
[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2
]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\sqrt{f-icfx}\,(d+icdx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2cx+i\,b^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2-\left(3\,c^3d^3x^2-6i\,c^2d^3x-3\,cd^3\right)\text{integral}\left(\frac{3i\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}a^2}{}\right.}{3\,c^3d^3x^2-6i\,c^2d^3x-3\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -((b^2*c*x + I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1))^2 - (3*c^3*d^3*x^2 - 6*I*c^2*d^3*x - 3*c*d^3)*integral((3*I*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x +
d)*sqrt(-I*c*f*x + f)*b^2 + 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*l
og(c*x + sqrt(c^2*x^2 + 1)))/(3*c^3*d^3*x^3 - 9*I*c^2*d^3*x^2 - 9*c*d^3*x +
 3*I*d^3), x))/(3*c^3*d^3*x^2 - 6*I*c^2*d^3*x - 3*c*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(f-I*c*f*x)**(1/2)/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(f-I*c*f*x)^(1/2)/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.576 $\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=774

$$-\frac{2ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}-\frac{4ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}-\frac{3bcdx^2}{}$$

```
[Out] (((8*I)/225)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*d*x*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (((16*I)/75)*b^2*d*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*d*x*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) + (((2*I)/125)*b^2*d*(d + I*c
*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*d*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (((2*I)
/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
+ c^2*x^2)^(3/2) - (3*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
+ b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) - (((4*I)/15)*b*c^2*d*x^3*(d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2)
 - (((2*I)/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*A
rcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((I/5)*
d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/c + (d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3)
/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rubi [A] time = 0.857992, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5821, 5684, 5682, 5675, 5661, 321, 215, 5717, 195, 194, 5679, 12, 1247, 698}

$$-\frac{2ibc^4dx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}-\frac{4ibc^2dx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}-\frac{3bcdx^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (((8*I)/225)*b^2*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*d*x*(d
 + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (((16*I)/75)*b^2*d*(d + I*c*d*x
```

```
)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*d*x*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) + (((2*I)/125)*b^2*d*(d + I*c
*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*d*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (((2*I)
/5)*b*d*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
+ c^2*x^2)^(3/2) - (3*b*c*d*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
+ b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) - (((4*I)/15)*b*c^2*d*x^3*(d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2)
 - (((2*I)/25)*b*c^4*d*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*A
rcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*d*x*(d + I*c*d*x)^(3
/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((I/5)*
d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/c + (d*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3)
/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (d+icdx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(d\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(d(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}dx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{id(d+icdx)^{3/2}}{}$$

$$= -\frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5\left(1+c^2x^2\right)^{3/2}} - \frac{4ibc^2dx^3(d}{}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{2ibdx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3/}}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2dx(d+icdx)^{3/2}(f-icfx)^{3}}{64\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2dx(d+icdx)^{3/2}(f-icfx)^{3}}{64\left(1+c^2x^2\right)}$$

$$= \frac{8ib^2d(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2dx(d+icdx)^{3/2}(f-icfx)^{3/2}$$

Mathematica [A] time = 3.26103, size = 1084, normalized size = 1.4

$$\frac{57600ia^2c^4d^2f\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^4 + 72000a^2c^3d^2f\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^3 + 115200ia^2c^2d^2f\sqrt{ic}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((-72000*I)*a*b*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (57600*I)*a
^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72000*I)*
b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^
2*c*d^2*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (115200
*I)*a^2*c^2*d^2*f*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 72000*a^2*c^3*d^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2
*x^2] + (57600*I)*a^2*c^4*d^2*f*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqr
t[1 + c^2*x^2] + 36000*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSin
h[c*x]^3 - 72000*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] + (4000*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Ar
cSinh[c*x]] - 4500*a*b*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*Arc
Sinh[c*x]] + (288*I)*b^2*d^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*A
rcSinh[c*x]] + 108000*a^2*d^(5/2)*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + S
qrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] - (12000*I)*a*b*d^2*f*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d^2*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + (20*I)*b*Sqrt[1 + c^2
*x^2] + (10*I)*b*Cosh[3*ArcSinh[c*x]] + (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b
*Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) - (1440*I)*a*b*d^2*f*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d^2*f*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1200*I)*b*c*x + (1200*I)*a*Sqrt[
1 + c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] + (600*I)*a*Cosh[3*ArcSinh[c*x]]
 - 75*b*Cosh[4*ArcSinh[c*x]] + (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh
[2*ArcSinh[c*x]] - (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*
x]] - (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.269, size = 0, normalized size = 0.

$$\int (d + icdx)^{\frac{5}{2}} \left(f - icfx\right)^{\frac{3}{2}} (a + b\text{Arcsinh}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(i\,b^2c^3d^2fx^3 + b^2c^2d^2fx^2 + i\,b^2cd^2fx + b^2d^2f\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(2i\,abc^3d^2fx^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((I*b^2*c^3*d^2*f*x^3 + b^2*c^2*d^2*f*x^2 + I*b^2*c*d^2*f*x + b^2*d
^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 +
 (2*I*a*b*c^3*d^2*f*x^3 + 2*a*b*c^2*d^2*f*x^2 + 2*I*a*b*c*d^2*f*x + 2*a*b*d
^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (
I*a^2*c^3*d^2*f*x^3 + a^2*c^2*d^2*f*x^2 + I*a^2*c*d^2*f*x + a^2*d^2*f)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.577 $\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=396

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\left(c^2x^2+1\right)^{3/2}}+\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}-\frac{b\sqrt{c^2x^2+1}(d+icdx)}{}$$

```
[Out] (b^2*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (15*b^2*x*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (9*b^2*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (3*b*c*x^2
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*
x^2)^(3/2)) - (b*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(8*c) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/4 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x])^3)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rubi [A] time = 0.481928, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{8bc\left(c^2x^2+1\right)^{3/2}}+\frac{3x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{8\left(c^2x^2+1\right)}-\frac{b\sqrt{c^2x^2+1}(d+icdx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (b^2*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 + (15*b^2*x*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (9*b^2*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) - (3*b*c*x^2
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(8*(1 + c^2*
x^2)^(3/2)) - (b*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*Sqrt[1 + c^2*x^2]*
(a + b*ArcSinh[c*x]))/(8*c) + (x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x])^2)/4 + (3*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a +
 b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) + ((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)*(a + b*ArcSinh[c*x])^3)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}x(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{\left(3(d+icdx)^{3/2}(f\right.}{}$$

$$= -\frac{b(d+icdx)^{3/2}(f-icfx)^{3/2}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{8c} + \frac{1}{4}x(d+$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} - \frac{3bcx^2(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a\right.}{8\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\left(1+c^2x^2\right)} -$$

$$= \frac{1}{32}b^2x(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2x(d+icdx)^{3/2}(f-icfx)^{3/2}}{64\left(1+c^2x^2\right)} -$$

Mathematica [A] time = 1.76358, size = 524, normalized size = 1.32

$$\frac{96a^2d^{3/2}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+64a^2c^3dfx^3\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+160a^2cdf}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (160*a^2*c*d*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 64
*a^2*c^3*d*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 32
*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 64*a*b*d*f*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - 4*a*b*d*f*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 96*a^2*d^(3/2)*f^(3/2)*S
qrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x]] + 32*b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*
x]] + b^2*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 8*
b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(12*a + 8*b*Sinh[2
*ArcSinh[c*x]] + b*Sinh[4*ArcSinh[c*x]]) - 4*b*d*f*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*ArcSinh[c*x]*(16*b*Cosh[2*ArcSinh[c*x]] + b*Cosh[4*ArcSinh[c*x]]
```

```
] - 4*a*(8*Sinh[2*ArcSinh[c*x]] + Sinh[4*ArcSinh[c*x]])))/(256*c*Sqrt[1 + c
^2*x^2])
```

Maple [F] time = 0.263, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{3}{2}}\left(f-icfx\right)^{\frac{3}{2}}(a+b\mathrm{Arcsinh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2c^2dfx^2+b^2df\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abc^2dfx^2+abdf\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^2*d*f*x^2 + b^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d*f*x^2 + a*b*d*f)*sqrt(I*c*d*x
```

```
+ d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^2*d*f*x^2 + a
^2*d*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")

[Out] Timed out
```

3.578 $\int \sqrt{d+icdx}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=508

$$\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{2ibfx\sqrt{d+icdx}\sqrt{f}}{3\sqrt{}}$$

```
[Out] (((-4*I)/9)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*f*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (((2*I)/27)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) + (((2*I)/3)*b*f*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*f*x^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSin
h[c*x]))/Sqrt[1 + c^2*x^2] + (f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x])^2)/2 - ((I/3)*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/c + (f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(
a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 0.649009, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43}

$$\frac{2ibc^2fx^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{c^2x^2+1}}-\frac{bcfx^2\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{c^2x^2+1}}+\frac{2ibfx\sqrt{d+icdx}\sqrt{f}}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-4*I)/9)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (b^2*f*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x])/4 - (((2*I)/27)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(1 + c^2*x^2))/c - (b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
ArcSinh[c*x])/(4*c*Sqrt[1 + c^2*x^2]) + (((2*I)/3)*b*f*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (b*c*f*x^2*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(2*Sqrt[1 + c^2*x^2])
 + (((2*I)/9)*b*c^2*f*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSin
h[c*x]))/Sqrt[1 + c^2*x^2] + (f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x])^2)/2 - ((I/3)*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/c + (f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(
```

```
a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[1 + c^2*x^2])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{d+icdx}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(f-icfx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2-icfx\sqrt{1+\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(icf\sqrt{d\right.}{}$$

$$= \frac{1}{2}fx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{if\sqrt{d+icdx}\sqrt{f-icf}}{}$$

$$= \frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+icdx}\sqrt{f-}}{2\sqrt{1}}$$

$$= \frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} + \frac{2ibfx\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-}\right.}{3\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b^2f\sqrt{d+icdx}\sqrt{f-icfx}\sinh^{-1}(cx)}{4c\sqrt{1+c^2x^2}} + \frac{2}{}$$

$$= -\frac{4ib^2f\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{1}{4}b^2fx\sqrt{d+icdx}\sqrt{f-icfx} - \frac{2ib^2f\sqrt{d}}{}$$

Mathematica [A] time = 1.81958, size = 705, normalized size = 1.39

$$\frac{108a^2\sqrt{d}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-72ia^2c^2fx^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-72ia^2f\sqrt{c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((108*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (72*I)*a^2*f*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (108*I)*b^2*f*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 108*a^2*c*f*x*Sqrt[d + I*c*d*
x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*f*x^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 36*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[
```

```
f - I*c*f*x]*ArcSinh[c*x]^3 - 54*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*
Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh
[3*ArcSinh[c*x]] + 108*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x +
Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 27*b^2*f*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f*Sqrt[d + I*c*d*x]*Sq
rt[f - I*c*f*x]*ArcSinh[c*x]^2*(6*a - (3*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[
3*ArcSinh[c*x]] + 3*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*f*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 6*b*f*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(-9*b*Cosh[2*ArcSinh[c*x]] + 2*((9*I)*b*c*x - (9*I)*
a*Sqrt[1 + c^2*x^2] - (3*I)*a*Cosh[3*ArcSinh[c*x]] + 9*a*Sinh[2*ArcSinh[c*x
]] + I*b*Sinh[3*ArcSinh[c*x]])))/(216*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int \left(f - icfx\right)^{\frac{3}{2}} \left(a + b\mathrm{Arcsinh}\left(cx\right)\right)^{2} \sqrt{d + icdx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-i\,b^{2}cfx + b^{2}f\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^{2}x^{2} + 1}\right)^{2} + \left(-2i\,abcfx + 2\,abf\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c*f*x + b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c*f*x + 2*a*b*f)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*f*x + a^2*f)*sqrt(
I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.579 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=436

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{}}{2}$$

```
[Out] ((-4*I)*b^2*f^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b
^2*f^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*f^2*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 ((4*I)*b*f^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]) + (b*c*f^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I)*f^2*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (f^2*x*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (f^2*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x])
```

Rubi [A] time = 0.632811, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((-4*I)*b^2*f^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b
^2*f^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*f^2*
Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 ((4*I)*b*f^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]) + (b*c*f^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I)*f^2*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (f^2*x*(1 + c^2*x^
2)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (f^2*S
qrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f -
```

I*c*f*x])

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^2\,dx, x, \sinh^{-1}(cx)\right)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int \left(c^2f^2(a+bx)^2 - 2ic^2f^2(a+bx)^2\sinh(x) - c^2f^2(a+bx)^2 \right.\right.}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2if^2\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int (a+bx)^2\sinh(\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{bcf^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{f}{}$$

$$= -\frac{b^2f^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{4ibf^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcf^2x^2\sqrt{1+}}{2\sqrt{d}}$$

$$= -\frac{4ib^2f^2\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2f^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2f^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] time = 2.18331, size = 532, normalized size = 1.22

$$\frac{12a^2\sqrt{d}f^{3/2}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)-16ia^2f\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-4a^2cfx\sqrt{c^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((32*I)*a*b*c*f*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (16*I)*a^2*f*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (32*I)*b^2*f*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*f*x*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^3 + 2*a*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*A
rcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((4*I
)*(4*b*c*x + a*(-4 + I*c*x)*Sqrt[1 + c^2*x^2]) + b*Cosh[2*ArcSinh[c*x]]) +
12*a^2*Sqrt[d]*f^(3/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*Sinh[2*ArcSinh[c*x]] + 2*b*f*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[
c*x]^2*(6*a - (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*d*S
qrt[1 + c^2*x^2])
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\left(f-icfx\right)^{\frac{3}{2}}\frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2cfx+ib^2f\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abcfx+iabf\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log}{cdx-id}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c
*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.580 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=752

$$-\frac{8b^2f^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8b^2f^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2f^3\left(c^2x^2+1\right)^{3/2}\text{Pol}}{c(d+icdx)^{3/2}}$$

```
[Out] ((-2*I)*a*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((2*I)*b^2*f^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) - ((2*I)*b^2*f^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*x*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((16*I)*b*f^3*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*
x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) - (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
 I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyL
og[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b
^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 1.12804, antiderivative size = 752, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261}

$$-\frac{8b^2f^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{8b^2f^3\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2f^3\left(c^2x^2+1\right)^{3/2}\text{Pol}}{c(d+icdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-2*I)*a*b*f^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((2*I)*b^2*f^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) - ((2*I)*b^2*f^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*x*(1 + c^2*x^2)*(
a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*f^3*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (I*f^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((16*I)*b*f^3*(1 + c^
2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) - (8*b*f^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*
x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)
) - (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
 I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*b^2*f^3*(1 + c^2*x^2)^(3/2)*PolyL
og[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b
^2*f^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(-\frac{4i\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{3f^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{icf^3x\left(a+b\sinh^{-1}\right.}{\sqrt{1+c^2x^2}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^3+cf^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3f^3\left(1+c^2x^2\right)^{3/2}\right)\int \frac{(a+b\sin}{\sqrt{1}}}{(d+icdx)^{3/2}(f-icf}$$

$$= \frac{if^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{(4}{}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{if^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{f^3\left(1+c^2\right.}{bc(d+}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^3\left(1+c^2x\right.}{c(d+icd}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3}}{(d+icdx)^{3/2}(f}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3}}{(d+icdx)^{3/2}(f}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3}}{(d+icdx)^{3/2}(f}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3}}{(d+icdx)^{3/2}(f}$$

$$= -\frac{2iabf^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2f^3x\left(1+c^2x^2\right)^{3}}{(d+icdx)^{3/2}(f}$$

Mathematica [A] time = 9.04469, size = 1174, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),
x]
```

```
[Out] ((I/3)*f*(-3*a^2*(-5*I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 +
c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 9*a^2*Sqrt[d]*Sq
rt[f]*(-I + c*x)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2
]) + 6*a*b*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]
/2]*(-(c*x) + (2 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*A
rcTan[Coth[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2]) + I*(-(c*x) + (-2 + Sqrt[
1 + c^2*x^2])*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/
2]] + I*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + (3*I)*a*b*(I - c*x)*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-4*I +
ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*Log[1 + c^2*x^2]) +
I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] +
 2*Log[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]) + I*b^2*(I - c*x)*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*((6 - 6*I)*ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sin
h[ArcSinh[c*x]/2]) + ArcSinh[c*x]^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2]) + 6*ArcSinh[c*x]*(I*Pi + 4*Log[1 - I/E^ArcSinh[c*x]])*(Cosh[ArcSin
h[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh
[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + 12*Pi*(Log[1 - I/E^ArcSinh[c*x
]] + 2*Log[1 + E^ArcSinh[c*x]] - 2*Log[Cosh[ArcSinh[c*x]/2]] - Log[Sin[(Pi
+ (2*I)*ArcSinh[c*x])/4]])*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2
])) + b^2*(I - c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((2*I)*ArcSinh[c*x]
^3*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) - 6*ArcSinh[c*x]*(Pi + c
*x - (4*I)*Log[1 - I/E^ArcSinh[c*x]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2]) + 6*(Sqrt[1 + c^2*x^2] + 2*Pi*Log[1 - I/E^ArcSinh[c*x]] + 4*Pi*L
og[1 + E^ArcSinh[c*x]] - 4*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 2*Pi*Log[Sin[(Pi
+ (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])
+ 24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh
[c*x]/2]) + 3*ArcSinh[c*x]^2*(((2 + 2*I) + Sqrt[1 + c^2*x^2])*Cosh[ArcSinh[
c*x]/2] + I*((-2 + 2*I) + Sqrt[1 + c^2*x^2])*Sinh[ArcSinh[c*x]/2]))))/(c*d^
2*(I - c*x)*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2
]))
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(cx))^2 \left(f - icfx\right)^{\frac{3}{2}} (d + icdx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(i\,b^2cfx-b^2f\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2i\,abcfx-2\,abf\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log}{c^2d^2x^2-2i\,cd^2x-d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c*f*x - b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c*f*x - 2*a*b*f)*sqrt(I*c*d*x + d)*sqrt
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (I*a^2*c*f*x - a^2*f)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.581 $\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=580

$$\frac{32b^2f^4\left(c^2x^2+1\right)^{5/2}\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] (-8*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(
3*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((8*I)/3)*b^2*f^4*(1 + c^
2*x^2)^(5/2)*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (((8*I)/3)*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*C
ot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (4*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSin
h[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Cs
c[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
 + (32*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b^2*f^4*(1 + c^2*
x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2))
```

Rubi [A] time = 1.20434, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5675, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{32b^2f^4\left(c^2x^2+1\right)^{5/2}\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{8f^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] (-8*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(
3*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((8*I)/3)*b^2*f^4*(1 + c^
2*x^2)^(5/2)*Cot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (((8*I)/3)*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*C
```

```
ot[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
+ (4*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSin
h[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^4*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Cs
c[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
 + (32*b*f^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b^2*f^4*(1 + c^2*
x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{f^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4f^4\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + \frac{4if^4\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(4if^4\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(4if^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{-ic+c\sinh(}\right.}{(d+icdx)^{5/2}(f-icfx)}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(f^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\,\text{cs}\right.}{c(d+icdx)^{5/2}(f-}$$

$$= \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{4if^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,\text{c}}{c(d+icdx)^{5/2}(f-icf}$$

$$= -\frac{4f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{8f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [B] time = 10.0471, size = 1609, normalized size = 2.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
```

x]

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((-4*I)/3)*a^2*f)/(d^3*(-I +
 c*x)^2) - (8*a^2*f)/(3*d^3*(-I + c*x))))/c + (a^2*f^(3/2)*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2)) + (
(I/3)*a*b*f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3
*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sq
rt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*Ar
cTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^
2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]
]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)
*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*
f*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/
2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2
]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[
ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sq
rt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*
ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2
]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/
2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*d^3*(I + c*x)
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^4) + ((I/3)*b^2*f*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I
)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*x]^2 - (2*A
rcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I)*ArcSinh[c
*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c
*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]
]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2
])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]
^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))
/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[Arc
Sinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*f*(I + c*x)*Sqrt[I*((-I)*d
 + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSin
h[c*x] - (7 + 7*I)*ArcSinh[c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*
I + ArcSinh[c*x]))/(1 + I*c*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^A
rcSinh[c*x]] - 28*Pi*Log[1 + E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/
2]] + 14*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^A
rcSinh[c*x]] - ((4*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[
c*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}(cx))^2 \left(f-icfx\right)^{\frac{3}{2}} (d+icdx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2cfx+ib^2f\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abcfx+iabf\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(\right.}{c^3d^3x^3-3ic^2d^3x^2-3cd^3x+id^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*f*x + I*b^2*f)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c*f*x + I*a*b*f)*sqrt(I*c*d*x + d)*sqrt(-
```

```
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*f*x + I*a^2*f)*sqrt(I*c*
d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I
*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(3/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(3/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.582 $\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=548

$$\frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\left(c^2x^2+1\right)^{5/2}}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-}{16(}$$

```
[Out] (b^2*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/108 + (245*b^2*x*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1152*(1 + c^2*x^2)^2) + (65*b^2*x*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1728*(1 + c^2*x^2)) - (115*b^2*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)*ArcSinh[c*x])/(1152*c*(1 + c^2*x^2)^(5/2)) - (
5*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16
*(1 + c^2*x^2)^(5/2)) - (5*b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b
*ArcSinh[c*x]))/(48*c*Sqrt[1 + c^2*x^2]) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (x*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*x*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*(1 + c^2*x^2)^2) + (5
*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(24*(1 +
 c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]
)^3)/(48*b*c*(1 + c^2*x^2)^(5/2))
```

Rubi [A] time = 0.611743, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5684, 5682, 5675, 5661, 321, 215, 5717, 195}

$$\frac{5(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{48bc\left(c^2x^2+1\right)^{5/2}}+\frac{5x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{24\left(c^2x^2+1\right)}+\frac{5x(d+icdx)^{5/2}(f-}{16(}$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] (b^2*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))/108 + (245*b^2*x*(d + I*c*d
*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1152*(1 + c^2*x^2)^2) + (65*b^2*x*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2))/(1728*(1 + c^2*x^2)) - (115*b^2*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)*ArcSinh[c*x])/(1152*c*(1 + c^2*x^2)^(5/2)) - (
5*b*c*x^2*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]))/(16
*(1 + c^2*x^2)^(5/2)) - (5*b*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b
*ArcSinh[c*x]))/(48*c*Sqrt[1 + c^2*x^2]) - (b*(d + I*c*d*x)^(5/2)*(f - I*c*
```

```
f*x)^(5/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(18*c) + (x*(d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/6 + (5*x*(d + I*c*d*x)^
(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(16*(1 + c^2*x^2)^2) + (5
*x*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(24*(1 +
 c^2*x^2)) + (5*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x]
)^3)/(48*b*c*(1 + c^2*x^2)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
```

```
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int (d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\frac{\left((d+icdx)^{5/2}(f-icfx)^{5/2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\left(1+c^2x^2\right)^{5/2}}$$

$$=\frac{1}{6}x(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{\left(5(d+icdx)^{5/2}(f\right.}{}$$

$$=-\frac{b(d+icdx)^{5/2}(f-icfx)^{5/2}\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{18c}+\frac{1}{6}x(d+$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}-\frac{5b(d+icdx)^{5/2}(f-icfx)^{5/2}\left(a+\right.}{48c\sqrt{1+c^2x^2}}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{65b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1728\left(1+c^2x^2\right)}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152\left(1+c^2x^2\right)^2}$$

$$=\frac{1}{108}b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}+\frac{245b^2x(d+icdx)^{5/2}(f-icfx)^{5/2}}{1152\left(1+c^2x^2\right)^2}$$

Mathematica [A] time = 2.24734, size = 735, normalized size = 1.34

$$\frac{2304a^2c^5d^2f^2x^5\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+7488a^2c^3d^2f^2x^3\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}+9504a^2cd^2f^2x\sqrt{c^2x^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9504*a^2*c*d^2*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
 + 7488*a^2*c^3*d^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^
2*x^2] + 2304*a^2*c^5*d^2*f^2*x^5*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[
1 + c^2*x^2] + 1440*b^2*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 3240*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSi
nh[c*x]] - 324*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSi
nh[c*x]] - 24*a*b*d^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[6*ArcSin
h[c*x]] + 4320*a^2*d^(5/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*
```

```
Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 1620*b^2*d^2*f^2*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 81*b^2*d^2*f^2*Sqrt[d + I*c
*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 4*b^2*d^2*f^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*Sinh[6*ArcSinh[c*x]] - 12*b*d^2*f^2*Sqrt[d + I*c*d*x]
*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*(270*b*Cosh[2*ArcSinh[c*x]] + 27*b*Cosh[4*A
rcSinh[c*x]] + 2*b*Cosh[6*ArcSinh[c*x]] - 540*a*Sinh[2*ArcSinh[c*x]] - 108*
a*Sinh[4*ArcSinh[c*x]] - 12*a*Sinh[6*ArcSinh[c*x]]) + 72*b*d^2*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a + 45*b*Sinh[2*ArcSinh[c*x]
] + 9*b*Sinh[4*ArcSinh[c*x]] + b*Sinh[6*ArcSinh[c*x]]))/(13824*c*Sqrt[1 + c
^2*x^2])
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{5}{2}} \left(f-icfx\right)^{\frac{5}{2}} (a+b\mathrm{Arcsinh}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(b^2c^4d^2f^2x^4 + 2\,b^2c^2d^2f^2x^2 + b^2d^2f^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + 2\left(abc^4d^2f^2x^4 + 2\,abc^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((b^2*c^4*d^2*f^2*x^4 + 2*b^2*c^2*d^2*f^2*x^2 + b^2*d^2*f^2)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^4*
d^2*f^2*x^4 + 2*a*b*c^2*d^2*f^2*x^2 + a*b*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-
I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c^4*d^2*f^2*x^4 + 2*a^2*c^
2*d^2*f^2*x^2 + a^2*d^2*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.583 $\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=774

$$\frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}+\frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}-3bcfx^2(\;$$

```
[Out] (((-8*I)/225)*b^2*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*f*x*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 - (((16*I)/75)*b^2*f*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*f*x*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (((2*I)/125)*b^2*f*(d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*f*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) + (((2*I
)/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
 + c^2*x^2)^(3/2) - (3*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) + (((4*I)/15)*b*c^2*f*x^3*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2
) + (((2*I)/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*
ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) - ((I/5)
*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/c + (f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3
)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rubi [A] time = 0.833405, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5821, 5684, 5682, 5675, 5661, 321, 215, 5717, 195, 194, 5679, 12, 1247, 698}

$$\frac{2ibc^4fx^5(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{25\left(c^2x^2+1\right)^{3/2}}+\frac{4ibc^2fx^3(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{15\left(c^2x^2+1\right)^{3/2}}-3bcfx^2(\;$$

Antiderivative was successfully verified.

```
[In] Int[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/225)*b^2*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/c + (b^2*f*x*(
d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))/32 - (((16*I)/75)*b^2*f*(d + I*c*d*
```

```
x)^(3/2)*(f - I*c*f*x)^(3/2))/(c*(1 + c^2*x^2)) + (15*b^2*f*x*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))/(64*(1 + c^2*x^2)) - (((2*I)/125)*b^2*f*(d + I*
c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2))/c - (9*b^2*f*(d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)*ArcSinh[c*x])/(64*c*(1 + c^2*x^2)^(3/2)) + (((2*I
)/5)*b*f*x*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1
 + c^2*x^2)^(3/2) - (3*b*c*f*x^2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a
 + b*ArcSinh[c*x]))/(8*(1 + c^2*x^2)^(3/2)) + (((4*I)/15)*b*c^2*f*x^3*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2
) + (((2*I)/25)*b*c^4*f*x^5*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*
ArcSinh[c*x]))/(1 + c^2*x^2)^(3/2) - (b*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) + (f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/4 + (3*f*x*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(8*(1 + c^2*x^2)) - ((I/5)
*f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/c + (f*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)*(a + b*ArcSinh[c*x])^3
)/(8*b*c*(1 + c^2*x^2)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5684

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n)/(2*p + 1), x] +
(Dist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 + c^
2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int (d+icdx)^{3/2}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int (f-icfx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left((d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(f\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\right.}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{\left(f(d+icdx)^{3/2}(f-icfx)^{3/2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\left(1+c^2x^2\right)^{3/2}}$$

$$= \frac{1}{4}fx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{if(d+icdx)^{3/2}}{}$$

$$= \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{5\left(1+c^2x^2\right)^{3/2}} + \frac{4ibc^2fx^3(d+}{}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{2ibfx(d+icdx)^{3/2}(f-icfx)^{3/2}}{5\left(1+c^2x^2\right)^{3}}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3}}{64\left(1+c^2x^2\right)}$$

$$= \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3/2} + \frac{15b^2fx(d+icdx)^{3/2}(f-icfx)^{3}}{64\left(1+c^2x^2\right)}$$

$$= -\frac{8ib^2f(d+icdx)^{3/2}(f-icfx)^{3/2}}{225c} + \frac{1}{32}b^2fx(d+icdx)^{3/2}(f-icfx)^{3}$$

Mathematica [A] time = 3.1153, size = 1084, normalized size = 1.4

$$\frac{-57600ia^2c^4df^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^4 + 72000a^2c^3df^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^3 - 115200ia^2c^2df^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((72000*I)*a*b*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (57600*I)*a^
2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72000*I)*b
^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180000*a^2
*c*d*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (115200*
I)*a^2*c^2*d*f^2*x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2]
+ 72000*a^2*c^3*d*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*
x^2] - (57600*I)*a^2*c^4*d*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt
[1 + c^2*x^2] + 36000*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^3 - 72000*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSin
h[c*x]] - (4000*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*Arc
Sinh[c*x]] - 4500*a*b*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcS
inh[c*x]] - (288*I)*b^2*d*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[5*Ar
cSinh[c*x]] + 108000*a^2*d^(3/2)*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sq
rt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] + 36000*b^2*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + (12000*I)*a*b*d*f^2*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]] + 1125*b^2*d*f^2*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1800*b*d*f^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (20*I)*b*Sqrt[1 + c^2*
x^2] - (10*I)*b*Cosh[3*ArcSinh[c*x]] - (2*I)*b*Cosh[5*ArcSinh[c*x]] + 40*b*
Sinh[2*ArcSinh[c*x]] + 5*b*Sinh[4*ArcSinh[c*x]]) + (1440*I)*a*b*d*f^2*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[5*ArcSinh[c*x]] + 60*b*d*f^2*Sqrt[d + I
*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((1200*I)*b*c*x - (1200*I)*a*Sqrt[1
+ c^2*x^2] - 1200*b*Cosh[2*ArcSinh[c*x]] - (600*I)*a*Cosh[3*ArcSinh[c*x]] -
 75*b*Cosh[4*ArcSinh[c*x]] - (120*I)*a*Cosh[5*ArcSinh[c*x]] + 2400*a*Sinh[2
*ArcSinh[c*x]] + (200*I)*b*Sinh[3*ArcSinh[c*x]] + 300*a*Sinh[4*ArcSinh[c*x]
] + (24*I)*b*Sinh[5*ArcSinh[c*x]]))/(288000*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.269, size = 0, normalized size = 0.

$$\int (d+icdx)^{\frac{3}{2}} \left(f-icfx\right)^{\frac{5}{2}} (a+b\operatorname{Arcsinh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-i\,b^2c^3df^2x^3+b^2c^2df^2x^2-i\,b^2cdf^2x+b^2df^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(-2i\,abc^3df^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="fricas")
```

```
[Out] integral((-I*b^2*c^3*d*f^2*x^3 + b^2*c^2*d*f^2*x^2 - I*b^2*c*d*f^2*x + b^2*
d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2
+ (-2*I*a*b*c^3*d*f^2*x^3 + 2*a*b*c^2*d*f^2*x^2 - 2*I*a*b*c*d*f^2*x + 2*a*b
*d*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) +
 (-I*a^2*c^3*d*f^2*x^3 + a^2*c^2*d*f^2*x^2 - I*a^2*c*d*f^2*x + a^2*d*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2,x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.584 $\int \sqrt{d+icdx}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=680

$$\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4ibc^2f^2x^3\sqrt{d+ic}}{}$$

```
[Out] (((-8*I)/9)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*f^2*x^3*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x])/32 - (((4*I)/27)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(1 + c^2*x^2))/c - (15*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) + (((4*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*f^2*x^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2
*x^2]) + (((4*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*f^2*x^3*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c
+ (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*
c*Sqrt[1 + c^2*x^2])
```

Rubi [A] time = 1.0629, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5821, 5682, 5675, 5661, 321, 215, 5717, 5679, 444, 43, 5742, 5758}

$$\frac{bc^3f^2x^4\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{8\sqrt{c^2x^2+1}}-\frac{1}{4}c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2+\frac{4ibc^2f^2x^3\sqrt{d+ic}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (((-8*I)/9)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/c + (15*b^2*f^2*x*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/64 - (b^2*c^2*f^2*x^3*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x])/32 - (((4*I)/27)*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*(1 + c^2*x^2))/c - (15*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*A
rcSinh[c*x])/(64*c*Sqrt[1 + c^2*x^2]) + (((4*I)/3)*b*f^2*x*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] - (3*b*c*f^2*x^
```

```
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2
*x^2]) + (((4*I)/9)*b*c^2*f^2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a +
b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2] + (b*c^3*f^2*x^4*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*(a + b*ArcSinh[c*x]))/(8*Sqrt[1 + c^2*x^2]) + (3*f^2*x*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/8 - (c^2*f^2*x^3*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^2)/4 - (((2*I)/3)*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/c
+ (5*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(a + b*ArcSinh[c*x])^3)/(24*b*
c*Sqrt[1 + c^2*x^2])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5682

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^n)/2, x] + (Dist[Sq
rt[d + e*x^2]/(2*Sqrt[1 + c^2*x^2]), Int[(a + b*ArcSinh[c*x])^n/Sqrt[1 + c^
2*x^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 + c^2*x^2]), Int[x*
(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e
, c^2*d] && GtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
```

```
:> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5679

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5742

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*Arc
Sinh[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 + c^2
*x^2]), Int[((f*x)^m*(a + b*ArcSinh[c*x])^n)/Sqrt[1 + c^2*x^2], x], x] - Di
st[(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 + c^2*x^2]), Int[(f*x)^(m + 1)
*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[e, c^2*d] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rubi steps

$$\int \sqrt{d+icdx}(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int(f-icfx)^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,d}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\left(f^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2-2icf^2x\right.}{\sqrt{}}$$

$$= \frac{\left(f^2\sqrt{d+icdx}\sqrt{f-icfx}\right)\int\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(2ic\right.}{}$$

$$= \frac{1}{2}f^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)^2 - \frac{1}{4}c^2f^2x^3\sqrt{d+icdx}$$

$$= \frac{4ibf^2x\sqrt{d+icdx}\sqrt{f-icfx}\left(a+b\sinh^{-1}(cx)\right)}{3\sqrt{1+c^2x^2}} - \frac{bcf^2x^2\sqrt{d+icdx}}{2}$$

$$= \frac{1}{4}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} + \frac{4}{}$$

$$= \frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b^2c^2f^2x^3\sqrt{d+icdx}\sqrt{f-icfx} - \frac{b}{}$$

$$= -\frac{8ib^2f^2\sqrt{d+icdx}\sqrt{f-icfx}}{9c} + \frac{15}{64}b^2f^2x\sqrt{d+icdx}\sqrt{f-icfx} - \frac{1}{32}b$$

Mathematica [A] time = 2.36589, size = 890, normalized size = 1.31

$$\frac{-1728a^2c^3f^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^3 - 4608ia^2c^2f^2\sqrt{icxd+d}\sqrt{f-icfx}\sqrt{c^2x^2+1}x^2 + 6912iabcf^2\sqrt{icxd+d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((6912*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (4608*I)*a^2*f^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (6912*I)*b^2*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 2592*a^2*c*f^2*x*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (4608*I)*a^2*c^2*f^2*
x^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 1728*a^2*c^3*f^
```

```
2*x^3*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 1440*b^2*f^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 - 1728*a*b*f^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (256*I)*b^2*f^2*Sqrt[d +
I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 108*a*b*f^2*Sqrt[d + I*c*
d*x]*Sqrt[f - I*c*f*x]*Cosh[4*ArcSinh[c*x]] + 4320*a^2*Sqrt[d]*f^(5/2)*Sqrt
[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*
f*x]] + 864*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]
] + (768*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]
] - 27*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sinh[4*ArcSinh[c*x]] + 1
2*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((576*I)*b*c*x - (
576*I)*a*Sqrt[1 + c^2*x^2] - 144*b*Cosh[2*ArcSinh[c*x]] - (192*I)*a*Cosh[3*
ArcSinh[c*x]] + 9*b*Cosh[4*ArcSinh[c*x]] + 288*a*Sinh[2*ArcSinh[c*x]] + (64
*I)*b*Sinh[3*ArcSinh[c*x]] - 36*a*Sinh[4*ArcSinh[c*x]]) + 72*b*f^2*Sqrt[d +
 I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^2*(60*a - (48*I)*b*Sqrt[1 + c^2*x^
2] - (16*I)*b*Cosh[3*ArcSinh[c*x]] + 24*b*Sinh[2*ArcSinh[c*x]] - 3*b*Sinh[4
*ArcSinh[c*x]]))/(6912*c*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int \left(f-icfx\right)^{\frac{5}{2}} \left(a+b\operatorname{Arcsinh}\left(cx\right)\right)^{2} \sqrt{d+icdx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2c^2f^2x^2 + 2i\,b^2cf^2x - b^2f^2\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 - \left(2\,abc^2f^2x^2 + 4i\,abcf^2x - 2\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 - (2*a*b*c^2*f^2*x^2 + 4*I
*a*b*c*f^2*x - 2*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) - (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.585 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx$

Optimal. Leaf size=615

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{11if^3}{}$$

```
[Out] (((-68*I)/9)*b^2*f^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
 - (3*b^2*f^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((
(2*I)/27)*b^2*f^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
+ (3*b^2*f^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (((22*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2]*(a
+ b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((2*I)/9)*b*c
^2*f^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[
c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f
^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x])
```

Rubi [A] time = 0.780721, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8, 2633}

$$\frac{5f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{icf^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3f^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{11if^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```


```
[Out] (((-68*I)/9)*b^2*f^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
 - (3*b^2*f^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((
(2*I)/27)*b^2*f^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
+ (3*b^2*f^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (((22*I)/3)*b*f^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*f^3*x^2*Sqrt[1 + c^2*x^2]*(a
```

```
+ b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((2*I)/9)*b*c
^2*f^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]) - (((11*I)/3)*f^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sq
rt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*f^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[
c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((I/3)*c*f^3*x^2*(1 + c^
2*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*f
^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f-icfx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2(cf-icf\sinh(x))^3\,dx, x, \sinh^{-1}(cx)\right)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \left(c^3f^3(a+bx)^2 - 3ic^3f^3(a+bx)^2\sinh(x) - 3c^3f^3(a+bx)\right.\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{f^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(if^3\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh^3(x\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{3bcf^3x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ibc^2f^3x^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right.}{9\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{3b^2f^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{6ibf^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcf^3x^2\sqrt{1}}{2\sqrt{d}}$$

$$= -\frac{56ib^2f^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2f^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{68ib^2f^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2f^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2f^3\left(1+c^2x^2\right)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [A] time = 3.60554, size = 723, normalized size = 1.18

$$-792ia^2f^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} + 72ia^2c^2f^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx} - 324a^2cf^2x\sqrt{c^2x^2+1}\sqrt{d+icd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[d + I*c*d*x],x]
```

```
[Out] ((1620*I)*a*b*c*f^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] - (792*I)*a^2*f^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (1620*I)*b^2*f^2*S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*f^2*x*Sqrt
[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (72*I)*a^2*c^2*f^2*x^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*f^2*Sqrt[d
+ I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*f^2*Sqrt[d + I*c*d*x]
```

```
*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] + (4*I)*b^2*f^2*Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f -
I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(-4*b*c*x*(-33 + c
^2*x^2) + 27*a*(-5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] + 3*a*Cosh[3*ArcSinh[c*x]
])) + 540*a^2*Sqrt[d]*f^(5/2)*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[
f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*f^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*f^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^2*(30*a - (45*I)*b*Sqrt[1 + c^2*x^2] + I*b*Cosh[3*ArcSi
nh[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) - (12*I)*a*b*f^2*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*d*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))^2 \left(f-icfx\right)^{\frac{5}{2}} \frac{1}{\sqrt{d+icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ib^2c^2f^2x^2-2\,b^2cf^2x-ib^2f^2\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2i\,abc^2f^2x^2-4\,abcf^2x-2i\,ab\right.}{cdx-i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*f^2*x^2 - 2*b^2*c*f^2*x - I*b^2*f^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c^2*f^2*x^2 -
 4*a*b*c*f^2*x - 2*I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*f^2*x^2 - 2*a^2*c*f^2*x - I*a^2*f^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*d*x - I*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-icfx+f\right)^{\frac{5}{2}}(b\,\mathrm{arsinh}\,(cx)+a)^2}{\sqrt{icdx+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((-I*c*f*x + f)^(5/2)*(b*arcsinh(c*x) + a)^2/sqrt(I*c*d*x + d), x)
```

3.586 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx$

Optimal. Leaf size=972

result too large to display

```
[Out] ((-8*I)*a*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b^2*f^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) + (b^2*f^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) - (b^2*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - ((8*I)*b^2*f^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*f^4*x^2*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((8
*I)*f^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (8*f^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^4*(1 + c^2*
x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
+ (f^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (5*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b
*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((32*I)*b*f^4*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (16*b*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16
*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
 I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*f^
4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 1.36351, antiderivative size = 972, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 19, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.514, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261, 5758, 5661, 321, 215}

$$-\frac{5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3 f^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x\left(c^2x^2+1\right)^2 f^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{8ib^2\left(c^2x^2+1\right)^2 f^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)^2\left(a+b\sinh\right.}{2(icxd+d)^{3/2}(f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2),x]
```

```
[Out] ((-8*I)*a*b*f^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b^2*f^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2)) + (b^2*f^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) - (b^2*f^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - ((8*I)*b^2*f^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]
)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*f^4*x^2*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((8
*I)*f^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (8*f^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[
c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*f^4*(1 + c^2*
x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (f^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (5*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b
*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((32*I)*b*f^4*(1 + c^2*x^2)^(
3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) - (16*b*f^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16
*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d
*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*b^2*f^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
 I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*f^
4*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{8i\left(if^4+cf^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{7f^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{4icf^4x\left(a+b\sinh^{-1}\cdots\right.}{\sqrt{1+c^2x^2}}\cdots\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(8i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(if^4+cf^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7f^4\left(1+c^2x^2\right)^{3/2}\right)\int \frac{(a+b\sin\cdots}{\sqrt{1\cdots}}}{(d+icdx)^{3/2}(f-icf\cdots}$$

$$= \frac{4if^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f^4x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \cdots$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcf^4x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{4if^4\left(\cdots\right.}{c\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2f^4x\left(1+c^2x^2\right)\cdots}{(d+icdx)^{3/2}(f\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2\cdots\right.}{4(d+icdx)^{3/2}(f-\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2x\cdots\right.}{4(d+icdx)^{3/2}(f-\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2x\cdots\right.}{4(d+icdx)^{3/2}(f-\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2x\cdots\right.}{4(d+icdx)^{3/2}(f-\cdots}$$

$$= -\frac{8iabf^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2f^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2f^4x\left(1+c^2x\cdots\right.}{4(d+icdx)^{3/2}(f-\cdots}$$

Mathematica [B] time = 11.6129, size = 2492, normalized size = 2.56

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(3/2), x]

[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((4*I)*a^2*f^2)/d^2 + (a^2*c*f^2*x)/(2*d^2) + (8*a^2*f^2)/(d^2*(-I + c*x))))/c - (15*a^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*d^(3/2)) + ((4*I)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]]) + I*(-(c*x) - 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] + (2*I)*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]]) + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) - (b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*((6*I)*Pi*ArcSinh[c*x] + (6 - 6*I)*ArcSinh[c*x]^2 + ArcSinh[c*x]^3 + 12*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - (24*I)*Pi*Log[1 + E^ArcSinh[c*x]] + (24*I)*Pi*Log[Cosh[ArcSinh[c*x]/2]] + (12*I)*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 24*PolyLog[2, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + (-6*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + 12*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2]))/(3*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (((2*I)/3)*b^2*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-6*Pi*ArcSinh[c*x] - 6*c*x*ArcSinh[c*x] + (6 + 6*I)*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + I*(-6*Pi*ArcSinh[c*x] - 6*c*x*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + (2*I)*ArcSinh[c*x]^3 + 3*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 12*Pi*Log[1 - I/E^ArcSinh[c*x]] + (24*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - 12*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*Sinh[ArcSinh[c*x]/2] + 24*PolyLog[2, I/E^ArcSinh[c*x]]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (b^2*f^2*Sqrt[I*((-I)*d

```
+ c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(96*PolyLog[2
, I/E^ArcSinh[c*x]]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + Sinh[
ArcSinh[c*x]/2]*(24*Pi*ArcSinh[c*x] + 48*c*x*ArcSinh[c*x] + (24 - 24*I)*Arc
Sinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 + (3*I)*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*
(2 + ArcSinh[c*x]^2)) - (3*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - 48*Pi*Log
[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - 96
*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*Log[S
in[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh[c*x]
]) + Cosh[ArcSinh[c*x]/2]*(3*Sqrt[1 + c^2*x^2]*(c*x + (8*I)*(2 + ArcSinh[c*
x]^2)) - 3*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] - I*(24*Pi*ArcSinh[c*x] + 48*c
*x*ArcSinh[c*x] - (24 + 24*I)*ArcSinh[c*x]^2 - (10*I)*ArcSinh[c*x]^3 - 48*P
i*Log[1 - I/E^ArcSinh[c*x]] - (96*I)*ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]]
 - 96*Pi*Log[1 + E^ArcSinh[c*x]] + 96*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 48*Pi*
Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (3*I)*ArcSinh[c*x]^2*Sinh[2*ArcSinh
[c*x]]))))/(12*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x
^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])) + (a*b*f^2*Sqrt[I*((-I
)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-(Sinh[A
rcSinh[c*x]/2]*((-16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + Cosh[2*ArcSinh[c*x
]] + 2*((8*I)*c*x + (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2 + (16*I)*ArcTan[T
anh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh[c*x]*Sinh[2*ArcSi
nh[c*x]]))) + Cosh[ArcSinh[c*x]/2]*(16*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + I*(
Cosh[2*ArcSinh[c*x]] + 2*((8*I)*c*x - (8*I)*ArcSinh[c*x] + 5*ArcSinh[c*x]^2
 + (16*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 8*Log[Sqrt[1 + c^2*x^2]] - ArcSinh
[c*x]*Sinh[2*ArcSinh[c*x]])))))/(4*c*d^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f
*x))]*Sqrt[1 + c^2*x^2]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2]))
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\left(f-icfx\right)^{\frac{5}{2}}(d+icdx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2c^2f^2x^2+2i\,b^2cf^2x-b^2f^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2\,abc^2f^2x^2+4i\,abcf^2x-2\,abf^2\right.}{c^2d^2x^2-2i\,cd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c^2*f^2*x^2 + 2*I*b^2*c*f^2*x - b^2*f^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*a*b*c^2*f^2*x^2 + 4*I
*a*b*c*f^2*x - 2*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sq
rt(c^2*x^2 + 1)) + (a^2*c^2*f^2*x^2 + 2*I*a^2*c*f^2*x - a^2*f^2)*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f))/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.587 $\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx$

Optimal. Leaf size=790

$$\frac{112b^2f^5\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2iabf^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5f^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^5}{}$$

```
[Out] ((2*I)*a*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - ((2*I)*b^2*f^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)
^(5/2)) + ((2*I)*b^2*f^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2)) - (28*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (I*f^5*(1 + c^2*x^2)
^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5
*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (((16*I)/3)*b^2*f^5*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((28*I)
/3)*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh
[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^5*(1 + c^2*x^2
)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*f^5*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[
c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b*f^5*(1 + c^2*
x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b^2*f^5*(1 + c^2*x^2)^(5/2)*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 1.36478, antiderivative size = 790, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {5712, 5833, 5675, 5717, 5653, 261, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{112b^2f^5\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2iabf^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5f^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{if^5}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),x]
```

```
[Out] ((2*I)*a*b*f^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - ((2*I)*b^2*f^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)
^(5/2)) + ((2*I)*b^2*f^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(5/2)*(f - I*c*f*x)^(5/2)) - (28*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*
x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (I*f^5*(1 + c^2*x^2)
^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (5
*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (((16*I)/3)*b^2*f^5*(1 + c^2*x^2)^(5/2)*Cot[Pi/4 +
(I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((28*I)
/3)*f^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh
[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*f^5*(1 + c^2*x^2
)^(5/2)*(a + b*ArcSinh[c*x])*Csc[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*
c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*f^5*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Cot[Pi/4 + (I/2)*ArcSinh[c*x]]*Csc[Pi/4 + (I/2)*ArcSinh[
c*x]]^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b*f^5*(1 + c^2*
x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*
x)^(5/2)*(f - I*c*f*x)^(5/2)) + (112*b^2*f^5*(1 + c^2*x^2)^(5/2)*PolyLog[2,
 (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(f-icfx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{5f^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icf^5x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{8f^5\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)^2\sqrt{1+c^2x^2}} + 12i\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(12if^5\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(-i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5f^5\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{if^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \cdots$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5f^5\left(1+c^2x^2\right.}{3bc(d+i\cdots}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{if^5\left(1+c^2x^2\right)^3\left(\cdots\right.}{c(d+icdx)^{5\cdots}}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\cdots}{(d+icdx)^{5/2}(f-i\cdots}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\cdots}{(d+icdx)^{5/2}(f-i\cdots}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\cdots}{(d+icdx)^{5/2}(f-i\cdots}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\cdots}{(d+icdx)^{5/2}(f-i\cdots}$$

$$= \frac{2iabf^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2f^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2f^5x\left(1+c^2x^2\right)^{5/2}\cdots}{(d+icdx)^{5/2}(f-i\cdots}$$

<u>Mathematica</u> [B] time = 13.4707, size = 2622, normalized size = 3.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f - I*c*f*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(d + I*c*d*x)^(5/2),
```

x]

[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*f^2)/d^3 - (((8*I)/3)*a^2*f^2)/(d^3*(-I + c*x)^2) - (28*a^2*f^2)/(3*d^3*(-I + c*x))))/c + (5*a^2*f^(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*d^(5/2)) + ((I/3)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] - I*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(4 + (3*I)*ArcSinh[c*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 3*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]) + 2*(I + ArcSinh[c*x] + 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) - (a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((-14 + (3*I)*ArcSinh[c*x])*ArcSinh[c*x] - 28*ArcTan[Tanh[ArcSinh[c*x]/2]] + (14*I)*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(84*ArcTan[Tanh[ArcSinh[c*x]/2]] - I*(8 - (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + 42*Log[Sqrt[1 + c^2*x^2]])) + 2*(4 - (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 + (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(-14*I + 3*ArcSinh[c*x]) + (28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(3*c*d^3*(I + c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^4) + ((I/3)*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 + I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(-I + c*x) + (2*I)*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + 4*PolyLog[2, I/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^2) - ((I/3)*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c^2*x^2] + ((13 - 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^3)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/((-I + c*x)*Sqrt[1 + c^2*x^2]) - (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(-2*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) + (4*I)*PolyLog[2, I/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Sqrt[1

```
 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]))))/(c*d^3*Sqrt[
-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c
*x]/2])^2) + (2*b^2*f^2*(I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(7*Pi*ArcSinh[c*x] - (7 + 7*I)*ArcSinh[
c*x]^2 - I*ArcSinh[c*x]^3 + (2*ArcSinh[c*x]*(-2*I + ArcSinh[c*x]))/(1 + I*c
*x) - 14*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] - 28*Pi*Log[1
+ E^ArcSinh[c*x]] + 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 14*Pi*Log[Sin[(Pi + (
2*I)*ArcSinh[c*x])/4]] + (28*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((4*I)*ArcSi
nh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^3 + (2*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/((-I)*Cosh[ArcSinh
[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*d^3*Sqrt[-(((-I)*d + c*d*x)*(I*f +
c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^
2) + ((I/6)*a*b*f^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[
-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])*(-3*C
osh[(5*ArcSinh[c*x])/2] + (3*I)*ArcSinh[c*x]*Cosh[(5*ArcSinh[c*x])/2] - Cos
h[(3*ArcSinh[c*x])/2]*(9 + (35*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (52*I)*
ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*
x]/2]*(20 - (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 - (156*I)*ArcTan[Coth[A
rcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) + (20*I)*Sinh[ArcSinh[c*x]/2]
- 24*ArcSinh[c*x]*Sinh[ArcSinh[c*x]/2] + (27*I)*ArcSinh[c*x]^2*Sinh[ArcSinh
[c*x]/2] + 156*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[ArcSinh[c*x]/2] + (78*I)*L
og[Sqrt[1 + c^2*x^2]]*Sinh[ArcSinh[c*x]/2] + (9*I)*Sinh[(3*ArcSinh[c*x])/2]
 + 35*ArcSinh[c*x]*Sinh[(3*ArcSinh[c*x])/2] + (9*I)*ArcSinh[c*x]^2*Sinh[(3*
ArcSinh[c*x])/2] + 52*ArcTan[Coth[ArcSinh[c*x]/2]]*Sinh[(3*ArcSinh[c*x])/2]
 + (26*I)*Log[Sqrt[1 + c^2*x^2]]*Sinh[(3*ArcSinh[c*x])/2] - (3*I)*Sinh[(5*A
rcSinh[c*x])/2] + 3*ArcSinh[c*x]*Sinh[(5*ArcSinh[c*x])/2]))/(c*d^3*(I + c*x
)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Ar
cSinh[c*x]/2])^4)
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \left(f - icfx\right)^{\frac{5}{2}} (d + icdx)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

```
[Out] int((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-i\,b^2c^2f^2x^2+2\,b^2cf^2x+i\,b^2f^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(-2i\,abc^2f^2x^2+4\,abcf^2x+\right.}{c^3d^3x^3-3i\,c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*f^2*x^2 + 2*b^2*c*f^2*x + I*b^2*f^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c^2*f^2*x^2
 + 4*a*b*c*f^2*x + 2*I*a*b*f^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*f^2*x^2 + 2*a^2*c*f^2*x + I*a^2*f^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d
^3*x + I*d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)**(5/2)*(a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f-I*c*f*x)^(5/2)*(a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.588 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=615

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11id^3}{\ldots}$$

```
[Out] ((((68*I)/9)*b^2*d^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
- (3*b^2*d^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((
2*I)/27)*b^2*d^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (3*b^2*d^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (((22*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(S
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (((2*I)/9)*b*c^
2*d^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c
*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^
3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Rubi [A] time = 0.785263, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8, 2633}

$$\frac{5d^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{6bc\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{icd^3x^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{3d^3x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{11id^3}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((((68*I)/9)*b^2*d^3*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])
- (3*b^2*d^3*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (((
2*I)/27)*b^2*d^3*(1 + c^2*x^2)^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
 (3*b^2*d^3*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]) - (((22*I)/3)*b*d^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(S
```

```
qrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (3*b*c*d^3*x^2*Sqrt[1 + c^2*x^2]*(a +
 b*ArcSinh[c*x]))/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (((2*I)/9)*b*c^
2*d^3*x^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]) + (((11*I)/3)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (3*d^3*x*(1 + c^2*x^2)*(a + b*ArcSinh[c
*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((I/3)*c*d^3*x^2*(1 + c^2
*x^2)*(a + b*ArcSinh[c*x])^2)/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (5*d^
3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(6*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^3\,dx, x, \sinh^{-1}(cx)\right)}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\mathrm{Subst}\left(\int \left(c^3d^3(a+bx)^2+3ic^3d^3(a+bx)^2\sinh(x)-3c^3d^3(a+bx)^2\,\mathrm{s}\right.\right.}{c^4\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{d^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(id^3\sqrt{1+c^2x^2}\right)\mathrm{Subst}\left(\int (a+bx)^2\sinh^3(x)\,\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{3bcd^3x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ibc^2d^3x^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= -\frac{3b^2d^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{6ibd^3x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3bcd^3x^2\sqrt{1+c}}{2\sqrt{d+}}$$

$$= \frac{56ib^2d^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2d^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3}{}$$

$$= \frac{68ib^2d^3\left(1+c^2x^2\right)}{9c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{3b^2d^3x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{27c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{3}{}$$

Mathematica [A] time = 3.35822, size = 723, normalized size = 1.18

$$\frac{792ia^2d^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-72ia^2c^2d^2x^2\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-324a^2cd^2x\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-1620*I)*a*b*c*d^2*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (792*I)*a^2*d^
2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (1620*I)*b^2*d^2*
Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 324*a^2*c*d^2*x*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - (72*I)*a^2*c^2*d^2*x^2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 180*b^2*d^2*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 162*a*b*d^2*Sqrt[d + I*c*d*x
```

```
]*Sqrt[f - I*c*f*x]*Cosh[2*ArcSinh[c*x]] - (4*I)*b^2*d^2*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Cosh[3*ArcSinh[c*x]] + 6*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]*ArcSinh[c*x]*(27*b*Cosh[2*ArcSinh[c*x]] + (2*I)*(4*b*c*x*(-33 + c
^2*x^2) + 27*a*(5 + (2*I)*c*x)*Sqrt[1 + c^2*x^2] - 3*a*Cosh[3*ArcSinh[c*x]]
)) + 540*a^2*d^(5/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f
]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - 81*b^2*d^2*Sqrt[d + I*c*d*x]*Sqrt[
f - I*c*f*x]*Sinh[2*ArcSinh[c*x]] + 18*b*d^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c
*f*x]*ArcSinh[c*x]^2*(30*a + (45*I)*b*Sqrt[1 + c^2*x^2] - I*b*Cosh[3*ArcSin
h[c*x]] - 9*b*Sinh[2*ArcSinh[c*x]]) + (12*I)*a*b*d^2*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]*Sinh[3*ArcSinh[c*x]])/(216*c*f*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\,(d+icdx)^{\frac{5}{2}}\,\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-i\,b^2c^2d^2x^2-2\,b^2cd^2x+i\,b^2d^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(-2i\,abc^2d^2x^2-4\,abcd^2x+2\right.}{cfx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c^2*d^2*x^2 - 2*b^2*c*d^2*x + I*b^2*d^2)*sqrt(I*c*d*x + d
)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (-2*I*a*b*c^2*d^2*x^2
 - 4*a*b*c*d^2*x + 2*I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c^2*d^2*x^2 - 2*a^2*c*d^2*x + I*a^2*d^2)*s
qrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(icdx+d)^{\frac{5}{2}}(b\operatorname{arsinh}(cx)+a)^2}{\sqrt{-icfx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((I*c*d*x + d)^(5/2)*(b*arcsinh(c*x) + a)^2/sqrt(-I*c*f*x + f), x)
```

3.589 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=436

$$\frac{d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c}}{2\sqrt{}}$$

```
[Out] ((4*I)*b^2*d^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b^
2*d^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*d^2*S
qrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) -
((4*I)*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]) + (b*c*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)*d^2*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (d^2*x*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (d^2*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Rubi [A] time = 0.668527, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {5712, 5831, 3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{d^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{2bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2id^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}-\frac{d^2x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{bcd^2x^2\sqrt{c}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```


```
[Out] ((4*I)*b^2*d^2*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (b^
2*d^2*x*(1 + c^2*x^2))/(4*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (b^2*d^2*S
qrt[1 + c^2*x^2]*ArcSinh[c*x])/(4*c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) -
((4*I)*b*d^2*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(Sqrt[d + I*c*d*x]*S
qrt[f - I*c*f*x]) + (b*c*d^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + ((2*I)*d^2*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - (d^2*x*(1 + c^2*x^2
)*(a + b*ArcSinh[c*x])^2)/(2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) + (d^2*Sq
rt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(2*b*c*Sqrt[d + I*c*d*x]*Sqrt[f - I
```

```
*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int (a+bx)^2(cd+icd\sinh(x))^2\,dx, x, \sinh^{-1}(cx)\right)}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \left(c^2d^2(a+bx)^2+2ic^2d^2(a+bx)^2\sinh(x)-c^2d^2(a+bx)^2\,\text{s}\right.\right.}{c^3\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{d^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(2id^2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int (a+bx)^2\sinh(x\right.}{c\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{bcd^2x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{d}{}$$

$$= -\frac{b^2d^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{4ibd^2x\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{bcd^2x^2\sqrt{1+}}{2\sqrt{d+}}$$

$$= \frac{4ib^2d^2\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{b^2d^2x\left(1+c^2x^2\right)}{4\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{b^2d^2\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{4c\sqrt{d+icdx}\sqrt{f-icfx}} -$$

Mathematica [A] time = 2.14799, size = 529, normalized size = 1.21

$$\frac{12a^2d^{3/2}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)+16ia^2d\sqrt{c^2x^2+1}\sqrt{d+icdx}\sqrt{f-icfx}-4a^2cdx\sqrt{c^2x^2+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((-32*I)*a*b*c*d*x*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x] + (16*I)*a^2*d*Sqrt[
d + I*c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + (32*I)*b^2*d*Sqrt[d + I*
c*d*x]*Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] - 4*a^2*c*d*x*Sqrt[d + I*c*d*x]*
Sqrt[f - I*c*f*x]*Sqrt[1 + c^2*x^2] + 4*b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*
c*f*x]*ArcSinh[c*x]^3 + 2*a*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*Cosh[2*
ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]*((-1
6*I)*b*c*x - 4*a*(-4*I + c*x)*Sqrt[1 + c^2*x^2] + b*Cosh[2*ArcSinh[c*x]]) +
 12*a^2*d^(3/2)*Sqrt[f]*Sqrt[1 + c^2*x^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqr
t[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - b^2*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]*Sinh[2*ArcSinh[c*x]] + 2*b*d*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*ArcSinh
[c*x]^2*(6*a + (8*I)*b*Sqrt[1 + c^2*x^2] - b*Sinh[2*ArcSinh[c*x]]))/(8*c*f*
Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.308, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))^2\,(d+icdx)^{\frac{3}{2}}\,\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(b^2cdx - i\,b^2d\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + (2\,abcdx - 2i\,abd)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log}{cfx + i\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 + (2*a*b*c*d*x - 2*I*a*b*d)*sqrt(I*c*d*x + d)*sqr
t(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I
*c*d*x + d)*sqrt(-I*c*f*x + f))/(c*f*x + I*f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.590 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=259

$$-\frac{2iabdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2ib^2d\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-$$

```
[Out] ((-2*I)*a*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
((2*I)*b^2*d*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I
)*b^2*d*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (I*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x])
```

Rubi [A] time = 0.510842, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {5712, 5821, 5675, 5717, 5653, 261}

$$-\frac{2iabdx\sqrt{c^2x^2+1}}{\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{d\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{id\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{2ib^2d\left(c^2x^2+1\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```


```
[Out] ((-2*I)*a*b*d*x*Sqrt[1 + c^2*x^2])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) +
((2*I)*b^2*d*(1 + c^2*x^2))/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]) - ((2*I
)*b^2*d*x*Sqrt[1 + c^2*x^2]*ArcSinh[c*x])/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f
*x]) + (I*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*Sqrt[d + I*c*d*x]*Sqrt
[f - I*c*f*x]) + (d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d
 + I*c*d*x]*Sqrt[f - I*c*f*x])
```


Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{\left(d\sqrt{1+c^2x^2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{\left(icd\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$

$$= \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{\left(2ibd\sqrt{1+\right.}{}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{d\sqrt{1+c^2x^2}\left(a+b\,\mathrm{s}\right.}{3bc\sqrt{d+icdx}\sqrt{}}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{id\left(1+c^2x^2\right)\left(a+b\sinh\right.}{c\sqrt{d+icdx}\sqrt{f-ic}}$$

$$= -\frac{2iabdx\sqrt{1+c^2x^2}}{\sqrt{d+icdx}\sqrt{f-icfx}} + \frac{2ib^2d\left(1+c^2x^2\right)}{c\sqrt{d+icdx}\sqrt{f-icfx}} - \frac{2ib^2dx\sqrt{1+c^2x^2}\sinh^{-1}(cx)}{\sqrt{d+icdx}\sqrt{f-icfx}} + $$

Mathematica [A] time = 1.11877, size = 315, normalized size = 1.22

$$\frac{3i\sqrt{d+icdx}\sqrt{f-icfx}\left(a^2\sqrt{c^2x^2+1}-2abcx+2b^2\sqrt{c^2x^2+1}\right)+3a^2\sqrt{d}\sqrt{f}\sqrt{c^2x^2+1}\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/Sqrt[f - I*c*f*x],x]
```

```
[Out] ((3*I)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-2*a*b*c*x + a^2*Sqrt[1 + c^2*x
^2] + 2*b^2*Sqrt[1 + c^2*x^2]) - (6*I)*b*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x
]*(b*c*x - a*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 3*b*Sqrt[d + I*c*d*x]*Sqrt[f
 - I*c*f*x]*(a + I*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x]^2 + b^2*Sqrt[d + I*c*d
*x]*Sqrt[f - I*c*f*x]*ArcSinh[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[f]*Sqrt[1 + c^2*x
^2]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]])/(3*
c*f*Sqrt[1 + c^2*x^2])
```

Maple [F] time = 0.323, size = 0, normalized size = 0.

$$\int (a+b\operatorname{Arcsinh}(cx))^2 \sqrt{d+icdx}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i\sqrt{icdx+d}\sqrt{-icfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2i\sqrt{icdx+d}\sqrt{-icfx+f}ab\log\left(cx+\sqrt{c^2x^2+1}\right)+i\sqrt{icdx}}{cfx+if}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
 + 1))^2 + 2*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*
x^2 + 1)) + I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c*f*x + I*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\left(icx+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)^2}{\sqrt{-f\left(icx-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(I*c*x + 1))*(a + b*asinh(c*x))**2/sqrt(-f*(I*c*x - 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.591 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rubi [A] time = 0.296079, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {5712, 5675}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^3)/(3*b*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x])
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}\sqrt{f-icfx}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+icdx}\sqrt{f-icfx}}$$
$$= \frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc\sqrt{d+icdx}\sqrt{f-icfx}}$$

Mathematica [B] time = 0.815277, size = 168, normalized size = 2.85

$$\frac{a^2\log\left(cdfx+\sqrt{d}\sqrt{f}\sqrt{d+icdx}\sqrt{f-icfx}\right)}{c\sqrt{d}\sqrt{f}}+\frac{ab\sqrt{c^2x^2+1}\sinh^{-1}(cx)^2}{c\sqrt{d+icdx}\sqrt{f-icfx}}+\frac{b^2\sqrt{c^2x^2+1}\sinh^{-1}(cx)^3}{3c\sqrt{d+icdx}\sqrt{f-icfx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (a*b*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2)/(c*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*
x]) + (b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^3)/(3*c*Sqrt[d + I*c*d*x]*Sqrt[f
- I*c*f*x]) + (a^2*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f -
 I*c*f*x]])/(c*Sqrt[d]*Sqrt[f])
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))^2\frac{1}{\sqrt{d+icdx}}\frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}ab\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{i\,cdx+}}{c^2dfx^2+df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
+ 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*d*f*x^2 + d*f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{asinh}\,(cx))^2}{\sqrt{d\,(icx+1)}\sqrt{-f\,(icx-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(sqrt(d*(I*c*x + 1))*sqrt(-f*(I*c*x - 1))),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.592 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=464

$$-\frac{2b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}}{c(d+icdx)^{3/2}(f-}$$

```
[Out] (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (f*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) + (f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*b*f*(1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) - (2*b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b^2*f*(1
 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2)) + (2*b^2*f*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*f*(1 + c^2*x^2)^(3
/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2))
```

Rubi [A] time = 0.753496, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180}

$$-\frac{2b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{2b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2f\left(c^2x^2+1\right)^{3/2}\text{PolyLog}}{c(d+icdx)^{3/2}(f-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (I*f*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*
f*x)^(3/2)) + (f*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) + (f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c
*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*b*f*(1 + c^2*x^2)^(3/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) - (2*b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (2*b^2*f*(1
```

```
 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(
f - I*c*f*x)^(3/2)) + (2*b^2*f*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*f*(1 + c^2*x^2)^(3
/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}\sqrt{f-icfx}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(f\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(icf\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf\left(1+c^2x^2\right.\right.}{(d+icdx}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2ibf\left(1+c^2x^2\right.\right.}{}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f\left(1+c^2x^2\right)^{3/2}}{c(d+icdx)}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f\left(1+c^2x^2\right)^{3/2}}{c(d+icdx)}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f\left(1+c^2x^2\right)^{3/2}}{c(d+icdx)}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{f\left(1+c^2x^2\right)^{3/2}}{c(d+icdx)}$$

Mathematica [A] time = 2.18953, size = 508, normalized size = 1.09

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(4b^2\sqrt{c^2x^2+1}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+\left(\cosh\left(\frac{1}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 + I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + (I*a^2 + a^2*c*x -
```

```
(4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I)*b^2*Pi*Sq
rt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*
Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi*Sqrt[1 + c^
2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]])*(Cosh[ArcSinh[c*x]/2] + I*Sin
h[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]]*(
Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]*(I*Cosh[ArcSinh[c*x]/2]*(2*a - b*Pi + (4*I)*b*Log[1 - I/E^ArcSinh[c*
x]]) + (2*a + b*Pi - (4*I)*b*Log[1 - I/E^ArcSinh[c*x]])*Sinh[ArcSinh[c*x]/2
])))/(c*d^2*f*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c
*x]/2]))
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(cx))^2 (d+icdx)^{-\frac{3}{2}} \frac{1}{\sqrt{f-icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(c^2d^2fx-i\,cd^2f\right)\text{integral}\left(\frac{-i\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}a^2-2\left(\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+}\right.}{c^3d^2fx^3-i\,c^2d^2fx^2+}\right.}{c^2d^2fx-i\,cd^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d^2*f*x - I*c*d^2*f)*integral((-I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)
*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 + I*sq
rt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*
d^2*f*x^3 - I*c^2*d^2*f*x^2 + c*d^2*f*x - I*d^2*f), x))/(c^2*d^2*f*x - I*c*
d^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.593 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx$

Optimal. Leaf size=942

$$-\frac{c^2f^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcf^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2f^2\left(c^2\right.}{3(}$$

```
[Out] (((-2*I)/3)*b^2*f^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (2*b^2*f^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2)) + (b^2*f^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f^2*x*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b*c*f^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*f^
2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (2*f^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b*f^2*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*
x)^(5/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])
/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b^2*f^2*(1 + c^2*x^2)^(
5/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 1.33188, antiderivative size = 942, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261, 5723, 5751, 288, 215}

$$-\frac{c^2f^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcf^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2f^2\left(c^2\right.}{3(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (((-2*I)/3)*b^2*f^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (2*b^2*f^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^
(5/2)) + (b^2*f^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (b*f^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f^2*x*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b*c*f^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(
5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*f^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*f^
2*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) + (2*f^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[
c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*b*f^2*(
1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*f^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcS
inh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*
x)^(5/2)) - (2*b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])
/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b^2*f^2*(1 + c^2*x^2)^(
5/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) - (b^2*f^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}\sqrt{f-icfx}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{f^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{2icf^2x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{c^2f^2x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2icf^2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(c\right.}{}$$

$$= \frac{2if^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{f^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{c^2f^2x^3\left(1+\right.}{3(d+i}$$

$$= \frac{bf^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ibf^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{bcf^2x^2}{3}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(}{3c(d+icdx)^{5/2}(f-icfx)}$$

$$= -\frac{2ib^2f^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2b^2f^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b^2f^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(}{3c(d+icdx)^{5/2}(f-icfx)}$$

Mathematica [A] time = 6.87509, size = 524, normalized size = 0.56

$$\sqrt{d+icdx}\sqrt{f-icfx}\left(-\frac{b^2\left(-4\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+(1-i)\sinh^{-1}(cx)^2-\frac{\left(\sinh^{-1}(cx)-2i\right)\sinh^{-1}(cx)}{cx-i}+2\left(2\sinh^{-1}(cx)-i\pi\right)\log\left(1-ie^{-\sinh^{-1}(cx)}\right)-\frac{}{\left(\cosh\right.}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*Sqrt[f - I*c*f*x]),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(-2*I + c*x))/(-I + c*x)^2 - (a*
b*((-I)*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]
/2]] - (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 - (3*I)*ArcSinh[c
*x] - (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*((-1
 + Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[
ArcSinh[c*x]/2]] + (I/2)*(-2 + (2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2]))*S
inh[ArcSinh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[Arc
Sinh[c*x]/2])^3) - (b^2*((1 - I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(-2*I + Arc
Sinh[c*x]))/(-I + c*x) + 2*((-I)*Pi + 2*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c
*x]] + I*Pi*(ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] + 4*Log[Cosh[ArcSinh[
c*x]/2]] + 2*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]) - 4*PolyLog[2, I/E^ArcS
inh[c*x]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] +
 I*Sinh[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])
/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c
*d^3*f)
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2\,(d + icdx)^{-\frac{5}{2}}\,\frac{1}{\sqrt{f - icfx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2cx - 2i\,b^2\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(3\,c^3d^3fx^2 - 6i\,c^2d^3fx - 3\,cd^3f\right)\text{integral}\left(-\frac{3\,\sqrt{i\,cdx+d}\sqrt{-i\,cf}}{}\right.}{3\,c^3d^3fx^2 - 6i\,c^2d^3fx - 3\,cd^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] ((b^2*c*x - 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^
2*x^2 + 1))^2 + (3*c^3*d^3*f*x^2 - 6*I*c^2*d^3*f*x - 3*c*d^3*f)*integral(-(
3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + (6*sqrt(I*c*d*x + d)*sqrt(-I*c
*f*x + f)*a*b + (2*b^2*c*x - 4*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^4*d^3*f*x^4 - 6*I*c^3
*d^3*f*x^3 - 6*I*c*d^3*f*x - 3*d^3*f), x))/(3*c^3*d^3*f*x^2 - 6*I*c^2*d^3*f
*x - 3*c*d^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.594 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=972

result too large to display

```
[Out] ((8*I)*a*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b^2*d^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b^2*d^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) - (b^2*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) + ((8*I)*b^2*d^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])
/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*d^4*x^2*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((8*
I)*d^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) + (8*d^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^4*(1 + c^2*x
^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) +
 (d^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (5*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b*
c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((32*I)*b*d^4*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (16*b*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*
b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16*b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^4
*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 1.36987, antiderivative size = 972, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 19, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.514, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261, 5758, 5661, 321, 215}

$$-\frac{5\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3 d^4}{2bc(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{b^2x\left(c^2x^2+1\right)^2 d^4}{4(icxd+d)^{3/2}(f-icfx)^{3/2}}-\frac{8ib^2\left(c^2x^2+1\right)^2 d^4}{c(icxd+d)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)^2\left(a+b\right.}{2(icxd+d)^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((8*I)*a*b*d^4*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((8*I)*b^2*d^4*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + (b^2*d^4*x*(1 + c^2*x^2)^2)/(4*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^
(3/2)) - (b^2*d^4*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/(4*c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) + ((8*I)*b^2*d^4*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])
/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b*c*d^4*x^2*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x]))/(2*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((8*
I)*d^4*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*
c*f*x)^(3/2)) + (8*d^4*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (8*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c
*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^4*(1 + c^2*x
^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) +
 (d^4*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(2*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (5*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(2*b*
c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((32*I)*b*d^4*(1 + c^2*x^2)^(3
/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f
- I*c*f*x)^(3/2)) - (16*b*d^4*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[
1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (16*
b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) - (16*b^2*d^4*(1 + c^2*x^2)^(3/2)*PolyLog[2,
I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^4
*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{8i\left(id^4-cd^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{7d^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4icd^4x\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(8i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^4-cd^4x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(7d^4\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-}\right.}{\sqrt{1+c^{}}}}{(d+icdx)^{3/2}(f-icfx)^{}}$$

$$= -\frac{4id^4\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d^4x\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - 7$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{bcd^4x^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{2(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^4\left(1+\right.}{c(d+}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{8ib^2d^4x\left(1+c^2x^2\right)^{3/2}\,\mathrm{s}}{(d+icdx)^{3/2}(f-i}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icf}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icf}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icf}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icf}$$

$$= \frac{8iabd^4x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{8ib^2d^4\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{b^2d^4x\left(1+c^2x^2\right)^2}{4(d+icdx)^{3/2}(f-icf}$$

Mathematica [B] time = 14.2886, size = 2323, normalized size = 2.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-4*I)*a^2*d^2)/f^2 + (a^2*c
*d^2*x)/(2*f^2) + (8*a^2*d^2)/(f^2*(I + c*x))))/c - (15*a^2*d^(5/2)*Log[c*d
*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(2*c*f
^(3/2)) - ((4*I)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*
Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] +
 Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[
c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] +
 I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])) - (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(
I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Ta
nh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 +
 c^2*x^2]])) + (ArcSinh[c*x]*(-4*I + ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt
[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/
2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*
Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-18*Pi*ArcSinh[c*x] -
(6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*
Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[
(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Po
lyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2]
)/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d
 + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[
ArcSinh[c*x]/2])^2) + (b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I
)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-96*c*x*ArcSinh[c*x])/Sqrt[1
+ c^2*x^2] + ((48 - 48*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] - ((20*I)*ArcSi
nh[c*x]^3)/Sqrt[1 + c^2*x^2] + 48*(2 + ArcSinh[c*x]^2) + (6*I)*c*x*(1 + 2*A
rcSinh[c*x]^2) - ((6*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]])/Sqrt[1 + c^2*x^2
] + (48*(2*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcS
inh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])
/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]
))/Sqrt[1 + c^2*x^2] + ((96*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1
 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))/(24*c*f^2*Sq
rt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2])^2) + (2*(-1)^(1/4)*b^2*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I
*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2]*(-3*(-1)^(1/4
)*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 2*(-9*(-1)^(1/4)*Pi*ArcSinh[c*x]
 + 3*(-1)^(1/4)*c*x*ArcSinh[c*x] - 3*Sqrt[2]*ArcSinh[c*x]^2 + (-1)^(3/4)*Ar
cSinh[c*x]^3 - 6*(-1)^(1/4)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c
*x]] + (6 + 6*I)*Sqrt[2]*Pi*Log[1 + E^ArcSinh[c*x]] + 6*(-1)^(1/4)*Pi*Log[-
```

```
Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 12*(-1)^(1/4)*Pi*Log[Cosh[ArcSinh[c*x]/
2]])) + (3*(-1)^(3/4)*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]^2) + 2*(9*(-1)^(3
/4)*Pi*ArcSinh[c*x] - 3*(-1)^(3/4)*c*x*ArcSinh[c*x] + 3*Sqrt[2]*ArcSinh[c*x
]^2 + (-1)^(1/4)*ArcSinh[c*x]^3 + 6*(-1)^(1/4)*(I*Pi + 2*ArcSinh[c*x])*Log[
1 + I/E^ArcSinh[c*x]] + (6 - 6*I)*Sqrt[2]*Pi*Log[1 + E^ArcSinh[c*x]] - 6*(-
1)^(3/4)*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 12*(-1)^(3/4)*Pi*Log[C
osh[ArcSinh[c*x]/2]]))*Sinh[ArcSinh[c*x]/2] - 24*(-1)^(1/4)*PolyLog[2, (-I)
/E^ArcSinh[c*x]]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*f^2
*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*
x]/2] - I*Sinh[ArcSinh[c*x]/2])) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(
-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Sinh[ArcSinh[c*x]/2]*(-16*Sq
rt[1 + c^2*x^2]*ArcSinh[c*x] + I*Cosh[2*ArcSinh[c*x]] + 2*(8*c*x + 8*ArcSin
h[c*x] + (5*I)*ArcSinh[c*x]^2 + 16*ArcTan[Tanh[ArcSinh[c*x]/2]] + (8*I)*Log
[Sqrt[1 + c^2*x^2]] - I*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]])) - Cosh[ArcSinh[
c*x]/2]*((16*I)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] + Cosh[2*ArcSinh[c*x]] - 2*(
(8*I)*c*x - (8*I)*ArcSinh[c*x] - 5*ArcSinh[c*x]^2 + (16*I)*ArcTan[Tanh[ArcS
inh[c*x]/2]] - 8*Log[Sqrt[1 + c^2*x^2]] + ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]]
))))/(4*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Co
sh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}(cx))^2 (d + icdx)^{\frac{5}{2}} \left(f - icfx\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2c^2d^2x^2-2i\,b^2cd^2x-b^2d^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(abc^2d^2x^2-2i\,abcd^2x-abd^2\right)}{c^2f^2x^2+2i\,cf^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c^2*d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(a*b*c^2*d^2*x^2 - 2*I
*a*b*c*d^2*x - a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt
(c^2*x^2 + 1)) + (a^2*c^2*d^2*x^2 - 2*I*a^2*c*d^2*x - a^2*d^2)*sqrt(I*c*d*x
 + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.595 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=752

$$\frac{8b^2d^3\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8b^2d^3\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^3\left(c^2x^2+1\right)^{3/2}\mathrm{PolyI}}{c(d+icdx)^{3/2}(f}$$

```
[Out] ((2*I)*a*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((2*I)*b^2*d^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((2*I)*b^2*d^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])
^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*(1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((16*I)*b*d^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^
2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 1.13277, antiderivative size = 752, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675, 5653, 261}

$$\frac{8b^2d^3\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{8b^2d^3\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^3\left(c^2x^2+1\right)^{3/2}\mathrm{PolyI}}{c(d+icdx)^{3/2}(f}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((2*I)*a*b*d^3*x*(1 + c^2*x^2)^(3/2))/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3
/2)) - ((2*I)*b^2*d^3*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((2*I)*b^2*d^3*x*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x])/((d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2)) - ((4*I)*d^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])
^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*x*(1 + c^2*x^2)*(a
 + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*d^3*(1
 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f
*x)^(3/2)) - (I*d^3*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x
)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]
)^3)/(b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((16*I)*b*d^3*(1 + c^2
*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(
3/2)*(f - I*c*f*x)^(3/2)) - (8*b*d^3*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x
])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
 + (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d +
I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (8*b^2*d^3*(1 + c^2*x^2)^(3/2)*PolyLo
g[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^
2*d^3*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)
^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{4i\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \frac{3d^3\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{icd^3x\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c^2x^2}}\right.}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(4i\left(1+c^2x^2\right)^{3/2}\right)\int\frac{\left(id^3-cd^3x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(3d^3\left(1+c^2x^2\right)^{3/2}\right)\int\frac{(a+b\sin}{\sqrt{1+}}}{(d+icdx)^{3/2}(f-icfx}$$

$$= -\frac{id^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \cdots$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{id^3\left(1+c^2x^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^3\left(1+c^2x^2\right.}{bc(d+ic}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}\sinh^{-1}(cx)}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{4id^3\left(1+c^2x^2\right)}{c(d+icdx}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-}$$

$$= \frac{2iabd^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2ib^2d^3\left(1+c^2x^2\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2ib^2d^3x\left(1+c^2x^2\right)^{3/2}}{(d+icdx)^{3/2}(f-}$$

Mathematica [B] time = 10.8145, size = 1526, normalized size = 2.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I)*a^2*d)/f^2 + (4*a^2*d)/
(f^2*(I + c*x))))/c - (3*a^2*d^(3/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d
*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(3/2)) - ((2*I)*a*b*d*Sqrt[I*((-
I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[Ar
cSinh[c*x]/2]*(-(c*x) + 2*ArcSinh[c*x] + Sqrt[1 + c^2*x^2]*ArcSinh[c*x] - I
*ArcSinh[c*x]^2 + 4*ArcTan[Coth[ArcSinh[c*x]/2]] - (2*I)*Log[Sqrt[1 + c^2*x
^2]]) - ((-I)*c*x - (2*I)*ArcSinh[c*x] + I*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] +
 ArcSinh[c*x]^2 + (4*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 2*Log[Sqrt[1 + c^2*x
^2]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]
*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])) - (a*b*
d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2
))]*(Cosh[ArcSinh[c*x]/2]*(8*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*(ArcSinh[c*x]
*(4*I + ArcSinh[c*x]) + 4*Log[Sqrt[1 + c^2*x^2]])) + (ArcSinh[c*x]*(-4*I +
ArcSinh[c*x]) - (8*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 4*Log[Sqrt[1 + c^2*x^2
]])*Sinh[ArcSinh[c*x]/2]))/(c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*S
qrt[1 + c^2*x^2]*(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])) - (b^2*d*
(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1
+ c^2*x^2))]*(-18*Pi*ArcSinh[c*x] - (6 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*
x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1
 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*L
og[Cosh[ArcSinh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]] - ((12*I)
*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSin
h[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*
x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + ((-1)^(1/4)*b^2*d
*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2)
)]*(Cosh[ArcSinh[c*x]/2]*(-3*(-1)^(1/4)*Sqrt[1 + c^2*x^2]*(2 + ArcSinh[c*x]
^2) + 2*(-9*(-1)^(1/4)*Pi*ArcSinh[c*x] + 3*(-1)^(1/4)*c*x*ArcSinh[c*x] - 3*
Sqrt[2]*ArcSinh[c*x]^2 + (-1)^(3/4)*ArcSinh[c*x]^3 - 6*(-1)^(1/4)*(Pi - (2*
I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + (6 + 6*I)*Sqrt[2]*Pi*Log[1 + E
^ArcSinh[c*x]] + 6*(-1)^(1/4)*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 1
2*(-1)^(1/4)*Pi*Log[Cosh[ArcSinh[c*x]/2]])) + (3*(-1)^(3/4)*Sqrt[1 + c^2*x^
2]*(2 + ArcSinh[c*x]^2) + 2*(9*(-1)^(3/4)*Pi*ArcSinh[c*x] - 3*(-1)^(3/4)*c*
x*ArcSinh[c*x] + 3*Sqrt[2]*ArcSinh[c*x]^2 + (-1)^(1/4)*ArcSinh[c*x]^3 + 6*(
-1)^(1/4)*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + (6 - 6*I)*Sqr
t[2]*Pi*Log[1 + E^ArcSinh[c*x]] - 6*(-1)^(3/4)*Pi*Log[-Cos[(Pi + (2*I)*ArcS
inh[c*x])/4]] + 12*(-1)^(3/4)*Pi*Log[Cosh[ArcSinh[c*x]/2]]))*Sinh[ArcSinh[c
*x]/2] - 24*(-1)^(1/4)*PolyLog[2, (-I)/E^ArcSinh[c*x]]*(I*Cosh[ArcSinh[c*x]
/2] + Sinh[ArcSinh[c*x]/2])))/(3*c*f^2*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x
))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \,(d + icdx)^{\frac{3}{2}} \left(f - icfx\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-i\,b^2cdx - b^2d\right)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 - 2\,(i\,abcdx + abd)\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}\,\log\left(\right.}{c^2f^2x^2 + 2i\,cf^2x - f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(((-I*b^2*c*d*x - b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c
*x + sqrt(c^2*x^2 + 1))^2 - 2*(I*a*b*c*d*x + a*b*d)*sqrt(I*c*d*x + d)*sqrt(
-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (-I*a^2*c*d*x - a^2*d)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f))/(c^2*f^2*x^2 + 2*I*c*f^2*x - f^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.596 $\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=544

$$\frac{4b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyL}}{c(d+icdx)^{3/2}(f}$$

```
[Out] ((-2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) + (2*d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
inh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*d^2*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (
-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*d
^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)
*(f - I*c*f*x)^(3/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.98788, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180, 5675}

$$\frac{4b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{4b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^{3/2}\text{PolyL}}{c(d+icdx)^{3/2}(f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```



```
[Out] ((-2*I)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f
 - I*c*f*x)^(3/2)) + (2*d^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I
*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcS
inh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (d^2*(1 + c^2*x^
2)^(3/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(
3/2)) + ((8*I)*b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcS
```

```
inh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b*d^2*(1 + c^2*
x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*
x)^(3/2)*(f - I*c*f*x)^(3/2)) + (4*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, (
-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (4*b^2*d
^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)
*(f - I*c*f*x)^(3/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*Arc
Sinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int\left(-\frac{2i\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(id^2-cd^2x\right)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(d^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c^2x^2}}}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2i\left(1+c^2x^2\right)^{3/2}\right)\int\left(\frac{id^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} - \right.}{(d+icdx)^{3/2}(f-icfx)}$$

$$= -\frac{d^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2d^2\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} +$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2}{}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{d^2}{}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d}{}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d}{}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d}{}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{2d}{}$$

Mathematica [A] time = 5.45369, size = 530, normalized size = 0.97

$$-\frac{b^2(cx-i)\sqrt{d+icdx}\sqrt{f-icfx}\left(-24i\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+i\sinh^{-1}(cx)^3-(6-6i)\sinh^{-1}(cx)^2-18\pi\sinh^{-1}(cx)-12\left(\pi-2i\sinh^{-1}(cx)\right)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)+24\right.}{\sqrt{c^2x^2+1}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)+i\sinh\left(\frac{1}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(3/2),x]
```

```
[Out] ((6*a^2*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x])/(I + c*x) - 3*a^2*Sqrt[d]*Sqrt
[f]*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]] - (b
^2*(-I + c*x)*Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*(-18*Pi*ArcSinh[c*x] - (6
 - 6*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 - 12*(Pi - (2*I)*ArcSinh[c*x])*Lo
g[1 + I/E^ArcSinh[c*x]] + 24*Pi*Log[1 + E^ArcSinh[c*x]] + 12*Pi*Log[-Cos[(P
i + (2*I)*ArcSinh[c*x])/4]] - 24*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (24*I)*Poly
Log[2, (-I)/E^ArcSinh[c*x]] - ((12*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (3*a*b*Sqrt[d + I*c*d*x]*Sqr
t[f - I*c*f*x]*(-(ArcSinh[c*x]^2*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])) + 4*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])
+ 2*(4*ArcTan[Tanh[ArcSinh[c*x]/2]] + I*Log[1 + c^2*x^2])*(I*Cosh[ArcSinh[c
*x]/2] + Sinh[ArcSinh[c*x]/2])))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] -
 I*Sinh[ArcSinh[c*x]/2])))/(3*c*f^2)
```

Maple [F] time = 0.268, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\,\sqrt{d+icdx}\left(f-icfx\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{icdx+d}\sqrt{-icfx+f}b^2\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\sqrt{icdx+d}\sqrt{-icfx+f}ab\log\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{icdx}}{c^2f^2x^2+2icf^2x-f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 2*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b*log(c*x + sqrt(c^2*x^2
 + 1)) + sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2)/(c^2*f^2*x^2 + 2*I*c*f^2
*x - f^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d(icx+1)}\,(a+b\,\text{asinh}\,(cx))^2}{\left(-f\,(icx-1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Integral(sqrt(d*(I*c*x + 1))*(a + b*asinh(c*x))**2/(-f*(I*c*x - 1))**(3/2),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.597 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=464

$$\frac{2b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}}{c(d+icdx)^{3/2}(f-}$$

```
[Out] ((-I)*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*b*d*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*b^2*d
*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) - (2*b^2*d*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*d*(1 + c^2*x^2)
^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2))
```

Rubi [A] time = 0.703858, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {5712, 5821, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 5693, 4180}

$$\frac{2b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{b^2d\left(c^2x^2+1\right)^{3/2}\text{PolyLog}}{c(d+icdx)^{3/2}(f-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-I)*d*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/2)*(f - I
*c*f*x)^(3/2)) + (d*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^
(3/2)*(f - I*c*f*x)^(3/2)) + (d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)
/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + ((4*I)*b*d*(1 + c^2*x^2)^(3/
2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f -
 I*c*f*x)^(3/2)) - (2*b*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log[1 +
E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) + (2*b^2*d
```

```
*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(3/2
)*(f - I*c*f*x)^(3/2)) - (2*b^2*d*(1 + c^2*x^2)^(3/2)*PolyLog[2, I*E^ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^2*d*(1 + c^2*x^2)
^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x
)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{3/2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}} + \frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\right)dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{\left(d\left(1+c^2x^2\right)^{3/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(icd\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ibd\left(1+c^2x^2\right)\right.}{(d+icdx)}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(2ibd\left(1+c^2x^2\right)\right.}{}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d\left(1+c^2x^2\right)^{3/2}\left(\right.}{c(d+icdx)^{3}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d\left(1+c^2x^2\right)^{3/2}\left(\right.}{c(d+icdx)^{3}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d\left(1+c^2x^2\right)^{3/2}\left(\right.}{c(d+icdx)^{3}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{d\left(1+c^2x^2\right)^{3/2}\left(\right.}{c(d+icdx)^{3}}$$

Mathematica [A] time = 2.21784, size = 511, normalized size = 1.1

$$\frac{\sqrt{d+icdx}\sqrt{f-icfx}\left(4b^2\sqrt{c^2x^2+1}\left(\cosh\left(\frac{1}{2}\sinh^{-1}(cx)\right)-i\sinh\left(\frac{1}{2}\sinh^{-1}(cx)\right)\right)\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+\left(\cosh\left(\frac{1}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((-1 - I)*b^2*Sqrt[1 + c^2*x^2]*ArcSin
h[c*x]^2*(Cosh[ArcSinh[c*x]/2] - Sinh[ArcSinh[c*x]/2]) + ((-I)*a^2 + a^2*c*
```

```
x + (4*I)*a*b*Sqrt[1 + c^2*x^2]*ArcTan[Tanh[ArcSinh[c*x]/2]] - (2*I)*b^2*Pi
*Sqrt[1 + c^2*x^2]*Log[1 + I/E^ArcSinh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^
2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqrt[1 + c^2*x^2]*Log[1 + c^2*x^2] + (2*I)
*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[c*x]/2]])*(Cosh[ArcSinh[c*x]/2] - I
*Sinh[ArcSinh[c*x]/2]) + 4*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[
c*x]]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + b*Sqrt[1 + c^2*x^2]
*ArcSinh[c*x]*((-I)*Cosh[ArcSinh[c*x]/2]*(2*a + 3*b*Pi - (4*I)*b*Log[1 + I/
E^ArcSinh[c*x]]) + (2*a - 3*b*Pi + (4*I)*b*Log[1 + I/E^ArcSinh[c*x]])*Sinh[
ArcSinh[c*x]/2])))/(c*d*f^2*(-I + c*x)*(I + c*x)*(Cosh[ArcSinh[c*x]/2] - I*
Sinh[ArcSinh[c*x]/2]))
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \left(f - icfx\right)^{-\frac{3}{2}} \frac{1}{\sqrt{d + icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}b^2 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(c^2df^2x + i\,cdf^2\right)\text{integral}\left(\frac{i\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}a^2 - 2\left(\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx\cdots}\right.}{c^3df^2x^3+i\,c^2df^2x^2\cdots}\right.}{c^2df^2x + i\,cdf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*log(c*x + sqrt(c^2*x^2 + 1))^2 +
(c^2*d*f^2*x + I*c*d*f^2)*integral((I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*
a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2 - I*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^3*d
*f^2*x^3 + I*c^2*d*f^2*x^2 + c*d*f^2*x + I*d*f^2), x))/(c^2*d*f^2*x + I*c*d
*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(3/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(3/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.598 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{b^2\left(c^2x^2+1\right)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b}{}$$

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - (2*b*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rubi [A] time = 0.423362, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {5712, 5687, 5714, 3718, 2190, 2279, 2391}

$$-\frac{b^2\left(c^2x^2+1\right)^{3/2}\operatorname{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}+\frac{x\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}-\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)
^(3/2)) + ((1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2)) - (2*b*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x])*Log
[1 + E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)) - (b^
2*(1 + c^2*x^2)^(3/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(c*(d + I*c*d*x)^(3/
2)*(f - I*c*f*x)^(3/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
```

```
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{3/2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2bc\left(1+c^2x^2\right)^{3/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{1+c^2x^2}\,dx}{(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(2b\left(1+c^2x^2\right)^{3/2}\right)\text{Subst}\left(\int(a+bx)\tanh(x)\,dx, x,\right.}{c(d+icdx)^{3/2}(f-icfx)^{3/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{\left(4b\left(1+c^2x^2\right)\right.}{c}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b\left(1+c^2x^2\right)^{3/}}{c}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b\left(1+c^2x^2\right)^{3/}}{c}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{3/2}} + \frac{\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{3/2}(f-icfx)^{3/2}} - \frac{2b\left(1+c^2x^2\right)^{3/}}{c}$$

Mathematica [B] time = 1.36882, size = 488, normalized size = 2.18

$$\frac{2b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+2b^2\sqrt{c^2x^2+1}\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+a^2cx-ab\sqrt{c^2x^2+1}\log\left(c^2x^2+1\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcSinh[c*x] - (2*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*ArcSinh[
c*x] + b^2*c*x*ArcSinh[c*x]^2 - b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + I*b^
2*Pi*Sqrt[1 + c^2*x^2]*Log[1 - I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*
ArcSinh[c*x]*Log[1 - I/E^ArcSinh[c*x]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 +
 I/E^ArcSinh[c*x]] - 2*b^2*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*Log[1 + I/E^ArcSi
nh[c*x]] + (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[1 + E^ArcSinh[c*x]] - a*b*Sqr
t[1 + c^2*x^2]*Log[1 + c^2*x^2] + I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[-Cos[(Pi +
 (2*I)*ArcSinh[c*x])/4]] - (4*I)*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Cosh[ArcSinh[
```

```
c*x]/2]] - I*b^2*Pi*Sqrt[1 + c^2*x^2]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]]
 + 2*b^2*Sqrt[1 + c^2*x^2]*PolyLog[2, (-I)/E^ArcSinh[c*x]] + 2*b^2*Sqrt[1 +
 c^2*x^2]*PolyLog[2, I/E^ArcSinh[c*x]])/(c*d*f*Sqrt[d + I*c*d*x]*Sqrt[f - I
*c*f*x])
```

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 (d + icdx)^{-\frac{3}{2}} \left(f - icfx\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2 \int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{(i\,cdx + d)^{\frac{3}{2}}\left(-i\,cfx + f\right)^{\frac{3}{2}}}\,dx - \frac{abc\sqrt{\frac{1}{c^4df}}\log\left(x^2 + \frac{1}{c^2}\right)}{df} + \frac{2\,abx\,\text{arsinh}\,(cx)}{\sqrt{c^2dfx^2 + df}df} + \frac{a^2x}{\sqrt{c^2dfx^2 + df}df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] b^2*integrate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(3/2)*(-I*c*f*x
 + f)^(3/2)), x) - a*b*c*sqrt(1/(c^4*d*f))*log(x^2 + 1/c^2)/(d*f) + 2*a*b*x
*arcsinh(c*x)/(sqrt(c^2*d*f*x^2 + d*f)*d*f) + a^2*x/(sqrt(c^2*d*f*x^2 + d*f
)*d*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{i\,cdx + d}\sqrt{-i\,cfx + f}b^2x\log\left(cx + \sqrt{c^2x^2 + 1}\right)^2 + \left(c^2d^2f^2x^2 + d^2f^2\right)\text{integral}\left(\frac{\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}a^2-2\left(\sqrt{c^2x^2+1}\sqrt{i\,cdx+d}\sqrt{-i\,cfx+\cdots}\right.}{c^4d^2f^2x^4+2\,c^2d\cdots}\right.}{c^2d^2f^2x^2 + d^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*x*log(c*x + sqrt(c^2*x^2 + 1))^2
+ (c^2*d^2*f^2*x^2 + d^2*f^2)*integral((sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f
)*a^2 - 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*b^2*c*x -
 sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*log(c*x + sqrt(c^2*x^2 + 1)))/(c
^4*d^2*f^2*x^4 + 2*c^2*d^2*f^2*x^2 + d^2*f^2), x))/(c^2*d^2*f^2*x^2 + d^2*f
^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.599 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx$

Optimal. Leaf size=743

$$-\frac{b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(\right.}{3c(d+icdx)^{5/2}(f-i}$$

```
[Out] ((-I/3)*b^2*f*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b^2*f*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*b*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f*x*(1 + c^2*
x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
(2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + (2*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (4*b*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b^2*f*(1
 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b^2*f*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*f*(1 + c^2*x^2
)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2))
```

Rubi [A] time = 0.908861, antiderivative size = 743, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261}

$$-\frac{b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2f\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(\right.}{3c(d+icdx)^{5/2}(f-i}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),x]
```

```
[Out] ((-I/3)*b^2*f*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (b^2*f*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
b*f*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
 I*c*f*x)^(5/2)) - ((I/3)*b*f*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(
(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*f*(1 + c^2*x^2)*(a + b*Ar
cSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (f*x*(1 + c^2*
x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) +
(2*f*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + (2*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*b*f*(1 + c^2*x^2)^(5/2)*
(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) - (4*b*f*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(
2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (b^2*f*(1
 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b^2*f*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c
*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*f*(1 + c^2*x^2
)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{3/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(f-icfx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}} - \frac{icfx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(f\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(icf\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{if\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{fx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2f\left(1+c^2x^2\right)\right. \cdots}{3(d+icd \cdots}$$

$$= \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{ibfx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{if\left(1+c^2 \cdots\right.}{3c(d+ \cdots}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sin \cdots\right.}{3c(d+icdx)^{5/2}(f-ic \cdots}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sin \cdots\right.}{3c(d+icdx)^{5/2}(f-ic \cdots}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sin \cdots\right.}{3c(d+icdx)^{5/2}(f-ic \cdots}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sin \cdots\right.}{3c(d+icdx)^{5/2}(f-ic \cdots}$$

$$= -\frac{ib^2f\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2fx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bf\left(1+c^2x^2\right)^{3/2}\left(a+b\sin \cdots\right.}{3c(d+icdx)^{5/2}(f-ic \cdots}$$

Mathematica [A] time = 8.25687, size = 754, normalized size = 1.01

$$\frac{ib^2\sqrt{c^2x^2+1}\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\left(6i\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+10i\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)-(1+4i)\sinh^{-1}(c \cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(3/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(((-I/6)*a^2)/(d^3*f^2*(-I + c
*x)^2) + (5*a^2)/(12*d^3*f^2*(-I + c*x)) + a^2/(4*d^3*f^2*(I + c*x))))/c +
((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSinh
[c*x] + (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 - (2
*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] - (2
*I)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d^2*f*(-I + c*
x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) + ((
I/6)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x^2
]*(7*Pi*ArcSinh[c*x] + ((2 + I*ArcSinh[c*x])*ArcSinh[c*x])/(-I + c*x) - (1
+ 4*I)*ArcSinh[c*x]^2 - 5*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x
]] + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - 16*Pi*Log[1 +
E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 16*Pi*Log[C
osh[ArcSinh[c*x]/2]] + 5*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + (6*I)*P
olyLog[2, (-I)/E^ArcSinh[c*x]] + (10*I)*PolyLog[2, I/E^ArcSinh[c*x]] + ((3*
I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcS
inh[c*x]/2]) + ((2*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*
x]/2] + I*Sinh[ArcSinh[c*x]/2])^3 + ((-4 + 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c
*x]/2])/((-I)*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])))/(c*d^2*f*Sqrt[
-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}(cx))^2 (d + icdx)^{-\frac{5}{2}} \left(f - icfx\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,b^{2}c^{2}x^{2}-2i\,b^{2}cx+b^{2}\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)^{2}+\left(3\,c^{4}d^{3}f^{2}x^{3}-3i\,c^{3}d^{3}f^{2}x^{2}+3\,c^{2}d^{3}f^{2}x-3i\,cd^{3}f^{2}\right.}{3\,c^{4}d^{3}f^{2}x^{3}-3i\,c^{3}d^{3}f^{2}x^{2}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] ((2*b^2*c^2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + (3*c^4*d^3*f^2*x^3 - 3*I*c^3*d^3*f^2*x^2 +
3*c^2*d^3*f^2*x - 3*I*c*d^3*f^2)*integral((-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c
*f*x + f)*a^2 - 2*(3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*b^2*c^
2*x^2 - 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^5*d^3*f^2*x^5 - 3*I*c^4*d^3*f^2*
x^4 + 6*c^3*d^3*f^2*x^3 - 6*I*c^2*d^3*f^2*x^2 + 3*c*d^3*f^2*x - 3*I*d^3*f^2
), x))/(3*c^4*d^3*f^2*x^3 - 3*I*c^3*d^3*f^2*x^2 + 3*c^2*d^3*f^2*x - 3*I*c*d
^3*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.600 $\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=794

$$-\frac{112b^2d^5\left(c^2x^2+1\right)^{5/2}\operatorname{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2iabd^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+$$

```
[Out] ((-2*I)*a*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + ((2*I)*b^2*d^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) - ((2*I)*b^2*d^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + (28*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (I*d^5*(1 + c^2*x^2
)^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
5*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (112*b*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]
)*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (112*b^2*d^5*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^5*(1 + c^2*x^2)^(5/2)*(a +
b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) + (((16*I)/3)*b^2*d^5*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((28*I)/
3)*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[
c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*d^5*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 1.39589, antiderivative size = 794, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {5712, 5833, 5675, 5717, 5653, 261, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{112b^2d^5\left(c^2x^2+1\right)^{5/2}\operatorname{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2iabd^5x\left(c^2x^2+1\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{5d^5\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] ((-2*I)*a*b*d^5*x*(1 + c^2*x^2)^(5/2))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + ((2*I)*b^2*d^5*(1 + c^2*x^2)^3)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) - ((2*I)*b^2*d^5*x*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/((d + I*c*d*x
)^(5/2)*(f - I*c*f*x)^(5/2)) + (28*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (I*d^5*(1 + c^2*x^2
)^3*(a + b*ArcSinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
5*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3*b*c*(d + I*c*d*x)^(5/2
)*(f - I*c*f*x)^(5/2)) + (112*b*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x]
)*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
- (112*b^2*d^5*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d
 + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (8*b*d^5*(1 + c^2*x^2)^(5/2)*(a +
b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) + (((16*I)/3)*b^2*d^5*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (
I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((28*I)/
3)*d^5*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[
c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((4*I)/3)*d^5*(1 + c^
2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^5\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{5d^5\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} + \frac{icd^5x\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{8d^5\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}} - \frac{12id}{}\right.}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(12id^5\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(5d^5\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{id^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{(1}{}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^5\left(1+c^2x^2\right)^3\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{5d^5\left(1+c^2x\right.}{3bc(d+}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^5\left(1+c^2x^2\right)^3}{c(d+icdx)}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-}$$

$$= -\frac{2iabd^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2ib^2d^5\left(1+c^2x^2\right)^3}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{2ib^2d^5x\left(1+c^2x^2\right)^{5/2}}{(d+icdx)^{5/2}(f-}$$

<u>Mathematica</u> [B] time = 13.6484, size = 2552, normalized size = 3.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(5/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
```

x]

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((I*a^2*d^2)/f^3 + (((8*I)/3)*
a^2*d^2)/(f^3*(I + c*x)^2) - (28*a^2*d^2)/(3*f^3*(I + c*x))))/c + (5*a^2*d^
(5/2)*Log[c*d*f*x + Sqrt[d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c
*x)]])/(c*f^(5/2)) - ((I/3)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSin
h[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSin
h[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*Arc
Sinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]])
+ 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]]
 + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcS
inh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 +
I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Si
nh[ArcSinh[c*x]/2])^4) + (a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f +
 c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[
c*x]/2])*(Cosh[(3*ArcSinh[c*x])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] +
(28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[Arc
Sinh[c*x]/2]*(8 + (6*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tan
h[ArcSinh[c*x]/2]] + 42*Log[Sqrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[
c*x] + 6*ArcSinh[c*x]^2 - (56*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt
[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) -
(28*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcS
inh[c*x]/2]))/(3*c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*
(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d^2*(-I + c
*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x
^2))]*((-1 - I)*ArcSinh[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I +
 c*x) - (2*I)*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3
*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[
c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]
] - (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[
ArcSinh[c*x]/2])^3 + (2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[Ar
cSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I
*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]
/2])^2) + ((I/3)*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f
 + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(((-6*I)*c*x*ArcSinh[c*x])/Sqrt[1 + c
^2*x^2] + ((13 + 13*I)*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (3*ArcSinh[c*x]^
3)/Sqrt[1 + c^2*x^2] + (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/((I + c*x)*Sqr
t[1 + c^2*x^2]) + (3*I)*(2 + ArcSinh[c*x]^2) + ((13*I)*(2*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^ArcSi
nh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*
x]/2]]) + (4*I)*PolyLog[2, (-I)/E^ArcSinh[c*x]]))/Sqrt[1 + c^2*x^2] + (4*Ar
cSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2]
 - I*Sinh[ArcSinh[c*x]/2])^3) - (2*(4 + 13*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x
```

```
]/2])/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]))))
/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])^2) + (2*b^2*d^2*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(-21*Pi*ArcSinh[c*x] - (7
 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*ArcSinh[c*x]*(2*I + ArcS
inh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x
]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x
])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*PolyLog[2, (-I)/E^ArcSinh
[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[
c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])
/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3))/(3*c*f^3*Sqrt[-(((-I)*
d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh
[ArcSinh[c*x]/2])^2) - ((I/6)*a*b*d^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I
*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcS
inh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(9 - (35*I)*ArcSinh[c*x] + 9*ArcSi
nh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]
)) + Cosh[ArcSinh[c*x]/2]*(20 + (24*I)*ArcSinh[c*x] + 27*ArcSinh[c*x]^2 + (
156*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + 78*Log[Sqrt[1 + c^2*x^2]]) - I*(3*(-I
 + ArcSinh[c*x])*Cosh[(5*ArcSinh[c*x])/2] + 2*(13 + (7*I)*ArcSinh[c*x] + 18
*ArcSinh[c*x]^2 + (104*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*(I + ArcSinh
[c*x])*Cosh[2*ArcSinh[c*x]] + 52*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2*x^2]
*(6 + (38*I)*ArcSinh[c*x] + 9*ArcSinh[c*x]^2 + (52*I)*ArcTan[Coth[ArcSinh[c
*x]/2]] + 26*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2])))/(c*f^3*(-I +
c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^4)
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \,(d + icdx)^{\frac{5}{2}} \left(f - icfx\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(i\,b^2c^2d^2x^2+2\,b^2cd^2x-i\,b^2d^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(2i\,abc^2d^2x^2+4\,abcd^2x-2i\,a\right.}{c^3f^3x^3+3i\,c^2f^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((I*b^2*c^2*d^2*x^2 + 2*b^2*c*d^2*x - I*b^2*d^2)*sqrt(I*c*d*x + d)
*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1))^2 + (2*I*a*b*c^2*d^2*x^2 +
 4*a*b*c*d^2*x - 2*I*a*b*d^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
+ sqrt(c^2*x^2 + 1)) + (I*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x - I*a^2*d^2)*sqrt
(I*c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*
x - I*f^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(5/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(5/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.601 $\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=584

$$-\frac{32b^2d^4\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

```
[Out] (8*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3
*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b*d^4*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (32*b^2*d^4*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^A
rcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*d^4*(1 +
 c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((8*I)/3)*b^2*d^4*(1 + c^2*x^2
)^(5/2)*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (((8*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((
2*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*Arc
Sinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rubi [A] time = 1.2055, antiderivative size = 584, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {5712, 5833, 5675, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{32b^2d^4\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{8d^4\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}\right.}{3c(d+icdx)^{5/2}(f-icfx)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```



```
[Out] (8*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^3)/(3
*b*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (32*b*d^4*(1 + c^2*x^2)^(5/
2)*(a + b*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) - (32*b^2*d^4*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^A
```

```
rcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (4*b*d^4*(1 +
 c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c
*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((8*I)/3)*b^2*d^4*(1 + c^2*x^2
)^(5/2)*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (((8*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi
/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (((
2*I)/3)*d^4*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*Arc
Sinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5675

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; F
reeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(d+icdx)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^4\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{d^4\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}} - \frac{4d^4\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}} - \frac{4id^4\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(4id^4\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{1+c^2x^2}}}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(4id^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{(a+bx)^2}{ic+c\sinh(x)}\right.}{(d+icdx)^{5/2}(f-icfx)^{5}}$$

$$= \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^4\left(1+c^2x^2\right)^{5/2}\right)\text{Subst}\left(\int (a+bx)^2\,\text{cs}\right.}{c(d+icdx)^{5/2}(f-}$$

$$= \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4bd^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\text{se}}{3c(d+icdx)^{5/2}(f-icf}$$

$$= \frac{4d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

$$= \frac{8d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{d^4\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^3}{3bc(d+icdx)^{5/2}(f-icfx)^{5/2}} +$$

Mathematica [B] time = 10.0731, size = 1617, normalized size = 2.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + I*c*d*x)^(3/2)*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),
```

x]

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((4*I)/3)*a^2*d)/(f^3*(I + c
*x)^2) - (8*a^2*d)/(3*f^3*(I + c*x))))/c + (a^2*d^(3/2)*Log[c*d*f*x + Sqrt[
d]*Sqrt[f]*Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]])/(c*f^(5/2)) - ((I/
3)*a*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 +
c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSi
nh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 +
 c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[A
rcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*A
rcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]])
+ 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 +
 c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d
*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^4) + (a
*b*d*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*
x^2))]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])*(Cosh[(3*ArcSinh[c*x
])/2]*((14*I - 3*ArcSinh[c*x])*ArcSinh[c*x] + (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] - 14*Log[Sqrt[1 + c^2*x^2]]) + Cosh[ArcSinh[c*x]/2]*(8 + (6*I)*ArcSi
nh[c*x] + 9*ArcSinh[c*x]^2 - (84*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 42*Log[S
qrt[1 + c^2*x^2]]) - (2*I)*(4 + (4*I)*ArcSinh[c*x] + 6*ArcSinh[c*x]^2 - (56
*I)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 28*Log[Sqrt[1 + c^2*x^2]] + Sqrt[1 + c^2
*x^2]*(ArcSinh[c*x]*(14*I + 3*ArcSinh[c*x]) - (28*I)*ArcTan[Tanh[ArcSinh[c*
x]/2]] + 14*Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(6*c*f^3*(1 + I
*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]/2] - I*Sin
h[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*d*(-I + c*x)*Sqrt[I*((-I)*d + c*d*x)]*Sq
rt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSinh[c*x]^2
- (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2*I)*ArcSi
nh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[1 + E^Arc
Sinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh[ArcSinh[
c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*Sinh[ArcS
inh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (2*(4 + Ar
cSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[
c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]
*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2) + (b^2*d*(-I + c*x)*Sqr
t[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(
-21*Pi*ArcSinh[c*x] - (7 - 7*I)*ArcSinh[c*x]^2 + I*ArcSinh[c*x]^3 + ((2*I)*
ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - 14*(Pi - (2*I)*ArcSinh[c*x])
*Log[1 + I/E^ArcSinh[c*x]] + 28*Pi*Log[1 + E^ArcSinh[c*x]] + 14*Pi*Log[-Cos
[(Pi + (2*I)*ArcSinh[c*x])/4]] - 28*Pi*Log[Cosh[ArcSinh[c*x]/2]] - (28*I)*P
olyLog[2, (-I)/E^ArcSinh[c*x]] - ((2*I)*(4 + 7*ArcSinh[c*x]^2)*Sinh[ArcSinh
[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) + (4*ArcSinh[c*x]
^2*Sinh[ArcSinh[c*x]/2])/(I*Cosh[ArcSinh[c*x]/2] + Sinh[ArcSinh[c*x]/2])^3)
)/(3*c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 + c^2*x^2]*(Cosh[
ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\,(d+icdx)^{\frac{3}{2}}\left(f-icfx\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2cdx-ib^2d\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(2\,abcdx-2i\,abd)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(\right.}{c^3f^3x^3+3i\,c^2f^3x^2-3\,cf^3x-i\,f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(((b^2*c*d*x - I*b^2*d)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*
x + sqrt(c^2*x^2 + 1))^2 + (2*a*b*c*d*x - 2*I*a*b*d)*sqrt(I*c*d*x + d)*sqrt
```

```
(-I*c*f*x + f)*log(c*x + sqrt(c^2*x^2 + 1)) + (a^2*c*d*x - I*a^2*d)*sqrt(I*
c*d*x + d)*sqrt(-I*c*f*x + f))/(c^3*f^3*x^3 + 3*I*c^2*f^3*x^2 - 3*c*f^3*x -
 I*f^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)**(3/2)*(a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+I*c*d*x)^(3/2)*(a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")

[Out] Exception raised: AttributeError
```

3.602 $\displaystyle\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=522

$$-\frac{4b^2d^3\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{-}\right.}{3c(d+icdx)^{5/2}}$$

```
[Out] (d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(
f - I*c*f*x)^(5/2)) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log
[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*
b^2*d^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (((4*I)/3)*b^2*d^3*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (I/2)*Arc
Sinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*d^3*(1 + c
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 1.15483, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {5712, 5833, 5831, 3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4b^2d^3\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^3\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{4bd^3\left(c^2x^2+1\right)^{5/2}\log\left(1+ie^{-}\right.}{3c(d+icdx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(
f - I*c*f*x)^(5/2)) + (4*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log
[1 + I/E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*
b^2*d^3*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)/E^ArcSinh[c*x]])/(3*c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*b*d^3*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*
c*f*x)^(5/2)) + (((4*I)/3)*b^2*d^3*(1 + c^2*x^2)^(5/2)*Tan[Pi/4 + (I/2)*Arc
Sinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + ((I/3)*d^3*(1 + c
```

```
^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2*Tan[Pi/4 + (I/2)*ArcSinh[c*x]])/(c*(d
+ I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d^3*(1 + c^2*x^2)^(5/2)*(a +
 b*ArcSinh[c*x])^2*Sec[Pi/4 + (I/2)*ArcSinh[c*x]]^2*Tan[Pi/4 + (I/2)*ArcSin
h[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5833

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*
x])^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && ILtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5831

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

```
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\sqrt{d+icdx}\left(a+b\sinh^{-1}(cx)\right)^2}{(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^3\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(-\frac{2d^3\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}}-\frac{id^3\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(id^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2d^3\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(i+cx)^2\sqrt{1+c^2x^2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{\left(id^3\left(1+c^2x^2\right)^{5/2}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^2}{ic+c\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2cd^3\left(1+c^2x^2\right)^{5/2}\right)\mathrm{S}\ldots}{(d\ldots}$$

$$= -\frac{\left(d^3\left(1+c^2x^2\right)^{5/2}\right)\mathrm{Subst}\left(\int (a+bx)^2\csc^2\left(\frac{\pi}{4}-\frac{ix}{2}\right)dx, x, \sinh^{-1}(cx)\right)}{2c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(d^3\left(1+\ldots\right.\right.}{}$$

$$= \frac{2bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\sec^2\left(\frac{\pi}{4}+\frac{1}{2}i\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{id^3\left(1+c^2x^2\right)^{5/2}\ldots}{}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2bd^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\sec^2\left(\ldots\right.}{3c(d+icdx)^{5/2}(f-icfx)^{5/\ldots}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sinh^{-1}(cx)\right)\ldots}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sinh^{-1}(cx)\right)\ldots}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sinh^{-1}(cx)\right)\ldots}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{d^3\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{4ib^2d^3\left(1+c^2x^2\right)^{5/2}\cot\left(\frac{\pi}{4}-\frac{1}{2}i\sinh^{-1}(cx)\right)\ldots}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 8.20737, size = 788, normalized size = 1.51

$$-\frac{ib^2(cx-i)\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\sqrt{-df\left(c^2x^2+1\right)}\left(4\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-(1+i)\sinh^{-1}(cx)^2-\frac{2(\sinh^{-1}(cx)\cdots}{cx\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + I*c*d*x]*(a + b*ArcSinh[c*x])^2)/(f - I*c*f*x)^(5/2),x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*((((2*I)/3)*a^2)/(f^3*(I + c*x
)^2) - a^2/(3*f^3*(I + c*x))))/c - ((I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt
[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*(Cosh[ArcSinh[c*x]/2] + I*S
inh[ArcSinh[c*x]/2])*(-(Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[C
oth[ArcSinh[c*x]/2]] + I*Log[Sqrt[1 + c^2*x^2]])) + Cosh[ArcSinh[c*x]/2]*(4
*I + 3*ArcSinh[c*x] - 6*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*I)*Log[Sqrt[1 + c
^2*x^2]]) + 2*(Sqrt[1 + c^2*x^2]*(I*ArcSinh[c*x] + (2*I)*ArcTan[Coth[ArcSin
h[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]) + 2*(1 + I*ArcSinh[c*x] + (2*I)*ArcTan
[Coth[ArcSinh[c*x]/2]] + Log[Sqrt[1 + c^2*x^2]]))*Sinh[ArcSinh[c*x]/2]))/(c
*f^3*(1 + I*c*x)*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*(Cosh[ArcSinh[c*x]
/2] - I*Sinh[ArcSinh[c*x]/2])^4) - ((I/3)*b^2*(-I + c*x)*Sqrt[I*((-I)*d + c
*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[-(d*f*(1 + c^2*x^2))]*((-1 - I)*ArcSin
h[c*x]^2 - (2*ArcSinh[c*x]*(2*I + ArcSinh[c*x]))/(I + c*x) - (2*I)*(Pi - (2
*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] - I*Pi*(3*ArcSinh[c*x] - 4*Log[
1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + 4*Log[Cosh
[ArcSinh[c*x]/2]]) + 4*PolyLog[2, (-I)/E^ArcSinh[c*x]] - (4*ArcSinh[c*x]^2*
Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])^3 + (
2*(4 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])))/(c*f^3*Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[1 +
 c^2*x^2]*(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])^2)
```

Maple [F] time = 0.324, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^2\sqrt{d+icdx}\left(f-icfx\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2cx-i\,b^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2-\left(3\,c^3f^3x^2+6i\,c^2f^3x-3\,cf^3\right)\text{integral}\left(\frac{-3i\,\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}a^2}{}\right.}{3\,c^3f^3x^2+6i\,c^2f^3x-3\,cf^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -((b^2*c*x - I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^2
*x^2 + 1))^2 - (3*c^3*f^3*x^2 + 6*I*c^2*f^3*x - 3*c*f^3)*integral((-3*I*sqr
t(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x +
 d)*sqrt(-I*c*f*x + f)*b^2 - 3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b)*
log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^3*f^3*x^3 + 9*I*c^2*f^3*x^2 - 9*c*f^3*x
- 3*I*f^3), x))/(3*c^3*f^3*x^2 + 6*I*c^2*f^3*x - 3*c*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(d+I*c*d*x)**(1/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(d+I*c*d*x)^(1/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.603 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=942

$$-\frac{c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2d^2\left(c^2\cdots\right.}{3(\cdots}$$

```
[Out] (((2*I)/3)*b^2*d^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (2*b^2*d^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + (b^2*d^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d^2*x*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b*c*d^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5
/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*d^2
*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (2*d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*b*d^2*(1
 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (2*b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(5
/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 1.30118, antiderivative size = 942, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 18, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261, 5723, 5751, 288, 215}

$$-\frac{c^2d^2\left(c^2x^2+1\right)\left(a+b\sinh^{-1}(cx)\right)^2x^3}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{bcd^2\left(c^2x^2+1\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)x^2}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2\left(c^2x^2+1\right)^2x}{3(icxd+d)^{5/2}(f-icfx)^{5/2}}+\frac{2d^2\left(c^2\cdots\right.}{3(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (((2*I)/3)*b^2*d^2*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (2*b^2*d^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(
5/2)) + (b^2*d^2*(1 + c^2*x^2)^(5/2)*ArcSinh[c*x])/(3*c*(d + I*c*d*x)^(5/2)
*(f - I*c*f*x)^(5/2)) + (b*d^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3
*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d^2*x*(1 + c^2*x
^2)^(3/2)*(a + b*ArcSinh[c*x]))/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b*c*d^2*x^2*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*(d + I*c*d*x)^(5
/2)*(f - I*c*f*x)^(5/2)) - (((2*I)/3)*d^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]
)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*x*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (c^2*d^2
*x^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (2*d^2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c
*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((4*I)/3)*b*d^2*(1
 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*
d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b*d^2*(1 + c^2*x^2)^(5/2)*(a + b*ArcSi
nh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x
)^(5/2)) + (2*b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/
(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d^2*(1 + c^2*x^2)^(5
/2)*PolyLog[2, I*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5
/2)) - (b^2*d^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5723

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSinh[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)
^FracPart[p])/(f*(m + 1)*(1 + c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 +
 c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, m, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &&
NeQ[m, -1]
```

Rule 5751

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcSinh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e*(p + 1))
, Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n, x], x] - Di
st[(b*f*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 + c^2*x^2)^
FracPart[p]), Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x]
)^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[
n, 0] && LtQ[p, -1] && GtQ[m, 1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+icdx}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int\left(\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}+\frac{2icd^2x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}-\frac{c^2d^2x^2\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d^2\left(1+c^2x^2\right)^{5/2}\right)\int\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{\left(2icd^2\left(1+c^2x^2\right)^{5/2}\right)\int\frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{\left(c\right.}{}$$

$$= -\frac{2id^2\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{d^2x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{c^2d^2x^3\left(1+\right.}{3(d+}$$

$$= \frac{bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2ibd^2x\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{bcd^2x^2}{3}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{bd^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh\right.}{3c(d+icdx)^{5/2}(f-icf}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2d^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2d^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2d^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{2ib^2d^2\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{b^2d^2\left(1+c^2x^2\right)^{5/2}\sinh^{-1}(cx}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Mathematica [A] time = 6.60242, size = 528, normalized size = 0.56

$$\sqrt{d+icdx}\sqrt{f-icfx}\left(-\frac{b^2\left(-4\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)+(1+i)\sinh^{-1}(cx)^2-\frac{\left(\sinh^{-1}(cx)+2i\right)\sinh^{-1}(cx)}{cx+i}+2\left(2\sinh^{-1}(cx)+i\pi\right)\log\left(1+ie^{-\sinh^{-1}(cx)}\right)-\frac{}{(\text{co}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(Sqrt[d + I*c*d*x]*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] (Sqrt[d + I*c*d*x]*Sqrt[f - I*c*f*x]*((a^2*(2*I + c*x))/(I + c*x)^2 - (a*b*
(I*Cosh[(3*ArcSinh[c*x])/2]*(ArcSinh[c*x] - 2*ArcTan[Coth[ArcSinh[c*x]/2]]
+ (I/2)*Log[1 + c^2*x^2]) + Cosh[ArcSinh[c*x]/2]*(-2 + (3*I)*ArcSinh[c*x] +
 (6*I)*ArcTan[Coth[ArcSinh[c*x]/2]] + (3*Log[1 + c^2*x^2])/2) + 2*(I + (-1
+ Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 2*(2 + Sqrt[1 + c^2*x^2])*ArcTan[Coth[A
rcSinh[c*x]/2]] - (I/2)*(2 + Sqrt[1 + c^2*x^2])*Log[1 + c^2*x^2])*Sinh[ArcS
inh[c*x]/2]))/(Sqrt[1 + c^2*x^2]*(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x
]/2])^3) - (b^2*((1 + I)*ArcSinh[c*x]^2 - (ArcSinh[c*x]*(2*I + ArcSinh[c*x]
))/(I + c*x) + 2*(I*Pi + 2*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + I*Pi*(
3*ArcSinh[c*x] - 4*Log[1 + E^ArcSinh[c*x]] - 2*Log[-Cos[(Pi + (2*I)*ArcSinh
[c*x])/4]] + 4*Log[Cosh[ArcSinh[c*x]/2]]) - 4*PolyLog[2, (-I)/E^ArcSinh[c*x
]] - (2*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh
[ArcSinh[c*x]/2])^3 - (2*(-2 + ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[
ArcSinh[c*x]/2] - I*Sinh[ArcSinh[c*x]/2])))/Sqrt[1 + c^2*x^2]))/(3*c*d*f^3)
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \left(f - icfx\right)^{-\frac{5}{2}} \frac{1}{\sqrt{d + icdx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2cx+2ib^2\right)\sqrt{icdx+d}\sqrt{-icfx+f}\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(3c^3df^3x^2+6ic^2df^3x-3cdf^3\right)\mathrm{integral}\left(-\frac{3\sqrt{icdx+d}\sqrt{-ic\ldots}}{\ldots}\right.}{3c^3df^3x^2+6ic^2df^3x-3cdf^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] ((b^2*c*x + 2*I*b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x + sqrt(c^
2*x^2 + 1))^2 + (3*c^3*d*f^3*x^2 + 6*I*c^2*d*f^3*x - 3*c*d*f^3)*integral(-(
3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + (6*sqrt(I*c*d*x + d)*sqrt(-I*c
*f*x + f)*a*b + (2*b^2*c*x + 4*I*b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*s
qrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^4*d*f^3*x^4 + 6*I*c^3
*d*f^3*x^3 + 6*I*c*d*f^3*x - 3*d*f^3), x))/(3*c^3*d*f^3*x^2 + 6*I*c^2*d*f^3
*x - 3*c*d*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(f-I*c*f*x)**(5/2)/(d+I*c*d*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(f-I*c*f*x)^(5/2)/(d+I*c*d*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.604 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=743

$$\frac{b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(\right.}{3c(d+icdx)^{5/2}(f-ic}$$

```
[Out] ((I/3)*b^2*d*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b^2*d*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + ((I/3)*b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2)) + (2*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d*(1 + c^2*x^2)^(5/2)*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (4*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2
*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b^2*d*(1
+ c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (b^2*d*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d*(1 + c^2*x^2)
^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2))
```

Rubi [A] time = 0.897444, antiderivative size = 743, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {5712, 5821, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191, 5693, 4180, 261}

$$\frac{b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,ie^{\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}-\frac{2b^2d\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(\right.}{3c(d+icdx)^{5/2}(f-ic}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),x]
```

```
[Out] ((I/3)*b^2*d*(1 + c^2*x^2)^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) -
 (b^2*d*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b
*d*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f -
I*c*f*x)^(5/2)) + ((I/3)*b*d*x*(1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/((
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - ((I/3)*d*(1 + c^2*x^2)*(a + b*Arc
Sinh[c*x])^2)/(c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (d*x*(1 + c^2*x
^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (
2*d*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/2)*(f - I
*c*f*x)^(5/2)) + (2*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2)/(3*c*(d +
 I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (((2*I)/3)*b*d*(1 + c^2*x^2)^(5/2)*(
a + b*ArcSinh[c*x])*ArcTan[E^ArcSinh[c*x]])/(c*(d + I*c*d*x)^(5/2)*(f - I*c
*f*x)^(5/2)) - (4*b*d*(1 + c^2*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2
*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b^2*d*(1
+ c^2*x^2)^(5/2)*PolyLog[2, (-I)*E^ArcSinh[c*x]])/(3*c*(d + I*c*d*x)^(5/2)*
(f - I*c*f*x)^(5/2)) - (b^2*d*(1 + c^2*x^2)^(5/2)*PolyLog[2, I*E^ArcSinh[c*
x]])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*d*(1 + c^2*x^2)
^(5/2)*PolyLog[2, -E^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f
*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
```

```
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5693

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d), Subst[Int[(a + b*x)^n*Sech[x], x], x, ArcSinh[c*x]], x
] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{3/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{(d+icdx)\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(1+c^2x^2\right)^{5/2}\int \left(\frac{d\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}+\frac{icdx\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\right)dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{\left(d\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(icd\left(1+c^2x^2\right)^{5/2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= -\frac{id\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{dx\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2d\left(1+c^2x\right.\right.}{3(d+}$$

$$= \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{ibdx\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{id\left(1+c\right.}{3c(d+}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-ic}$$

$$= \frac{ib^2d\left(1+c^2x^2\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{b^2dx\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{bd\left(1+c^2x^2\right)^{3/2}\left(a+b\sin\right.}{3c(d+icdx)^{5/2}(f-ic}$$

Mathematica [A] time = 8.20823, size = 757, normalized size = 1.02

$$-\frac{ib^2\sqrt{c^2x^2+1}\sqrt{i(cdx-id)}\sqrt{-i(cfx+if)}\left(-10i\text{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-6i\text{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)-(1-4i)\sinh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(3/2)*(f - I*c*f*x)^(5/2)),
x]
```

```
[Out] (Sqrt[I*d*(-I + c*x)]*Sqrt[(-I)*f*(I + c*x)]*(a^2/(4*d^2*f^3*(-I + c*x)) +
((I/6)*a^2)/(d^2*f^3*(I + c*x)^2) + (5*a^2)/(12*d^2*f^3*(I + c*x))))/c - ((
I/3)*a*b*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*(4*c*x*ArcSinh[c
*x] - (2*I)*ArcSinh[c*x]*Cosh[2*ArcSinh[c*x]] + Sqrt[1 + c^2*x^2]*(1 + (2*I
)*ArcTan[Tanh[ArcSinh[c*x]/2]] + 2*c*x*(ArcTan[Tanh[ArcSinh[c*x]/2]] + (2*I
)*Log[Sqrt[1 + c^2*x^2]]) - 4*Log[Sqrt[1 + c^2*x^2]])))/(c*d*f^2*(I + c*x)*
Sqrt[-(((-I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))]) - ((I/6
)*b^2*Sqrt[I*((-I)*d + c*d*x)]*Sqrt[(-I)*(I*f + c*f*x)]*Sqrt[1 + c^2*x^2]*(
-9*Pi*ArcSinh[c*x] + ((2 - I*ArcSinh[c*x])*ArcSinh[c*x])/(I + c*x) - (1 - 4
*I)*ArcSinh[c*x]^2 + 3*(Pi + (2*I)*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]]
- 5*(Pi - (2*I)*ArcSinh[c*x])*Log[1 + I/E^ArcSinh[c*x]] + 16*Pi*Log[1 + E^A
rcSinh[c*x]] + 5*Pi*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] - 16*Pi*Log[Cosh
[ArcSinh[c*x]/2]] - 3*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] - (10*I)*Pol
yLog[2, (-I)/E^ArcSinh[c*x]] - (6*I)*PolyLog[2, I/E^ArcSinh[c*x]] - ((2*I)*
ArcSinh[c*x]^2*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh[c*x]/2] - I*Sinh[ArcSinh
[c*x]/2])^3 + (I*(4 - 5*ArcSinh[c*x]^2)*Sinh[ArcSinh[c*x]/2])/(Cosh[ArcSinh
[c*x]/2] - I*Sinh[ArcSinh[c*x]/2]) - ((3*I)*ArcSinh[c*x]^2*Sinh[ArcSinh[c*x
]/2])/(Cosh[ArcSinh[c*x]/2] + I*Sinh[ArcSinh[c*x]/2])))/(c*d*f^2*Sqrt[-(((-
I)*d + c*d*x)*(I*f + c*f*x))]*Sqrt[-(d*f*(1 + c^2*x^2))])
```

Maple [F] time = 0.269, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 (d + icdx)^{-\frac{3}{2}} \left(f - icfx\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,b^2c^2x^2+2i\,b^2cx+b^2\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+\left(3\,c^4d^2f^3x^3+3i\,c^3d^2f^3x^2+3\,c^2d^2f^3x+3i\,cd^2\right.}{3\,c^4d^2f^3x^3+3i\,c^3d^2f^3x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] ((2*b^2*c^2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + (3*c^4*d^2*f^3*x^3 + 3*I*c^3*d^2*f^3*x^2 +
3*c^2*d^2*f^3*x + 3*I*c*d^2*f^3)*integral((3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*
f*x + f)*a^2 - 2*(-3*I*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a*b + (2*b^2*c^
2*x^2 + 2*I*b^2*c*x + b^2)*sqrt(c^2*x^2 + 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*
x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(3*c^5*d^2*f^3*x^5 + 3*I*c^4*d^2*f^3*
x^4 + 6*c^3*d^2*f^3*x^3 + 6*I*c^2*d^2*f^3*x^2 + 3*c*d^2*f^3*x + 3*I*d^2*f^3
), x))/(3*c^4*d^2*f^3*x^3 + 3*I*c^3*d^2*f^3*x^2 + 3*c^2*d^2*f^3*x + 3*I*c*d
^2*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(3/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(3/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.605 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx$

Optimal. Leaf size=386

$$-\frac{2b^2\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

```
[Out] -(b^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*(1 + c^2*x^2)^(5/2)*(a + b*Arc
Sinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*b*(1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E
^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rubi [A] time = 0.531256, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.27, Rules used = {5712, 5690, 5687, 5714, 3718, 2190, 2279, 2391, 5717, 191}

$$-\frac{2b^2\left(c^2x^2+1\right)^{5/2}\text{PolyLog}\left(2,-e^{2\sinh^{-1}(cx)}\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2\left(c^2x^2+1\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}}+\frac{2x\left(c^2x^2+1\right)^2\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),x]
```


```
[Out] -(b^2*x*(1 + c^2*x^2)^2)/(3*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (b*(
1 + c^2*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*
f*x)^(5/2)) + (x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x])^2)/(3*(d + I*c*d*x)^(5/
2)*(f - I*c*f*x)^(5/2)) + (2*x*(1 + c^2*x^2)^2*(a + b*ArcSinh[c*x])^2)/(3*(
d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) + (2*(1 + c^2*x^2)^(5/2)*(a + b*Arc
Sinh[c*x])^2)/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (4*b*(1 + c^2
*x^2)^(5/2)*(a + b*ArcSinh[c*x])*Log[1 + E^(2*ArcSinh[c*x])])/(3*c*(d + I*c
*d*x)^(5/2)*(f - I*c*f*x)^(5/2)) - (2*b^2*(1 + c^2*x^2)^(5/2)*PolyLog[2, -E
^(2*ArcSinh[c*x])])/(3*c*(d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2))
```

Rule 5712

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_
) + (g_.)*(x_))^(q_), x_Symbol] :> Dist[((d + e*x)^q*(f + g*x)^q)/(1 + c^2*
x^2)^q, Int[(d + e*x)^(p - q)*(1 + c^2*x^2)^q*(a + b*ArcSinh[c*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 + e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Rule 5690

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*d*(p +
1)), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*Arc
Sinh[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2
*(p + 1)*(1 + c^2*x^2)^FracPart[p]), Int[x*(1 + c^2*x^2)^(p + 1/2)*(a + b*A
rcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 5687

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[(x*(a + b*ArcSinh[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist
[(b*c*n*Sqrt[1 + c^2*x^2])/(d*Sqrt[d + e*x^2]), Int[(x*(a + b*ArcSinh[c*x])
^(n - 1))/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[n, 0]
```

Rule 5714

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+icdx)^{5/2}(f-icfx)^{5/2}}\,dx = \frac{\left(1+c^2x^2\right)^{5/2}\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{5/2}}\,dx}{(d+icdx)^{5/2}(f-icfx)^{5/2}}$$

$$= \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{\left(2\left(1+c^2x^2\right)^{5/2}\right)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(1+c^2x^2\right)^{3/2}}\,dx}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} - \frac{\left(2bc\left(1+c^2\right.\right.}{3(d+}$$

$$= \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{2x\left(1+c^2x^2\right)^2\left(\right.}{3(d+icdx)^5}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\,\mathrm{s}\right.}{3(d+icdx)^{5/2}(f}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\,\mathrm{s}\right.}{3(d+icdx)^{5/2}(f}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\,\mathrm{s}\right.}{3(d+icdx)^{5/2}(f}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\,\mathrm{s}\right.}{3(d+icdx)^{5/2}(f}$$

$$= -\frac{b^2x\left(1+c^2x^2\right)^2}{3(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{b\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{3c(d+icdx)^{5/2}(f-icfx)^{5/2}} + \frac{x\left(1+c^2x^2\right)\left(a+b\,\mathrm{s}\right.}{3(d+icdx)^{5/2}(f}$$

<u>Mathematica</u> [A] time = 7.82093, size = 642, normalized size = 1.66

$$\frac{-b^2\left(-16\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,-ie^{-\sinh^{-1}(cx)}\right)-16\left(c^2x^2+1\right)^{3/2}\mathrm{PolyLog}\left(2,ie^{-\sinh^{-1}(cx)}\right)+2\sqrt{c^2x^2+1}\left(\left(6\sinh^{-1}(c\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/((d + I*c*d*x)^(5/2)*(f - I*c*f*x)^(5/2)),
x]
```

```
[Out] (4*a^2*c*x*(3 + 2*c^2*x^2) - b^2*(c*x - 6*c*x*ArcSinh[c*x]^2 + (4*I)*Pi*Arc
Sinh[c*x]*Cosh[3*ArcSinh[c*x]] + 2*ArcSinh[c*x]^2*Cosh[3*ArcSinh[c*x]] - (2
*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh
[3*ArcSinh[c*x]]*Log[1 - I/E^ArcSinh[c*x]] + (2*I)*Pi*Cosh[3*ArcSinh[c*x]]*
```

```
Log[1 + I/E^ArcSinh[c*x]] + 4*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]]*Log[1 + I/E
^ArcSinh[c*x]] - (8*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[1 + E^ArcSinh[c*x]] - (2
*I)*Pi*Cosh[3*ArcSinh[c*x]]*Log[-Cos[(Pi + (2*I)*ArcSinh[c*x])/4]] + (8*I)*
Pi*Cosh[3*ArcSinh[c*x]]*Log[Cosh[ArcSinh[c*x]/2]] + (2*I)*Pi*Cosh[3*ArcSinh
[c*x]]*Log[Sin[(Pi + (2*I)*ArcSinh[c*x])/4]] + 2*Sqrt[1 + c^2*x^2]*(((-3*I)
*Pi + 6*ArcSinh[c*x])*Log[1 - I/E^ArcSinh[c*x]] + I*((2*I)*ArcSinh[c*x] + 6
*Pi*ArcSinh[c*x] - (3*I)*ArcSinh[c*x]^2 + 3*(Pi - (2*I)*ArcSinh[c*x])*Log[1
 + I/E^ArcSinh[c*x]] - 12*Pi*Log[1 + E^ArcSinh[c*x]] - 3*Pi*Log[-Cos[(Pi +
(2*I)*ArcSinh[c*x])/4]] + 12*Pi*Log[Cosh[ArcSinh[c*x]/2]] + 3*Pi*Log[Sin[(P
i + (2*I)*ArcSinh[c*x])/4]])) - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, (-I)/E^Ar
cSinh[c*x]] - 16*(1 + c^2*x^2)^(3/2)*PolyLog[2, I/E^ArcSinh[c*x]] + Sinh[3*
ArcSinh[c*x]] - 2*ArcSinh[c*x]^2*Sinh[3*ArcSinh[c*x]]) + 2*a*b*(Sqrt[1 + c^
2*x^2]*(2 - 3*Log[1 + c^2*x^2]) - Cosh[3*ArcSinh[c*x]]*Log[1 + c^2*x^2] + 2
*ArcSinh[c*x]*(3*c*x + Sinh[3*ArcSinh[c*x]])))/(12*d^2*f^2*Sqrt[d + I*c*d*x
]*Sqrt[f - I*c*f*x]*(c + c^3*x^2))
```

Maple [F] time = 0.278, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}(cx))^2 (d+icdx)^{-\frac{5}{2}} \left(f-icfx\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}abc\left(\frac{1}{c^4 d^{\frac{5}{2}} f^{\frac{5}{2}} x^2 + c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}} - \frac{2\log\left(c^2x^2+1\right)}{c^2 d^{\frac{5}{2}} f^{\frac{5}{2}}}\right) + \frac{2}{3}ab\left(\frac{x}{\left(c^2dfx^2+df\right)^{\frac{3}{2}}df} + \frac{2x}{\sqrt{c^2dfx^2+df}d^2f^2}\right)\text{arsinh}(cx) + \frac{1}{3}a^2\Bigg(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/3*a*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 + c^2*d^(5/2)*f^(5/2)) - 2*log(c^2*x^
2 + 1)/(c^2*d^(5/2)*f^(5/2))) + 2/3*a*b*(x/((c^2*d*f*x^2 + d*f)^(3/2)*d*f)
```

```
+ 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2))*arcsinh(c*x) + 1/3*a^2*(x/((c^2*d*
f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(c^2*d*f*x^2 + d*f)*d^2*f^2)) + b^2*inte
grate(log(c*x + sqrt(c^2*x^2 + 1))^2/((I*c*d*x + d)^(5/2)*(-I*c*f*x + f)^(5
/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,b^2c^2x^3+3\,b^2x\right)\sqrt{i\,cdx+d}\sqrt{-i\,cfx+f}\,\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3\left(c^4d^3f^3x^4+2\,c^2d^3f^3x^2+d^3f^3\right)\text{integral}\left(\frac{3\sqrt{i\,cdx+d}\sqrt{-}}{}\right.}{3\left(c^4d^3f^3x^4+2\,c^2d^3f^3x^2+d^3f^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*b^2*c^2*x^3 + 3*b^2*x)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(c^4*d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
*integral(1/3*(3*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f)*a^2 + 2*(3*sqrt(I*c*d
*x + d)*sqrt(-I*c*f*x + f)*a*b - (2*b^2*c^3*x^3 + 3*b^2*c*x)*sqrt(c^2*x^2 +
 1)*sqrt(I*c*d*x + d)*sqrt(-I*c*f*x + f))*log(c*x + sqrt(c^2*x^2 + 1)))/(c^
6*d^3*f^3*x^6 + 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 + d^3*f^3), x))/(c^4*
d^3*f^3*x^4 + 2*c^2*d^3*f^3*x^2 + d^3*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(d+I*c*d*x)**(5/2)/(f-I*c*f*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(d+I*c*d*x)^(5/2)/(f-I*c*f*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.606 $\int \left(d+ex^2\right)^4 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=312

$$\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}e^4x^9\left(a\right.$$

```
[Out] -(b*(315*c^8*d^4 - 420*c^6*d^3*e + 378*c^4*d^2*e^2 - 180*c^2*d*e^3 + 35*e^4
)*Sqrt[1 + c^2*x^2])/(315*c^9) - (4*b*e*(105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*(1 + c^2*x^2)^(3/2))/(945*c^9) - (2*b*e^2*(63*c^4*d^2 -
 90*c^2*d*e + 35*e^2)*(1 + c^2*x^2)^(5/2))/(525*c^9) - (4*b*(9*c^2*d - 7*e)
*e^3*(1 + c^2*x^2)^(7/2))/(441*c^9) - (b*e^4*(1 + c^2*x^2)^(9/2))/(81*c^9)
+ d^4*x*(a + b*ArcSinh[c*x]) + (4*d^3*e*x^3*(a + b*ArcSinh[c*x]))/3 + (6*d^
2*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcSinh[c*x]))/7 +
(e^4*x^9*(a + b*ArcSinh[c*x]))/9
```

Rubi [A] time = 0.350649, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {194, 5704, 12, 1799, 1850}

$$\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{9}e^4x^9\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -(b*(315*c^8*d^4 - 420*c^6*d^3*e + 378*c^4*d^2*e^2 - 180*c^2*d*e^3 + 35*e^4
)*Sqrt[1 + c^2*x^2])/(315*c^9) - (4*b*e*(105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*(1 + c^2*x^2)^(3/2))/(945*c^9) - (2*b*e^2*(63*c^4*d^2 -
 90*c^2*d*e + 35*e^2)*(1 + c^2*x^2)^(5/2))/(525*c^9) - (4*b*(9*c^2*d - 7*e)
*e^3*(1 + c^2*x^2)^(7/2))/(441*c^9) - (b*e^4*(1 + c^2*x^2)^(9/2))/(81*c^9)
+ d^4*x*(a + b*ArcSinh[c*x]) + (4*d^3*e*x^3*(a + b*ArcSinh[c*x]))/3 + (6*d^
2*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcSinh[c*x]))/7 +
(e^4*x^9*(a + b*ArcSinh[c*x]))/9
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(d+ex^2\right)^4 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= d^4x\left(a+b\sinh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\sinh^{-1}(cx)\right)$$

$$= -\frac{b\left(315c^8d^4-420c^6d^3e+378c^4d^2e^2-180c^2de^3+35e^4\right)\sqrt{1+c^2x^2}}{315c^9}-\frac{4be\left(105c^6\right.}{}$$

Mathematica [A] time = 0.343265, size = 260, normalized size = 0.83

$$\frac{315ax\left(378d^2e^2x^4+420d^3ex^2+315d^4+180de^3x^6+35e^4x^8\right)-\frac{b\sqrt{c^2x^2+1}\left(c^8\left(23814d^2e^2x^4+44100d^3ex^2+99225d^4+8100de^3x^6+1225e^4x^8\right)-8\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[1 + c^2*x^2]*(4480*e^4 - 320*c^2*e^3*(81*d + 7*e*x^2) + 48
*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) - 8*c^6*e*(11025*d^3 + 3969*
d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*e*x^
2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x*(315
*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*ArcSin
h[c*x])/99225
```

Maple [A] time = 0.014, size = 451, normalized size = 1.5

$$\frac{1}{c}\left(\frac{a}{c^8}\left(\frac{e^4c^9x^9}{9}+\frac{4\,c^9de^3x^7}{7}+\frac{6\,c^9d^2e^2x^5}{5}+\frac{4\,c^9d^3ex^3}{3}+c^9d^4x\right)+\frac{b}{c^8}\left(\frac{\mathrm{Arcsinh}\,(cx)\,e^4c^9x^9}{9}+\frac{4\,\mathrm{Arcsinh}\,(cx)\,c^9de^3x^7}{7}+\frac{6\,\mathrm{A}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^8*(1/9*e^4*c^9*x^9+4/7*c^9*d*e^3*x^7+6/5*c^9*d^2*e^2*x^5+4/3*c^9*d
^3*e*x^3+c^9*d^4*x)+b/c^8*(1/9*arcsinh(c*x)*e^4*c^9*x^9+4/7*arcsinh(c*x)*c^
9*d*e^3*x^7+6/5*arcsinh(c*x)*c^9*d^2*e^2*x^5+4/3*arcsinh(c*x)*c^9*d^3*e*x^3
+arcsinh(c*x)*c^9*d^4*x-1/9*e^4*(1/9*c^8*x^8*(c^2*x^2+1)^(1/2)-8/63*c^6*x^6
*(c^2*x^2+1)^(1/2)+16/105*c^4*x^4*(c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(c^2*x^2
+1)^(1/2)+128/315*(c^2*x^2+1)^(1/2))-4/7*c^2*d*e^3*(1/7*c^6*x^6*(c^2*x^2+1)
^(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*
(c^2*x^2+1)^(1/2))-6/5*c^4*d^2*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*
x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-4/3*c^6*d^3*e*(1/3*c^2*x^2*(c
^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-c^8*d^4*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.22702, size = 560, normalized size = 1.79

$$\frac{1}{9}ae^4x^9+\frac{4}{7}ade^3x^7+\frac{6}{5}ad^2e^2x^5+\frac{4}{3}ad^3ex^3+\frac{4}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^3e+\frac{2}{25}\left(15x^5\operatorname{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/9
*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c
^4))*b*d^3*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4
*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 + 4/245*
(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x
^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*d*e^3
 + 1/2835*(315*x^9*arcsinh(c*x) - (35*sqrt(c^2*x^2 + 1)*x^8/c^2 - 40*sqrt(c
^2*x^2 + 1)*x^6/c^4 + 48*sqrt(c^2*x^2 + 1)*x^4/c^6 - 64*sqrt(c^2*x^2 + 1)*x
^2/c^8 + 128*sqrt(c^2*x^2 + 1)/c^10)*c)*b*e^4 + a*d^4*x + (c*x*arcsinh(c*x)
 - sqrt(c^2*x^2 + 1))*b*d^4/c
```

Fricas [A] time = 2.48825, size = 803, normalized size = 2.57

$$\frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\left(35\,bc^9e^4x^9+180\,bc^9de^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^2
*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9 +
 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315*b*
c^9*d^4*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8
*d^4 - 88200*b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 - 25920*b*c^2*d*e^3 + 100*(8
1*b*c^8*d*e^3 - 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 - 16
20*b*c^6*d*e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e - 7938*b*c^6*d^2
*e^2 + 3240*b*c^4*d*e^3 - 560*b*c^2*e^4)*x^2)*sqrt(c^2*x^2 + 1))/c^9
```

Sympy [A] time = 24.2907, size = 593, normalized size = 1.9

$$\begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{asinh}(cx) + \frac{4bd^3ex^3\operatorname{asinh}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{asinh}(cx)}{5} + \frac{4bde^3x^7\operatorname{asinh}(cx)}{7} + \frac{be^4x^9 \ldots \\ a\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*asinh(c*x) + 4*b*d**3*e*x**3*asinh(c*x)/
3 + 6*b*d**2*e**2*x**5*asinh(c*x)/5 + 4*b*d*e**3*x**7*asinh(c*x)/7 + b*e**4
*x**9*asinh(c*x)/9 - b*d**4*sqrt(c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 + 1)/(81*
c) + 8*b*d**3*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 + 1)/(25*c**3) + 24*b*d*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) + 8*
b*e**4*x**6*sqrt(c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
+ 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) - 16*b*e**
4*x**4*sqrt(c**2*x**2 + 1)/(945*c**5) + 64*b*d*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7) + 64*b*e**4*x**2*sqrt(c**2*x**2 + 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**4*x + 4*d**3*e*x**3/3 + 6*d**
2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

Giac [A] time = 1.84178, size = 547, normalized size = 1.75

$$\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)bd^4 + ad^4x + \frac{1}{2835}\left(315\,ax^9 + \left(315\,x^9\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{35\left(c^2x^2+1\right)^{\frac{9}{2}} - 180 \ldots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^4 + a*d^4*x + 1/
2835*(315*a*x^9 + (315*x^9*log(c*x + sqrt(c^2*x^2 + 1)) - (35*(c^2*x^2 + 1)
^(9/2) - 180*(c^2*x^2 + 1)^(7/2) + 378*(c^2*x^2 + 1)^(5/2) - 420*(c^2*x^2 +
 1)^(3/2) + 315*sqrt(c^2*x^2 + 1))/c^9)*b)*e^4 + 4/245*(35*a*d*x^7 + (35*x^
7*log(c*x + sqrt(c^2*x^2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^
(5/2) + 35*(c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*b*d)*e^3 + 2/25
```

```
*(15*a*d^2*x^5 + (15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5
/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*b*d^2)*e^2 + 4/9*
(3*a*d^3*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) -
 3*sqrt(c^2*x^2 + 1))/c^3)*b*d^3)*e
```

3.607 $\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=221

$$d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+d^3x\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\sinh^{-1}(cx)\right)-\frac{be\left(c^2x^2+1\right)}{}$$

```
[Out] -(b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 + c^2*x^2])/(
35*c^7) - (b*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/2))/(105
*c^7) - (3*b*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(175*c^7) - (b*e^3*(1
 + c^2*x^2)^(7/2))/(49*c^7) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*(a + b
*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a + b*Arc
Sinh[c*x]))/7
```

Rubi [A] time = 0.260728, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {194, 5704, 12, 1799, 1850}

$$d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+d^3x\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\sinh^{-1}(cx)\right)-\frac{be\left(c^2x^2+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -(b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Sqrt[1 + c^2*x^2])/(
35*c^7) - (b*e*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*(1 + c^2*x^2)^(3/2))/(105
*c^7) - (3*b*(7*c^2*d - 5*e)*e^2*(1 + c^2*x^2)^(5/2))/(175*c^7) - (b*e^3*(1
 + c^2*x^2)^(7/2))/(49*c^7) + d^3*x*(a + b*ArcSinh[c*x]) + d^2*e*x^3*(a + b
*ArcSinh[c*x]) + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]))/5 + (e^3*x^7*(a + b*Arc
Sinh[c*x]))/7
```


Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right) dx = d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+$$

$$= d^3x\left(a+b\sinh^{-1}(cx)\right)+d^2ex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\sinh^{-1}(cx)\right)+$$

$$= -\frac{b\left(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3\right)\sqrt{1+c^2x^2}}{35c^7}-\frac{be\left(35c^4d^2-42c^2de+15e^2\right)(}{105c^7}$$

Mathematica [A] time = 0.272513, size = 187, normalized size = 0.85

$$a\left(d^2ex^3+d^3x+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)-\frac{b\sqrt{c^2x^2+1}\left(c^6\left(1225d^2ex^2+3675d^3+441de^2x^4+75e^3x^6\right)-2c^4e\left(1225d^2+294de\right.\right.}{3675c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 + c^2*x^2
]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2
 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^
6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)*Arc
Sinh[c*x]
```

Maple [A] time = 0.005, size = 316, normalized size = 1.4

$$\frac{1}{c}\left(\frac{a}{c^6}\left(\frac{e^3c^7x^7}{7}+\frac{3\,c^7de^2x^5}{5}+c^7d^2ex^3+xc^7d^3\right)+\frac{b}{c^6}\left(\frac{\mathrm{Arcsinh}\,(cx)\,e^3c^7x^7}{7}+\frac{3\,\mathrm{Arcsinh}\,(cx)\,c^7de^2x^5}{5}+\mathrm{Arcsinh}\,(cx)\,c^7d^2e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b/c^
6*(1/7*arcsinh(c*x)*e^3*c^7*x^7+3/5*arcsinh(c*x)*c^7*d*e^2*x^5+arcsinh(c*x)
*c^7*d^2*e*x^3+arcsinh(c*x)*c^7*x*d^3-1/7*e^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2
)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*
x^2+1)^(1/2))-3/5*c^2*d*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^
2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-c^4*d^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(
1/2)-2/3*(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.24061, size = 387, normalized size = 1.75

$$\frac{1}{7}\,ae^3x^7+\frac{3}{5}\,ade^2x^5+ad^2ex^3+\frac{1}{3}\left(3\,x^3\,\mathrm{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bd^2e+\frac{1}{25}\left(15\,x^5\,\mathrm{arsinh}\,(cx)-\left(\frac{3\,\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arcsinh(c*x) - c
*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(15*
x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c
^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsinh(c*x) - (5*
```

```
sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 +
1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arcsinh(c*
x) - sqrt(c^2*x^2 + 1))*b*d^3/c
```

Fricas [A] time = 2.54864, size = 559, normalized size = 2.53

$$\frac{525\,ac^7e^3x^7 + 2205\,ac^7de^2x^5 + 3675\,ac^7d^2ex^3 + 3675\,ac^7d^3x + 105\left(5\,bc^7e^3x^7 + 21\,bc^7de^2x^5 + 35\,bc^7d^2ex^3 + 35\,bc^7d^3x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 + 3
675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*
e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 + 1)) - (75*b*c^6*e^3*x^6 +
3675*b*c^6*d^3 - 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 -
10*b*c^4*e^3)*x^4 - 240*b*e^3 + (1225*b*c^6*d^2*e - 588*b*c^4*d*e^2 + 120*b
*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))/c^7
```

Sympy [A] time = 8.74923, size = 389, normalized size = 1.76

$$\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{asinh}(cx) + bd^2ex^3\operatorname{asinh}(cx) + \frac{3bde^2x^5\operatorname{asinh}(cx)}{5} + \frac{be^3x^7\operatorname{asinh}(cx)}{7} - \frac{bd^3\sqrt{c^2x^2+1}}{c} - \\ a\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*asinh(c*x) + b*d**2*e*x**3*asinh(c*x) + 3*b*d*e**2*x**5*asinh(c*x)/
5 + b*e**3*x**7*asinh(c*x)/7 - b*d**3*sqrt(c**2*x**2 + 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c
**3) + 4*b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**4*sqrt(c
**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(c**2*x**2 + 1)/(24
5*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/
```

```
7), True))
```

Giac [A] time = 1.8443, size = 396, normalized size = 1.79

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)bd^3+ad^3x+\frac{1}{245}\left(35\,ax^7+\left(35\,x^7\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{5\left(c^2x^2+1\right)^{\frac{7}{2}}-21\left(c^2x^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^3 + a*d^3*x + 1/
245*(35*a*x^7 + (35*x^7*log(c*x + sqrt(c^2*x^2 + 1)) - (5*(c^2*x^2 + 1)^(7/
2) - 21*(c^2*x^2 + 1)^(5/2) + 35*(c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1)
)/c^7)*b)*e^3 + 1/25*(15*a*d*x^5 + (15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (
3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)
*b*d)*e^2 + 1/3*(3*a*d^2*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*
x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*b*d^2)*e
```

3.608 $\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=147

$$d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\left(15c^4d^2-10c^2de+3e^2\right)}{15c^5}-$$

```
[Out] -(b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/(15*c^5) - (2*b*(5
*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(45*c^5) - (b*e^2*(1 + c^2*x^2)^(5/2))
/(25*c^5) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c*x]))/3
 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rubi [A] time = 0.143094, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {194, 5704, 12, 1247, 698}

$$d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\left(15c^4d^2-10c^2de+3e^2\right)}{15c^5}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Sqrt[1 + c^2*x^2])/(15*c^5) - (2*b*(5
*c^2*d - 3*e)*e*(1 + c^2*x^2)^(3/2))/(45*c^5) - (b*e^2*(1 + c^2*x^2)^(5/2))
/(25*c^5) + d^2*x*(a + b*ArcSinh[c*x]) + (2*d*e*x^3*(a + b*ArcSinh[c*x]))/3
 + (e^2*x^5*(a + b*ArcSinh[c*x]))/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)\,dx = d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-(bc$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{15}$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{30}$$

$$= -\frac{b\left(15c^4d^2-10c^2de+3e^2\right)\sqrt{1+c^2x^2}}{15c^5}-\frac{2b\left(5c^2d-3e\right)e\left(1+c^2x^2\right)^{3/2}}{45c^5}-\frac{be^2\left(1+c^2\right.}{25c^5}$$

Mathematica [A] time = 0.163037, size = 125, normalized size = 0.85

$$\frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right)-\frac{b\sqrt{c^2x^2+1}\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)-4c^2e\left(25d+3ex^2\right)+24e^2\right)}{c^5}+15bx\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 + c^2*x^2]*(24*e^2 -
4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 + 1
5*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSinh[c*x])/225
```

Maple [A] time = 0.005, size = 204, normalized size = 1.4

$$\frac{1}{c}\left(\frac{a}{c^4}\left(\frac{e^2c^5x^5}{5}+\frac{2\,c^5dex^3}{3}+xc^5d^2\right)+\frac{b}{c^4}\left(\frac{\text{Arcsinh}\,(cx)\,e^2c^5x^5}{5}+\frac{2\,\text{Arcsinh}\,(cx)\,c^5dex^3}{3}+\text{Arcsinh}\,(cx)\,c^5xd^2-\frac{e^2}{5}\left(\frac{c^4x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arcsinh(c
*x)*e^2*c^5*x^5+2/3*arcsinh(c*x)*c^5*d*e*x^3+arcsinh(c*x)*c^5*x*d^2-1/5*e^2
*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^
2+1)^(1/2))-2/3*c^2*d*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2
))-d^2*c^4*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.17582, size = 243, normalized size = 1.65

$$\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+\frac{2}{9}\left(3\,x^3\,\text{arsinh}\,(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)bde+\frac{1}{75}\left(15\,x^5\,\text{arsinh}\,(cx)-\left(\frac{3\,\sqrt{c^2x^2+1}x^4}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 +
 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arcsinh(c*x) -
 (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d
^2/c
```

Fricas [A] time = 2.36344, size = 370, normalized size = 2.52

$$\frac{45 \, ac^{5}e^{2}x^{5} + 150 \, ac^{5}dex^{3} + 225 \, ac^{5}d^{2}x + 15 \left(3 \, bc^{5}e^{2}x^{5} + 10 \, bc^{5}dex^{3} + 15 \, bc^{5}d^{2}x\right) \log\left(cx + \sqrt{c^{2}x^{2} + 1}\right) - \left(9 \, bc^{4}e^{2}x^{4} + 225\right.}{225 \, c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^5
*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))
- (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 - 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4
*d*e - 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 2.78751, size = 240, normalized size = 1.63

$$\begin{cases} ad^{2}x + \frac{2adex^{3}}{3} + \frac{ae^{2}x^{5}}{5} + bd^{2}x \operatorname{asinh}(cx) + \frac{2bdex^{3} \operatorname{asinh}(cx)}{3} + \frac{be^{2}x^{5} \operatorname{asinh}(cx)}{5} - \frac{bd^{2}\sqrt{c^{2}x^{2}+1}}{c} - \frac{2bdex^{2}\sqrt{c^{2}x^{2}+1}}{9c} - \frac{be^{2}x^{4}\sqrt{c^{2}x^{2}+1}}{25c} + \frac{4bde\sqrt{\ldots}}{9 \ldots} \\ a\left(d^{2}x + \frac{2dex^{3}}{3} + \frac{e^{2}x^{5}}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asinh(c*x)
+ 2*b*d*e*x**3*asinh(c*x)/3 + b*e**2*x**5*asinh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 + 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**2*sq
rt(c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 + 1)/(75*c**5), Ne(c,
 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [A] time = 1.62516, size = 262, normalized size = 1.78

$$\left(x \log\left(cx + \sqrt{c^{2}x^{2} + 1}\right) - \frac{\sqrt{c^{2}x^{2} + 1}}{c}\right)bd^{2} + ad^{2}x + \frac{1}{75}\left(15 \, ax^{5} + \left(15 \, x^{5} \log\left(cx + \sqrt{c^{2}x^{2} + 1}\right) - \frac{3\left(c^{2}x^{2} + 1\right)^{\frac{5}{2}} - 10\left(c^{2}x^{2} + \ldots\right.}{c^{5}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d^2 + a*d^2*x + 1/
75*(15*a*x^5 + (15*x^5*log(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5/2
) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))/c^5)*b)*e^2 + 2/9*(3*a*d
*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3*sqrt(
c^2*x^2 + 1))/c^3)*b*d)*e
```

3.609 $\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=81

$$dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\left(3c^2d-e\right)}{3c^3}-\frac{be\left(c^2x^2+1\right)^{3/2}}{9c^3}$$

```
[Out] -(b*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/(3*c^3) - (b*e*(1 + c^2*x^2)^(3/2))/(9
*c^3) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```

Rubi [A] time = 0.0697441, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5704, 444, 43}

$$dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{b\sqrt{c^2x^2+1}\left(3c^2d-e\right)}{3c^3}-\frac{be\left(c^2x^2+1\right)^{3/2}}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```


```
[Out] -(b*(3*c^2*d - e)*Sqrt[1 + c^2*x^2])/(3*c^3) - (b*e*(1 + c^2*x^2)^(3/2))/(9
*c^3) + d*x*(a + b*ArcSinh[c*x]) + (e*x^3*(a + b*ArcSinh[c*x]))/3
```


Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)\,dx = dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-(bc)\int \frac{x\left(d+\frac{ex^2}{3}\right)}{\sqrt{1+c^2x^2}}\,dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{d+\frac{ex}{3}}{\sqrt{1+c^2x}}\,dx,x\right.$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)-\frac{1}{2}(bc)\,\text{Subst}\left(\int\left(\frac{3c^2d-e}{3c^2\sqrt{1+c^2x}}\right.\right.$$

$$= -\frac{b\left(3c^2d-e\right)\sqrt{1+c^2x^2}}{3c^3}-\frac{be\left(1+c^2x^2\right)^{3/2}}{9c^3}+dx\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\,\text{s}\right.$$

Mathematica [A] time = 0.0691694, size = 71, normalized size = 0.88

$$\frac{1}{9}\left(3ax\left(3d+ex^2\right)-\frac{b\sqrt{c^2x^2+1}\left(c^2\left(9d+ex^2\right)-2e\right)}{c^3}+3bx\sinh^{-1}(cx)\left(3d+ex^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))/c^3
 + 3*b*x*(3*d + e*x^2)*ArcSinh[c*x])/9
```

Maple [A] time = 0.005, size = 109, normalized size = 1.4

$$\frac{1}{c}\left(\frac{a}{c^2}\left(\frac{c^3x^3e}{3}+c^3dx\right)+\frac{b}{c^2}\left(\frac{\text{Arcsinh}\,(cx)\,c^3x^3e}{3}+\text{Arcsinh}\,(cx)\,c^3dx-\frac{e}{3}\left(\frac{c^2x^2}{3}\sqrt{c^2x^2+1}-\frac{2}{3}\sqrt{c^2x^2+1}\right)-c^2d\sqrt{c^2x^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(a/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b/c^2*(1/3*arcsinh(c*x)*c^3*x^3*e+arcsin
h(c*x)*c^3*d*x-1/3*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-
c^2*d*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.12198, size = 123, normalized size = 1.52

$$\frac{1}{3}aex^3 + \frac{1}{9}\left(3x^3\operatorname{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)be + adx + \frac{\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sq
rt(c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*
b*d/c
```

Fricas [A] time = 2.47913, size = 208, normalized size = 2.57

$$\frac{3ac^3ex^3 + 9ac^3dx + 3\left(bc^3ex^3 + 3bc^3dx\right)\log\left(cx + \sqrt{c^2x^2+1}\right) - \left(bc^2ex^2 + 9bc^2d - 2be\right)\sqrt{c^2x^2+1}}{9c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
sqrt(c^2*x^2 + 1)) - (b*c^2*e*x^2 + 9*b*c^2*d - 2*b*e)*sqrt(c^2*x^2 + 1))/c
^3
```

Sympy [A] time = 0.668448, size = 109, normalized size = 1.35

$$\begin{cases} adx + \frac{aex^3}{3} + bdx\operatorname{asinh}(cx) + \frac{bex^3\operatorname{asinh}(cx)}{3} - \frac{bd\sqrt{c^2x^2+1}}{c} - \frac{bex^2\sqrt{c^2x^2+1}}{9c} + \frac{2be\sqrt{c^2x^2+1}}{9c^3} & \text{for}\ c \neq 0 \\ a\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asinh(c*x) + b*e*x**3*asinh(c*x)/3 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x**2*sqrt(c**2*x**2 + 1)/(9*c) + 2*b*e*sqrt
(c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

Giac [A] time = 1.55539, size = 146, normalized size = 1.8

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)bd+adx+\frac{1}{9}\left(3\,ax^3+\left(3\,x^3\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\left(c^2x^2+1\right)^{\frac{3}{2}}-3\,\sqrt{c^2x^2+1}}{c^3}\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d + a*d*x + 1/9*(3
*a*x^3 + (3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3*sqr
t(c^2*x^2 + 1))/c^3)*b)*e
```

3.610 $\int \left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rubi [A] time = 0.0141884, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5653, 261}

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \left(a + b\sinh^{-1}(cx)\right)\,dx = ax + b\int \sinh^{-1}(cx)\,dx$$
$$= ax + bx\sinh^{-1}(cx) - (bc)\int \frac{x}{\sqrt{1+c^2x^2}}\,dx$$
$$= ax - \frac{b\sqrt{1+c^2x^2}}{c} + bx\sinh^{-1}(cx)$$

Mathematica [A] time = 0.0081827, size = 30, normalized size = 1.

$$ax - \frac{b\sqrt{c^2x^2+1}}{c} + bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Maple [A] time = 0.01, size = 31, normalized size = 1.

$$ax + \frac{b}{c}\left(\mathrm{Arcsinh}\,(cx)\,cx - \sqrt{c^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(c*x),x)
```

```
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.22996, size = 41, normalized size = 1.37

$$ax + \frac{\left(cx\,\mathrm{arsinh}\,(cx) - \sqrt{c^2x^2+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Fricas [A] time = 2.34865, size = 95, normalized size = 3.17

$$\frac{bcx\log\left(cx+\sqrt{c^2x^2+1}\right)+acx-\sqrt{c^2x^2+1}b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
```

```
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A] time = 0.151141, size = 26, normalized size = 0.87

$$ax+b\left(\begin{cases}x\operatorname{asinh}(cx)-\frac{\sqrt{c^2x^2+1}}{c} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

Giac [A] time = 1.46019, size = 55, normalized size = 1.83

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

3.611 $\int \frac{a+b\sinh^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=485

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sq
rt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 0.832494, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 + (Sq
rt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*S
```

```
qrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{d+ex^2}\,dx = \int\left(\frac{\sqrt{-d}\left(a+b\sinh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\left(a+b\sinh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}-\sqrt{e}\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{c\sqrt{-d}+\sqrt{e}\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \sinh^{-1}(cx)\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b}{}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b}{}$$

Mathematica [A] time = 0.426022, size = 434, normalized size = 0.89

$$\frac{b\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)-b\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}-c\sqrt{-d}}\right)-b\sqrt{d}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)+b\sqrt{d}\text{PolyLog}\left(2,\frac{\sqrt{e}e}{\sqrt{e-}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2),x]
```

```
[Out] (2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*Arc
Sinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) +
 e])] + b*Sqrt[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh
```

```
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - b*Sqrt[d]*Po
lyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + b
*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Maple [C] time = 0.424, size = 224, normalized size = 0.5

$$a \arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{bc}{2}\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d-2e\right)\_Z^2+e\right)}\frac{1}{\_R1\left(\_R1^2e+2c^2d-e\right)}\left(\mathrm{Arcsinh}\left(cx\right)\ln\left(\frac{1}{\_R1}\left(\_R1-c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/2*c*b*sum(1/_R1/(_R1^2*e+2*c^2*d-e)
*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-(c^2*x^2+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/2*c*b*sum(_R1/(_
R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{arsinh}\left(cx\right)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asinh}(cx)}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

3.612 $\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=707

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

```
[Out] -(a + b*ArcSinh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSinh
[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^2*Sq
rt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e
]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x
^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e
]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)
*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[
e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) +
e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sq
rt[e])
```

Rubi [A] time = 1.07483, antiderivative size = 707, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5706, 5801, 725, 204, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+c\sqrt{-d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```


```
[Out] -(a + b*ArcSinh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSinh
[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTan[(Sqrt[e] - c^2*Sq
```

```
rt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e
]) - (b*c*ArcTan[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x
^2])])/(4*d*Sqrt[c^2*d - e]*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 - (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e
]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)
*Sqrt[e]) + ((a + b*ArcSinh[c*x])*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[
e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) +
e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sq
rt[e])
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
 - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\sinh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$=-\frac{e\int \frac{a+b\sinh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\sinh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{4d}-\frac{e\int \frac{a+b\sinh^{-1}(cx)}{-de-e^2x^2}\,dx}{2d}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{(bc)\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-ex\right)\sqrt{1+c^2x^2}}\,dx}{4d}-\frac{(bc)\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+ex\right)\sqrt{1\cdots}}}{4d}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}}+\frac{\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}\,dx}{4(-d)^{3/2}}-(bc)\,\mathrm{Sub}\cdots$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2\cdots}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2\cdots}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2\cdots}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2\cdots}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2\cdots}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

$$=-\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\sinh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2x^2}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}-\frac{bc\tan^{-1}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d-e}\sqrt{1+c^2\cdots}}\right)}{4d\sqrt{c^2d-e}\sqrt{e}}$$

Mathematica [C] time = 1.60643, size = 622, normalized size = 0.88

$$\frac{1}{2}\left(\frac{b\left(i\left(2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}-ic\sqrt{d}}\right)+2\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{\sqrt{e-c^2d}+ic\sqrt{d}}\right)+\sinh^{-1}(cx)\left(-\sinh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{-\sqrt{e-c^2d}+ic\sqrt{d}}\right)\cdots\right.\right.\right.\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(-2*Sqrt[d]*(-(ArcSinh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*ArcTan[(Sqrt
[e] - I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + (2*I)*Sqrt[d]*(ArcSinh[c*x]/(Sqrt[d] + I*Sqrt[e]*x) + (c*ArcTanh[(I*Sq
rt[e] - c^2*Sqrt[d]*x)/(Sqrt[c^2*d - e]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d - e
]) + I*(ArcSinh[c*x]*(-ArcSinh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(
I*c*Sqrt[d] - Sqrt[-(c^2*d) + e])] + Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(I*c*
Sqrt[d] + Sqrt[-(c^2*d) + e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/((
-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*
x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]))]) - I*(ArcSinh[c*x]*(-ArcSinh[c*x]
+ 2*(Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e])
] + Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])]))
+ 2*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) +
 e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 + e])])))/(4*d^(3/2)*Sqrt[e]))/2
```

Maple [C] time = 0.668, size = 1745, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
+1/2*c^2*b*arcsinh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*c*b/d*sum(1/_R1/(_R1^2*e+
2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x
-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e)*_Z^2+e))+1/4*c*b/
d*sum(_R1/(_R1^2*e+2*c^2*d-e)*(arcsinh(c*x)*ln((_R1-c*x-(c^2*x^2+1)^(1/2))/
_R1)+dilog((_R1-c*x-(c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d-2*e
)*_Z^2+e))+c^5*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((
c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))*
d/(c^2*d-e)/e^3+c^3*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arct
anh((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1
/2))/(c^2*d-e)/e^3*(d*c^2*(c^2*d-e))^(1/2)-c^3*b*(-(2*c^2*d-2*(d*c^2*(c^2*d
-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^
2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/(c^2*d-e)/e^2-1/2*c*b*(-(2*c^2*d-2*(d*c^2*(
c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*
```

```
(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/(c^2*d-e)/e^2*(d*c^2*(c^2*d-e))^(1/2
)-c^3*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*
x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/e^3-c*b*(
-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctanh((c*x+(c^2*x^2+1)^(1
/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2))/d/e^3*(d*c^2*(c^2*
d-e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan
h((c*x+(c^2*x^2+1)^(1/2))*e/((-2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)+e)*e)^(1/2
))/d/e^2+c^5*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+
(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))*d/(c^
2*d-e)/e^3-c^3*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*
x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/(c^
2*d-e)/e^3*(d*c^2*(c^2*d-e))^(1/2)-c^3*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2
)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e)
)^(1/2)-e)*e)^(1/2))/(c^2*d-e)/e^2+1/2*c*b*((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1
/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-
e))^(1/2)-e)*e)^(1/2))/d/(c^2*d-e)/e^2*(d*c^2*(c^2*d-e))^(1/2)-c^3*b*((2*c^
2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/
((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/e^3+c*b*((2*c^2*d+2*(d*c^2
*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/2))*e/((2*c^2*d+2*
(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/e^3*(d*c^2*(c^2*d-e))^(1/2)+1/2*c*b*
((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2)*arctan((c*x+(c^2*x^2+1)^(1/
2))*e/((2*c^2*d+2*(d*c^2*(c^2*d-e))^(1/2)-e)*e)^(1/2))/d/e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\operatorname{arsinh}(cx)+a}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asinh}(cx)}{\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

3.613 $\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=559

$$-\frac{2bd^2ex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{4bd^2e\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c^3}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{6bde^2}{}$$

```
[Out] 2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b*d*e^
2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
45*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + (
12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) - (2*b*e^3*x
^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a + b*ArcSinh[c*
x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]
)^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

Rubi [A] time = 0.965765, antiderivative size = 559, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30}

$$-\frac{2bd^2ex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}+\frac{4bd^2e\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c^3}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{6bde^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) - (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c + (4*b*d^2*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c^3) - (16*b*d*e^
2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c^5) + (32*b*e^3*Sqrt[1 + c^2
*x^2]*(a + b*ArcSinh[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 + c^2*x^2]*(a
 + b*ArcSinh[c*x]))/(3*c) + (8*b*d*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh
```

```
[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2
45*c^5) - (6*b*d*e^2*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + (
12*b*e^3*x^4*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(245*c^3) - (2*b*e^3*x
^6*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(49*c) + d^3*x*(a + b*ArcSinh[c*
x])^2 + d^2*e*x^3*(a + b*ArcSinh[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcSinh[c*x]
)^2)/5 + (e^3*x^7*(a + b*ArcSinh[c*x])^2)/7
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
```

```
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \int \left(d^3 \left(a+b\sinh^{-1}(cx)\right)^2 + 3d^2ex^2 \left(a+b\sinh^{-1}(cx)\right)^2 + 3de^2x^4 \left(a+b\sinh^{-1}(cx\right.\right.$$

$$= d^3 \int \left(a+b\sinh^{-1}(cx)\right)^2 \, dx + \left(3d^2e\right) \int x^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx + \left(3de^2\right) \int x^4 \left(\right.$$

$$= d^3x \left(a+b\sinh^{-1}(cx)\right)^2 + d^2ex^3 \left(a+b\sinh^{-1}(cx)\right)^2 + \frac{3}{5}de^2x^5 \left(a+b\sinh^{-1}(cx)\right)^2$$

$$= -\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{2bd^2ex^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{3c} - \frac{6bd}{}$$

$$= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx\right.}{c}$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 - \frac{12b^2e^3x^5}{1225c^2} + \frac{2}{343}b^2e^3x^7$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{6}{125}b^2de^2x^5$$

$$= 2b^2d^3x - \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} - \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 - \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{}{1}$$

Mathematica [A] time = 0.595735, size = 443, normalized size = 0.79

$$\frac{11025a^2c^7x\left(35d^2ex^2+35d^3+21de^2x^4+5e^3x^6\right)-210ab\sqrt{c^2x^2+1}\left(c^6\left(1225d^2ex^2+3675d^3+441de^2x^4+75e^3x^6\right)-2c^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[1 + c^2*x^2]*(-240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(122
5*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*
e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(-25200*e^3 + 840*c^2*e^2*(147*d + 5*e*x
^2) - 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3 + 428
75*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*(35*d^
3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 + c^2*x^2]*(-240*e^
3 + 24*c^2*e^2*(49*d + 5*e*x^2) - 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*
x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))*ArcSi
nh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6
)*ArcSinh[c*x]^2)/(385875*c^7)
```

Maple [B] time = 0.115, size = 1166, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b^
2/c^6*(d^3*c^6*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+
1/9*c^4*d^2*e*(9*arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^
(1/2)+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*
c*x)-3*c^4*d^2*e*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+1/1125*d*e^2*c^2*(675*arcsinh(c*x)^2*c^5*x^5-270*arcsinh(c*x)*(c^2*x^2+1)
^(1/2)*c^4*x^4+2250*arcsinh(c*x)^2*c^3*x^3+54*c^5*x^5-1140*arcsinh(c*x)*c^2
*x^2*(c^2*x^2+1)^(1/2)+3375*arcsinh(c*x)^2*c*x+380*c^3*x^3-4470*arcsinh(c*x
)*(c^2*x^2+1)^(1/2)+4470*c*x)-2/9*d*e^2*c^2*(9*arcsinh(c*x)^2*c^3*x^3-6*arc
sinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)+3*d*e^2*c^2*(arcsinh(c*x)^2*c*x-2*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/385875*e^3*(55125*arcsinh(c*x)^2*c^7*x^7-
15750*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^6*x^6+231525*arcsinh(c*x)^2*c^5*x^5+
2250*c^7*x^7-73710*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x^4+385875*arcsinh(c*
x)^2*c^3*x^3+14742*c^5*x^5-158970*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+38
5875*arcsinh(c*x)^2*c*x+52990*c^3*x^3-453810*arcsinh(c*x)*(c^2*x^2+1)^(1/2)
+453810*c*x)-1/1125*e^3*(675*arcsinh(c*x)^2*c^5*x^5-270*arcsinh(c*x)*(c^2*x
^2+1)^(1/2)*c^4*x^4+2250*arcsinh(c*x)^2*c^3*x^3+54*c^5*x^5-1140*arcsinh(c*x
)*c^2*x^2*(c^2*x^2+1)^(1/2)+3375*arcsinh(c*x)^2*c*x+380*c^3*x^3-4470*arcsin
h(c*x)*(c^2*x^2+1)^(1/2)+4470*c*x)+1/9*e^3*(9*arcsinh(c*x)^2*c^3*x^3-6*arcs
inh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcsi
nh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-e^3*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c
```

```
^2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^6*(1/7*arcsinh(c*x)*e^3*c^7*x^7+3/5*arcsinh
(c*x)*c^7*d*e^2*x^5+arcsinh(c*x)*c^7*d^2*e*x^3+arcsinh(c*x)*c^7*x*d^3-1/7*e
^3*(1/7*c^6*x^6*(c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(c^2*x^2+1)^(1/2)+8/35*c^2*x
^2*(c^2*x^2+1)^(1/2)-16/35*(c^2*x^2+1)^(1/2))-3/5*c^2*d*e^2*(1/5*c^4*x^4*(c
^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)^(1/2)+8/15*(c^2*x^2+1)^(1/2))-c^4*
d^2*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-d^3*c^6*(c^2*x^
2+1)^(1/2)))
```

Maxima [A] time = 1.25873, size = 923, normalized size = 1.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e^3*x^7*arcsinh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arcsin
h(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arcsinh(c*x)^2 + a^2*d^2*e*x^3
 + b^2*d^3*x*arcsinh(c*x)^2 + 2/3*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 +
1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b
^2*d^2*e + 2/25*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqr
t(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*
(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2
 + 2/245*(35*x^7*arcsinh(c*x) - (5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x
^2 + 1)*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c
)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 + 1)*x^6/c^2 - 6*sqrt(c^2*x^2 + 1)
*x^4/c^4 + 8*sqrt(c^2*x^2 + 1)*x^2/c^6 - 16*sqrt(c^2*x^2 + 1)/c^8)*c*arcsin
h(c*x) - (75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^3 - 1680*x)/c^6)*b^2*e^3 + 2
*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsin
h(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c
```

Fricas [A] time = 2.58636, size = 1338, normalized size = 2.39

$$1125\left(49\,a^2+2\,b^2\right)c^7e^3x^7+189\left(49\left(25\,a^2+2\,b^2\right)c^7de^2-20\,b^2c^5e^3\right)x^5+35\left(1225\left(9\,a^2+2\,b^2\right)c^7d^2e-1176\,b^2c^5de^2+240$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7*
d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e - 1176*b^2
*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^7*d
*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2
+ 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e + 2352*b^2*c
^3*d*e^2 - 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7*d*e^2
*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3*x^6 +
3675*b^2*c^6*d^3 - 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 - 240*b^2*e^3 +
9*(49*b^2*c^6*d*e^2 - 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e - 588*b^2*c
^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2
+ 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 - 2450*a*b*c^4*d^2*e + 1
176*a*b*c^2*d*e^2 - 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 - 10*a*b*c^4*e^3)*x^4
 + (1225*a*b*c^6*d^2*e - 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2)*sqrt(c^2
*x^2 + 1))/c^7
```

Sympy [A] time = 18.8257, size = 989, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*asinh(c*x) + 2*a*b*d**2*e*x**3*asinh(c*x) + 6*a*b*d
*e**2*x**5*asinh(c*x)/5 + 2*a*b*e**3*x**7*asinh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 + 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 + 1)/(49*
c) + 4*a*b*d**2*e*sqrt(c**2*x**2 + 1)/(3*c**3) + 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 + 1)/(25*c**3) + 12*a*b*e**3*x**4*sqrt(c**2*x**2 + 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 + 1)/(245*c**5) + 32*a*b*e**3*sqrt(c**2*x**2 + 1)/(245*c**7) + b**2*d**3*
x*asinh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*asinh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*asinh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*asinh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 + 1)*asinh(c*x)/(49*c) - 4*b**2*d**2*e*x/(3*c*
*2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(1225*c**2) + 4*b**2
*d**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 8*b**2*d*e**2*x**2*sqrt(c
```

```
**2*x**2 + 1)*asinh(c*x)/(25*c**3) + 12*b**2*e**3*x**4*sqrt(c**2*x**2 + 1)*
asinh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**5) - 32*b**2*e**3*x/(245*c
**6) + 32*b**2*e**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(245*c**7), Ne(c, 0)), (
a**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), True))
```

Giac [A] time = 3.02436, size = 986, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b*d^3 + (x*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2
*x^2 + 1))/c^2))*b^2*d^3 + a^2*d^3*x + 1/25725*(3675*a^2*x^7 + 210*(35*x^7*
log(c*x + sqrt(c^2*x^2 + 1)) - (5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5
/2) + 35*(c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))/c^7)*a*b + (3675*x^7*l
og(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((75*c^6*x^7 - 126*c^4*x^5 + 280*c^2*x^
3 - 1680*x)/c^7 - 105*(5*(c^2*x^2 + 1)^(7/2) - 21*(c^2*x^2 + 1)^(5/2) + 35*
(c^2*x^2 + 1)^(3/2) - 35*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^
8))*b^2)*e^3 + 1/375*(225*a^2*d*x^5 + 30*(15*x^5*log(c*x + sqrt(c^2*x^2 + 1
)) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1)
)/c^5)*a*b*d + (225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((9*c^4*x^5 -
20*c^2*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2
) + 15*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^6))*b^2*d)*e^2 + 1
/9*(9*a^2*d^2*x^3 + 6*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1)) - ((c^2*x^2 + 1)^
(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*a*b*d^2 + (9*x^3*log(c*x + sqrt(c^2*x^2 +
 1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2
 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*d^2)*e
```

3.614 $\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \,dx$

Optimal. Leaf size=329

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{8bde\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}-\frac{2be^2x}{}$$

```
[Out] 2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75
*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (8*b*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*x])^2 +
 (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x])^2)/5
```

Rubi [A] time = 0.581781, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30}

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{4bdex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{8bde\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}-\frac{2be^2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```


```
[Out] 2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[1
 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (8*b*d*e*Sqrt[1 + c^2*x^2]*(a + b*Arc
Sinh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75
*c^5) - (4*b*d*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + (8*b*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt
[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(25*c) + d^2*x*(a + b*ArcSinh[c*x])^2 +
 (2*d*e*x^3*(a + b*ArcSinh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcSinh[c*x])^2)/5
```


Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
```

```
0 || IGtQ[n, 0])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2 dx = \int \left(d^2\left(a+b\sinh^{-1}(cx)\right)^2 + 2dex^2\left(a+b\sinh^{-1}(cx)\right)^2 + e^2x^4\left(a+b\sinh^{-1}(cx)\right)^2\right.$$

$$= d^2\int\left(a+b\sinh^{-1}(cx)\right)^2 dx + (2de)\int x^2\left(a+b\sinh^{-1}(cx)\right)^2 dx + e^2\int x^4\left(a+\right.$$

$$= d^2x\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{2}{3}dex^3\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5\left(a+b\sinh^{-1}(cx)\right)^2$$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{4bdex^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c} - \frac{2be}{}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + \frac{8bde\sqrt{1}}{}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sin\right.}{c}$$

$$= 2b^2d^2x - \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 - \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{1+c^2}}{}$$

Mathematica [A] time = 0.426275, size = 289, normalized size = 0.88

$$\frac{225a^2c^5x\left(15d^2+10dex^2+3e^2x^4\right) - 30ab\sqrt{c^2x^2+1}\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)-4c^2e\left(25d+3ex^2\right)+24e^2\right) - 30b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 + c^2*x^2]
*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)) + 2*b^2*c*x*(360*e^2 - 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*d*e
*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) +
 b*Sqrt[1 + c^2*x^2]*(24*e^2 - 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50
*d*e*x^2 + 9*e^2*x^4)))*ArcSinh[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e*x^2 +
 3*e^2*x^4)*ArcSinh[c*x]^2)/(3375*c^5)
```

Maple [B] time = 0.071, size = 620, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b^2/c^4*(d^2*c^4*(
arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+2/27*c^2*d*e*(9*
arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+27*arcsinh(
c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-2*c^2*d*e*(a
rcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/3375*e^2*(675*a
rcsinh(c*x)^2*c^5*x^5-270*arcsinh(c*x)*(c^2*x^2+1)^(1/2)*c^4*x^4+2250*arcsi
nh(c*x)^2*c^3*x^3+54*c^5*x^5-1140*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+33
75*arcsinh(c*x)^2*c*x+380*c^3*x^3-4470*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+4470*
c*x)-2/27*e^2*(9*arcsinh(c*x)^2*c^3*x^3-6*arcsinh(c*x)*c^2*x^2*(c^2*x^2+1)^
(1/2)+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+42*
c*x)+e^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x))+2*a*b
/c^4*(1/5*arcsinh(c*x)*e^2*c^5*x^5+2/3*arcsinh(c*x)*c^5*d*e*x^3+arcsinh(c*x
)*c^5*x*d^2-1/5*e^2*(1/5*c^4*x^4*(c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(c^2*x^2+1)
^(1/2)+8/15*(c^2*x^2+1)^(1/2))-2/3*c^2*d*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2
/3*(c^2*x^2+1)^(1/2))-d^2*c^4*(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.19348, size = 579, normalized size = 1.76

$$\frac{1}{5}\,b^2e^2x^5\operatorname{arsinh}(cx)^2+\frac{1}{5}\,a^2e^2x^5+\frac{2}{3}\,b^2dex^3\operatorname{arsinh}(cx)^2+\frac{2}{3}\,a^2dex^3+b^2d^2x\operatorname{arsinh}(cx)^2+\frac{4}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5*arcsinh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arcsinh(
c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arcsinh(c*x)^2 + 4/9*(3*x^3*arcsinh(c*
x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*b*d*e - 4/2
7*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*arcsinh(c*x) -
 (c^2*x^3 - 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arcsinh(c*x) - (3*sqrt(c^2*x^2
 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c)*a
*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 + 1)*x^4/c^2 - 4*sqrt(c^2*x^2 + 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 + 1)/c^6)*c*arcsinh(c*x) - (9*c^4*x^5 - 20*c^2*x^3 + 1
20*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2
*d^2*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c
```

Fricas [A] time = 2.92329, size = 845, normalized size = 2.57

$$\frac{27\left(25\,a^2+2\,b^2\right)c^5e^2x^5+10\left(25\left(9\,a^2+2\,b^2\right)c^5de-12\,b^2c^3e^2\right)x^3+225\left(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c^5d^2x\right)\log\left(cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e -
12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^2*
c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 + 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^2 -
 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c^5*d
*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 - 100*b^2
*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e - 6*b^2*c^2*e^2)*x^2)*sqrt(c^2*x^
2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*
d^2 - 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e - 6*a*b*c^2*e^2)*x^2
)*sqrt(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 6.18093, size = 595, normalized size = 1.81

$$\begin{cases} a^2d^2x+\frac{2a^2dex^3}{3}+\frac{a^2e^2x^5}{5}+2abd^2x\operatorname{asinh}(cx)+\frac{4abdex^3\operatorname{asinh}(cx)}{3}+\frac{2abe^2x^5\operatorname{asinh}(cx)}{5}-\frac{2abd^2\sqrt{c^2x^2+1}}{c}-\frac{4abdex^2\sqrt{c^2x^2+1}}{9c}-\frac{2abe^2x^4}{25} \\ a^2\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*asinh(c*x) + 4*a*b*d*e*x**3*asinh(c*x)/3 + 2*a*b*e**2*x**5*asinh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 + 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 + 1)/(25*c) + 8*a*b*d*e*sqrt(c**2*x**2 +
 1)/(9*c**3) + 8*a*b*e**2*x**2*sqrt(c**2*x**2 + 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*asinh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*asin
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 + 1)*asinh(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**2) - 8*b**
2*e**2*x**3/(225*c**2) + 8*b**2*d*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3)
 + 8*b**2*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**3) + 16*b**2*e**2
```

```
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(75*c**5), Ne(c,
 0)), (a**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Giac [A] time = 2.55152, size = 656, normalized size = 1.99

$$2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)abd^2+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b*d^2 + (x*log(c
*x + sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2
*x^2 + 1))/c^2))*b^2*d^2 + a^2*d^2*x + 1/1125*(225*a^2*x^5 + 30*(15*x^5*log
(c*x + sqrt(c^2*x^2 + 1)) - (3*(c^2*x^2 + 1)^(5/2) - 10*(c^2*x^2 + 1)^(3/2)
 + 15*sqrt(c^2*x^2 + 1))/c^5)*a*b + (225*x^5*log(c*x + sqrt(c^2*x^2 + 1))^2
 + 2*c*((9*c^4*x^5 - 20*c^2*x^3 + 120*x)/c^5 - 15*(3*(c^2*x^2 + 1)^(5/2) -
10*(c^2*x^2 + 1)^(3/2) + 15*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))
/c^6))*b^2)*e^2 + 2/27*(9*a^2*d*x^3 + 6*(3*x^3*log(c*x + sqrt(c^2*x^2 + 1))
 - ((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*a*b*d + (9*x^3*log(c*x
+ sqrt(c^2*x^2 + 1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2 + 1)^(3/2)
- 3*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2*d)*e
```

3.615 $\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=153

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{4be\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\right.$$

```
[Out] 2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*x*(
a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

Rubi [A] time = 0.277206, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5706, 5653, 5717, 8, 5661, 5758, 30}

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{4be\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x - (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 + c^2*x^
2]*(a + b*ArcSinh[c*x]))/c + (4*b*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))
/(9*c^3) - (2*b*e*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c) + d*x*(
a + b*ArcSinh[c*x])^2 + (e*x^3*(a + b*ArcSinh[c*x])^2)/3
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \int \left(d\left(a+b\sinh^{-1}(cx)\right)^2 + ex^2\left(a+b\sinh^{-1}(cx)\right)^2\right)dx$$

$$= d\int \left(a+b\sinh^{-1}(cx)\right)^2 dx + e\int x^2\left(a+b\sinh^{-1}(cx)\right)^2 dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)^2 + \frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^2 - (2bcd)\int \frac{x\left(a+b\sinh^{-1}(cx)\right.}{\sqrt{1+c^2x^2}}$$

$$= -\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{2bex^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c} + dx\left(a\right.$$

$$= 2b^2dx + \frac{2}{27}b^2ex^3 - \frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + \frac{4be\sqrt{1+c^2x^2}\left(a+b\sinh\right.}{9c^3}$$

$$= 2b^2dx - \frac{4b^2ex}{9c^2} + \frac{2}{27}b^2ex^3 - \frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} + \frac{4be\sqrt{1+c^2x^2}\left(a\right.}{9c}$$

Mathematica [A] time = 0.213184, size = 164, normalized size = 1.07

$$\frac{9a^2c^3x\left(3d+ex^2\right) - 6ab\sqrt{c^2x^2+1}\left(c^2\left(9d+ex^2\right)-2e\right) - 6b\sinh^{-1}(cx)\left(b\sqrt{c^2x^2+1}\left(c^2\left(9d+ex^2\right)-2e\right) - 3ac^3x\left(3d+e\right.\right.}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x
^2)) + 2*b^2*c*x*(-6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x^2
) + b*Sqrt[1 + c^2*x^2]*(-2*e + c^2*(9*d + e*x^2)))*ArcSinh[c*x] + 9*b^2*c^
3*x*(3*d + e*x^2)*ArcSinh[c*x]^2)/(27*c^3)
```

Maple [A] time = 0.046, size = 271, normalized size = 1.8

$$\frac{1}{c}\left(\frac{a^2}{c^2}\left(\frac{c^3x^3e}{3}+c^3dx\right) + \frac{b^2}{c^2}\left(c^2d\left((\mathrm{Arcsinh}\,(cx))^2\,cx - 2\,\mathrm{Arcsinh}\,(cx)\,\sqrt{c^2x^2+1} + 2\,cx\right) + \frac{e}{27}\left(9\,(\mathrm{Arcsinh}\,(cx))^2\,c^3x^3 - \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b^2/c^2*(c^2*d*(arcsinh(c*x)^2*c*x-2*a
rcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/27*e*(9*arcsinh(c*x)^2*c^3*x^3-6*arc
sinh(c*x)*c^2*x^2*(c^2*x^2+1)^(1/2)+27*arcsinh(c*x)^2*c*x+2*c^3*x^3-42*arcs
inh(c*x)*(c^2*x^2+1)^(1/2)+42*c*x)-e*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^
2*x^2+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3*arcsinh(c*x)*c^3*x^3*e+arcsinh(c*x)*c
^3*d*x-1/3*e*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))-c^2*d*(c
^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.17482, size = 294, normalized size = 1.92

$$\frac{1}{3}\,b^2ex^3\operatorname{arsinh}(cx)^2+\frac{1}{3}\,a^2ex^3+b^2dx\operatorname{arsinh}(cx)^2+\frac{2}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abe-\frac{2}{27}\left(3\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*x^3*arcsinh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arcsinh(c*x)^2 + 2/9
*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c
^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4
)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 + 1
)*arcsinh(c*x)/c) + a^2*d*x + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*
d/c
```

Fricas [A] time = 2.79202, size = 454, normalized size = 2.97

$$\frac{\left(9\,a^2+2\,b^2\right)c^3ex^3+9\left(b^2c^3ex^3+3\,b^2c^3dx\right)\log\left(cx+\sqrt{c^2x^2+1}\right)^2+3\left(9\left(a^2+2\,b^2\right)c^3d-4\,b^2ce\right)x+6\left(3\,abc^3ex^3+9\,abc^3\right.}{27\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d - 4*b^2*c*e)*x + 6*(3*a*
b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d - 2*b^2*e)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d
 - 2*a*b*e)*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 1.62599, size = 279, normalized size = 1.82

$$\begin{cases} a^2dx + \frac{a^2ex^3}{3} + 2abdx\operatorname{asinh}(cx) + \frac{2abex^3\operatorname{asinh}(cx)}{3} - \frac{2abd\sqrt{c^2x^2+1}}{c} - \frac{2abex^2\sqrt{c^2x^2+1}}{9c} + \frac{4abe\sqrt{c^2x^2+1}}{9c^3} + b^2dx\operatorname{asinh}^2(cx) + 2b^2dx \\ a^2\left(dx + \frac{ex^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*asinh(c*x) + 2*a*b*e*x**3*a
sinh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 +
 1)/(9*c) + 4*a*b*e*sqrt(c**2*x**2 + 1)/(9*c**3) + b**2*d*x*asinh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*asinh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x
)/(9*c) - 4*b**2*e*x/(9*c**2) + 4*b**2*e*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*
c**3), Ne(c, 0)), (a**2*(d*x + e*x**3/3), True))
```

Giac [B] time = 2.07283, size = 373, normalized size = 2.44

$$2\left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)abd + \left(x\log\left(cx + \sqrt{c^2x^2+1}\right)^2 + 2c\left(\frac{x}{c} - \frac{\sqrt{c^2x^2+1}\log\left(cx + \sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b*d + (x*log(c*x
 + sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x
^2 + 1))/c^2))*b^2*d + a^2*d*x + 1/27*(9*a^2*x^3 + 6*(3*x^3*log(c*x + sqrt(
c^2*x^2 + 1)) - ((c^2*x^2 + 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))/c^3)*a*b + (9*x
^3*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*c*((c^2*x^3 - 6*x)/c^3 - 3*((c^2*x^2
+ 1)^(3/2) - 3*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1))/c^4))*b^2)*e
```

3.616 $\int \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=46

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rubi [A] time = 0.0636252, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5653, 5717, 8}

$$-\frac{2b\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx &= x\left(a+b\sinh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx\\ &= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.0589254, size = 74, normalized size = 1.61

$$x\left(a^2+2b^2\right)-\frac{2ab\sqrt{c^2x^2+1}}{c}+\frac{2b\sinh^{-1}(cx)\left(acx-b\sqrt{c^2x^2+1}\right)}{c}+b^2x\sinh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Maple [A] time = 0.027, size = 72, normalized size = 1.6

$$\frac{1}{c}\left(cxa^2+b^2\left((\mathrm{Arcsinh}\,(cx))^2\,cx-2\,\mathrm{Arcsinh}\,(cx)\,\sqrt{c^2x^2+1}+2\,cx\right)+2\,ab\left(\mathrm{Arcsinh}\,(cx)\,cx-\sqrt{c^2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(c*x*a^2+b^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

Maxima [A] time = 1.10004, size = 97, normalized size = 2.11

$$b^2 x \operatorname{arsinh}(cx)^2 + 2\,b^2\left(x - \frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right) + a^2x + \frac{2\left(cx\operatorname{arsinh}(cx) - \sqrt{c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Fricas [B] time = 2.74014, size = 212, normalized size = 4.61

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2 + \left(a^2+2\,b^2\right)cx - 2\,\sqrt{c^2x^2+1}ab + 2\left(abcx - \sqrt{c^2x^2+1}b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

Sympy [A] time = 0.319782, size = 82, normalized size = 1.78

$$\begin{cases} a^2x + 2abx\operatorname{asinh}(cx) - \frac{2ab\sqrt{c^2x^2+1}}{c} + b^2x\operatorname{asinh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c \neq 0 \\ a^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
```

```
, 0)), (a**2*x, True))
```

Giac [B] time = 1.7064, size = 150, normalized size = 3.26

$$2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

3.617 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=739

$$-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{}$$

```
[Out] ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcSin
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 1.31507, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5706, 5799, 5561, 2190, 2531, 2282, 6589}

$$-\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{e-c^2d}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\sinh^{-1}(cx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcSinh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcSin
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcSinh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-
(c^2*d) + e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\left(a+b\sinh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\left(a+b\sinh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right) dx$$

$$= -\frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}-\sqrt{e}\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{c\sqrt{-d}+\sqrt{e}\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \sinh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}+\sqrt{-c^2d+e}-\sqrt{e}e^x}\,dx, x, \sinh^{-1}\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} +$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} +$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} +$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}e^{\sinh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} +$$

Mathematica [A] time = 0.681279, size = 985, normalized size = 1.33

$$\frac{2\sqrt{-d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)a^2 - 2b\sqrt{d}\sinh^{-1}(cx)\log\left(\frac{e^{\sinh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{e-c^2d}}+1\right)a + 2b\sqrt{d}\sinh^{-1}(cx)\log\left(\frac{e^{\sinh^{-1}(cx)}\sqrt{e}}{\sqrt{e-c^2d}-c\sqrt{-d}}+1\right)a + 2b\sqrt{d}\sinh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (2*a^2*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Lo
g[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) + e])] - b^2*Sqr
```

```
t[d]*ArcSinh[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) + e])] + 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/
(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 +
(Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] + 2*a*b*Sqrt
[d]*ArcSinh[c*x]*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) + e])] + b^2*Sqrt[d]*ArcSinh[c*x]^2*Log[1 - (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*a*b*Sqrt[d]*ArcSinh[c*x]*Log[1 + (Sqrt[
e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - b^2*Sqrt[d]*ArcSinh
[c*x]^2*Log[1 + (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])]
 + 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*
Sqrt[-d] - Sqrt[-(c^2*d) + e])] - 2*b*Sqrt[d]*(a + b*ArcSinh[c*x])*PolyLog[
2, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) + e])] - 2*a*b*S
qrt[d]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) +
e]))] - 2*b^2*Sqrt[d]*ArcSinh[c*x]*PolyLog[2, -((Sqrt[e]*E^ArcSinh[c*x])/(c
*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] + 2*a*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^Arc
Sinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*ArcSinh[c*x]*
PolyLog[2, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])] - 2*
b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 + e])] + 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E^ArcSinh[c*x])/(-(c*Sqrt[-d])
+ Sqrt[-(c^2*d) + e])] + 2*b^2*Sqrt[d]*PolyLog[3, -((Sqrt[e]*E^ArcSinh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e]))] - 2*b^2*Sqrt[d]*PolyLog[3, (Sqrt[e]*E
^ArcSinh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d^2]*Sqrt[e])
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Arcsinh}\,(cx))^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d), x)
```

3.618 $\int \frac{(d+ex^2)^3}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=670

result too large to display

```
[Out] (d^3*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (3*d^2*e*Cosh[
a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (3*d*e^2*Cosh[a/b]*C
oshIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) - (5*e^3*Cosh[a/b]*CoshInteg
ral[(a + b*ArcSinh[c*x])/b])/(64*b*c^7) + (3*d^2*e*Cosh[(3*a)/b]*CoshIntegr
al[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (9*d*e^2*Cosh[(3*a)/b]*CoshInte
gral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (9*e^3*Cosh[(3*a)/b]*CoshInt
egral[(3*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (3*d*e^2*Cosh[(5*a)/b]*Cosh
Integral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (5*e^3*Cosh[(5*a)/b]*Cos
hIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) + (e^3*Cosh[(7*a)/b]*Cosh
Integral[(7*(a + b*ArcSinh[c*x]))/b])/(64*b*c^7) - (d^3*Sinh[a/b]*SinhInteg
ral[(a + b*ArcSinh[c*x])/b])/(b*c) + (3*d^2*e*Sinh[a/b]*SinhIntegral[(a + b
*ArcSinh[c*x])/b])/(4*b*c^3) - (3*d*e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSi
nh[c*x])/b])/(8*b*c^5) + (5*e^3*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])
/b])/(64*b*c^7) - (3*d^2*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x
]))/b])/(4*b*c^3) + (9*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c
*x]))/b])/(16*b*c^5) - (9*e^3*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[
c*x]))/b])/(64*b*c^7) - (3*d*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSi
nh[c*x]))/b])/(16*b*c^5) + (5*e^3*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcS
inh[c*x]))/b])/(64*b*c^7) - (e^3*Sinh[(7*a)/b]*SinhIntegral[(7*(a + b*ArcSi
nh[c*x]))/b])/(64*b*c^7)
```

Rubi [A] time = 1.34432, antiderivative size = 658, normalized size of antiderivative = 0.98, number of steps used = 42, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5706, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{3d^2e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3d^2e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*d^2*e*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*
Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(8*b*c^5) - (5*e^3*Cosh[a/b]*Co
```

```
shIntegral[a/b + ArcSinh[c*x]])/(64*b*c^7) + (3*d^2*e*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (9*d*e^2*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) + (9*e^3*Cosh[(3*a)/b]*CoshInte
gral[(3*a)/b + 3*ArcSinh[c*x]])/(64*b*c^7) + (3*d*e^2*Cosh[(5*a)/b]*CoshInt
egral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) - (5*e^3*Cosh[(5*a)/b]*CoshInte
gral[(5*a)/b + 5*ArcSinh[c*x]])/(64*b*c^7) + (e^3*Cosh[(7*a)/b]*CoshIntegra
l[(7*a)/b + 7*ArcSinh[c*x]])/(64*b*c^7) + (d^3*Cosh[a/b]*CoshIntegral[(a +
b*ArcSinh[c*x])/b])/(b*c) + (3*d^2*e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c
*x]])/(4*b*c^3) - (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b
*c^5) + (5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(64*b*c^7) - (3*
d^2*e*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) + (9*
d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) - (9
*e^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(64*b*c^7) - (3*
d*e^2*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) + (5
*e^3*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(64*b*c^7) - (e^
3*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcSinh[c*x]])/(64*b*c^7) - (d^3*S
inh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d^3}{a+b\sinh^{-1}(cx)}+\frac{3d^2ex^2}{a+b\sinh^{-1}(cx)}+\frac{3de^2x^4}{a+b\sinh^{-1}(cx)}+\frac{e^3x^6}{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d^3\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx+\left(3d^2e\right)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx+\left(3de^2\right)\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx+e^3$$

$$= \frac{d^3\,\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{\left(3d^2e\right)\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx\right.}{c^3}$$

$$= \frac{\left(3d^2e\right)\mathrm{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^3}+\frac{\left(3de^2\right)\mathrm{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}\right.\right.}{c^5}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(3d^2e\right)\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x\right.}{4c^3}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(3d^2e\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \right.}{4c^3}$$

$$= -\frac{3d^2e\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}-\frac{5e^3\cosh\left(\frac{a}{b}\right)\mathrm{Ch}}{64}$$

Mathematica [A] time = 0.974046, size = 444, normalized size = 0.66

$$\frac{\cosh\left(\frac{a}{b}\right)\left(-48c^4d^2e+64c^6d^3+24c^2de^2-5e^3\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3e\cosh\left(\frac{3a}{b}\right)\left(16c^4d^2-12c^2de+3e^2\right)\text{Chi}\left(3\left(\frac{a}{b}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((64*c^6*d^3 - 48*c^4*d^2*e + 24*c^2*d*e^2 - 5*e^3)*Cosh[a/b]*CoshIntegral[
a/b + ArcSinh[c*x]] + 3*e*(16*c^4*d^2 - 12*c^2*d*e + 3*e^2)*Cosh[(3*a)/b]*C
oshIntegral[3*(a/b + ArcSinh[c*x])] + 12*c^2*d*e^2*Cosh[(5*a)/b]*CoshIntegr
al[5*(a/b + ArcSinh[c*x])] - 5*e^3*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcS
inh[c*x])] + e^3*Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcSinh[c*x])] - 64*c^
6*d^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 48*c^4*d^2*e*Sinh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]] - 24*c^2*d*e^2*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] + 5*e^3*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 48*c^4*
d^2*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 36*c^2*d*e^2*Sin
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 9*e^3*Sinh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcSinh[c*x])] - 12*c^2*d*e^2*Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c*x])] + 5*e^3*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c*x])] - e^3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcSinh[c*x])])/(64*b*c^7)
```

Maple [A] time = 0.221, size = 654, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/128/c^6*e^3/b*exp(-7*a/b)*Ei(1,-7*arcsinh(c*x)-7*a/b)-1/128/c^6*e^3
/b*exp(7*a/b)*Ei(1,7*arcsinh(c*x)+7*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a
/b)*d^3+3/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^2*e-3/16/c^4/b*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)*d*e^2+5/128/c^6/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e
^3-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^3+3/8/c^2/b*exp(-a/b)*Ei(1,-ar
csinh(c*x)-a/b)*d^2*e-3/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e^2+5/
128/c^6/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^3-3/8/c^2*e/b*exp(3*a/b)*Ei(1
,3*arcsinh(c*x)+3*a/b)*d^2+9/32/c^4*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*
a/b)*d-9/128/c^6*e^3/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-3/8/c^2*e/b*ex
p(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d^2+9/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-
```

```
3*arcsinh(c*x)-3*a/b)*d-9/128/c^6*e^3/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*
a/b)-3/32/c^4*e^2/b*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)*d+5/128/c^6*e^3/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-3/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*a
rcsinh(c*x)-5*a/b)*d+5/128/c^6*e^3/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^3}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)/(b*arcsinh(c*x) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^3}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**3/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^3}{b \operatorname{arsinh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3/(b*arcsinh(c*x) + a), x)
```

3.619 $\int \frac{(d+ex^2)^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=388

$$-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{C}}{}$$

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (d*e*Cosh[a/b]
*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(2*b*c^3) + (e^2*Cosh[a/b]*CoshInteg
ral[(a + b*ArcSinh[c*x])/b])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3
*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegral[(5*
(a + b*ArcSinh[c*x]))/b])/(16*b*c^5) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*A
rcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(8*b*c^
5) - (d*e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b*c^3)
 + (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5
) - (e^2*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b*c^5)
```

Rubi [A] time = 0.786215, antiderivative size = 380, normalized size of antiderivative = 0.98, number of steps used = 27, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5706, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2bc^3}+\frac{de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}-\frac{3e^2\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(d*e*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(2*b*c^3) + (e^2*Cosh[a/b
]*CoshIntegral[a/b + ArcSinh[c*x]])/(8*b*c^5) + (d*e*Cosh[(3*a)/b]*CoshInte
gral[(3*a)/b + 3*ArcSinh[c*x]])/(2*b*c^3) - (3*e^2*Cosh[(3*a)/b]*CoshIntegr
al[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) + (e^2*Cosh[(5*a)/b]*CoshIntegral[
(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) + (d^2*Cosh[a/b]*CoshIntegral[(a + b*
ArcSinh[c*x])/b])/(b*c) + (d*e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(2*b*c^3) - (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(8*b*c^5) - (d
*e*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(2*b*c^3) + (3*e^2
*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(16*b*c^5) - (e^2*Si
```

```
nh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c*x]])/(16*b*c^5) - (d^2*Sinh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^2}{a+b\sinh^{-1}(cx)}\,dx &= \int \left(\frac{d^2}{a+b\sinh^{-1}(cx)}+\frac{2dex^2}{a+b\sinh^{-1}(cx)}+\frac{e^2x^4}{a+b\sinh^{-1}(cx)}\right)dx\\
&= d^2\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx+(2de)\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx+e^2\int \frac{x^4}{a+b\sinh^{-1}(cx)}\,dx\\
&= \frac{d^2\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^3}\\
&= \frac{(2de)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\sinh^{-1}(cx)\right)}{c^3}+\frac{e^2\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}+\frac{\cosh}{16(a}\right.\right.}{c^5}\\
&= \frac{d^2\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{(de)\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\text{si}\right.}{2c^3}\\
&= \frac{d^2\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\mathbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh(}{a+}\right.}{2c^3}\\
&= -\frac{de\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8bc^5}+\frac{de\cosh\left(\frac{3a}{b}\right)\mathbf{Chi}\left(\frac{3a}{b}\right.}{2bc^3}
\end{aligned}
$$

Mathematica [A] time = 0.541353, size = 253, normalized size = 0.65

$$
\frac{2\cosh\left(\frac{a}{b}\right)\left(8c^4d^2-4c^2de+e^2\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\cosh\left(\frac{3a}{b}\right)\left(8c^2d-3e\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-16c^4d^2\sinh\left(\frac{a}{b}\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]]
 + (8*c^2*d - 3*e)*e*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + e
^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c*x])] - 16*c^4*d^2*Sinh[a/b
]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d*e*Sinh[a/b]*SinhIntegral[a/b +
 ArcSinh[c*x]] - 2*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 8*c^2*d
```

```
*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] + 3*e^2*Sinh[(3*a)/b]
*SinhIntegral[3*(a/b + ArcSinh[c*x])] - e^2*Sinh[(5*a)/b]*SinhIntegral[5*(a
/b + ArcSinh[c*x])])/(16*b*c^5)
```

Maple [A] time = 0.131, size = 380, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e^2}{32\,c^4b}\mathrm{e}^{-5\,\frac{a}{b}}\mathrm{Ei}\left(1,-5\,\mathrm{Arcsinh}\,(cx)-5\,\frac{a}{b}\right)-\frac{e^2}{32\,c^4b}\mathrm{e}^{5\,\frac{a}{b}}\mathrm{Ei}\left(1,5\,\mathrm{Arcsinh}\,(cx)+5\,\frac{a}{b}\right)-\frac{d^2}{2\,b}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*arcsinh(c*x)-5*a/b)-1/32/c^4*e^2/b
*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d*e-1/16/c^4/b*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d^2+1/4/c^2
/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arcsinh
(c*x)-a/b)*e^2-1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arcsinh(c*x)-3*a/b)*d+3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a
/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^2}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{a + b \operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{b \operatorname{arsinh}(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a), x)
```

3.620 $\int \frac{d+ex^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=180

$$-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\right.}{4bc^3}$$

```
[Out] (d*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (e*Cosh[a/b]*Cos
hIntegral[(a + b*ArcSinh[c*x])/b])/(4*b*c^3) + (e*Cosh[(3*a)/b]*CoshIntegra
l[(3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b
*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b
])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(
4*b*c^3)
```

Rubi [A] time = 0.367419, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5706, 5657, 3303, 3298, 3301, 5669, 5448}

$$-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] -(e*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e*Cosh[(3*a)/b
]*CoshIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) + (d*Cosh[a/b]*CoshInte
gral[(a + b*ArcSinh[c*x])/b])/(b*c) + (e*Sinh[a/b]*SinhIntegral[a/b + ArcSi
nh[c*x]])/(4*b*c^3) - (e*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x
]])/(4*b*c^3) - (d*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{a+b\sinh^{-1}(cx)}\,dx = \int \left(\frac{d}{a+b\sinh^{-1}(cx)} + \frac{ex^2}{a+b\sinh^{-1}(cx)}\right) dx$$

$$= d\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx + e\int \frac{x^2}{a+b\sinh^{-1}(cx)}\,dx$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{e\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right) dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+ \cdots\right.}{bc}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh \cdots\right.}{4c^3}$$

$$= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b} \cdots\right.}{a+bx \cdots}\right.}{4c^3}$$

$$= -\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{\cdots}\right.}{bc}$$

Mathematica [A] time = 0.240057, size = 126, normalized size = 0.7

$$\frac{\cosh\left(\frac{a}{b}\right)\left(4c^2d-e\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 4c^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) \cdots}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d - e)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e*Cosh[(3*a)/b]
*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c^2*d*Sinh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]] + e*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - e*Sinh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Maple [A] time = 0.086, size = 178, normalized size = 1.

$$\frac{1}{c}\left(-\frac{e}{8c^2b}e^{-3\frac{a}{b}}\text{Ei}\left(1,-3\,\text{Arcsinh}(cx)-3\frac{a}{b}\right) - \frac{e}{8c^2b}e^{3\frac{a}{b}}\text{Ei}\left(1,3\,\text{Arcsinh}(cx)+3\frac{a}{b}\right) - \frac{d}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{Arcsinh}(cx)+\frac{a}{b}\right) + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/8/c^2*e/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8/c^2*e/b*exp(3
*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x
)-a/b)*d+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{b \operatorname{arsinh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2 + d}{b \operatorname{arsinh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{a + b \operatorname{asinh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{b \operatorname{arsinh}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

3.621 $\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rubi [A] time = 0.0699068, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5657, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$

$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sin$$

$$= \frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A] time = 0.0187903, size = 45, normalized size = 0.83

$$\frac{\cosh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a/b +
ArcSinh[c*x]])/(b*c)
```

Maple [A] time = 0.031, size = 56, normalized size = 1.

$$\frac{1}{c}\left(-\frac{1}{2b}e^{\frac{a}{b}}\text{Ei}\left(1,\text{Arcsinh}(cx)+\frac{a}{b}\right)-\frac{1}{2b}e^{-\frac{a}{b}}\text{Ei}\left(1,-\text{Arcsinh}(cx)-\frac{a}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x)),x)
```

```
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsinh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b \operatorname{arsinh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{asinh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

3.622 $\int \frac{1}{(d+ex^2)(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)\,(a+b\sinh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.040101, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]

[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\,(a+b\sinh^{-1}(cx))}\,dx = \int \frac{1}{(d+ex^2)\,(a+b\sinh^{-1}(cx))}\,dx$$

Mathematica [A] time = 0.677592, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)\,(a+b\sinh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)\left(b\operatorname{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{aex^2+ad+\left(bex^2+bd\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)(b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

3.623 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.0401727, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.29035, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.305, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 (a+b\mathrm{Arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 (b\,\mathrm{arsinh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\mathrm{arsinh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*
d^2)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2\left(b\operatorname{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)), x)
```

3.624 $\int \frac{(d+ex^2)^2}{(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=495

$$\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(cx))}{b}\right)}{2b^2c^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Ch}}{16}$$

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/
(b^2*c) + (d*e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(2*b^2*c^3)
- (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(8*b^2*c^5) - (3*d*e
*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(2*b^2*c^3) + (9*e
^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(16*b^2*c^5) - (
5*e^2*CoshIntegral[(5*(a + b*ArcSinh[c*x]))/b]*Sinh[(5*a)/b])/(16*b^2*c^5)
+ (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (d*e*Cosh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(2*b^2*c^3) + (e^2*Cosh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c*x])/b])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*SinhI
ntegral[(3*(a + b*ArcSinh[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cosh[(3*a)/b]*Sin
hIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*
SinhIntegral[(5*(a + b*ArcSinh[c*x]))/b])/(16*b^2*c^5)
```

Rubi [A] time = 0.874567, antiderivative size = 483, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5706, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{2b^2c^3}-\frac{3de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{2b^2c^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{8b^2c^5}+\frac{9e^2\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^4*Sqrt[1 + c^2*x^2])/(b*c*(a
 + b*ArcSinh[c*x])) - (d^2*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(b^2
*c) + (d*e*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(2*b^2*c^3) - (e^2*C
```

```
oshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(8*b^2*c^5) - (3*d*e*CoshIntegra
l[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(2*b^2*c^3) + (9*e^2*CoshIntegra
l[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^5) - (5*e^2*CoshIntegr
al[(5*a)/b + 5*ArcSinh[c*x]]*Sinh[(5*a)/b])/(16*b^2*c^5) + (d^2*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (d*e*Cosh[a/b]*SinhIntegral[a/b
+ ArcSinh[c*x]])/(2*b^2*c^3) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*
x]])/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(2*b^2*c^3) - (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*
x]])/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcSinh[c
*x]])/(16*b^2*c^5)
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int\left(\frac{d^2}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{2dex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{e^2x^4}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx$$

$$=d^2\int\frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+(2de)\int\frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+e^2\int\frac{x^4}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$=-\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(cd^2\right)\int\frac{x}{\sqrt{1+c^2x^2}\left(a+b\right.}}{b}$$

$$=-\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{d^2\,\mathrm{Subst}\left(\int\frac{\sinh(x)}{a+bx}\right.}{b}$$

$$=-\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subs}}{}$$

$$=-\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^4\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{d^2\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}\right.}{b^2c}$$

Mathematica [A] time = 2.02929, size = 356, normalized size = 0.72

$$-\frac{2\sinh\left(\frac{a}{b}\right)\left(8c^4d^2-4c^2de+e^2\right)\mathbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+3e\sinh\left(\frac{3a}{b}\right)\left(8c^2d-3e\right)\mathbf{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-16c^4d^2\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((16*b*c^4*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (32*b*c^4*d*e*x^2
*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (16*b*c^4*e^2*x^4*Sqrt[1 + c^2*x
^2])/(a + b*ArcSinh[c*x]) + 2*(8*c^4*d^2 - 4*c^2*d*e + e^2)*CoshIntegral[a/
b + ArcSinh[c*x]]*Sinh[a/b] + 3*(8*c^2*d - 3*e)*e*CoshIntegral[3*(a/b + Arc
Sinh[c*x])]*Sinh[(3*a)/b] + 5*e^2*CoshIntegral[5*(a/b + ArcSinh[c*x])]*Sinh
[(5*a)/b] - 16*c^4*d^2*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 8*c^2*d
*e*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] - 2*e^2*Cosh[a/b]*SinhIntegra
l[a/b + ArcSinh[c*x]] - 24*c^2*d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])] + 9*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])] - 5*e
^2*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[c*x])])/(16*b^2*c^5)
```

Maple [B] time = 0.221, size = 1036, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/32*(16*c^5*x^5-16*c^4*x^4*(c^2*x^2+1)^(1/2)+20*c^3*x^3-12*c^2*x^2*(c
^2*x^2+1)^(1/2)+5*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+5/32/
c^4*e^2/b^2*exp(5*a/b)*Ei(1,5*arcsinh(c*x)+5*a/b)-1/32/c^4*e^2/b*(16*c^5*x^
5+20*c^3*x^3+16*c^4*x^4*(c^2*x^2+1)^(1/2)+5*c*x+12*c^2*x^2*(c^2*x^2+1)^(1/2
)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-5/32/c^4*e^2/b^2*exp(-5*a/b)*Ei(1,-
5*arcsinh(c*x)-5*a/b)+1/2*(c*x-(c^2*x^2+1)^(1/2))*d^2/b/(a+b*arcsinh(c*x))+
1/2*d^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/4*(c*x-(c^2*x^2+1)^(1/2))*d*e
/c^2/b/(a+b*arcsinh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)+1
/16*(c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+b*arcsinh(c*x))+1/16/c^4*e^2/b^2*e
xp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsi
nh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4/c^2/b*d*e*(c*x+(
c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-arcsin
h(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/16/
c^4/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(4*c^3*x^3-4*c^2*x^2*(c^2
*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*d*e/c^2/b/(a+b*arcsinh(c*x))-3/32*(4
*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e^2/c^4/b/(a+
b*arcsinh(c*x))+3/4/c^2*e/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)*d-9/32/
c^4*e^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/4/c^2*e/b*(4*c^3*x^3+3*
c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))*d+3/3
```

```
2/c^4*e^2/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2))
/(a+b*arcsinh(c*x))-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)*d
+9/32/c^4*e^2/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3e^2x^7 + \left(2\,c^3de + ce^2\right)x^5 + cd^2x + \left(c^3d^2 + 2\,cde\right)x^3 + \left(c^2e^2x^6 + \left(2\,c^2de + e^2\right)x^4 + \left(c^2d^2 + 2\,de\right)x^2 + d^2\right)\sqrt{c^2x^2+1}}{abc^3x^2 + \sqrt{c^2x^2+1}abc^2x + abc + \left(b^2c^3x^2 + \sqrt{c^2x^2+1}b^2c^2x + b^2c\right)\log\left(cx + \sqrt{c^2x^2+1}\right)} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^7 + (2*c^3*d*e + c*e^2)*x^5 + c*d^2*x + (c^3*d^2 + 2*c*d*e)*x^3
 + (c^2*e^2*x^6 + (2*c^2*d*e + e^2)*x^4 + (c^2*d^2 + 2*d*e)*x^2 + d^2)*sqrt
(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^
3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1)))
+ integrate((5*c^5*e^2*x^8 + 2*(3*c^5*d*e + 5*c^3*e^2)*x^6 + (c^5*d^2 + 12*
c^3*d*e + 5*c*e^2)*x^4 + c*d^2 + 2*(c^3*d^2 + 3*c*d*e)*x^2 + (5*c^3*e^2*x^6
 + 3*(2*c^3*d*e + c*e^2)*x^4 - c*d^2 + (c^3*d^2 + 2*c*d*e)*x^2)*(c^2*x^2 +
1) + (10*c^4*e^2*x^7 + (12*c^4*d*e + 13*c^2*e^2)*x^5 + 2*(c^4*d^2 + 7*c^2*d
*e + 2*e^2)*x^3 + (c^2*d^2 + 4*d*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c
^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 +
 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sq
rt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{b^2\,\mathrm{arsinh}\,(cx)^2 + 2\,ab\,\mathrm{arsinh}\,(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*
x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+e x^2\right)^2}{\left(a+b \operatorname{asinh}(c x)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^2}{\left(b \operatorname{arsinh}(c x)+a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

3.625 $\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=247

$$\frac{e\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(}{}\right.}{4b^2c^3}$$

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*x
^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*S
inh[a/b])/(b^2*c) + (e*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b
^2*c^3) - (3*e*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b
^2*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e*C
osh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) + (3*e*Cosh[(3*a
)/b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Rubi [A] time = 0.47568, antiderivative size = 239, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5706, 5655, 5779, 3303, 3298, 3301, 5665}

$$\frac{e\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4b^2c^3}-\frac{e\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x^2*Sqrt[1 + c^2*x
^2])/(b*c*(a + b*ArcSinh[c*x])) - (d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[
a/b])/(b^2*c) + (e*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b])/(4*b^2*c^3)
- (3*e*CoshIntegral[(3*a)/b + 3*ArcSinh[c*x]]*Sinh[(3*a)/b])/(4*b^2*c^3) +
(d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c) - (e*Cosh[a/b]*SinhI
ntegral[a/b + ArcSinh[c*x]])/(4*b^2*c^3) + (3*e*Cosh[(3*a)/b]*SinhIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b^2*c^3)
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
0 || IGtQ[n, 0])
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^2} + \frac{ex^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\right) dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + e\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b} + \frac{e\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{d\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{bc} - \frac{e\,\mathrm{Su}}{}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh\right.}{bc}$$

$$= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{ex^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{d\,\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c} + \frac{e\,\mathrm{Chi}\left(\frac{a}{b}\right.}{}$$

Mathematica [A] time = 0.934752, size = 190, normalized size = 0.77

$$-\frac{\sinh\left(\frac{a}{b}\right)\left(4c^2d-e\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) - 4c^2d\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + \frac{4bc^2d\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)} + \frac{4bc^2ex^2\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)} + 3e\sinh}{4b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((4*b*c^2*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e*x^2*Sqrt[
1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*c^2*d - e)*CoshIntegral[a/b + ArcSi
nh[c*x]]*Sinh[a/b] + 3*e*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b]
 - 4*c^2*d*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e*Cosh[a/b]*SinhInt
egral[a/b + ArcSinh[c*x]] - 3*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh
[c*x])])/(4*b^2*c^3)
```

Maple [A] time = 0.115, size = 438, normalized size = 1.8

$$\frac{1}{c}\left(\frac{e}{8\,bc^2\,(a+b\mathrm{Arcsinh}\,(cx))}\left(4\,c^3x^3-4\,c^2x^2\sqrt{c^2x^2+1}+3\,cx-\sqrt{c^2x^2+1}\right)+\frac{3\,e}{8\,c^2b^2}\mathrm{e}^{3\,\frac{a}{b}}\mathrm{Ei}\left(1,3\,\mathrm{Arcsinh}\,(cx)+3\,\frac{a}{b}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(4*c^3*x^3-4*c^2*x^2*(c^2*x^2+1)^(1/2)+3*c*x-(c^2*x^2+1)^(1/2))*e/
c^2/b/(a+b*arcsinh(c*x))+3/8/c^2*e/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b
)-1/8/c^2*e/b*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+1)^(1/2
))/(a+b*arcsinh(c*x))-3/8/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)
+1/2*(c*x-(c^2*x^2+1)^(1/2))*d/b/(a+b*arcsinh(c*x))+1/2*d/b^2*exp(a/b)*Ei(1
,arcsinh(c*x)+a/b)-1/8*(c*x-(c^2*x^2+1)^(1/2))*e/c^2/b/(a+b*arcsinh(c*x))-1
/8/c^2*e/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d*(c*x+(c^2*x^2+1)^(1/2)
)/(a+b*arcsinh(c*x))-1/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/8/c^2/b*
e*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c^2/b^2*e*exp(-a/b)*Ei(1,-
arcsinh(c*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3ex^5+\left(c^3d+ce\right)x^3+cdx+\left(c^2ex^4+\left(c^2d+e\right)x^2+d\right)\sqrt{c^2x^2+1}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{3\,c^5ex^6+}{abc^5x^4+\left(c^2x^2+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^5 + (c^3*d + c*e)*x^3 + c*d*x + (c^2*e*x^4 + (c^2*d + e)*x^2 + d)
*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b
^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 +
1))) + integrate((3*c^5*e*x^6 + (c^5*d + 6*c^3*e)*x^4 + (2*c^3*d + 3*c*e)*x
^2 + (3*c^3*e*x^4 + (c^3*d + c*e)*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (6*c^4*e
*x^5 + (2*c^4*d + 7*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c
^5*x^4 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 +
 (c^2*x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c
^2*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*
b*c^2*x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^2+d}{b^2\operatorname{arsinh}(cx)^2+2\,ab\operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^2+d}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

3.626 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(a + b*ArcS
inh[c*x])/b]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c)
```

Rubi [A] time = 0.188053, antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5655, 5779, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[a/b + ArcSi
nh[c*x]]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/
(b^2*c)
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
```

```
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Mathematica [A] time = 0.142299, size = 71, normalized size = 0.84

$$\frac{-\frac{b\sqrt{c^2x^2+1}}{a+b\sinh^{-1}(cx)}-\sinh\left(\frac{a}{b}\right)\textbf{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{a}{b}\right)\textbf{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Maple [A] time = 0.043, size = 118, normalized size = 1.4

$$\frac{1}{c}\left(\frac{1}{2\,b\,(a+b\mathrm{Arcsinh}\,(cx))}\left(cx-\sqrt{c^2x^2+1}\right)+\frac{1}{2\,b^2}\mathrm{e}^{\frac{a}{b}}\mathrm{Ei}\left(1,\mathrm{Arcsinh}\,(cx)+\frac{a}{b}\right)-\frac{1}{2\,b\,(a+b\mathrm{Arcsinh}\,(cx))}\left(cx+\sqrt{c^2x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(c*x-(c^2*x^2+1)^(1/2))/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(1
,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2
*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{abc^3x^2+\sqrt{c^2x^2+1}abc^2x+abc+\left(b^2c^3x^2+\sqrt{c^2x^2+1}b^2c^2x+b^2c\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}+\int\frac{}{abc^4x^4+\left(c^2x^2+1\right)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

3.627 $\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0382917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 11.3646, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.216, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)\left(a+b\text{Arcsinh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^{3}x^{3}+cx+\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}}{abc^{3}ex^{4}+\left(c^{3}d+ce\right)abx^{2}+abcd+\left(b^{2}c^{3}ex^{4}+\left(c^{3}d+ce\right)b^{2}x^{2}+b^{2}cd+\left(b^{2}c^{2}ex^{3}+b^{2}c^{2}dx\right)\sqrt{c^{2}x^{2}+1}\right)\log\left(cx+\sqrt{c^{2}x^{2}+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e*x^4 + (c^3*d + c*e)*a*b*x
^2 + a*b*c*d + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*c^2*
e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b
*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*e*x^6 - (c^5*
d - 2*c^3*e)*x^4 - (2*c^3*d - c*e)*x^2 + (c^3*e*x^4 - (c^3*d - 3*c*e)*x^2 +
 c*d)*(c^2*x^2 + 1) - c*d + (2*c^4*e*x^5 - (2*c^4*d - 5*c^2*e)*x^3 - (c^2*d
 - 2*e)*x)*sqrt(c^2*x^2 + 1))/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*a*b*
x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + 2*(c^3*d^2 + c*d*
e)*a*b*x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c^2*x
^2 + 1) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6 + (c^5*d^2 + 4*c
^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*b^2*x^2 + (b^2*c^
3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4
*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2
*d*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4
*e^2*x^7 + (2*c^4*d*e + c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2
```

```
*d*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2ex^2 + a^2d + \left(b^2ex^2 + b^2d\right)\text{arsinh}\,(cx)^2 + 2\left(abex^2 + abd\right)\text{arsinh}\,(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2 + d\right)(b\,\text{arsinh}\,(cx) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

3.628 $\int \frac{1}{(d+ex^2)^2 \left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0372013, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 27.4453, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^2 \left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.344, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 (a+b\mathrm{Arcsinh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*e^2*x^6 + (2*c^3*d*e + c*e^
2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2 + (b^2*c^3*e^2*x^6 + (
2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e)*b^2*x^2 + (b^2
*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1))*log(c*
x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2
*x)*sqrt(c^2*x^2 + 1)) - integrate((3*c^5*e*x^6 - (c^5*d - 6*c^3*e)*x^4 - (
2*c^3*d - 3*c*e)*x^2 + (3*c^3*e*x^4 - (c^3*d - 5*c*e)*x^2 + c*d)*(c^2*x^2 +
 1) - c*d + (6*c^4*e*x^5 - (2*c^4*d - 11*c^2*e)*x^3 - (c^2*d - 4*e)*x)*sqrt
(c^2*x^2 + 1))/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x^8 + (3*c
^5*d^2*e + 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^
2)*a*b*x^4 + a*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2 + (a*b*c^3*e^3*x^8
 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)*(c^2*x^2 +
1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e +
 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4
 + b^2*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*b^2*x^2 + (b^2*c^3*e^3*x^8 + 3*b^2*c
^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^
2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^
2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3 + 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 +
```

```
1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*
e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*a*b*x^5 + (c^4*d^3
 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2e^2x^4 + 2\,a^2dex^2 + a^2d^2 + \left(b^2e^2x^4 + 2\,b^2dex^2 + b^2d^2\right)\operatorname{arsinh}\left(cx\right)^2 + 2\left(abe^2x^4 + 2\,abdex^2 + abd^2\right)\operatorname{arsinh}\left(cx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*
e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)
*arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2 + d\right)^2 (b \operatorname{arsinh}\left(cx\right) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

3.629 $\int \left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=672

$$-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee\cdots}{\cdots}$$

```
[Out] d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) + (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rubi [A] time = 1.8788, antiderivative size = 672, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5706, 5653, 5779, 3308, 2180, 2204, 2205, 5663, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcSinh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcSinh[c*x]])/5 + (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
```

```
qrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3) + (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5) + (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[
b]])/(64*c^5) + (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcSinh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) + (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) + (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\sinh^{-1}(cx)}\,dx = \int \left(d^2\sqrt{a+b\sinh^{-1}(cx)}+2dex^2\sqrt{a+b\sinh^{-1}(cx)}+e^2x^4\sqrt{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d^2\int \sqrt{a+b\sinh^{-1}(cx)}\,dx+(2de)\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx+e^2\int x^4\sqrt{a+b\sin$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{2}\Big(b$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}-\frac{\Big(bd}{}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}+\frac{\Big(bd}{}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}+\frac{d^2\,S}{}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}c}{}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}c}{}$$

$$= d^2x\sqrt{a+b\sinh^{-1}(cx)}+\frac{2}{3}dex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}c}{}$$

Mathematica [A] time = 6.17694, size = 535, normalized size = 0.8

$$-\frac{be^{-\frac{5a}{b}}\left(450e^{\frac{6a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)\left(be\left(4c^2d-e\right)\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\sqrt{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{b^2}}+8ac^4d^2\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -(b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcSinh[c*x]] + 8*b*c^4*d^2*Ar
cSinh[c*x]*Sqrt[a/b + ArcSinh[c*x]] + b*(4*c^2*d - e)*e*Sqrt[-((a + b*ArcSi
nh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, a/b + ArcSinh[
```

```
c*x]] + 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c*x]
)^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcSinh[c*x]))/b] + 25*Sqrt[3]*b*(8*c^2*d -
 3*e)*e*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a + b*ArcSinh[c*x])^2/
b^2)]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] + 450*E^((4*a)/b)*(8*a*c^4*d^
2*Sqrt[-((a + b*ArcSinh[c*x])/b)] + 8*b*c^4*d^2*ArcSinh[c*x]*Sqrt[-((a + b*
ArcSinh[c*x])/b)] + b*e*(-4*c^2*d + e)*Sqrt[a/b + ArcSinh[c*x]]*Sqrt[-((a +
 b*ArcSinh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)] - b*e*E^((8
*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)]*
(25*Sqrt[3]*(8*c^2*d - 3*e)*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b] + 9*Sqrt
[5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcSinh[c*x]))/b])))/(7200*c^5*E^((5
*a)/b)*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [F] time = 0.35, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 \sqrt{a+b\operatorname{Arcsinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)^2 \sqrt{b\operatorname{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{asinh}(cx)} \left(d + ex^2\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)^2 \sqrt{b \operatorname{arsinh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a), x)
```

3.630 $\int (d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\operatorname{E}}{}$$

```
[Out] d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.93161, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5706, 5653, 5779, 3308, 2180, 2204, 2205, 5663, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{\pi}\sqrt{b}ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}ee^{-\frac{3a}{b}}\operatorname{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```


```
[Out] d*x*Sqrt[a + b*ArcSinh[c*x]] + (e*x^3*Sqrt[a + b*ArcSinh[c*x]])/3 + (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) + (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```


Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}\,dx = \int \left(d\sqrt{a+b\sinh^{-1}(cx)}+ex^2\sqrt{a+b\sinh^{-1}(cx)}\right)dx$$

$$= d\int \sqrt{a+b\sinh^{-1}(cx)}\,dx + e\int x^2\sqrt{a+b\sinh^{-1}(cx)}\,dx$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{2}(bcd)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh}}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}-\frac{(bd)\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-}\right.}{2c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{(bd)\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-}\right.}{4c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{d\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\,\mathrm{s}}\right.}{2c}$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-$$

$$= dx\sqrt{a+b\sinh^{-1}(cx)}+\frac{1}{3}ex^3\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-$$

Mathematica [A] time = 2.77962, size = 319, normalized size = 0.99

$$\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},-\frac{3(a+b\sinh \cdots}{b}\cdots\right)\cdots\right)}{72c^3\sqrt{-\frac{(a+b\sinh \cdots}{b \cdots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcSinh[c*x]]*(9*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]] +
Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b] -
9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x])/b)
] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(a +
b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2
)])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\sqrt{a+b\text{Arcsinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\sqrt{b\,\text{arsinh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \operatorname{asinh}(cx)} \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right) \sqrt{b \operatorname{arsinh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a), x)
```

3.631 $\int \sqrt{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rubi [A] time = 0.260122, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5653, 5779, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\sinh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
```

```
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(cx)}\,dx = x\sqrt{a+b\sinh^{-1}(cx)}-\frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}-\frac{b\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{2c}$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{b\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{4c}-\frac{b\,\mathrm{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(c$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{2c}-\frac{\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a$$
$$= x\sqrt{a+b\sinh^{-1}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] time = 0.0947891, size = 101, normalized size = 0.99

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\frac{\mathrm{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}-\frac{e^{\frac{2a}{b}}\mathrm{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\mathrm{Arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\,\text{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\,\text{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

3.632 $\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Rubi [A] time = 0.0564397, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]

[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Mathematica [A] time = 7.0475, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2),x]
```

```
[Out] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\sqrt{a+b\mathrm{Arcsinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(cx\right)}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \operatorname{arsinh}(cx) + a}}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d), x)
```

3.633 $\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.0539159, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]

[Out] Defer[Int][Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 15.9533, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sinh^{-1}(cx)}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2,x]
```

```
[Out] Integrate[Sqrt[a + b*ArcSinh[c*x]]/(d + e*x^2)^2, x]
```

Maple [A] time = 0.365, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)^{2}}\sqrt{a+b\mathrm{Arcsinh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}}{\left(ex^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b \operatorname{asinh}(cx)}}{\left(d + ex^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x))/(d + e*x**2)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \operatorname{arsinh}(cx) + a}}{\left(ex^2 + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a)/(e*x^2 + d)^2, x)
```

3.634 $\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=427

$$-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\sqrt{\frac{\pi}{3}}b^{3/2}ee$$

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 + c
^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sqrt
[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a +
b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*e*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sq
rt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)
/b))
```

Rubi [A] time = 1.2637, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5706, 5653, 5717, 5657, 3307, 2180, 2205, 2204, 5663, 5758, 5669, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}-\frac{3\sqrt{\pi}b^{3/2}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+\sqrt{\frac{\pi}{3}}b^{3/2}ee$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + (b*e*Sqrt[1 + c
^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 + c^2*x^2]*Sqrt
[a + b*ArcSinh[c*x]])/(6*c) + d*x*(a + b*ArcSinh[c*x])^(3/2) + (e*x^3*(a +
b*ArcSinh[c*x])^(3/2))/3 + (3*b^(3/2)*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*A
rcSinh[c*x]]/Sqrt[b]])/(32*c^3) + (b^(3/2)*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*c*E^(a/b)) - (3*b^(3/2)*e*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sq
```

```
rt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)
/b))
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5663

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcSinh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5758

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b
*ArcSinh[c*x])^n)/(e*m), x] + (-Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m -
 2)*(a + b*ArcSinh[c*x])^n)/Sqrt[d + e*x^2], x], x] - Dist[(b*f*n*Sqrt[1 +
c^2*x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSinh[c*x])^(n
- 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[n, 0]
 && GtQ[m, 1] && IntegerQ[m]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx = \int \left(d\left(a+b\sinh^{-1}(cx)\right)^{3/2}+ex^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}\right)dx$$

$$= d\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx + e\int x^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx$$

$$= dx\left(a+b\sinh^{-1}(cx)\right)^{3/2} + \frac{1}{3}ex^3\left(a+b\sinh^{-1}(cx)\right)^{3/2} - \frac{1}{2}(3bcd)\int \frac{x\sqrt{a+b\sinh^{-1}}}{\sqrt{1+c^2x^2}}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} - \frac{bex^2\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{6c} + dx\left(a+b\right.$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+}}{}$$

$$= -\frac{3bd\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c} + \frac{be\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{3c^3} - \frac{bex^2\sqrt{1+}}{}$$

Mathematica [A] time = 4.19707, size = 770, normalized size = 1.8

$$\frac{aee^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{3}{2},-\frac{3(a+b\sin}{b}\right.\right.}{72c^3\sqrt{-\frac{(a+b\sin}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]
])/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-
((a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcSinh[c*x]]*(
9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, a/b + ArcSinh[c*x]
] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c*x]))/b
] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, -((a + b*ArcSinh[c*x]
)/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (3*(
a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2
/b^2)]) + (Sqrt[b]*d*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x
^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*c) + (Sqrt[b]*e*(-9*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x
]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b]
 - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]
*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (-2*a + b)*Sqrt[3*
Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(
3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcSinh[c*x]]*(-Cosh[3*ArcSinh[c*x]] + 2*A
rcSinh[c*x]*Sinh[3*ArcSinh[c*x]])))/(288*c^3)
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \left(e x^{2}+d\right) \left(a +b \operatorname{Arcsinh}\left(c x \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(e x^{2} + d\right)} {\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{\frac{3}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{asinh}(cx))^{\frac{3}{2}} \left(d + ex^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2)*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^2 + d\right)(b\operatorname{arsinh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2), x)
```

3.635 $\int \left(a+b\sinh^{-1}(cx)\right)^{3/2} dx$

Optimal. Leaf size=135

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(\right.$$

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rubi [A] time = 0.251194, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5653, 5717, 5657, 3307, 2180, 2205, 2204}

$$\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rule 5653

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[
1 + c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5717

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(
```

```
1 + c^2*x^2)^FracPart[p]), Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])
^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n,
 0] && NeQ[p, -1]
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\sinh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{1}{4}\left(3b^2\right)\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx}}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{(3b)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,\right.}{4c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{(3b)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,\right.}{8c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{(3b)\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{\right.}}{4c}$$

$$= -\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}{2c}+x\left(a+b\sinh^{-1}(cx)\right)^{3/2}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c}}{\sqrt{b}}\right.}{8c}$$

Mathematica [A] time = 0.160703, size = 251, normalized size = 1.86

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(cx)}\left(\frac{\text{Gamma}\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}}-\frac{e^{\frac{2a}{b}}\text{Gamma}\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\left(2cx\sinh^{-1}(cx)-3\sqrt{c^2x^2+1}\right)\sqrt{\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{arsinh}\,(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{asinh}\,(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

3.636 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi [A] time = 0.0654058, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]

[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Mathematica [A] time = 2.73994, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2), x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\left(a+b\operatorname{Arcsinh}\left(cx\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(3/2)/(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

3.637 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi [A] time = 0.0645926, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 8.91439, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Maple [A] time = 0.392, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2}\left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}{(ex^2 + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

3.638 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=608

$$-\frac{\sqrt{\pi}dee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a}\cdots\right)}{4\sqrt{b}c^3}$$

```
[Out] (d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
- (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c
^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt
[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*Sqrt[P
i/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (d
^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b)) -
 (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a/
b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^5
*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erfi
[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rubi [A] time = 1.19517, antiderivative size = 608, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5706, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{\sqrt{\pi}dee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}-\frac{\sqrt{\pi}dee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}dee^{-\frac{3a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{a}\cdots\right)}{4\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
- (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c
^3) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt
[b]*c^5) + (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
```

```
])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqr
t[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (e^2*E^((5*a)/b)*Sqrt[P
i/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (d
^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b)) -
 (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a/
b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^5
*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(4*Sqrt[b]*c^3*E^((3*a)/b)) - (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*A
rcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erfi
[(Sqrt[5]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \left(\frac{d^2}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2dex^2}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{e^2x^4}{\sqrt{a+b\sinh^{-1}(cx)}}\right)dx$$

$$= d^2\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+(2de)\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx+e^2\int \frac{x^4}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}\right.}{c^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{2bc}+\frac{d^2\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\sinh^{-1}(cx)\right)}{2bc}$$

$$= \frac{d^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}+\frac{d^2\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right.}{bc}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{(de)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\text{s}\right.}{4c^3}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{(de)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\right.}{2bc^3}$$

$$= \frac{d^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}-\frac{dee^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^3}+\frac{e^2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}c^5}$$

Mathematica [A] time = 1.17055, size = 530, normalized size = 0.87

$$e^{-\frac{5a}{b}}\left(-30e^{\frac{6a}{b}}\left(8c^4d^2-4c^2de+e^2\right)\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+240c^4d^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\text{Gamma}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-30*(8*c^4*d^2 - 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gam
ma[1/2, a/b + ArcSinh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]
```

```
*Gamma[1/2, (-5*(a + b*ArcSinh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - 15
*Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a
+ b*ArcSinh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - 120*c^2*d*e*E^((4*a)/b)*Sqrt[-
((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] + 30*e^2*E^
((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])
/b)] - 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (
3*(a + b*ArcSinh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcSinh[
c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b] - 3*Sqrt[5]*e^2*E^((10*a)/b)*S
qrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c*x]))/b])/(480*c^5*E^
((5*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.324, size = 0, normalized size = 0.

$$\int \left(e x^2 + d\right)^2 \frac{1}{\sqrt{a + b \operatorname{Arcsinh}(cx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2 + d\right)^2}{\sqrt{b \operatorname{arsinh}(cx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^2}{\sqrt{a + b \operatorname{asinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^2}{\sqrt{b \operatorname{arsinh}\left(cx\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arcsinh(c*x) + a), x)
```

3.639 $\int \frac{d+ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sin}}{\sqrt{b}}\right.}{8\sqrt{b}c^3}$$

```
[Out] (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
(e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
+ (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]
)/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*
Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8
*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rubi [A] time = 0.571743, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5706, 5657, 3307, 2180, 2205, 2204, 5669, 5448}

$$-\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{3}}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sin}}{\sqrt{b}}\right.}{8\sqrt{b}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
(e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
+ (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]]
)/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*
Sqrt[b]*c*E^(a/b)) - (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8
*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]
])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5669

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m*Cosh[x], x], x, ArcSinh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \left(\frac{d}{\sqrt{a+b\sinh^{-1}(cx)}}+\frac{ex^2}{\sqrt{a+b\sinh^{-1}(cx)}}\right)dx$$

$$= d\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx + e\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

$$= \frac{d\,\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} + \frac{e\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{d\,\mathrm{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \cdots$$

$$= \frac{d\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{d\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} - \cdots$$

$$= \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{e\,\mathrm{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(\cdots\right.}{8c^3}$$

$$= \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{e\,\mathrm{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+\cdots}\right.}{4bc^3}$$

$$= \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} - \frac{ee^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \cdots$$

Mathematica [A] time = 0.66194, size = 218, normalized size = 0.76

$$\frac{e^{-\frac{3a}{b}}\left(-3e^{\frac{4a}{b}}\left(4c^2d-e\right)\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+3e^{\frac{2a}{b}}\left(4c^2d-e\right)\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\cdots}{}\right.\right.}{24c^3\sqrt{a+b\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-3*(4*c^2*d - e)*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + Arc
Sinh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c*x]))/b] + 3*(4*c^2*d - e)*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x
])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*e*E^((6*a)/b)*Sqrt[a
/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)
/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int (ex^2 + d)\frac{1}{\sqrt{a + b\text{Arcsinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{b\,\text{arsinh}\,(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{a + b\operatorname{asinh}(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{b\operatorname{arsinh}(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/sqrt(b*arcsinh(c*x) + a), x)
```

3.640 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rubi [A] time = 0.107028, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5657, 3307, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 5657

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[a/b - x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, n}, x]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^{-1}(cx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}\\ &= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(cx)\right)}{2bc}\\ &= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(cx)}\right)}{bc}\\ &= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}\end{aligned}$$

Mathematica [A] time = 0.103813, size = 101, normalized size = 1.15

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(cx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2c\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\text{Arcsinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\text{arsinh}\left(cx\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\,\mathrm{asinh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\,\mathrm{arsinh}\,(cx) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

3.641 $\int \frac{1}{(d+ex^2)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [A] time = 0.0601469, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \frac{1}{\left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.133467, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)\sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]),x]
```

```
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Maple [A] time = 0.186, size = 0, normalized size = 0.

$$\int \frac{1}{e x^2 + d} \frac{1}{\sqrt{a + b \operatorname{Arcsinh}(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e x^2 + d)\sqrt{b \operatorname{arsinh}(cx) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\operatorname{asinh}(cx)}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*asinh(c*x))*(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arcsinh(c*x) + a)), x)
```

3.642 $\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi [A] time = 0.0570992, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]

[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx = \int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.249384, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\sinh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcSinh[c*x]]), x]
```

Maple [A] time = 0.361, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2}\frac{1}{\sqrt{a+b\mathrm{Arcsinh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2\sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 \sqrt{b \operatorname{arsinh}(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arcsinh(c*x) + a)), x)
```

3.643 $\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

```
[Out] (-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(
Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3*E^((3*a
)/b))
```

Rubi [A] time = 0.691273, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5706, 5655, 5779, 3308, 2180, 2204, 2205, 5665}

$$\frac{\sqrt{\pi}ee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{3\pi}ee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}-\frac{\sqrt{\pi}ee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}ee^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (2*e*x^2*Sqrt[1 +
 c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (e*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(
Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) + (d*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b)) - (e*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b)) + (e*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3*E^((3*a
```

```
)/b))
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5665

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Di
st[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)^(n + 1), S
inh[x]^(m - 1)*(m + (m + 1)*Sinh[x]^2), x], x], x, ArcSinh[c*x]], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}+\frac{ex^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx + e\int \frac{x^2}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2cd)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b} + \frac{(2e)\,\mathrm{S}}{}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(cx)\right)}{bc} + \frac{d}{}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-}}\right.}{b^2c}$$

$$= -\frac{2d\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{2ex^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}} - \frac{de^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{ee^{a/b}\sqrt{\pi}\,\mathrm{er}}{}$$

Mathematica [A] time = 1.44713, size = 303, normalized size = 0.87

$$e^{-3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}\left(\left(4c^2d-e\right)e^{\frac{4a}{b}+3\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+\left(4c^2d-e\right)e^{\frac{2a}{b}+3\sinh^{-1}(cx)}\sqrt{-\frac{a+b}{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] ((4*c^2*d - e)*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*Gamma[
1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*e*E^(3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSi
nh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + (4*c^2*d - e)*E^((2*
a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*
ArcSinh[c*x])/b)] + E^((3*a)/b)*(-((1 + E^(2*ArcSinh[c*x]))*(4*c^2*d*E^(2*A
rcSinh[c*x]) + e*(-1 + E^(2*ArcSinh[c*x]))^2)) + Sqrt[3]*e*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])
)/(4*b*c^3*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\left(a+b\text{Arcsinh}\left(cx\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\left(b\,\text{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{(a + b\operatorname{asinh}(cx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(a + b*asinh(c*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{(b\operatorname{arsinh}(cx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arcsinh(c*x) + a)^(3/2), x)
```

3.644 $\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rubi [A] time = 0.251165, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5655, 5779, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 5655

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[1 + c
^2*x^2]*(a + b*ArcSinh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c/(b*(n + 1)
), Int[(x*(a + b*ArcSinh[c*x])^(n + 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[d^p/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]^m
*Cosh[x]^(2*p + 1), x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[e, c^2*d] && IntegerQ[2*p] && GtQ[p, -1] && IGtQ[m, 0] && (Integer
Q[p] || GtQ[d, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\sinh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}+\frac{2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}(cx)\right)}{bc}+\frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx,x,\sinh^{-1}\right.}{bc}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^{-1}(cx)}\right)}{b^2c}+\frac{2\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\sinh^{-1}(cx)}}-\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

Mathematica [A] time = 0.11453, size = 137, normalized size = 1.18

$$\frac{e^{-\frac{a+b\sinh^{-1}(cx)}{b}}\left(e^{\frac{2a}{b}+\sinh^{-1}(cx)}\sqrt{\frac{a}{b}+\sinh^{-1}(cx)}\,\text{Gamma}\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(cx)\right)+e^{\sinh^{-1}(cx)}\sqrt{-\frac{a+b\sinh^{-1}(cx)}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{a+b\sin\ldots}{}\right.\right.}{bc\sqrt{a+b\sinh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsinh}(cx) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

3.645 $\int \frac{1}{(d+ex^2)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [A] time = 0.0673352, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]

[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.141332, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A] time = 0.184, size = 0, normalized size = 0.

$$\int \frac{1}{e x^{2}+d} (a +b \operatorname{Arcsinh}\left(c x \right))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e x^{2} + d\right)\left(b \operatorname{arsinh}\left(c x\right) + a\right)^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**(3/2)*(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arcsinh(c*x) + a)^(3/2)), x)
```

3.646 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi [A] time = 0.0652315, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]

[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.272277, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\sinh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcSinh[c*x])^(3/2)), x]
```

Maple [A] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2} \left(a+b\mathrm{Arcsinh}\left(cx\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^2 (b\,\mathrm{arsinh}\,(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arcsinh(c*x) + a)^(3/2)), x)
```

3.647 $\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Rubi [A] time = 0.0231611, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right) \, dx = \int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right) \, dx$$

Mathematica [A] time = 4.96801, size = 0, normalized size = 0.

$$\int \sqrt{d+ex^2} \left(a+b\sinh^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x]), x]
```

Maple [A] time = 0.636, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}(cx))\sqrt{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex^2 + d}(b \operatorname{arsinh}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asinh}(cx))\sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2+d}(b\,\operatorname{arsinh}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a), x)
```

3.648 $\int \frac{a+b \sinh^{-1}(cx)}{\sqrt{d+ex^2}} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0245021, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 3.7548, size = 0, normalized size = 0.

$$\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.447, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{arsinh}\,(cx)+a}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{asinh}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\operatorname{arsinh}(cx)+a}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.649 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1 +
 c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Rubi [A] time = 0.101097, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {191, 5704, 12, 444, 63, 217, 206}

$$\frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcSinh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTanh[(Sqrt[e]*Sqrt[1 +
 c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - (bc)\int \frac{x}{d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\,dx$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2d}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{1+c^2x^2}\right)}{cd}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

Mathematica [C] time = 0.111725, size = 75, normalized size = 1.07

$$\frac{x\left(2\left(a+b\sinh^{-1}(cx)\right)-bcx\sqrt{\frac{ex^2}{d}+1}F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right)\right)}{2d\sqrt{d+ex^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(-(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x
^2)/d)]) + 2*(a + b*ArcSinh[c*x])))/(2*d*Sqrt[d + e*x^2])
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (a+b\text{Arcsinh}\,(cx))\left(ex^2+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75295, size = 736, normalized size = 10.51

$$\left[\frac{4\sqrt{ex^2+d}bex\log\left(cx+\sqrt{c^2x^2+1}\right)+4\sqrt{ex^2+d}aex+\left(bex^2+bd\right)\sqrt{e}\log\left(8c^4e^2x^4+c^4d^2+6c^2de+8\left(c^4de+c^2e^2\right)x^2-\right.}{4\left(de^2x^2+d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 4*sqrt(e*x^2 +
 d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e
 + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 +
 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*x^2
+ d)*b*e*x*log(c*x + sqrt(c^2*x^2 + 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b*e*x^
2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)))/(d*e
^2*x^2 + d^2*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{asinh}\,(cx)}{\left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{arsinh}\,(cx) + a}{\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.650 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}-\frac{bc\sqrt{c^2x^2+1}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcS
inh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt
[d + e*x^2]) - (2*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])
])/(3*d^2*Sqrt[e])
```

Rubi [A] time = 0.168782, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {192, 191, 5704, 12, 571, 78, 63, 217, 206}

$$\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}-\frac{bc\sqrt{c^2x^2+1}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```


```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcS
inh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcSinh[c*x]))/(3*d^2*Sqrt
[d + e*x^2]) - (2*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])
])/(3*d^2*Sqrt[e])
```


Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-(bc)\int \frac{x\left(3d+2ex^2\right)}{3d^2\sqrt{1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x\left(3d+2ex^2\right)}{\sqrt{1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d^2}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\,\mathrm{Subst}\left(\int \frac{3d+2ex}{\sqrt{1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6d^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(bc)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+c^2x}\sqrt{\cdots}}\right.}{3d^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}+\cdots}}\right.}{3c\cdots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,d\cdots\right.}{3cd^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{3d\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\sinh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right.}{3d^2\sqrt{e}}$$

Mathematica [C] time = 0.335032, size = 139, normalized size = 0.95

$$\frac{ax\left(3d+2ex^2\right)-bcx^2\left(d+ex^2\right)\sqrt{\frac{ex^2}{d}+1}F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right)-\frac{bcd\sqrt{c^2x^2+1}\left(d+ex^2\right)}{c^2d-e}+bx\sinh^{-1}(cx)\left(3d+2ex^2\right)}{3d^2\left(d+ex^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^2))/(c^2*d - e)) + a*x*(3*d + 2*e*x^2)
 - b*c*x^2*(d + e*x^2)*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x
^2), -((e*x^2)/d)] + b*x*(3*d + 2*e*x^2)*ArcSinh[c*x])/(3*d^2*(d + e*x^2)^(
3/2))
```

Maple [F] time = 0.325, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))\left(ex^2 + d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{ex^2 + d}d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}}d}\right) + b\int \frac{\log\left(cx + \sqrt{c^2x^2 + 1}\right)}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

Fricas [B] time = 3.29745, size = 1503, normalized size = 10.29

$$\left[\frac{\left(bc^2d^3 + \left(bc^2de^2 - be^3\right)x^4 - bd^2e + 2\left(bc^2d^2e - bde^2\right)x^2\right)\sqrt{e}\log\left(8\,c^4e^2x^4 + c^4d^2 + 6\,c^2de + 8\left(c^4de + c^2e^2\right)x^2 - 4\left(2\,c^3ex\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b
*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e +
 c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d + c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2
+ d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3*(b*c^2*d^2*e - b*d
*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(2*(a*c^2*d*e^2 -
 a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqr
t(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*
e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 -
 b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e
^2*x^4 + c*d*e + (c^3*d*e + c*e^2)*x^2)) + (2*(b*c^2*d*e^2 - b*e^3)*x^3 + 3
*(b*c^2*d^2*e - b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
(2*(a*c^2*d*e^2 - a*e^3)*x^3 + 3*(a*c^2*d^2*e - a*d*e^2)*x - (b*c*d*e^2*x^2
 + b*c*d^2*e)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c
^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 - d^3*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.651 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal. Leaf size=227

$$\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}-\frac{8b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(15*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (2*b*c*(3*c
^2*d - 2*e)*Sqrt[1 + c^2*x^2])/(15*d^2*(c^2*d - e)^2*Sqrt[d + e*x^2]) + (x*
(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x]))/
(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d + e*
x^2]) - (8*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*
d^3*Sqrt[e])
```

Rubi [A] time = 0.82047, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {192, 191, 5704, 12, 6715, 949, 78, 63, 217, 206}

$$\frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{2bc\sqrt{c^2x^2+1}\left(3c^2d-2e\right)}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}}-\frac{8b\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2+1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] -(b*c*Sqrt[1 + c^2*x^2])/(15*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (2*b*c*(3*c
^2*d - 2*e)*Sqrt[1 + c^2*x^2])/(15*d^2*(c^2*d - e)^2*Sqrt[d + e*x^2]) + (x*
(a + b*ArcSinh[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcSinh[c*x]))/
(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcSinh[c*x]))/(15*d^3*Sqrt[d + e*
x^2]) - (8*b*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*
d^3*Sqrt[e])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5704

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSinh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2*x^2], x], x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[e, c^2*d] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{\left(d+ex^2\right)^{7/2}}\,dx = \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - (bc)\int \frac{x\left(15d^2+20d\right.}{15d^3\sqrt{1+c^2x^2}}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x\left(15d^2+20dex^2+8e^2\right.}{\sqrt{1+c^2x^2}\left(d+ex^2\right)^5}}{15d^3}$$

$$= \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\sinh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\operatorname{Subst}\left(\int \frac{15d^2+20d}{\sqrt{1+c^2x}}\right.}{30d^3}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sinh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\sinh^{-1}\right.}{15d^3\sqrt{d+ex}}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sin\right.}{15d^2\left(d+\right.}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sin\right.}{15d^2\left(d+\right.}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sin\right.}{15d^2\left(d+\right.}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{15d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{2bc\left(3c^2d-2e\right)\sqrt{1+c^2x^2}}{15d^2\left(c^2d-e\right)^2\sqrt{d+ex^2}} + \frac{x\left(a+b\sinh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\sin\right.}{15d^2\left(d+\right.}$$

Mathematica [C] time = 0.426648, size = 191, normalized size = 0.84

$$\frac{ax\left(15d^2+20dex^2+8e^2x^4\right) - 4bcx^2\sqrt{\frac{ex^2}{d}+1}\left(d+ex^2\right)^2 F_1\left(1;\frac{1}{2},\frac{1}{2};2;-c^2x^2,-\frac{ex^2}{d}\right) - \frac{bcd\sqrt{c^2x^2+1}\left(d+ex^2\right)\left(c^2d\left(7d+6ex^2\right)-e\left(5d+4ex\right.\right.}{\left(e-c^2d\right)^2}}{15d^3\left(d+ex^2\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 + c^2*x^2]*(d + e*x^
2)*(-(e*(5*d + 4*e*x^2)) + c^2*d*(7*d + 6*e*x^2)))/(-(c^2*d) + e)^2 - 4*b*c
*x^2*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2),
 -((e*x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcSinh[c*x])/(15*d^
3*(d + e*x^2)^(5/2))
```

Maple [F] time = 0.412, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx)) \left(ex^2 + d\right)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{15}\,a\left(\frac{8\,x}{\sqrt{ex^2+d}d^3} + \frac{4\,x}{\left(ex^2+d\right)^{\frac{3}{2}}d^2} + \frac{3\,x}{\left(ex^2+d\right)^{\frac{5}{2}}d}\right) + b\int \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*x
^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(e*x^2 + d)^(7
/2), x)
```

Fricas [B] time = 4.29059, size = 2782, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(b*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^
5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*
(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d
+ c*e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^2*e^3
 - 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e
^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)
*log(c*x + sqrt(c^2*x^2 + 1)) + (8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*
x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e
- 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2 + 2*(3*b*
c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e^3)*x^2)*sq
rt(c^2*x^2 + 1))*sqrt(e*x^2 + d))/(c^4*d^8*e - 2*c^2*d^7*e^2 + d^6*e^3 + (c
^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2*c^2*d^5*e^4
+ d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 - 2*c^2*d^6*e^3 + d^5*e^4)*x^2), 1/15*(4*(b
*c^4*d^5 - 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*
d^3*e^2 + 3*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*
e - 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 + c*d*e +
(c^3*d*e + c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^5 +
20*(b*c^4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e - 2*b*
c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1)) +
(8*(a*c^4*d^2*e^3 - 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 - 2*a*c^
2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x
 - (7*b*c^3*d^4*e - 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 - 2*b*c*d*e^4)*x^4 +
 (13*b*c^3*d^3*e^2 - 9*b*c*d^2*e^3)*x^2)*sqrt(c^2*x^2 + 1))*sqrt(e*x^2 + d)
)/(c^4*d^8*e - 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 - 2*c^2*d^4*e^5 + d^3
*e^6)*x^6 + 3*(c^4*d^6*e^3 - 2*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2
- 2*c^2*d^6*e^3 + d^5*e^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x**2+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsinh}(cx) + a}{\left(ex^2 + d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

3.652 $\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi [A] time = 0.0420453, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx=\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Mathematica [A] time = 13.7045, size = 0, normalized size = 0.

$$\int \sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.242, size = 0, normalized size = 0.

$$\int (a+b\mathrm{Arcsinh}\,(cx))^2\,\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\,\mathrm{arsinh}\,(cx)^2+2\,ab\,\mathrm{arsinh}\,(cx)+a^2\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+b\,\mathrm{asinh}\,(cx))^2\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arsinh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2, x)
```

3.653 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi [A] time = 0.0448606, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]

[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 10.8624, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^2/Sqrt[d + e*x^2], x]
```

Maple [A] time = 0.224, size = 0, normalized size = 0.

$$\int (a + b\text{Arcsinh}\,(cx))^2 \frac{1}{\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{arsinh}\,(cx)^2 + 2\,ab\,\text{arsinh}\,(cx) + a^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/sqrt(e*x^2 + d), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

3.654 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi [A] time = 0.0490404, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 3.00758, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(3/2), x]
```

Maple [A] time = 0.174, size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(cx))^2 \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2 \operatorname{arsinh}(cx)^2 + 2\,ab \operatorname{arsinh}(cx) + a^2\right)\sqrt{ex^2 + d}}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{asinh}(cx))^2}{\left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x**2)**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{arsinh}(cx) + a)^2}{\left(ex^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

3.655 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi [A] time = 0.0475373, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]

[Out] Defer[Int][(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 5.72008, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Maple [A] time = 0.182, size = 0, normalized size = 0.

$$\int (a + b\mathrm{Arcsinh}\,(cx))^2 \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2\left(\frac{2\,x}{\sqrt{ex^2 + d}d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}} d}\right) + \int \frac{b^2 \log\left(cx + \sqrt{c^2x^2 + 1}\right)^2}{\left(ex^2 + d\right)^{\frac{5}{2}}} + \frac{2\,ab \log\left(cx + \sqrt{c^2x^2 + 1}\right)}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c*x + sqrt(
c^2*x^2 + 1))/(e*x^2 + d)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2 \operatorname{arsinh}\left(cx\right)^2 + 2\,ab \operatorname{arsinh}\left(cx\right) + a^2\right)\sqrt{ex^2 + d}}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsinh}(cx) + a)^2}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

3.656 $\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Rubi [A] time = 0.0472286, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A] time = 1.17245, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x]), x]
```

Maple [A] time = 0.218, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\mathrm{Arcsinh}\,(cx)}\sqrt{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}}{b\,\mathrm{arsinh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{b\,\mathrm{arsinh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a), x)
```

3.657 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.0498483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.03433, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\mathrm{Arcsinh}\left(cx\right)}\frac{1}{\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex^{2}+d}\left(b\,\operatorname{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^{2}+d}}{aex^{2}+ad+\left(bex^{2}+bd\right)\operatorname{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{asinh}(cx))\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex^2 + d}(b\operatorname{arsinh}(cx) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)), x)
```

3.658 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.0542436, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.489, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\mathrm{Arcsinh}\left(cx\right)}\left(ex^{2}+d\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^{2}+d}}{ae^{2}x^{4}+2\,adex^{2}+ad^{2}+\left(be^{2}x^{4}+2\,bdex^{2}+bd^{2}\right)\mathrm{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*
b*d*e*x^2 + b*d^2)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arsinh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)), x)
```

3.659 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi [A] time = 0.053293, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.60287, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])), x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\mathrm{Arcsinh}\left(cx\right)}\left(ex^{2}+d\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^{2}+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arsinh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex^{2}+d}}{ae^{3}x^{6}+3\,ade^{2}x^{4}+3\,ad^{2}ex^{2}+ad^{3}+\left(be^{3}x^{6}+3\,bde^{2}x^{4}+3\,bd^{2}ex^{2}+bd^{3}\right)\mathrm{arsinh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3
 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asinh}(cx)) \left(d + ex^2\right)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x**2)**(5/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2 + d\right)^{\frac{5}{2}} (b \operatorname{arsinh}(cx) + a)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)), x)
```

3.660 $\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi [A] time = 0.04383, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]

[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 4.41893, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\operatorname{Arcsinh}\left(cx\right)\right)^{2}}\sqrt{ex^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(c^{2}x^{2}+1\right)^{\frac{3}{2}}\sqrt{ex^{2}+d}+\left(c^{3}x^{3}+cx\right)\sqrt{ex^{2}+d}}{abc^{3}x^{2}+\sqrt{c^{2}x^{2}+1}abc^{2}x+abc+\left(b^{2}c^{3}x^{2}+\sqrt{c^{2}x^{2}+1}b^{2}c^{2}x+b^{2}c\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)}+\int\frac{}{abc^{5}ex^{6}+\left(c^{5}d+2\,c^{3}e\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*sqrt(e*x^2 + d) + (c^3*x^3 + c*x)*sqrt(e*x^2 + d))/(a
*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*
x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((2*c
^3*e*x^4 + c^3*d*x^2 - c*d)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (4*c^4*e*x^5 +
2*(c^4*d + 2*c^2*e)*x^3 + (c^2*d + e)*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)
+ (2*c^5*e*x^6 + (c^5*d + 4*c^3*e)*x^4 + 2*(c^3*d + c*e)*x^2 + c*d)*sqrt(e*
x^2 + d))/(a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*e)*a*b*
x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*e*
x^6 + (c^5*d + 2*c^3*e)*b^2*x^4 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d + (b^2*
c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x +
 (c^4*d + c^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) +
 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{b^2 \operatorname{arsinh}(cx)^2+2\,ab \operatorname{arsinh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*asinh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arcsinh(c*x) + a)^2, x)
```

3.661 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0473418, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 6.22324, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\mathrm{Arcsinh}\,(cx))^2}\frac{1}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3x^3+cx+\left(c^2x^2+1\right)^{\frac{3}{2}}}{\sqrt{c^2x^2+1}\sqrt{ex^2+d}abc^2x+\left(\sqrt{c^2x^2+1}\sqrt{ex^2+d}b^2c^2x+\left(b^2c^3x^2+b^2c\right)\sqrt{ex^2+d}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+\left(abc^3x^2+ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*a
*b*c^2*x + (sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d)*b^2*c^2*x + (b^2*c^3*x^2 + b^
2*c)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*x^2 + a*b*c)*
sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 + 2*c^3*d*x^2 + (c^2*x^2 + 1)*((c^3
*d - 2*c*e)*x^2 - c*d) + c*d + sqrt(c^2*x^2 + 1)*(2*(c^4*d - c^2*e)*x^3 + (
c^2*d - e)*x))/((a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 +
d) + 2*(a*b*c^4*e*x^5 + a*b*c^2*d*x + (c^4*d + c^2*e)*a*b*x^3)*sqrt(c^2*x^2
 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c^2*x^2 + 1)*sqrt
(e*x^2 + d) + 2*(b^2*c^4*e*x^5 + b^2*c^2*d*x + (c^4*d + c^2*e)*b^2*x^3)*sqr
t(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e*x^6 + (c^5*d + 2*c^3*e)*b^2*x^4
 + (2*c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x
^2 + 1)) + (a*b*c^5*e*x^6 + (c^5*d + 2*c^3*e)*a*b*x^4 + (2*c^3*d + c*e)*a*b
*x^2 + a*b*c*d)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\operatorname{arsinh}(cx)^2+2\left(abex^2+abd\right)\operatorname{arsinh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arcsinh(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+b\operatorname{asinh}(cx))^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*sqrt(d + e*x**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{ex^2+d}(b\operatorname{arsinh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arcsinh(c*x) + a)^2), x)
```

3.662 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0514789, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 16.3942, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.152, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\text{Arcsinh}\,(cx))^2} \left(ex^2 + d\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(
c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c^2*x^2
+ 1)*sqrt(e*x^2 + d) + (b^2*c^3*e*x^4 + (c^3*d + c*e)*b^2*x^2 + b^2*c*d)*sq
rt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e*x^4 + (c^3*d + c*e
)*a*b*x^2 + a*b*c*d)*sqrt(e*x^2 + d)) - integrate((2*c^5*e*x^6 - (c^5*d - 4
*c^3*e)*x^4 - 2*(c^3*d - c*e)*x^2 + (2*c^3*e*x^4 - (c^3*d - 4*c*e)*x^2 + c*
d)*(c^2*x^2 + 1) - c*d + (4*c^4*e*x^5 - 2*(c^4*d - 4*c^2*e)*x^3 - (c^2*d -
3*e)*x)*sqrt(c^2*x^2 + 1))/((a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*
d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e +
c^2*e^2)*a*b*x^5 + a*b*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*a*b*x^3)*sqrt(c^2*
x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^3*e^2*x^6 + 2*b^2*c^3*d*e*x^4 + b^2*c^3*
d^2*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^2*x^7 + (2*c^4*d*e +
c^2*e^2)*b^2*x^5 + b^2*c^2*d^2*x + (c^4*d^2 + 2*c^2*d*e)*b^2*x^3)*sqrt(c^2*
x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^2*x^8 + 2*(c^5*d*e + c^3*e^2)*b^2*x^6
 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + 2*(c^3*d^2 + c*d*e)*
b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^5*e^2*x^8 +
 2*(c^5*d*e + c^3*e^2)*a*b*x^6 + (c^5*d^2 + 4*c^3*d*e + c*e^2)*a*b*x^4 + a*
b*c*d^2 + 2*(c^3*d^2 + c*d*e)*a*b*x^2)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^2x^4+2\,a^2dex^2+a^2d^2+\left(b^2e^2x^4+2\,b^2dex^2+b^2d^2\right)\operatorname{arsinh}\left(cx\right)^2+2\left(abe^2x^4+2\,abdex^2+abd^2\right)\operatorname{arsinh}\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*
x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*
x^2 + a*b*d^2)*arcsinh(c*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\operatorname{asinh}\left(cx\right)\right)^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arsinh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arcsinh(c*x) + a)^2), x)
```

3.663 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi [A] time = 0.0500434, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 28.172, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A] time = 0.153, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\operatorname{Arcsinh}(cx))^2} \left(ex^2 + d\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/((a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^
3 + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^2*c^2*e^2*x^5 +
2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2
*c^3*e^2*x^6 + (2*c^3*d*e + c*e^2)*b^2*x^4 + b^2*c*d^2 + (c^3*d^2 + 2*c*d*e
)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^3*e^2*x^6
 + (2*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 + (c^3*d^2 + 2*c*d*e)*a*b*x^2)*s
qrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d - 8*c^3*e)*x^4 - 2*(c^3*d
 - 2*c*e)*x^2 + (4*c^3*e*x^4 - (c^3*d - 6*c*e)*x^2 + c*d)*(c^2*x^2 + 1) - c
*d + (8*c^4*e*x^5 - 2*(c^4*d - 7*c^2*e)*x^3 - (c^2*d - 5*e)*x)*sqrt(c^2*x^2
 + 1))/((a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*
c^3*d^3*x^2)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*
e^2 + c^2*e^3)*a*b*x^7 + a*b*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*a*b*x^5
+ (c^4*d^3 + 3*c^2*d^2*e)*a*b*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + ((b^
2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2
)*(c^2*x^2 + 1)*sqrt(e*x^2 + d) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 + c^2*e
^3)*b^2*x^7 + b^2*c^2*d^3*x + 3*(c^4*d^2*e + c^2*d*e^2)*b^2*x^5 + (c^4*d^3
+ 3*c^2*d^2*e)*b^2*x^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (b^2*c^5*e^3*x^
10 + (3*c^5*d*e^2 + 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e + 6*c^3*d*e^2 + c*e^3
```

```
)*b^2*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 + (2*c^
3*d^3 + 3*c*d^2*e)*b^2*x^2)*sqrt(e*x^2 + d))*log(c*x + sqrt(c^2*x^2 + 1)) +
 (a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 + 2*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*e + 6*c
^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 + 6*c^3*d^2*e + 3*c*d*e^2)*a*b*x^4 + a
*b*c*d^3 + (2*c^3*d^3 + 3*c*d^2*e)*a*b*x^2)*sqrt(e*x^2 + d)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^3x^6+3\,a^2de^2x^4+3\,a^2d^2ex^2+a^2d^3+\left(b^2e^3x^6+3\,b^2de^2x^4+3\,b^2d^2ex^2+b^2d^3\right)\operatorname{arsinh}(cx)^2+2\left(abe^3x^6+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2 +
 a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*arcs
inh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3)*
arcsinh(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\operatorname{arsinh}(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arcsinh(c*x) + a)^2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```